Business Card Math Tables

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This is a collection of small math tables -- each one will fit roughly on a business card. I was doing a little programming experiment and I made a small table of numbers with the output (the first table was logarithms to 3 figures). Then something made me connect the output I was getting with putting the table into a small space -- say, about the size of a credit card or business card. I realized that when I'm places it's not convenient to have a calculator, some small tables would be handy and allow an approximate calculation. Yes, this is geeky, but sometimes engineers and scientists need such things. My typical example is when we're camping -- a calculator isn't handy, yet I want to make a simple numerical calculation. Using these tables, you can make a 1%-type calculation with a pencil and paper. Warning: if you haven't done this in a long time (or never have), you're going to want to practice a bit, as it's awfully easy to make mistakes.

If you're familiar with using log and trig tables, head to the Table Summary section to get a description, then just use the tables. Otherwise, go to the Table Explanations section for more details on what the tables are and what they can do for you, along with some examples. There are also tips and techniques for doing math without a calculator handy. Before the early 1970's, this stuff was pretty common knowledge, but since the advent of calculators, many students have never been exposed to the various techniques. I'm certainly not advocating the return to pre-calculator days, but some knowledge of this stuff can be useful even with a calculator -- especially the parts about estimating the magnitude of an answer and approximations.

These small tables are intended to be printed and cut out. You can use some clear packing tape as a poor-man's laminating material. Print the tables on thin paper, then put a length of tape over them to protect them. I cut out two tables next to each other, fold them over, and use Elmer's School Glue Stick to glue them back-to-back. All these little tables fit into a small, clear plastic badge holder and are about 1/8" thick, so they're easy to slip into a pocket.

Notation: in this document, *log* means the base 10 logarithm and *ln* means the natural logarithm.

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Table Explanations

Anyone who has had to slog through a manual calculation without a calculator knows they'd always rather do it with the calculator. But sometimes a calculator isn't handy -- like when you've been marooned on a desert island and your calculator batteries are dead (wasn't that always a popular argument against becoming dependent on calculators?).

I'll assume you know the basics of using logarithms, trig tables, etc., so I'm just going to briefly discuss the tables and how they can benefit you.

Let's look at the table for logarithms:

Log 1[.1]10

	0	1	2	3	4	5	6	7	8	9
1	000	041	079	114	146	176	204	230	255	279
2	301	322	342	362	380	398	415	431	447	462
3	477	491	505	519	531	544	556	568	580	591
4	602	613	623	633	643	653	663	672	681	690
5	699	708	716	724	732	740	748	756	763	771
6	778	785	792	799	806	813	820	826	833	839
7	845	851	857	863	869	875	881	886	892	898
8	903	908	914	919	924	929	934	940	944	949
9	954	959	964	968	973	978	982	987	991	996

ln(10) = 2.3026, log(pi) = 0.497, log(e) = 0.4343

The 1[.1]10 means the table's abscissa (i.e., x values) run from 1 to 10 in steps of 0.1. The tabulated values are f(x), the ordinates. Here, f(x) = log(x).

The table's values are actually $1000 \log(x)$. This saves space by not having to display the decimal points. Since there are three significant figures to the logarithms, the best you can hope to calculate is about 1 part out of 1000, or 0.1%. Because of roundoff error, the practical uncertainties for typical calculations are in the range of 0.5% to a few percent.

Some examples:

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> Calculate log(37): go down the left-most column with the most significant digit (here, 3), then read across that row to the column headed by the second digit. The number for 37 is 568. This means the logarithm of 3.7 is 0.568. The characteristic is 1, so the final logarithm is 1.568. The characteristic is easily found by inspection of the number.
>
> Calculate log(0.0872): For 87, we get the log 940. Using linear interpolation, we get the correction 0.8. Thus, the log of 0.0872 is $\bar{2}.9408$.

If you're not familiar with the $\bar{2}.9408$ notation, it means the logarithm is 0.9408 - 2. This avoids having to do the subtraction to get the actual logarithm -- which is handy in chained calculations where a number of logarithms will be added, as in the following example:

> Calculate $47.5 \times 912 \times 0.0872 \times 1.92$: using the table, we write down the column of logarithms and sum them:

$$\begin{array}{r} 1.676 \\ 2.960 \\ \bar{2}.941 \\ \underline{0.283} \\ 3.860 \end{array}$$

> When summing the leftmost column, you simply subtract the number with an overbar from the sum. Using the antilog table, we get the answer as 7240. The correct answer is 7252.

Once you have the logarithm of the result, there are two ways to get the number that has that logarithm. You can inverse interpolate in the logarithm table or use the antilogarithm table.

This introduction should serve to get one started using the log table. You can read more about calculations with logarithms from many math and laboratory handbooks from the 1960's or before.

The other tables

The other tables should be straightforward to figure out. The only twist is that some of them just display the three digits of the mantissa. This means you have to figure out where the decimal point is.

For example, in the table of squares, consider the entry for 41. This can be used to find the square of 0.041, 0.41, 4.1, 41, 4100, etc. If the number is 4.1, then 4.1^2 is slightly over 16. Using the algebraic aid shown elsewhere, it's $(4 + 0.1)^2$ or 16 + 2(0.4) + 0.01 = 16.81. Shoot, you didn't even need a table! Still, looking up 41 in the squares table gives 168. We thus know the answer from the table is 16.8.

Here's another example:

> Calculate 79^2. Since 80^2 is 6400 (figure this from $(8\times10)^2$ or 64×100), we know the answer should be under 6400. The table gives us 624, so we know the result is 6240. Again, an algebraic substitution makes it easy to calculate the exact result: $(80 - 1)^2 = 6400 - 160 + 1 = 6400 - 200 + 40 + 1 = 6241$.

Oh, one other subtlety. In the table for the tangent in radians, you no doubt know that the tangent of $\pi/4$ is 1. Thus, the table "rolls over" to numbers above one if the abscissa is $\pi/4$ or larger. Since we want only three digits in the table, we rely on you knowing about this rollover and adding the necessary 1 when using the table.

Final example

Let's put this all together and use the tables to perform a "real" calculation. The radius of a drop of oil in the Millikan oil drop experiment (a famous physics experiment used to calculate the elementary unit of charge) is calculated by the equation

$$r^2 = \frac{9\eta v_1}{2g(\rho - \rho_{air})}$$

If we have the measurements

$$\begin{aligned} \eta &= 8.9\times10^{-4} \\ v_1 &= 2.1\times10^{-4} \\ g &= 9.81 \\ \rho &= 817 \\ \rho_{air} &= 1.204 \end{aligned}$$

calculate r (the units are SI, but aren't germane to the calculation). First, let's estimate about what the result will be. We approximate the numerator by $10(10\times10^{-4})(2\times10^{-4})$ or 2×10^{-6}. The denominator is approximated by 2(10)(800) = 16000. We write this as

$$\frac{20\times10^{-7}}{16\times10^{3}} \approx 10^{-10}$$

Taking the square root yields $r = 10^{-5}$.

