3

1. $(x_A - x_D)^2 + (y_A - y_D)^2 = r_{AD}^2$ — ①

$(x_B - x_D)^2 + (y_A - y_D)^2 = r_{BD}^2$ — ②

$(x_C - x_D)^2 + (y_C - y_D)^2 = r_{CD}^2$ — ③

Substituting ③ from ① and ②, we get

$$2(x_c - x_A)x_D + 2(y_C - y_A)y_D = (r_{AB}^2 - r_{CD}^2) - (x_A^2 - x_C^2) - (y_A^2 - y_C^2)$$

$$2(x_c - x_B)x_D + 2(y_C - y_B)y_D = (r_{BD}^2 - r_{CD}^2) - (x_B^2 - x_C^2) - (y_B^2 - y_C^2)$$

$$2\begin{bmatrix} x_c - x_A & y_C - y_A \\ x_c - x_B & y_C - y_B \end{bmatrix}\begin{bmatrix} x_D \\ y_D \end{bmatrix} = \begin{bmatrix} (r_{AD}^2 - r_{CD}^2) - (x_A^2 - x_C^2) - (y_A - y_C)^2 \\ (r_{BD}^2 - r_{CD}^2) - (x_B^2 - x_C^2) - (y_B - y_C^2) \end{bmatrix}$$

$$2\begin{bmatrix} -2 & 10 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x_D \\ y_D \end{bmatrix} = \begin{bmatrix} 15 \\ 8 \end{bmatrix}$$

$x_D = 1.3125$

$y_D = 1.0125$

$\therefore (x_D, y_D) = (1.3125, 1.0125)$

- Node E.

$$\Rightarrow 2\begin{bmatrix} x_D - x_A & y_D - y_A \\ x_D - x_C & y_D - y_C \end{bmatrix}\begin{bmatrix} x_E \\ y_E \end{bmatrix} = \begin{bmatrix} 21.5 \\ -82.5 \end{bmatrix}$$

$$\Rightarrow \quad x_E = 2.36$$
$$y_E = 5.67$$

$$\therefore E(2.36, 5.67)$$

- Node G

Similar to A, E, J we find G by

$$2\begin{bmatrix} x_J - x_A & y_J - y_A \\ x_J - x_E & y_J - y_A \end{bmatrix}\begin{bmatrix} x_G \\ y_G \end{bmatrix} = \begin{bmatrix} (r_{AG}^2 - r_{JG}^2) - (x_A^2 - x_J^2) - (y_A^2 - y_J^2) \\ (r_{EG}^2 - r_{JG}^2) - (x_E^2 - x_J^2) - (y_E^2 - y_J^2) \end{bmatrix}$$

$$2\begin{bmatrix} 6 & 8 \\ 7.64 & 0.33 \end{bmatrix}\begin{bmatrix} x_G \\ y_G \end{bmatrix} = \begin{bmatrix} 75.75 \\ 58.012 \end{bmatrix}$$

$$(x_G, y_G) = G(3.71, 1.95)$$

- Node F.

$$\Rightarrow 2\begin{bmatrix} x_J - x_A & y_J - y_A \\ x_J - x_G & y_J - y_G \end{bmatrix}\begin{bmatrix} x_F \\ y_F \end{bmatrix} = \begin{bmatrix} (r_{AF}^2 - r_{JF}^2) - (x_A^2 - x_J^2) - (y_A^2 - y_J^2) \\ (r_{AF}^2 - r_{JF}^2) - (x_G^2 - x_J^2) - (y_G^2 - y_J^2) \end{bmatrix}$$

$$\Rightarrow 2\begin{bmatrix} 6 & 8 \\ 6.29 & 4.05 \end{bmatrix}\begin{bmatrix} x_F \\ y_F \end{bmatrix} = \begin{bmatrix} 156 \\ 118.4 \end{bmatrix}$$

$x_F = 6.06$ $\qquad$ $F(6.06, 5.2)$

$y_F = 5.2$

Q.3 The distance between the Anchore node can be found and is shown below.

