



# LOCALIZATION

# INTRODUCTION: WHY, WHAT, HOW?

# LOCALIZATION

Means for a node to determine its physical position (with respect to some coordinate system) or reference location

Using the help of

- GPS
- Anchor nodes that know their position
- Directly adjacent
- Over multiple hops

Using different means to determine distances/angles locally

# WSN AND LOCATION AWARENESS

- WSNs are often deployed to:
  - monitor phenomena in the physical world,
  - establish spatial relationships between the nodes, and
  - detect objects and events; hence offering many convenient services.
- For example WSNs deployed in a forest to raise alarms whenever wildfires occur. For such alarms to be actionable, the location of the event that triggered the alarm must be reported as well.

There are many applications where knowledge of sensor location is a fundamental requirement, without which the network will have limited value.

# WHY LOCALIZATION?

Location information could be useful for applications in ad hoc networks, e.g.:

- For sensor networks that collect data or report events, it's important to know where the data readings or events occurred.
- For networks that control robots, cars, or airplanes, it's important to know where the robots, cars,...are.

Location information can be used in location-aware algorithms, for tasks like:

- Routing point-to-point messages
- Network-wide broadcasting
- Aggregating data on the way to a central location
- Establishing spanning trees
- Implementing virtual geography-based infrastructure
- Allocating nearby resources

# WHAT IS NODE LOCALIZATION?

Localization is the process of determining the physical coordinates of a sensor node or the spatial relationships among objects.

There are many techniques to achieve this goal, a sensor node can collect information from the surroundings to estimate its position.

For example, from ranging measurements such as RSS, the distances or ranges between a number of transmitters and a received node could be measured and used to find the position of the receiver node.

There is a wide range of Localization techniques.

Ultimately, detection and estimation theory, and distributed estimation are basis for such techniques.

# LOCALIZATION APPROACHES

Localization can be generally divided into two classes, **range-based** techniques and **range-free** techniques.

- ❖ Range-based techniques are based on distance measurement using ranging techniques (for example, the received signal strength, angle of arrival, and time of arrival).
- ❖ They require presence of at least three special nodes, called “anchor nodes” whose positions are clearly known by all the nodes in the network.
- ❖ The position of unknown nodes can be estimated using the distance measurements between them and anchor nodes.
- ❖ In contrast, range-free techniques do not require distance measurements, they use connectivity information to determine the positions.
- ❖ Triangulation, Trilateration, Iterative and Collaborative Multilateration are some examples of range-based localization.
- ❖ Ad-Hoc Positioning System, Approximate Point in Triangulation and Localization Based on Multidimensional Scaling fall in the category of range-free localization.

# CHALLENGES

Sensor localization faces many challenges:

- ❖ physical layer measurement errors,
- ❖ computational constraints,
- ❖ lack of GPS data,
- ❖ low-end sensor nodes, and varying configurations.

In addition, different applications have different requirements.

A WSN localization system will come across various challenges to fulfill all kinds of requirements.

# PHYSICAL LAYER MEASUREMENTS

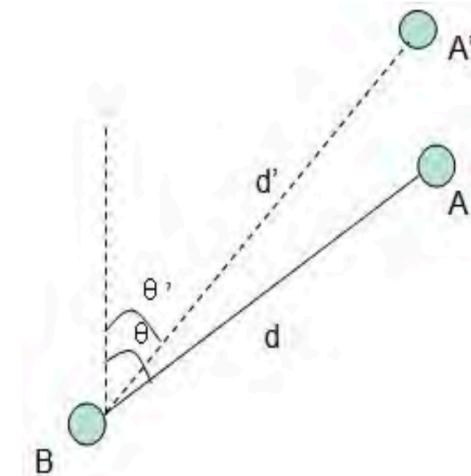
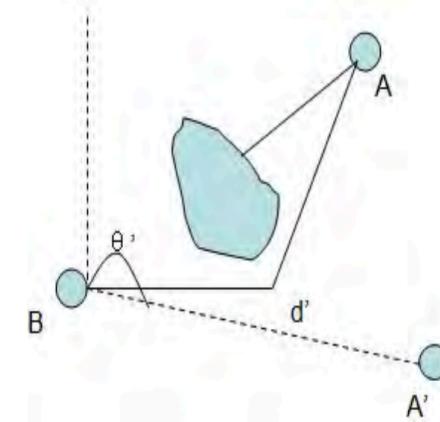
Sensor nodes can use ranging techniques/hardware to estimate their positions.

- The ranging measurements can be time, angle or received signal strength.
- These measurements can affect the accuracy of the estimated positions greatly.
- For line-of-sight communication, small error in the measurement leads to large deviation in the estimated position.

See figure: Node A transmits a signal to node B in order to estimate its position using received signal strength.

If the estimated angle measurement varies slightly from the actual angle, and the range is large, then the estimated position differs significantly different from the one.

In addition, if there is an obstacle between A and B, then multipath propagation can induce even larger error in the estimates.



# COMPUTATIONAL CONSTRAINTS

- A node uses information such as **distance, time, orientation or connectivity** computed in collaboration with neighbors to estimate its positions.
- In order to find the exact location, a node should combine as much information as possible to perform position estimation algorithms, this may necessitate a costly computational complexity, that are beyond the cost constraints of the sensor.
- Centralizing the localization on a well resourced node may mitigate the hardware cost, but introduces transmission cost, power consumption, and bandwidth reduction.
- Therefore, localization techniques must explore cost effective ways to estimate positions

# LACK OF GPS

GPS(Global Positioning System) is a widely used technology in determining position in many applications, such as navigation, and phones.