The numerator sum of logarithms is

$$\begin{array}{l} 0.954 \\ \bar{4}.949 \\ \underline{\bar{4}.322} \\ \bar{6}.225 \text{ or } -5.775 \end{array}$$

The denominator sum of logarithms is

$$\begin{array}{l} 0.301 \\ 0.991 \\ \underline{2.912} \\ 4.204 \end{array}$$

The log of the result is -5.775 - 4.204 or -9.979. Since we want the square root, we divide by 2 to get -4.989. Calculating the antilog (note we rewrite this log as 0.011 - 5 so we can look it up in the table), we get the answer $\underline{1.023\times10}^{-5}$. The exact answer is 1.025×10^{-5}.

This problem is a bit special for me, as when I was a student, we did this experiment. I sweated over the calculation with logarithms and a slide rule (the formula used was substantially more complicated than the above one) and never was able to get the correct answer for the lab. My teacher told me that I simply would have to learn to do these calculations because they are part of the job. He, of course, was right.

Other Table Techniques

How to get a natural logarithm

We can write

$$x = 10^{\log(x)} = e^{\ln(10)\log(x)}.$$

Taking the natural log of both sides, we get

$$\ln(x) = \log(x)\ln(10) = 2.3206\log(x)$$

To calculate the natural logarithm of a number x, get the base 10 log from the table and multiply it by 2.32 to get the natural log of x. You might want to use the distributive law and write 2.32 as (2 + 0.3 + 0.02), as you only then need to multiply by 2 and 3.

> Find ln(0.00432): Rewrite it as log(4.32) - 3. Using the table, this is 0.635 - 3 = -2.365. The natural log we are after is thus (-2.365)(2.32). I calculated this by squaring 2.34†, which is roughly half way between the two numbers. This gave the result -5.47. The correct answer is -5.44.
>
> Alternatively, using -2.365(2 + 0.3 + 0.02), we get -4.730 - 0.709 - 0.0047 = -5.444.

Interpolation

Interpolation is a method used to get the value of a tabulated function at a particular abscissa that is not in the table. We'll look at linear interpolation. This is where you draw a line between the nearest tabulated points, then pick out the value for your abscissa that lies on the line.

The procedure is: you get the difference between the two ordinates lying on either side of your abscissa and scale that by the abscissa's position relative to the two ordinate's abscissas.

An example makes things clearer. Suppose we want the sine of 34.7°. Looking in the table for sines for values in degrees, we get sin(34°) = 0.559 and sine(35°) = 0.574. You drop the decimal points and subtract the numbers: 574 - 559 = 15. You want 0.7 of this value (i.e., seven tenths along the line); this is 10 -- you do the multiplication in your head. Thus, you add 10 to the 559 to get the sine you're after as 0.569. The actual value is 0.5693.

If this is new to you, you'll probably want to write the intermediate numbers down until you get a feel for what's going on. After a while, you'll just do it in your head.

The other example we should look at is when the derivative of the tabulated function is negative, such as in the reciprocal. Suppose we want the reciprocal of 0.0278. We enter the table to get 370 at 27 and 357 at 28. The difference is -13 and we want 8/10ths of that value or -10. We add the -10 to 370 to get 360. To get the decimal place, note that 1/0.02 = 1/(2/100) = 50 and we have a slightly larger denominator. Thus, the answer is 36.0. The correct answer is 35.97.

Inverse interpolation works the same way. This is where you have a value in the table and you want to get the abscissa corresponding to it. Let's use another example, which was common when people used log tables, as antilog tables weren't as common. Suppose we got a result with a logarithm of 2.719. What is the value we're looking for? We find 719 in the log table between the entries for 52 and 53. The table values corresponding to these abscissas are 716 and 724. The difference is 8 and we're 3/8 along that distance. The abscissa corresponding to this is 52 + 3/8 or, finally, 5.24. With the exponent of 2, the final answer is 524. The correct value is $10^{2.719}$ = 523.6.

Calculating quickly

Because numerical calculations are drudgery without calculators or computers, people had to be clever and found many tricks to speed things up or get an approximate answer.

Quick, what's the square root of 75? Well, you know it's between 8 and 9, but you probably can't do much better than that. But here's a shortcut that gets you the answer. You probably know that 0.75 is the decimal equivalent of 3/4. Take the square root of that to get $\sqrt{3}/2$. Look familiar? If you've taken basic trig, you know the sine of 60° is $\sqrt{3}/2$ and you've probably gotten it's value as 0.8660 many times (or you do the division of 1.732/2 in your head because you know the square roots of 2 and 3). Thus, you then immediately know that the square root of 75 is 8.660.

If you didn't know or follow the preceding example, don't worry about it. What I was trying to show is that by gaining calculation experience, you'll find you'll use that knowledge in the future in slightly

† If you have a*b where a and b are close, this can be rewritten as $(c + \delta)(c - \delta) = c^2 - \delta^2$ where c = (a + b)/2 and δ = (a - b)/2. Since δ is small, the δ^2 term can be neglected. A little tidbit from that high school algebra you thought you'd never use. ☺

unexpected ways (Feynman discussed this a bit in his entertaining book *Surely You're Joking, Mr. Feynman*). In working with 30-60-90 and 45-45-90 triangles when studying your basic math and physics, you naturally see $\sqrt{3}/2$ and $1/\sqrt{2}$ a lot and come to know their decimal values.

Here are some of the things that, if you know them, find repeated use in doing calculations by hand:

1. The multiplication table. You must know this cold, backwards and forwards.
2. The squares of numbers up to 20.
3. The reciprocals of numbers up to 15 or 20 and a few special values above 20.
4. How to figure any fraction whose denominator is a power of two up to 6 (i.e., 64). See the section on fractions below.
5. Square roots of a few numbers like 2, 3, etc.

Estimating the order of magnitude

If there's one thing I think is most important for folks doing calculations, regardless of the method being used, it's being able to accurately estimate the order of magnitude of the answer. This means you get a number, say, 30 and you then think "OK, now I know the answer should come out somewhere between 3 and 300. In fact, I think it's one of the most important arithmetic lessons a student should come out of high school or college with -- the wise person will use it for the rest of his or her life.

The method is pretty simple -- just round the numbers in the calculation to "nicer" numbers that are easier to do arithmetic with.

Here's an example

$$\frac{572.02\,(0.401657)}{87.1\sqrt{66.93}} \approx \frac{600\,(0.4)}{90\sqrt{70}} = \frac{240}{90\cdot 8} = \frac{240}{720} = \frac{1}{3} \quad \text{or about } 0.33$$

When you get experienced doing these approximations, you do them in your head as follows: "572 is about 600, and 600 times 0.4 is 240. 87 is about 100, so we have 2.4. The square root is about 8, so the answer is about 0.3." Do you see the basic pattern? It's mostly to convert to one digit numbers that you can then quickly use your knowledge of integer multiplication and division.

It's pretty simple -- and it gets simpler the more frequently you do it. The key is that you have to be rock-solid doing multiplications. The benefit is that if you get knee-deep in dealing with all those significant figures and make a mistake and get, say 372, your order of magnitude calculation tells you immediately that you made a mistake, as the answer should have been near 30. ***This is true whether you're doing the calculation with log tables, a slide rule, a calculator, or a computer.***

There are times when this approximation is not so easy to do. Here's an example. Suppose you had a term that was tan(3477.5). You simply can't change that tangent to an order of magnitude until you reduce the argument to lie within the range 0 to 2π. In fact, you won't even know the sign of your answer until you do.