- Distance between A and B.

$$d_{AB} = \sqrt{(4-(-1))^2 + (-2-(3))^2}$$

$$= \sqrt{(5)^2 + (-5)^2}$$

$$= \sqrt{25+25}$$

$$= \sqrt{50}$$

$$d_{AB} = 7.07$$

- Distance between A and C is

$$d_{AC} = \sqrt{(4-2)^2 + (-2-8)^2}$$

$$= \sqrt{(2)^2 + (-10)^2}$$

$$= \sqrt{4+100}$$

$$= \sqrt{104}$$

$$= 10.198$$

- Distance between A and J is

$$d_{AJ} = \sqrt{(4-10)^2 + (-2-6)^2}$$

$$= \sqrt{(-6)^2 + (-8)^2}$$

$$= \sqrt{36+64}$$

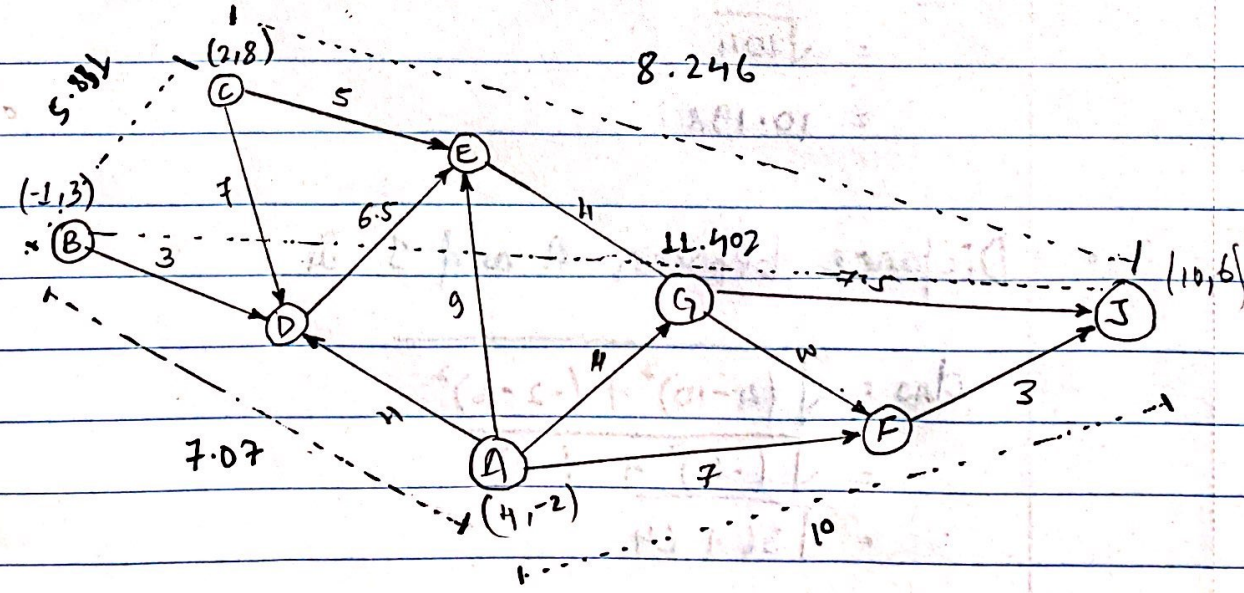
$d_{AJ} = 10$

- Distance between B and C is

$$d_{BC} = \sqrt{(-1-2)^2 + (3-8)^2}$$
$$= \sqrt{(-3)^2 + (-5)^2}$$
$$= \sqrt{9 + 25}$$
$$= \sqrt{34}$$
$$d_{BC} = 5.831$$

- Distance between C and J is

$$d_{CJ} = \sqrt{(10-2)^2 + (6-8)^2}$$
$$= \sqrt{(8)^2 + (-2)^2}$$
$$= \sqrt{64 + 4}$$
$$= \sqrt{68}$$
$$d_{CJ} = 8.246$$



- Distance between node 13 and J is

$$d_{BJ} = \sqrt{(11)^2 + (3)^2}$$

$$= \sqrt{(121) + 9}$$

$$d_{BJ} = \sqrt{130}$$

$$d_{BJ} = 11.402$$