It can provide reasonably accurate positioning in outdoor with clear line-of-sight situations,

It is highly unreliable in indoor situations.

Due to its formfactor requirement, power consumption, and cost, It is not feasible to use GPS in all of the nodes in a WSN.

However, it is possible to equip a selected set of network nodes with GPS capabilities. These anchor nodes may then use the GPS as a mean to assist other nodes with positioning.

# LOW-END SENSOR NODE

Wireless sensor nodes tend to be equipped with low-end components to meet unit cost constraints.

These imperfect components pose several challenges for localization in WSNs.

In addition to errors in range measurement, some hardware errors can also be introduced into the measurement processes.

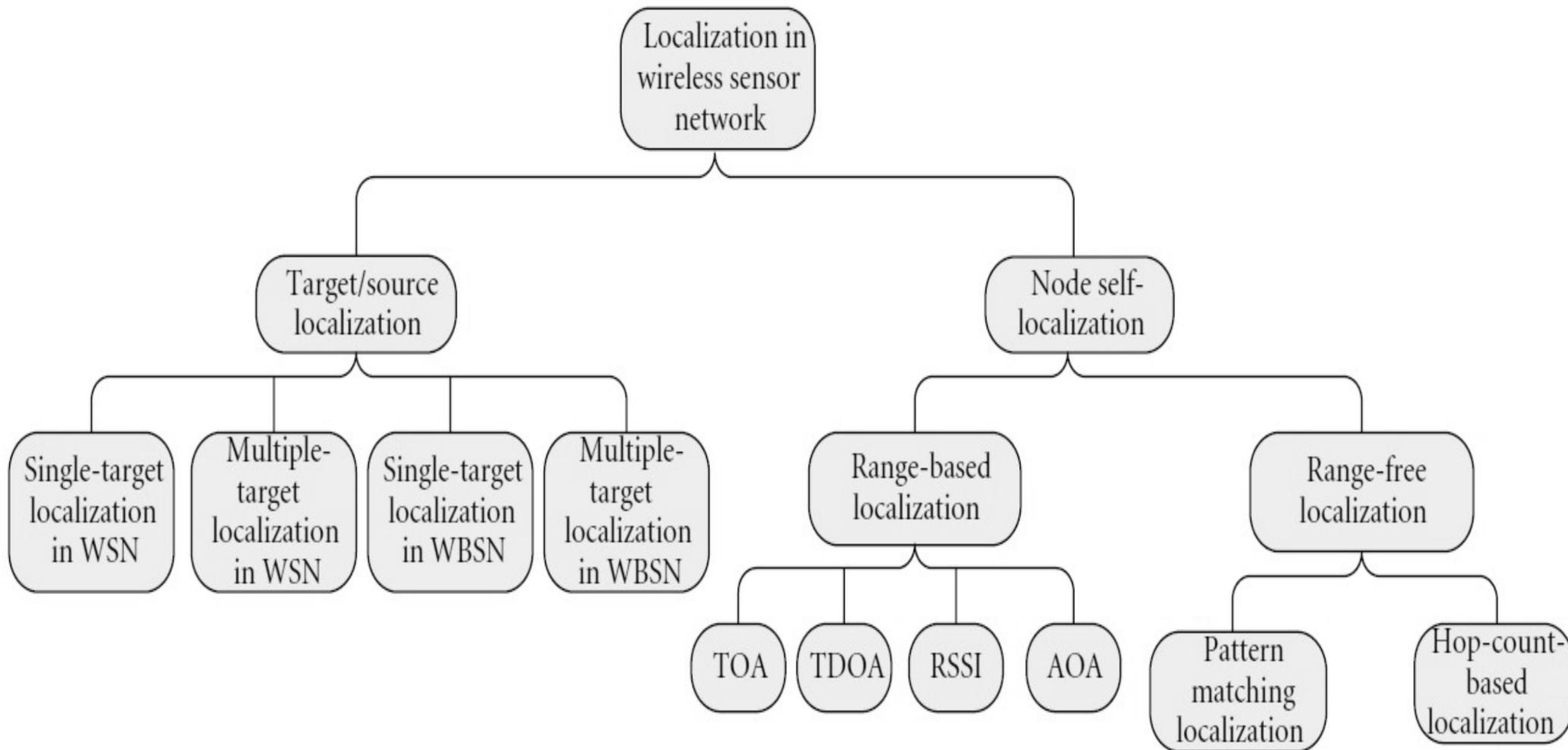
These errors are random, and hence we can do little to avoid them.

Low-end component can lead to frequent node failure, and hence affect the reliability of the localization process.

# WHAT SENSOR NODE LOCALIZATION?

- Output: nodes' location.
  - Global location, e.g., what GPS gives.
  - Relative location.
- Input:
  - Connectivity, hop count.
    - Nodes with  $k$  hops away are within Euclidean distance  $k$ .
    - Nodes without a link must be at least distance 1 away.
  - Distance measurement of an incoming link.
  - Angle measurement of an incoming link.
  - Combinations of the above.

# || HOW? WIDE RANGE OF POSSIBILITIES



# SELF-LOCALIZATION

- Given distances or angle measurements, find the locations of the sensors.
- **Anchor-based**
  - Some nodes know their locations, either by a GPS or as pre-specified.
- **Anchor-free**
  - Relative location only.
  - A harder problem, need to solve the global structure. Nowhere to start.
- **Range-based**
  - Use range information (distance estimation).
- **Range-free**
  - No distance estimation, use connectivity information such as hop count.



# RANGING TECHNIQUES

Estimating distances

# REQUIRED LOCALIZATION MEASUREMENTS

In order for a node to estimate its position it needs to know distances (or angles) to other nodes.