A final point: with practice, you'll find that e.g. in a numerator, you'll round one term up to an easy-to-use number and you'll round another number down so that the roundings approximately cancel each other. This helps you get a closer estimate.

The back of the envelope

Scientists and engineers often do "back of the envelope" calculations. This term is well-known, as an opened envelope is often handy on one's desk and is quickly grabbed to write on. The process is to do a "simple" calculation, estimating the order of magnitude of something.

Some people get quite expert at doing these types of calculations and being creative in estimating

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unknown quantities. You can read a bit more about them at http://en.wikipedia.org/wiki/Fermi_calculation. They're named after the physicist Fermi, but people were doing them long before Fermi. Seemingly impossible-to-answer questions that can be estimated with these techniques also are often asked in job interviews. They're an excellent way to find out about how someone thinks and works when faced with an "impossible" task. For example, an interviewer will surprise you with a question like "Tell me how many gallons of water come out of the Mississippi River every second".

Here's how you might reason out a solution:

> "OK, I need to know the area of the cross-section of the river. Call it a rectangle. This rectangle moves a certain distance in one second, which depends on the flow velocity of the river. This gives me a box whose volume equals the discharge volume in one second. I need to estimate the three dimensions of the box. For the depth, I'll take 20 feet. For the width, I'll take 2 miles or 10,000 feet. For the flow velocity, I'll estimate 5 feet per second. Thus, the box volume is $5 \times 20 \times 10^4$ ft^3 or 10^6 ft^3/s. I know there's roughly 10 gallons per cubic foot, so I get 10 million gallons per second."

Wikipedia's web page http://en.wikipedia.org/wiki/Mississippi_River gives the average flow at about 0.5 million cubic feet per second (the page has two different numbers), so my estimate was a bit high. But you can see I was within an order of magnitude -- and I honestly didn't look at the answer before doing the problem (and I've never seen the Mississippi River or been near it).

Fractions

It's still relatively common in the US for people to measure distances in fractions of an inch. When I work in the shop, I much prefer working in decimal inches or metric, so I find I'm always converting others' fractional measurements. Since, as an amateur machinist, I often measure things to the nearest thousandth of an inch, it's nice to be able to convert fractions in my head to decimals. Here's how I do it.

First, you need to know the following fractions:

1/2	0.5	0.500
1/4	0.25	0.250
1/8	0.125	0.125
1/16	0.0625	0.063
1/32	0.03125	0.031
1/64	0.015625	0.016

I only need to remember the 3 decimal place expansions on the right.

Now, I also know all the decimal equivalents of the eighths of an inch -- this just came from frequent use. If I have a number such as 37/64 to convert to a decimal, here's how I think it through. 36/64 is 18/32 or 9/16, which is 1/16 beyond 8/16 or 1/2. Thus, 9/16 is 0.5 + 0.063 = 0.563. The value we wanted is 1/64 beyond this, so we get 0.563 + 0.016 = 0.579. The actual value is 0.5781, so I'm within a thousandth of an inch, which is almost always just fine for things dimensioned in fractions.

By knowing fractions, you'll make connections like the following. In the above table, 1/16 is 0.0625. But 0.625 is the decimal equivalent of 5/8. Thus, 1/10th of 5/8 is 1/16, something I didn't realize until I looked at the table while I was writing this.

Now, if you see a term in a calculation that is 625 or pretty close to it, you can substitute 5/8 for it. Sometimes multiplying by 5's and 2's multiple times is easy to do in your head. Here's an example:

Calculate $\frac{17.7}{62.5}$. Recognizing the 62.5, we replace it by $100\left(\frac{5}{8}\right)$. Thus, we have

$$\frac{17.7(8)}{100(5)} = \frac{0.177(8)}{5} = 0.0354(8) = 0.2832$$

We know this is pretty close, because we estimated the answer as a bit less than 20/60 = 0.33.

Other shortcuts can be picked out analogously. If you see 25 in a calculation's numerator, you can put an appropriate 4 (0.4, 4, 40, etc.) in the denominator and thus be able to divide by 2 twice. Alternatively, you think, "25 is 100/4" and substitute the numbers that are easier to do arithmetic with.

Here are some fractions (reciprocals, actually) that you might want to get familiar with:

1/2	0.5	1/13	$0.\underline{076923}$
1/3	$0.333\underline{3}$	1/14	$0.0\underline{714285}$
1/4	0.25	1/15	$0.0666\underline{6}$
1/5	0.2	1/16	0.0625
1/6	$0.166\underline{6}$	1/17	0.058823
1/7	$0.\underline{142857}$	1/18	$0.05555\underline{5}$
1/8	0.125	1/19	0.052631
1/9	0.111	1/20	0.05
1/10	0.1	1/25	0.04
1/11	$0.09\underline{09}$	1/75	$0.01333\underline{3}$
1/12	$0.083\underline{3}$		

The underlined portion indicates that digit group repeats indefinitely. Besides the 4 and 0.25 "trick" above, you can see that 5 and 0.2, 2 and 0.5, and 8 and 0.125 will frequently be of use.

Approximations

I'm going to give an example of a calculation you might do by hand, then explain the approximations used in it. This one is from *An Introduction to Scientific Research* by E. Bright Wilson, McGraw-Hill, 1952.

$$\begin{aligned}
\frac{75 \times 21}{88} &= \frac{(80-5)(20+1)}{(90-2)} \\
&\approx \frac{80(1-0.06)\times 20(1+0.05)}{90(1-0.02)} \\
&\approx \left[\frac{80\times 20}{90}\right](1-0.06)(1+0.05)(1+0.02) \\
&\approx 17.8(1+0.01) \\
&= 17.8+0.178 \\
&\approx 18.0
\end{aligned}$$

The first step is pretty obvious, but the next step (to the right of the first $\approx$) might be mysterious. The thinking for the first term in the numerator was "5 is about 6% of 80". The remaining steps make use of two of the most-used approximations in doing manual calculations.

First, we have

$$\frac{1}{1\pm\delta} \approx 1\mp\delta$$

This approximation is better the smaller δ is. Note how the signs change. Relate this to what happened above. Math, science, and engineering students learn to use this approximation so much they do it in their sleep. The formula comes from the geometric series:

$$\frac{1}{1-\delta} = 1+\delta+\delta^2+\delta^3+\dots \quad (\delta\neq 1)$$

and, since δ is small, we drop the quadratic and higher terms in δ.

Secondly, we have when ϵ and δ are much less than 1:

$$(1+\delta)(1+\epsilon) \approx 1+\delta+\epsilon$$

This comes from algebraic expansion and noting that $\delta\epsilon$ is much smaller than the other terms.

These two approximations are used everywhere when people do calculations and they are probably the most useful to memorize.

Thus, in the above calculation's third line, you can see the three products on the right were replaced with (1 - 0.06 + 0.05 + 0.02) or (1 + 0.01).

