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Title: Stable central limit theorems for super Ornstein-Uhlenbeck processes, II
Authors: Ren, Song, Sun and Zhao
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Summary:

This paper is a completion of the authors' previous paper [20]. A superprocess X with an O-U spatial motion is considered with a branching mechanism which is essentially a small perturbation of a stable branching mechanism. The authors investigate the behavior (scaling limit) of $X_t(f)$ where f is not identically zero and has polynomial growth as $|x| \to \infty$.

In [20] they obtained some related results for particular type of functions. Since a general f can be decomposed into a sum of three of such functions belonging to three different function spaces, this would yield a scaling limit for a general f as well, provided that one knows the independence of the scaling limits obtained. (Although the scaling is not exactly the same for the three classes, this simply means one has to consider two different cases for f.)

Realizing this, in [20] the authors mentioned that

> We leave the question of the independence of the limit stable random variables to a future project.

In the paper under review they fulfilled this promise and proved the missing ingredient, that is the independence of the random variables, and as a corollary they gave the above mentioned result for a general f with polynomial growth.

Evaluation:

This work gives a nice contribution to the literature about scaling limits (LLNs and CLTs) for superprocesses. At the same time I do not feel that it should be published by such a prestigeous probability journal as ECP because of the following reasons:

(1) The setting is quite restricted as the spatial motion has to be precisely an inward O-U process, and the branching mechanism has to be "almost exactly" a stable one.
(2) This case is what is often called an "incremental contribution." That is, the achievements in the much longer paper [20] have now been complemented by filling a (crucial) gap, hence obtaining an anticipated stronger result.

I do believe though that the paper definitely deserves to be published in a decent probability journal.