## Distance estimation:

- Received Signal Strength Indicator (RSSI)
  - The further away, the weaker the received signal.
  - Mainly used for RF signals.
- Time of Arrival (ToA) or Time Difference of Arrival (TDoA)
  - Signal propagation time translates to distance.
  - RF, acoustic, infrared and ultrasound.

## Angle estimation:

- Angle of Arrival (AoA)
  - Determining the direction of propagation of a radio-frequency wave incident on an antenna array.
- Directional Antenna
- Special hardware, e.g., laser transmitter and receivers.

# DISTANCE ESTIMATION BASED RECEIVED SIGNAL STRENGTH

The method of measuring received signal strength **relies on that a transmitted signal power decays with distance.**

If we can estimate the attenuation the transmitted signal experiences before arriving at the receiver, we can use the amount of attenuation to estimate the distance between the transmitter and receiver.

Received Signal Strength Indicator (RSSI) is a common feature in wireless devices that can measure the power of the incoming radio signal.

The RSSI is a mapping of the power into quantized levels. The mapping between the RSSI value and signal power varies from vendor to vendor. The distance can be calculated according to the received signal power.

In the simplest case of no attenuations due to slow and fast fading, the Friis transmission equation for free space gives

$$\frac{P_r}{P_t} = G_t G_r \frac{\lambda^2}{(4\pi)^2 d^2},$$

The unknown

In reality, the signal power is affected by multi-path propagation, noise and so on, so this equation gives a rough ideal approximation

Where,

$P_r$  = Power at the receiving antenna

$P_t$  = Output power of transmitting antenna

$G_t$  = Gain of the transmitting antenna

$G_r$  = Gain of the receiving antenna

$\lambda$  = Wavelength

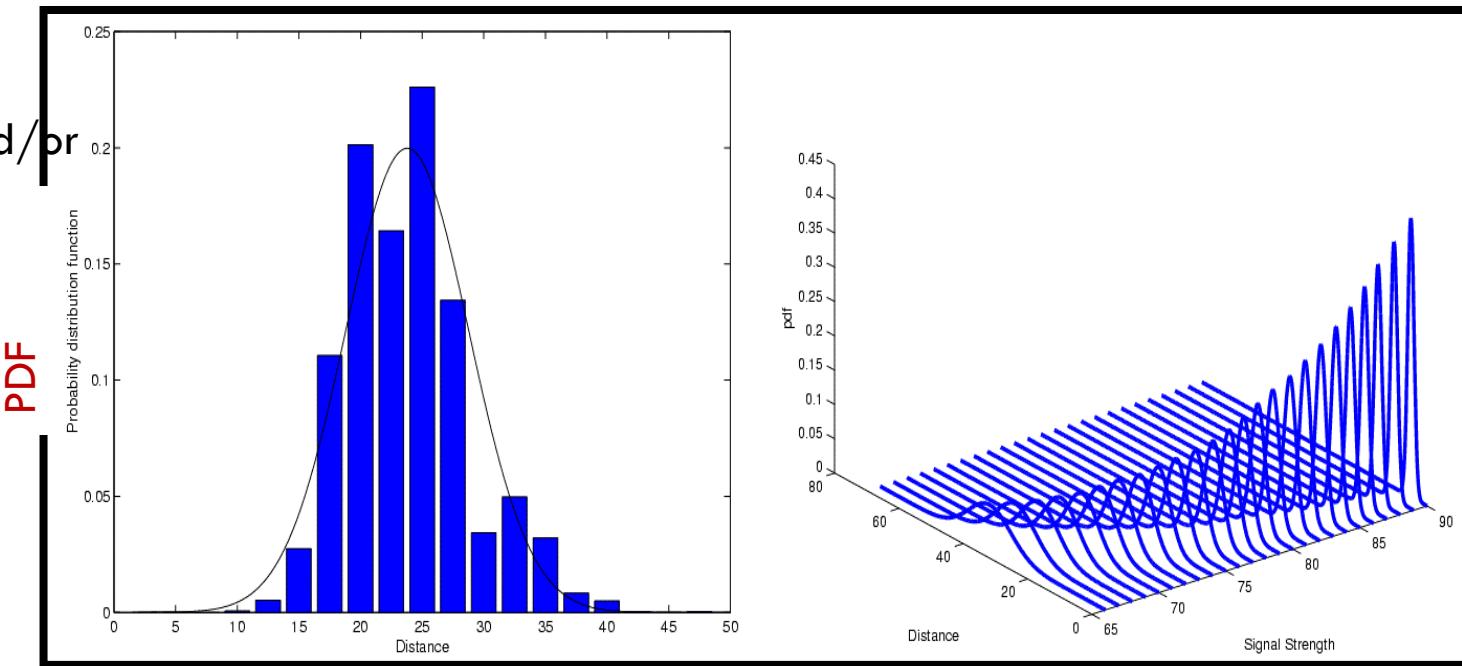
$d$  = Distance between the antennas

# RSSI BASED DISTANCE ESTIMATION

Nothing is easy: the relationship between RSSI and distance is inconsistent, never deterministic and/or Precise. It is highly affected by random measures