As mentioned elsewhere in a footnote, if you're calculating the product of two numbers a and b that are close together, you can just square the number c where c is the mean of a and b. This is because of the approximation

$$ab = (c-\delta)(c+\delta) = c^2-\delta^2 \approx c^2$$

because δ is assumed small. Here, $\delta = \frac{b-a}{2.}$

Algebraic identities

Suppose you want to calculate 56^2. You could do the long multiplication, but there's an easier way based on the algebraic identity $\boxed{(a+b)^2 = a^2+2ab+b^2}$. We write $56^2 = (50 + 6)^2 = 2500 + 2\cdot50\cdot6 + 6^2 = 2500 + 600 + 36 = 3136$.

The distributive law is always useful: $\boxed{a(b+c) = ab+ac}$. Example: $84\cdot13 = 84(10 + 3) = 840 + 252 = 1092$.

With practice, you'll find you can do such decompositions in your head.

Oh, one other tip: when adding or subtracting numbers, feel free to round up or down to a more convenient number and add or subtract the correction afterwards. For example,

$$-37-88 = -(37+80+8) = -(117+8) = -125.$$

Units

When I work in my shop and do design work, I often convert back and forth between metric and customary US measurements, such as pounds and inches. Some of these can be done in your head and, if you do them a lot, it's worth memorizing some numbers and techniques.

Inches to mm

The inch is defined to be exactly 25.4 mm or 2.54 cm. This is close to 25, which we recognize the equivalent of dividing by 4 (see the section on Fractions). Or, divide by two twice. The error is 4 parts out of 254, which is about 1 out of 125 or a bit less than 1%. Let's examine this is more detail:

$$25.4 = 25+0.4 = 25\left(1+\frac{0.4}{25}\right) = 25(1+0.016)$$

From this, we see we can get a good approximation by the following:

1. Divide the number of inches by 4 (e.g., divide by 2 twice).
2. Multiply by 100.

3. Increase the number by 1.5%.

Example: Convert 3.788 inches to mm. Dividing by 2 gives 1.894; another division by 2 gives 0.947. Multiplying by 100 gives 94.7. To increase by 1.5%, we can do it exactly by adding 0.947 (the 1% part) and adding half of this (0.473) to get 96.12 mm. Compare this to 96.2152 mm. We're off by about 1 part in 1000 or 0.1%. Since 94.7 is about 95, which is near 100, increasing by 1.5% is nearly the same as adding 1.5. If we did that (in our heads), we would have gotten 96.2 mm. The error of this approximation is about 5% of 1.5%

mm to inches

The inverse of 25.4 mm/in works out to be 0.039370079 inches per mm. For a first approximation, I always use 0.04 inches per mm -- multiply the number of mm by 4 and shift the decimal point to the left two places. From the previous section, we know the correction is to **subtract** 1.5% (or 1.6% if we want to be exact).

Example: What's the bore in inches of a 105 mm howitzer? Multiply by 4 to get 420 and divide by 100 to get 4.2. 1.5% of this is $0.042+\frac{1}{2}(0.042) = 0.042+0.021 = 0.063$. Thus, we get 4.137 inches. The exact answer is 4.1338 inches.

kg to pounds

There are 2.2046 pounds per kg. We obviously round this to 2.2 pounds per kg. If we want a first order correction, we increase by $100\left(\frac{0.0046}{2.2}\right)$ or 0.2% (2 parts out of 1000).

Example: convert 19.77 kg to pounds. Multiply by 2 to get 39.54 and add 3.954 to this to get 43.494 pounds. To handle the first order correction, multiply the first two digits by 2 to get 86. Divide this by 1000 to get 0.086 and add. We get 43.58 pounds. The exact answer is 43.585 pounds.

pounds to kg

We need to divide by 2.2 and, if needed, use the linear correction of **subtracting** 0.2%. Now 2.2 is 2 + 2/10 or 2 + 1/5 or 11/5. Thus, dividing by 2.2 is the same as multiplying by 5/11.

The multiplication by 5 is easy, but the division by 11 is problematic. Probably the best you can do is divide by 10, then correct down by 10% (this should be a pretty obvious approximation by now so I won't derive it).

In fact, there's probably no approximation better than simply doing a long division -- fortunately, dividing by 22 isn't terribly challenging.

Example: If I weigh 200 pounds, what is my weight in kg? I did the long division 200/2.2 and it wasn't hard to see the answer was the repeating decimal $90.\underline{90}$. If we correct downwards with the 0.2%, we get 90.90 - 0.18 (i.e., just multiply 90 by 2 and shift the decimal point left 3 places) or 90.90 - 0.2 + 0.02 = 90.72 kg. The exact answer is 90.719 kg.

Densities

When I make things in my shop, they are often out of metal. The metals I use the most are steel, brass, and aluminum. I can rattle off their densities in pounds per cubic inch as 0.283, 0.307, and 0.1. The first two I remember as the cubic inch displacement of some popular Chevy engines -- the 283 and the 307. Aluminum is remembered just because it's a round decimal number. I would have preferred to memorize the specific gravities, but these are the numbers that stuck.

I also know that water's specific gravity is 1 when it's at it's densest, around 4 °C. This is 1 g/cc or 1000 kg/m^3. For more careful work, one can remember the density at 20 °C is 0.998. A gallon of water weighs 8.3 lb and a cubic foot of water weighs 62.4 lbs -- these are numbers that engineers in the US often remember.

Because density information for a variety of materials one might encounter is scattered all over, here's a short table made from the numbers I've collected over the years. Specific gravity is essentially the same as the density in g/cc for non-precise work; multiply the specific gravity by 0.03605 to get lb/in^3.

ABS plastic	1.06-1.08	Garbage	0.5	Rock salt	2.2
Acetone	0.79	Glass	2.4-2.8	Rubber	1.2
Alcohol, ethyl	0.79	Glycerine	1.26	Rubber, Buna N	1.0
Alcohol, methyl	0.792	Gold	19.3	Rubber, hard	1.2
Asphalt	1.1-1.5	Granite	2.5-3	Rubber, natural	0.93
Bakelite	1.36	Graphite	2.2	Rubber, Neoprene	1.25
Balsa wood	0.11-0.14	Gravel, dry, 1/4 to 2 inch	1.7	Rubber, Viton	1.85
Bamboo	0.3-0.4	Gravel, wet, 1/4 to 2 inch	2.0	Salt, fine	1.2
Basalt	2.8-3.2	Hay, compressed	0.1	Sand	1.2-1.6
Beech	0.7-0.9	Hydrochloric acid (38%)	1.19	Sand and gravel, dry	1.7
Beeswax	0.96	Ice at 0 °C	0.917	Sand and gravel, wet	2.0
Bentonite (dry)	0.59	Ice, crushed	0.6	Sand, rammed	1.7
Birch plywood, 1/4" thick	0.34	Iron	7.9	Sand, wet	1.92
Bone	1.8-2	Kapton polyimide	1.42	Sand, wet, packed	2.08
Brick	1.6-1.7	Kel-F	2.10	Sandstone	2.14-2.36
Bronze	8.16	Kerosene	0.82	Sawdust	0.15-0.27
Butter	0.87	Lead, cast	11.3	Sewage, sludge	0.72
Cardboard	0.7	Leather	0.95	Silicon	2.3
Cast iron	7	Linoleum	1.2	Silicon carbide	2.72
Cedar, red	0.38	Machine oil	0.9	Slate	2.6-3.3
Cement, Portland	1.6	Magnesium oxide	2.80	Snow, compacted	0.2-0.4
Chalk (dry)	1.9-2.8	Mahogany, Honduras	0.66	Snow, fresh-fallen	0.08-0.19
Charcoal	0.2	Manure	0.4	Soap powder	0.37
Charcoal, wood	0.4	Maple (wood)	0.62-0.75	Soap, solid	0.8
Cherry (wood)	0.7-0.9	Marble	2.5-2.8	Stone, crushed	1.6
Clay	1.8-2.6	Melamine	1.5-1.78	Sugar, granulated	0.85
Clay, dry excavated	1.1	Mercury	13.55	Sugar, powdered	0.80
Clay, fire	1.36	Mica	2.6-3.2	Sulfur	2.0
Coal, anthracite	1.4-1.8	Milk	1.03	Tallow	0.94
Coal, bituminous	1.2-1.5	Milk, powdered	0.45	Tar	1.1
Concrete, 8% water, with crushed rock	2.0	Mineral spirits	0.66	Teflon	2.1-2.3
		Molybdenum	10.19	Tin, cast	7.4
Concrete, 8%, reinforced	2.2	Mortar, wet	2.4	Toluene	0.87
Copper	8.9	Mylar	1.4	Tung oil	0.94
Cork	0.22-0.26	Naphtha	0.85	Tungsten	19.3
Corn (grain)	0.75	Nickel, rolled	8.67	Turf	0.40
Corundum, 90% Al2O3	3.2	Nitric acid, 91%	1.5	Turpentine	0.86
Cotton	1.5	Nylon	1.14	Vermiculite	0.13
Crude oil	0.76-0.85	Oak	0.8	Walnut	0.6-0.7
Diamond	3.0-3.5	Paper	0.7-1.15	Walnut, black	0.61
Drywall board (gypsum)	0.8	Pine (wood)	0.35-0.5	Water, deuterium oxide	1.1086
Earth, dry	1.4	Plexiglas	1.18	Water, sea	1.01-1.03
Earth, moist, excavated	1.44	Polycarbonate	1.2	Wax, paraffin	0.9
Earth, packed	1.5	Polyethylene	0.93	Wheat, cracked	0.7
Earth, soft, loose mud	1.73	Polyethylene, high density	0.941-0.965	Wool cloth	0.24
Epoxy	1.11	Polyethylene, low density	0.91-0.925	Wool, felt	0.3
Epoxy with 65% by weight glass cloth	2.0	Polypropylene	0.90	Xylene	0.89
		Polystyrene	1.04-1.08	Zinc	7.1
Ethylene glycol	1.10	Propane	0.5	Zinc, cast	7.05
Fir, Douglas	0.53	PVC, plasticized	1.15-1.35		
Firebrick	2.1	Quartz	2.6		
Flour, wheat	0.6	Redwood	0.45		

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Densities of gases at 0 °C and 760 mmHg in kg/m^3 = 10^{-3} g/cc

Acetylene	1.173	Hydrogen sulfide	1.44
Air	1.293	Krypton	3.74
Ammonia	0.771	Neon	0.900
Argon	1.783	Nitric oxide (NO)	1.27
Butane	2.53	Nitrogen	1.251
Carbon dioxide	1.977	Nitrous oxide (N_2O)	1.86
Carbon monoxide	1.25	Oxygen	1.429
CCl_3F, Refrigerant 11	5.81	Ozone	2.139
Chlorine	3.22	Propane	1.86
Fluorine	1.60	Sulfur dioxide	2.71
Helium	0.1785	Water (saturated)	0.005
Hydrogen	0.08988	Xenon	5.55
Hydrogen chloride	1.54		

Lowest density solids

Foam, formaldehyde-urea (mipora)	0.02	Sugarbeet pulp, wet	0.21
Polystyrene foam (Styrofoam)	0.04	Cork	0.22-0.26
Hay, fresh mowed	0.05	Bark	0.24
Carbon, powdered	0.08	Cork, solid	0.24
Hay, loose	0.08	Fiberboard, light	0.24
Hair felt	0.1	Wood refuse	0.24
Hay, compressed	0.1	Wool, cloth	0.24
Mineral wool blanket	0.1	Broadcloth	0.25
Wool, loose	0.1	Magnesia (85%)	0.25
Aerogel, silica	0.11	Alfalfa, ground	0.26
Balsa	0.11-0.14	Bran	0.26
Vermiculite	0.13	Peanuts, not shelled	0.27
Sawdust	0.15	Sawdust	0.27
Cork, ground	0.16	Sugarcane	0.27
Snow, freshly fallen	0.16	Charcoal, pine	0.28-0.44
Soap, chips or flakes	0.16	Oats, rolled	0.3
Cottonseed, hulls	0.19	Wool, felt	0.3
Pressed wood, pulp board	0.19	Wool, felt	0.30
Corkboard	0.2	Bamboo	0.31-0.40
Plastics, foamed	0.2	Basswood	0.32-0.59
Charcoal	0.21		
Sugarbeet pulp, dry	0.21		

Highest density solids

Brass, 80 Cu, 20 Zn	8.60	Constantan (cupronickel 55-45)	8.9
Cadmium	8.65	Nickel	8.9
Nickel, rolled	8.67	Copper, rolled	8.91
Copper, cast	8.69	Molybdenum carbide (Mo2C)	9.0
Bronze, 90 Cu, 10 Sn	8.78	Solder, 35 Sn, 65 Pb	9.50
Holmium	8.795	Bismuth	9.75
Nickel silver	8.8	Molybdenum	10.19
Solder, 50 Sn, 50 Pb	8.85	Silver	10.42-10.53
Nickel	8.89	Antimony	10.7
Solder, 50-50	8.89	Lead	11.35
Cobalt	8.9	Thorium	11.7

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Thallium	11.850	Uranium	18.8
Palladium	12.0	Tungsten, filament	19.3
Rhodium	12.4	Gold	19.32
Mercury	13.546	Plutonium	19.84
Tantalum carbide (TaC)	14.1	Rhenium	21.0
Tungsten carbide (WC)	15.2	Platinum	21.37
Tantalum	16.6	Iridium	22.4
Tungsten carbide (W2C)	17.3	Osmium	22.59
Tungsten	18.6-19.1		

When one compiles a table of numbers such as these densities, one is struck by the variation in the (supposedly same) numbers amongst different references. It's the old story of "A man with one watch knows what time it is; a man with two watches is never quite sure". If you have to do careful work, you'd be wise to search out the original literature. These numbers came from the following references:

Bolz & Tuve, *Handbook of Tables for Applied Engineering Science*, 2nd ed., CRC Press, 1973

Emsley, *The Elements*, Oxford, 1989

Glover, *Pocket Ref*, Sequoia Publishing, 1993

Koshkin & Shirkevich, *Handbook of Elementary Physics*, 3rd ed., MIR Publishers, 1977

Oberg, Jones, Horton, , *Machinery's Handbook*, 21st ed., Industrial Press, 1979

By the way, the *Handbook of Elementary Physics* is a superb little book. I bought my copy in 1978 in a bookstore in Palo Alto, CA (near Stanford University) for a couple of dollars along with the also superb *Handbook of Physics* by Yavorsky and Detlaf (also published by MIR) for $9. These books have paid for themselves hundreds of times over in the last 30 years.