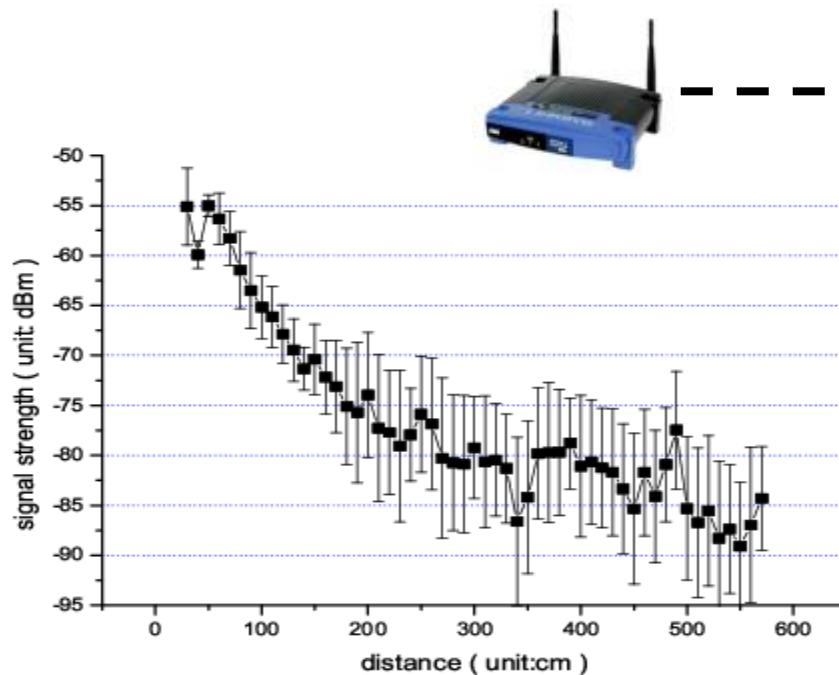
## Received Signal Strength Indicator

- Send out signal of known strength, use received signal strength and path loss coefficient to estimate distance
- Problem: Highly error-prone process – Shown: PDF for a **fixed RSSI**



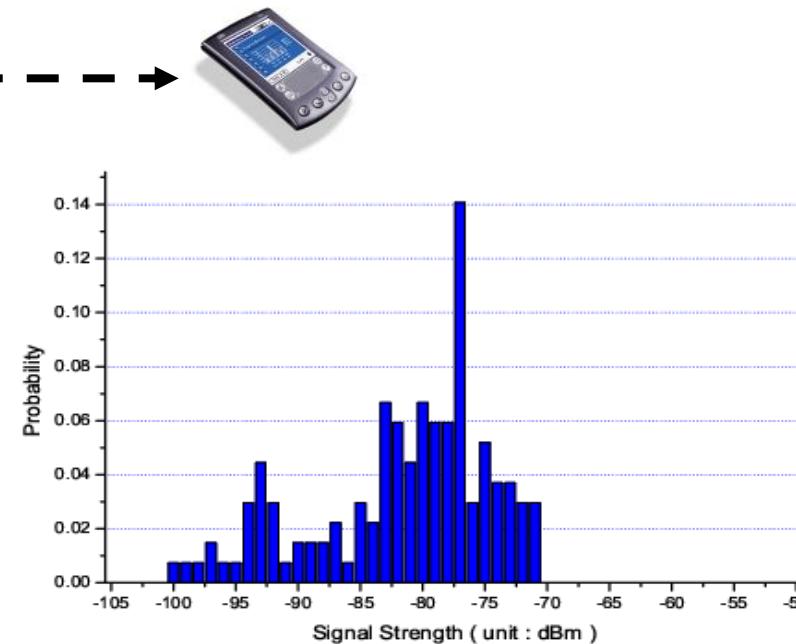
$$P_{\text{recv}} = c \frac{P_{\text{tx}}}{d^\alpha} \Leftrightarrow d = \sqrt[\alpha]{\frac{c P_{\text{tx}}}{P_{\text{recv}}}}$$

# “CHEAP AND UBIQUITOUS: RECEIVED-SIGNAL-STRENGTH” ?



Signal attenuation along with distance  
**nonlinear**

$d_1 < d_2$  but sometimes  $\text{RSSI}_1 < \text{RSSI}_2$



Signal Distribution at a fixed location  
**noisy**

# CLASSICAL PATH LOSS MODEL

$$P_{rx} = P_{tx} + K - 10\eta \log_{10}\left(\frac{d}{d_0}\right) + \psi + \alpha(t)$$

rx power (in [dBm])

tx power (in [dBm])

path loss coefficient

constant (free space atten., antenna gain,...)

actual distance

slow fading (shadowing)

fast fading

reference distance

free space + shadowing

$\rightarrow \begin{cases} K = 20 \log_{10}(\lambda / 4\pi d_0) \\ \eta = 2 \\ \psi \rightarrow N(0, \sigma_\psi^2) \end{cases}$

Least Mean Square criterion to estimate  $K$  and  $\eta$

# DISTANCE ESTIMATION USING TIME OF ARRIVAL

Determine the distance between the sensor nodes using signal propagation time measurement and known propagation velocity.

The chief principle here is that the longer the distance between two nodes the longer time it takes the signal to travel between them.

ToA has two types, **one-way** ToA and **two-way** ToA.

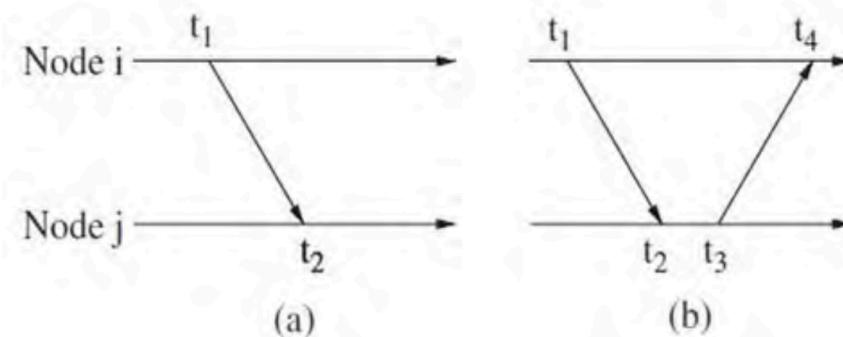
One-way ToA measures the one-way signal propagation time and requires the sender and the receiver to be **synchronized** with each other.

The difference between the sending time and receiving times is calculated as shown in the figure. The distance between the nodes  $i$  and  $j$  can be determined as

$$d_{ij} = (t_2 - t_1) \times v$$

where  $t_1$  and  $t_2$  are the sending and receive times of the signal (measured at the sender and receiver, respectively).

Here, the receiver calculates the distance and uses it to determine its location.



One-way and two-way ToA ranging measurement scheme.

# TWO-WAY TOA

In two-way ToA, in addition to the first signal, the receiver then transmits a response signal back to transmitter.

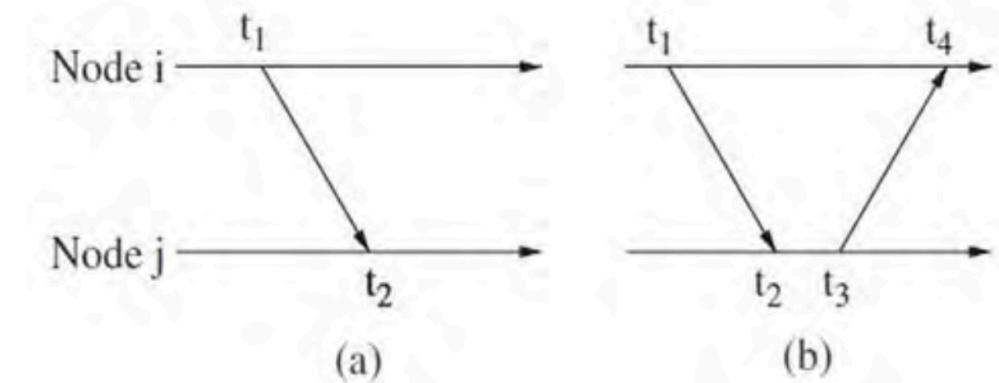
So we have four time points and the transmitter uses them, together with signal velocity, to measure the distance.

$$d_{ij} = \frac{(t_4 - t_1) - (t_3 - t_2)}{2} \times v,$$

where  $t_3$  and  $t_4$  are the sending and receive times of the response signal.

Hence the transmitter is calculating the receiver's location. Note that a third message is necessary to inform the receiver about its location.

Moreover this two-way technique **does not require synchronization** of the sender and the receiver, hence making it a preferred approach.



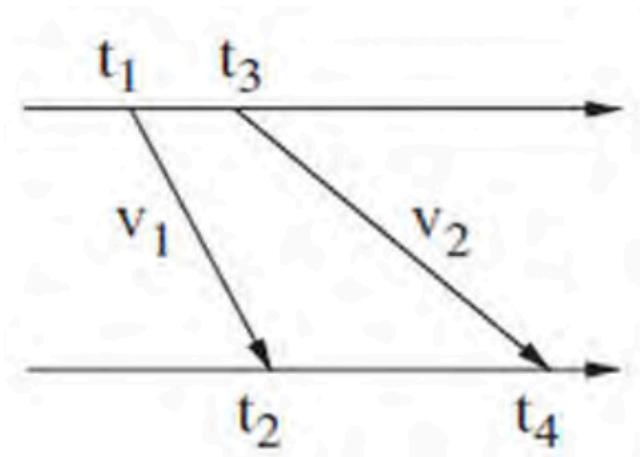
One-way and two-way ToA ranging measurement scheme.

# TDOA LOCALIZATION

Here,

- ✓ the time difference of arrival (TDoA) approach uses two signals that travel with different velocities.
- ✓ The transmitting node sends a signal with speed  $v_1$  at time  $t_1$ , the receiving node receives this signal at time  $t_2$ .
- ✓ After a time delay  $t_{\text{delay}} = t_3 - t_1$ , the transmitter sends another signal with velocity  $v_2$ , the receiver gets this signal at time  $t_4$ .
- ✓ The distance between transmitter and receiver can be estimated using these measurements.

$$d_{ij} = (t_4 - t_2 - t_{\text{delay}}) \times (v_1 - v_2) .$$

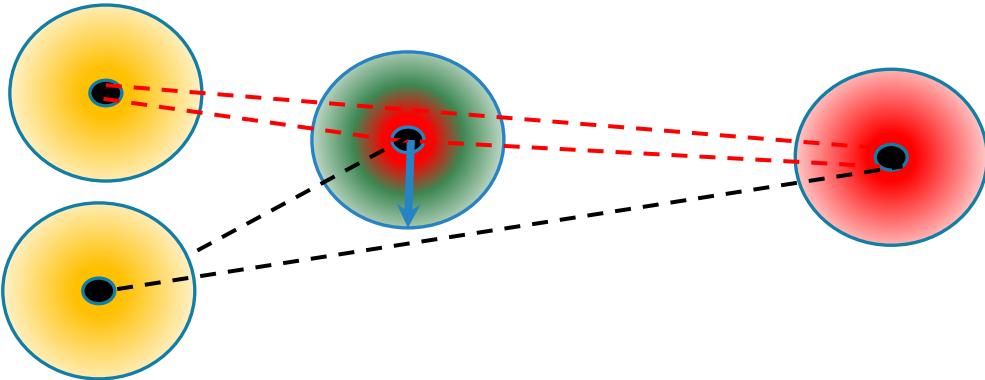


TDoA measurement scheme.