Some Examples

Multiplication and division

1. What is $\frac{34.5(678)}{9012}$? As explained in the section on estimating the order of magnitude, we get $\frac{30(700)}{9000}=\frac{3\cdot 7}{9}=\frac{21}{9}=2.3$. We get the logs from the table and combine them: 1.537 + 2.831 - 3.954 = 0.414. The antilog of this from the table is <u>2.59</u>. The correct answer is <u>2.595</u>.
2. In SI, what is Planck's constant (6.626×10^{-34}) times the charge of the electron (1.602×10^{-19})? We estimate the mantissa of the answer as a little more than 1.5 times 6.6 or 6.6 + 3.3 or 10. The exponents add to get -34 - 20 + 1 = -53. The log of 6.626 is 819 plus the interpolation correction of 1.75 or 820.75 or 0.821 (the interpolation correction is 1/4th of 7). The log of 1.602 is 0.204. The log of the mantissa of the result is thus 0.821 + 0.204 or 1.025. The antilog of 0.025 is 1.055 (the interpolation is 1/2 of 3). Multiply by 10 because the result's mantissa was slightly over 1; thus, we get 10.55. The calculated answer is 10.55×10^{-53} or <u>1.055×10^{-52}</u>. The correct answer is <u>1.061×10^{-52}</u>. We're low by about half a percent.

Powers

1. What is $64^{2.7}$? First, estimate the magnitude of the answer. It will be on the order of 60^3 or $6^3\times 1000$ or 200×1000 (because 36*6 is a little over 200). Here's a better estimate: since 64 is 2^6, the answer is around $2^{2.7(6)} = 2^{16.2}$. This is a little over 2^{16}, which is about 65000. You know your powers of 2, right? They're easy to learn -- just pick a few key exponents like 10

(about 1000), 16 (65,000), 20 (2^{20} is about a million), and 30 (2^{30} is about a billion). You can figure the rest by division and multiplication

Get the logarithm of 64 in the table as 1.806. Multiply it by 2.7 to get 4.876. Look up the antilog of 0.876 in the A table to get 7.51 (mentally do the interpolation as 6/10ths of 17 is about 10). Multiply by 10^4 to get 75100. The correct value is 75281, so we're about 0.25% too low.

2. What is $e^{-3.4}$? This is $2.72^{-3.4}$ and this can be done with logarithms, just as in the previous example (I'm assuming you've used e enough to know it's decimal value is about 2.718). We first calculate $2.72^{3.4}$ and then calculate the reciprocal. The log of 2.72 is 434; we multiply this by 3.4 to get 1.476. The antilog of this is 29.9. The reciprocal table gives us 0.0335. The correct answer is 0.03337.

Trigonometry

1. In estimating the height of a tree, I used a protractor to measure the angle to the top of the tree. The angle was 43.2° and the distance from the center of the base of the tree was 22.5 m. How tall is the tree? The height is 22.5*tan(43.2°). Using the degree to radian table, we convert 43.2° to 0.7536 radians using linear interpolation. In the tangent table, we get the tangent as 932 plus the linearly interpolated part of 18*0.4 = 7.2. Thus, the tangent is 0.9392. You can now either convert the numbers to logs and add them or just long multiply them. You can see the answer will be about 6% less than 22.5. Rounding to 0.939, you can use 6.1% and only have to multiply 61*225 instead of 9392*225. There is an added subtraction, however. I get 1.3725 m as the correction, leading to the answer of 21.13 m. The exact answer is 21.129.
2. What is the tangent of 0.163°? Convert to radians and the answer will be approximately equal to the angle. Since tan(0.1) = 0.100 and since 0.1 radians is 5.7 degrees, you can see the approximation is fine for angles 6 degrees and less (which was well-known to slide rule makers who put an ST scale on their rules -- this was the scale used to calculate the sine and tangents of small (< 6 °) angles). Use the degree to radian table to convert 1.63° to 0.2844 radians, then divide by 10 to get 0.02844. the correct answer is 0.028449.
3. What's the tangent of 89°? We know it will be a number substantially greater than 1. It's the same as the cotangent of 1°. Since cot(x) is cos(x)/sin(x) and the cosine of 1° is about 1, we know we just have to calculate the reciprocal of the sine of 1°. Convert the 1° to radians: use 10° to get 0.175 and divide by 10. We want 1/0.0175. Since 1/0.02 is 50, we use the reciprocal table to get that the tangent is 57.2. The actual value is 57.3.
4. What's the tangent of 1.48 radians? One way to solve this is to use the formula

$$\tan\left(x+\frac{\pi}{4}\right)=\frac{1+\tan x}{1-\tan x}$$

(This formula comes from the tangent addition formula.) This lets us find tangents for angles greater than $\pi/4$. Since $\pi/4$ = 0.783 and x = 1.48 - 0.785 = 0.695, the tangent from the table is 0.837. Thus, we calculate 1.837/0.163. Change it to 18.4/1.63: the logs are 1.264 - 0.212 = 1.052. The antilog of 052 is 112.6, so the answer is 11.26 and we round to 11.3. The correct answer is 10.98. We're off by about 3%, which isn't too bad when you consider the slope of the tangent at 1.48 is about 120.

Another way to solve this is to use the sine and cosine tables. We convert the argument to degrees: 57.3 + 27.5 = 84.8 (here, I've remembered that 1 radian is 57.3 degrees). The sine over the cosine is 996/89. We make the denominator 90 and increase the numerator by about the same 1% to get 1000.6/90 or 100/9. Using the reciprocal table for 9, we get 100*0.111 or 11.1.

Reciprocals

1. What's $(37.9)^{-1}$? The answer will be a little larger than 1/40, which is 1/10th of 1/4 or 0.025. Looking in the Reciprocal table for 37, we get 270. Linearly interpolating for the 0.9 gives 0.9(-7) = -6.3, so we get 270 - 6.3 = 263.7; so the answer is 0.02637. The exact answer is 0.026385.

Miscellaneous

1. I want to cut some Monel on the lathe with a high speed steel bit. The recommended surface speed is 280 surface feet per minute. The material is 1.25" bar stock. What RPM should I set the lathe to? The formula is RPM = 12*SFPM/(π*D). Use the πD table to see that 1.25π is 377 + 31/2 = 383, so the answer is 3.83. In the R table, get 1/3.83 = 0.261. We split the multiplication by 12 into (10 + 2)*0.261 = 2.61 + 0.52 = 3.13. Multiply 280 by (3 + 0.1) to get 840 + 28 for 870 rpm, rounded to two places. An experienced machinist would simplify the formula to 3*SFPM/D, call D the same as 1 in this case, and multiply 3 by 280 in his head to get 840 and round off to 800 rpm. Cutting speed formulas are only a rough guideline about where to start. The sound and "feel" when making the cut would drive subsequent speed adjustments.