- One advantage of TDoA is that it does not require time synchronization between the transmitter and the receiver.
- The estimation of TDoA may have a better accuracy compared to ToA, but it can require additional hardware.

# RANGE BASED DISTANCE ESTIMATION

Poorer configuration  
Erroneous Estimate

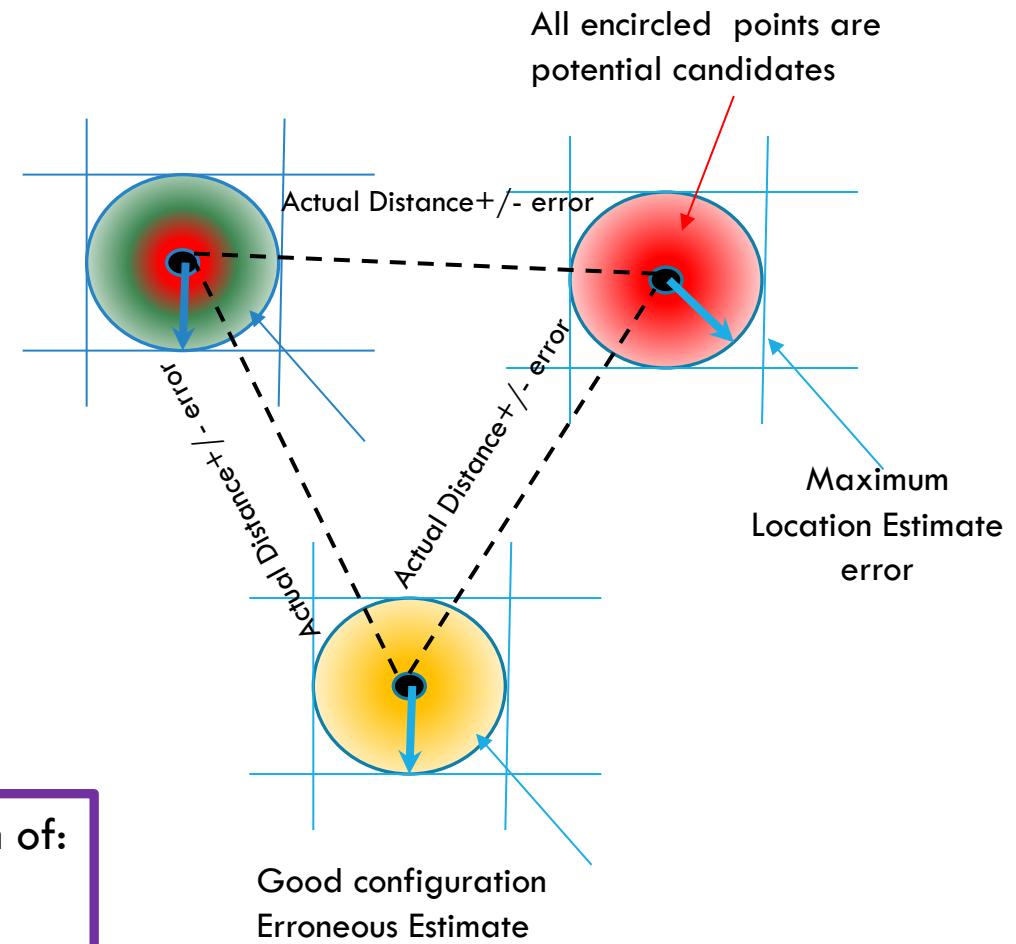


Poor configuration  
Erroneous Estimate

Poor/Poorer configuration  
Erroneous Estimate

Distance estimation error is a function of:

- ✓ Environmental conditions,
- ✓ Radio hardware,
- ✓ Time synchronization
- ✓ Relative positioning arrangements

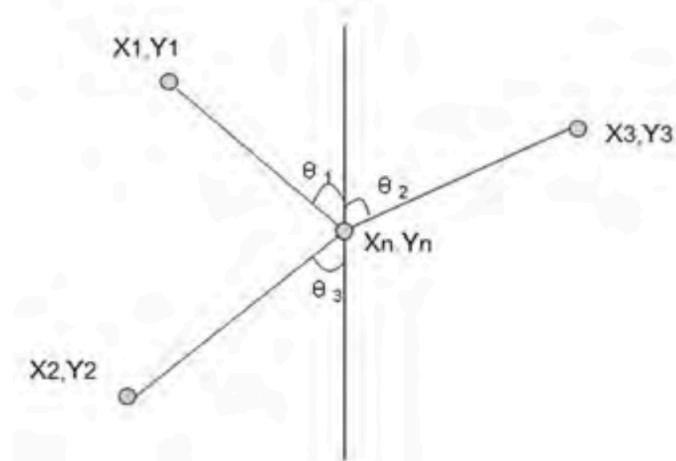
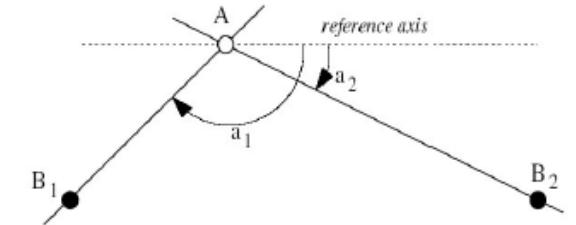


# ANGLE OF ARRIVAL

Here,

- ✓ A node can estimate its position by measuring angles of arriving signals using an array of antennas or microphones.
- ✓ Measurements are made from at least three anchor nodes as shown in the Figure.
- ✓ This mechanism can be used to estimate the sensor localization. Its accuracy depends on the accuracy of directional antennas.
- ✓ Also, this method can easily get corrupted in **NLoS** environment due to multi-path fading and scattering, which prevent the accurate measurement of the angles.