Table Summary

The tables are given to approximately three significant figures. Some of the tables just display three digits of the mantissa, such as the x/π table. You'll have to figure out where the decimal point is in the answer.

Some tables have "rollovers" in them. This is where the table value (as a three digit integer) went past 1000 (this happens, for example, in the tangent table). You'll have to pay attention to the values in the table and where they roll over -- otherwise, you'll use the wrong number.

Since the tangent table in radians is limited to less than 1 radian, you'll have to remember a formula to get tangents outside the table. A useful formula is

$$\tan\left(x+\frac{\pi}{4}\right)=\frac{1+\tan x}{1-\tan x}$$

In the tables, the notation a[b]c means the table's abscissas run from a to c in steps of b.

Table	Entries are:	Example
Logarithms	The values are 1000*log(x). Table is monotonic.	log(1.1) = 0.041
Antilogarithms	The values are 10^x. Table is monotonic.	$10^{0.22}$ = 1.66
Sine, degrees	The values are 1000*sin(x) for x in degrees. Table is monotonic.	sin(31°) = 0.515
Cosine, degrees	The values are 1000*cos(x) for x in degrees. Table is monotonic.	cos(31°) = 0.857
Tangent, radians	The values are 1000*tan(x) for x in radians. Table is monotonic but "rolls	tan(0.25) = 0.255

	over" to 1 at $\pi/4$ = 0.78.	
Reciprocal	The mantissas of 1/x.	1/27 = 0.0370
Square	The mantissa of x^2.	410^2 = 1680*100 = 168000
Square root	The values are $1000\sqrt{x}$. Note there are two tables, one for $\sqrt{x}$ and one for $\sqrt{10x}$. Table is monotonic.	$\sqrt{87.9}$ = 9.37
Circumference of circle	The mantissa of πx.	14π = 44.0
Area of circle	The mantissa of $\pi\frac{x^2}{4}$. This lets you work with circle diameters.	Area of a circle with a diameter of 83.7 m is 5500 m^2.
Volume of sphere	The mantissa of $\pi\frac{x^3}{6}$. This lets you work with sphere diameters.	Volume of a 1 m sphere is 0.524 m^3.
Degrees to radians	The values are $1000\left(\frac{\pi}{180}\right)x$. Table is monotonic.	5° is 0.087 rad
Radians to degrees	The values are $1000\left(\frac{180}{\pi}\right)x$. Table is monotonic.	0.1 rad is 0.57°
x/π	The mantissa of x/π.	$\frac{8}{\pi}$ = 2.55
Cube root	The values are $1000\sqrt[3]{x}$. Note there need to be three tables to cover all possible cube root mantissas. Table is monotonic.	$\sqrt[3]{87.5}$ = 4.44
Cube	The mantissa of x^3.	2.1^3 = 9.26

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Tables

Log 1[.1]10

	0	1	2	3	4	5	6	7	8	9
1	000	041	079	114	146	176	204	230	255	279
2	301	322	342	362	380	398	415	431	447	462
3	477	491	505	519	531	544	556	568	580	591
4	602	613	623	633	643	653	663	672	681	690
5	699	708	716	724	732	740	748	756	763	771
6	778	785	792	799	806	813	820	826	833	839
7	845	851	857	863	869	875	881	886	892	898
8	903	908	914	919	924	929	934	940	944	949
9	954	959	964	968	973	978	982	987	991	996

ln(10) = 2.3026, log(pi) = 0.497, log(e) = 0.4343

Antilog 0[.01]1

	0	1	2	3	4	5	6	7	8	9
0	100	102	105	107	110	112	115	117	120	123
1	126	129	132	135	138	141	145	148	151	155
2	158	162	166	170	174	178	182	186	191	195
3	200	204	209	214	219	224	229	234	240	245
4	251	257	263	269	275	282	288	295	302	309
5	316	324	331	339	347	355	363	372	380	389
6	398	407	417	427	437	447	457	468	479	490
7	501	513	525	537	550	562	575	589	603	617
8	631	646	661	676	692	708	724	741	759	776
9	794	813	832	851	871	891	912	933	955	977

Sine 0[1]90 degrees

	0	1	2	3	4	5	6	7	8	9
0	000	017	035	052	070	087	105	122	139	156
1	174	191	208	225	242	259	276	292	309	326
2	342	358	375	391	407	423	438	454	469	485
3	500	515	530	545	559	574	588	602	616	629
4	643	656	669	682	695	707	719	731	743	755
5	766	777	788	799	809	819	829	839	848	857
6	866	875	883	891	899	906	914	921	927	934
7	940	946	951	956	961	966	970	974	978	982
8	985	988	990	993	995	996	998	999	999	

Cosine 0[1]90 degrees

	0	1	2	3	4	5	6	7	8	9
0			999	999	998	996	995	993	990	988
1	985	982	978	974	970	966	961	956	951	946
2	940	934	927	921	914	906	899	891	883	875
3	866	857	848	839	829	819	809	799	788	777
4	766	755	743	731	719	707	695	682	669	656
5	643	629	616	602	588	574	559	545	530	515
6	500	485	469	454	438	423	407	391	375	358
7	342	326	309	292	276	259	242	225	208	191
8	174	156	139	122	105	087	070	052	035	017

Tangent 0[.01]1 radians

	0	1	2	3	4	5	6	7	8	9
0	000	010	020	030	040	050	060	070	080	090
1	100	110	121	131	141	151	161	172	182	192
2	203	213	224	234	245	255	266	277	288	298
3	309	320	331	343	354	365	376	388	399	411
4	423	435	447	459	471	483	495	508	521	533
5	546	559	573	586	599	613	627	641	655	670
6	684	699	714	729	745	760	776	792	809	825
7	842	860	877	895	913	932	950	970	989	009
8	030	050	072	093	116	138	162	185	210	235
9	260	286	313	341	369	398	428	459	491	524

Reciprocal [mantissa]

	0	1	2	3	4	5	6	7	8	9
1	100	909	833	769	714	667	625	588	556	526
2	500	476	455	435	417	400	385	370	357	345
3	333	323	313	303	294	286	278	270	263	256
4	250	244	238	233	227	222	217	213	208	204
5	200	196	192	189	185	182	179	175	172	169
6	167	164	161	159	156	154	152	149	147	145
7	143	141	139	137	135	133	132	130	128	127
8	125	123	122	120	119	118	116	115	114	112
9	111	110	109	108	106	105	104	103	102	101

Square [mantissa]

	0	1	2	3	4	5	6	7	8	9
1	100	121	144	169	196	225	256	289	324	361
2	400	441	484	529	576	625	676	729	784	841
3	900	961	102	109	116	123	130	137	144	152
4	160	168	176	185	194	202	212	221	230	240
5	250	260	270	281	292	302	314	325	336	348
6	360	372	384	397	410	422	436	449	462	476
7	490	504	518	533	548	563	578	593	608	624
8	640	656	672	689	706	722	740	757	774	792
9	810	828	846	865	884	903	922	941	960	980