- A measures the direction of an incoming link by radio array.
- By using 2 anchors, A can determine its position.



Angle of Arrival ranging measurement scheme.

# || TRIANGULATION ERROR

Consider the situation depicted in the figure.

- ✓ A source is located on the right bisector of the line of length  $d$  joining two reference points.
- ✓ The angle is measured with an error of  $1^\circ$ , with nominal angles of  $45^\circ$  and  $85^\circ$ . Compute the relative errors for the two situations and comment.

Solution:

$$\tan\theta = 2R/d,$$

$$R = (d/2) \tan\theta.$$

$R_1$  the distance computed when the measurement error

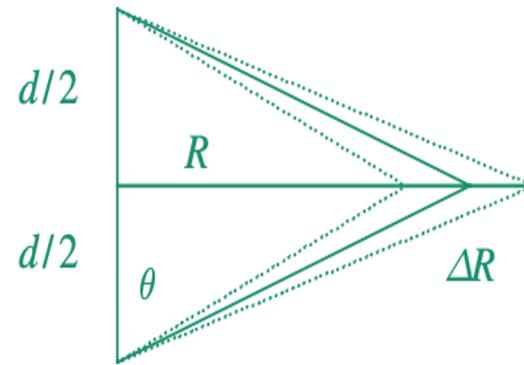
Causes the angle to be larger than the nominal value by  $\Delta\theta$ .

$$R_1 = (d/2) \tan(\theta + \Delta\theta)$$

$$\Delta R = (d/2) [\tan(\theta + \Delta\theta) - \tan\theta].$$

The relative error is given by

$$[\tan(\theta + \Delta\theta) - \tan\theta]/\tan\theta. \quad \text{For } \theta = 45^\circ, \text{ and } \Delta\theta = 1^\circ \text{ this is } 0.036, \quad \text{while for } 85^\circ \text{ it is } 0.25.$$



Source location uncertainty.

Thus, in estimating a new location using angle measurements there is greater sensitivity when the distance is large compared to the baseline.



## RANGING BASED LOCALIZATION

How these distance estimation techniques are used to localize nodes?

# || TRIANGULATION (ANGLES-BASED)

The position of the unknown node is estimated by measuring the angle of arrival of signals from **anchor nodes**.

Statistical methods (e.g, maximum likelihood) is used to minimize the estimation error.

Example:

Three anchor nodes with known locations  $x = [x_i, y_i]^T$  where  $i=1,2,3$  and one unknown node at location  $x_r = [x_r, y_r]^T$ . The actual angles between the **unknown node** and **anchors** are

$$\theta(X_r) = [\theta_1(X_r), \dots, \theta_N(X_r)]^T,$$

where  $N=3$  and

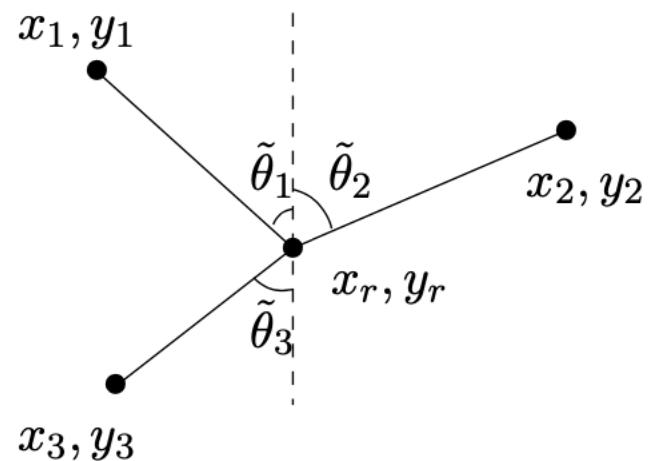
$$\theta_i(X_r) = \arctan \frac{x_r - x_i}{y_r - y_i}.$$

But due to some noise or errors in the measurement process, the measured angles do not perfectly reflect the actual angles and are represented as  $Y = [\tilde{\theta}_1, \dots, \tilde{\theta}_N]^T$ , so that we have

$$Y = \theta(X_r) + \underline{n}, \quad (10.5)$$

where  $\underline{n} = [n_1, n_2, n_3]^T$  is Gaussian noise with zero mean and covariance given by

$$R = \mathbb{E}\{n \cdot n^T\} = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}. \quad (10.6)$$



# TRIANGULATION

If we use ML criterion to estimate a sensor's location, the location estimation is achieved by minimizing the following error covariance

$$\begin{aligned} C(X_r) &= \left[ \theta(\hat{X}_r) - Y \right]^T R^{-1} \left[ \theta(\hat{X}_r) - Y \right] \\ &= \sum_{i=1}^3 \frac{\left( \theta_i(\hat{X}_r) - \tilde{\theta}_i \right)^2}{\sigma_i^2}, \end{aligned}$$

and hence

$$\hat{X}_r = \operatorname{argmin}_{X_r} C(X_r)$$

To find the position estimation ( $\hat{X}_r = [\hat{x}_r, \hat{y}_r]^T$ ), we have to minimize  $C(X_r)$ . This can be achieved by taking a derivative of  $C(X_r)$  and equating it to zero. Since this is the minimization of a non-linear least-square, we apply the Newton-Gauss method:

$$\begin{aligned} \hat{X}_{r,i+1} &= \hat{X}_{r,i} \\ &+ (\theta_X(\hat{X}_{r,i})^T S^{-1} \theta_X(\hat{X}_{r,i}))^{-1} \theta_X(\hat{X}_{r,i})^T S^{-1} [Y - \theta_X(\hat{X}_{r,i})], \end{aligned} \tag{10.10}$$

where  $\theta_X(\hat{X}_{r,i})$  is the matrix of the partial derivatives of  $\theta$  with respect to its arguments and evaluated at  $\hat{X}_{r,i}$ . As  $i$  tends to  $\infty$ ,  $\hat{X}_{r,i}$  tends to the minimum of  $C(X_r)$ . Equation (10.10) requires an initial estimate that is close to the true minimum of the cost function.