Square root 1[.1]10

	0	1	2	3	4	5	6	7	8	9
1	100	105	110	114	118	122	126	130	134	138
2	141	145	148	152	155	158	161	164	167	170
3	173	176	179	182	184	187	190	192	195	197
4	200	202	205	207	210	212	214	217	219	221
5	224	226	228	230	232	235	237	239	241	243
6	245	247	249	251	253	255	257	259	261	263
7	265	266	268	270	272	274	276	277	279	281
8	283	285	286	288	290	292	293	295	297	298
9	300	302	303	305	307	308	310	311	313	315

Square root 10[1]100

	0	1	2	3	4	5	6	7	8	9
1	316	332	346	361	374	387	400	412	424	436
2	447	458	469	480	490	500	510	520	529	539
3	548	557	566	574	583	592	600	608	616	624
4	632	640	648	656	663	671	678	686	693	700
5	707	714	721	728	735	742	748	755	762	768
6	775	781	787	794	800	806	812	819	825	831
7	837	843	849	854	860	866	872	877	883	889
8	894	900	906	911	917	922	927	933	938	943
9	949	954	959	964	970	975	980	985	990	995

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Circumference of circle [mantissa]

	0	1	2	3	4	5	6	7	8	9
1	314	346	377	408	440	471	503	534	565	597
2	628	660	691	723	754	785	817	848	880	911
3	942	974	531	367	681	996	131	162	194	225
4	257	288	319	351	382	414	445	477	508	539
5	571	602	634	665	696	728	759	791	822	854
6	885	916	948	979	106	420	735	105	136	168
7	199	231	262	293	325	356	388	419	450	482
8	513	545	576	608	639	670	702	733	765	796
9	827	859	890	922	953	985	159	473	788	110

Area of circle [mantissa]

	0	1	2	3	4	5	6	7	8	9
1	785	950	113	133	154	177	201	227	254	284
2	314	346	380	415	452	491	531	573	616	661
3	707	755	804	855	908	962	102	108	113	119
4	126	132	139	145	152	159	166	173	181	189
5	196	204	212	221	229	238	246	255	264	273
6	283	292	302	312	322	332	342	353	363	374
7	385	396	407	419	430	442	454	466	478	490
8	503	515	528	541	554	567	581	594	608	622
9	636	650	665	679	694	709	724	739	754	770

Volume of sphere [mantissa]

	0	1	2	3	4	5	6	7	8	9
1	524	697	905	115	144	177	214	257	305	359
2	419	485	558	637	724	818	920	103	115	128
3	141	156	172	188	206	224	244	265	287	311
4	335	361	388	416	446	477	510	544	579	616
5	654	695	736	780	824	871	920	970	102	108
6	113	119	125	131	137	144	151	157	165	172
7	180	187	195	204	212	221	230	239	248	258
8	268	278	289	299	310	322	333	345	357	369
9	382	395	408	421	435	449	463	478	493	508

Deg to rad 0[1]100

	0	1	2	3	4	5	6	7	8	9
0	000	017	035	052	070	087	105	122	140	157
1	175	192	209	227	244	262	279	297	314	332
2	349	367	384	401	419	436	454	471	489	506
3	524	541	559	576	593	611	628	646	663	681
4	698	716	733	750	768	785	803	820	838	855
5	873	890	908	925	942	960	977	995	012	030
6	047	065	082	100	117	134	152	169	187	204
7	222	239	257	274	292	309	326	344	361	379
8	396	414	431	449	466	484	501	518	536	553
9	571	588	606	623	641	658	676	693	710	728

Rad to deg 0[.01]1

	0	1	2	3	4	5	6	7	8	9
0	000	006	011	017	023	029	034	040	046	052
1	057	063	069	074	080	086	092	097	103	109
2	115	120	126	132	138	143	149	155	160	166
3	172	178	183	189	195	201	206	212	218	223
4	229	235	241	246	252	258	264	269	275	281
5	286	292	298	304	309	315	321	327	332	338
6	344	350	355	361	367	372	378	384	390	395
7	401	407	413	418	424	430	435	441	447	453
8	458	464	470	476	481	487	493	498	504	510
9	516	521	527	533	539	544	550	556	561	567

x/pi [mantissa]

	0	1	2	3	4	5	6	7	8	9
1	318	350	382	414	446	477	509	541	573	605
2	637	668	700	732	764	796	828	859	891	923
3	955	987	102	105	108	111	115	118	121	124
4	127	131	134	137	140	143	146	150	153	156
5	159	162	166	169	172	175	178	181	185	188
6	191	194	197	201	204	207	210	213	216	220
7	223	226	229	232	236	239	242	245	248	251
8	255	258	261	264	267	271	274	277	280	283
9	286	290	293	296	299	302	306	309	312	315

Cube root 1[.1]10

	0	1	2	3	4	5	6	7	8	9
1	100	103	106	109	112	114	117	119	122	124
2	126	128	130	132	134	136	138	139	141	143
3	144	146	147	149	150	152	153	155	156	157
4	159	160	161	163	164	165	166	168	169	170
5	171	172	173	174	175	177	178	179	180	181
6	182	183	184	185	186	187	188	189	189	190
7	191	192	193	194	195	196	197	197	198	199
8	200	201	202	202	203	204	205	206	206	207
9	208	209	210	210	211	212	213	213	214	215

Cube root 10[1]100

	0	1	2	3	4	5	6	7	8	9
1	215	222	229	235	241	247	252	257	262	267
2	271	276	280	284	288	292	296	300	304	307
3	311	314	317	321	324	327	330	333	336	339
4	342	345	348	350	353	356	358	361	363	366
5	368	371	373	376	378	380	383	385	387	389
6	391	394	396	398	400	402	404	406	408	410
7	412	414	416	418	420	422	424	425	427	429
8	431	433	434	436	438	440	441	443	445	446
9	448	450	451	453	455	456	458	459	461	463

Cube root 100[10]1000

	0	1	2	3	4	5	6	7	8	9
1	464	479	493	507	519	531	543	554	565	575
2	585	594	604	613	621	630	638	646	654	662
3	669	677	684	691	698	705	711	718	724	731
4	737	743	749	755	761	766	772	777	783	788
5	794	799	804	809	814	819	824	829	834	839
6	843	848	853	857	862	866	871	875	879	884
7	888	892	896	900	905	909	913	917	921	924
8	928	932	936	940	944	947	951	955	958	962
9	965	969	973	976	980	983	986	990	993	997

Cube [mantissa]

	0	1	2	3	4	5	6	7	8	9
1	100	133	173	220	274	338	410	491	583	686
2	800	926	106	122	138	156	176	197	220	244
3	270	298	328	359	393	429	467	507	549	593
4	640	689	741	795	852	911	973	104	111	118
5	125	133	141	149	157	166	176	185	195	205
6	216	227	238	250	262	275	287	301	314	329
7	343	358	373	389	405	422	439	457	475	493
8	512	531	551	572	593	614	636	659	681	705
9	729	754	779	804	831	857	885	913	941	970

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