# TRILATERATION

Trilateration is the process of calculating a node's position based on the measured distance between this node and other anchor nodes whose positions are known.

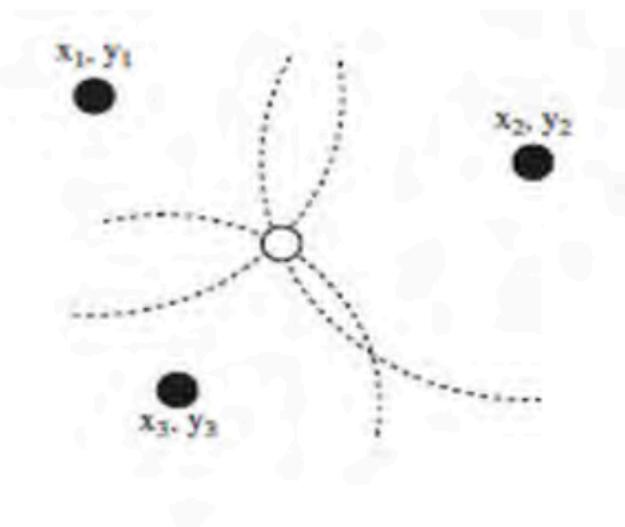
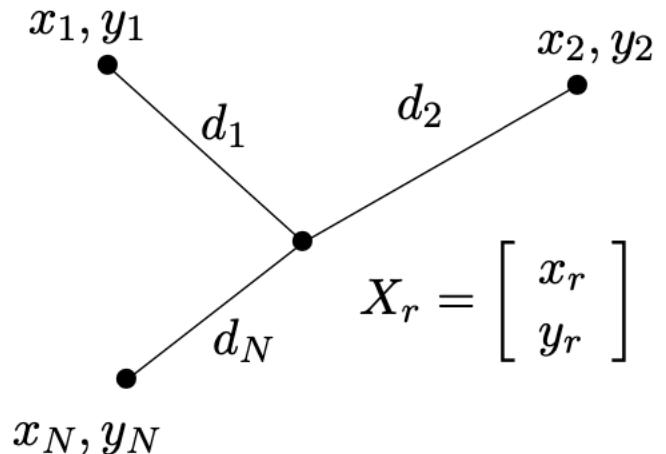
Obviously, for a given distance, this node must be positioned on the circumference of a circle centered at an anchor node with a radius given by the distance between these two nodes. Anchor nodes are located at  $x_i = (x_i, y_i)$  ( $i = 1, \dots, N$ ) and the unknown node is at location  $X_r = [x_r, y_r]^T$ .

The distance measurements are assumed to be corrupted by noise and given by

$$\tilde{d}_i = d_i + n_i, \quad i = 1, \dots, N.$$

From simple trigonometry,

$$\begin{cases} (x_1 - x_r)^2 + (y_1 - y_r)^2 = \tilde{d}_1^2 \\ \vdots \\ (x_N - x_r)^2 + (y_N - y_r)^2 = \tilde{d}_N^2. \end{cases}$$



# TRILATERATION

After subtracting the above N equations we arrive at the following system of equations

$$A \cdot X_r = Y,$$

where

$$A \in \mathbb{R}^{(N-1) \times 2}, \quad X_r \in \mathbb{R}^2, \quad Y \in \mathbb{R}^{(N-1) \times 1},$$

$$A = 2 \cdot \begin{bmatrix} (x_N - x_1) & (y_N - y_1) \\ \vdots & \vdots \\ (x_N - x_{N-1}) & (y_N - y_{N-1}) \end{bmatrix}$$

$$Y = \begin{bmatrix} \tilde{d}_1^2 - \tilde{d}_N^2 - x_1^2 - y_1^2 + x_N^2 + y_N^2 \\ \vdots \\ \tilde{d}_{N-1}^2 - \tilde{d}_N^2 - x_{N-1}^2 - y_{N-1}^2 + x_N^2 + y_N^2 \end{bmatrix}.$$

We can then get the estimation of position as follows:

$$\hat{X} = LY,$$

$$\text{where } L = (A^T A)^{-1} A^T.$$

## SIMILAR FORMULATION

- k beacons at positions  $(x_i, y_i)$
- Assume node 0 has position  $(x_0, y_0)$
- Distance measurement between node 0 and beacon i is  $r_i$
- Error:

$$f_i = r_i - \sqrt{(x_i - \textcolor{red}{x}_0)^2 + (y_i - \textcolor{red}{y}_0)^2}$$

- The objective function is

$$F(x_0, y_0) = \min \sum f_i^2$$

- This is a non-linear optimization problem

## LINEARIZATION AND MIN MEAN LEAST SQUARE ESTIMATION

- Ideally, we would like the error to be 0

$$f_i = r_i - \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} = 0$$

- Re-arrange:

$$(x_0^2 + y_0^2) + x_0(-2x_i) + y_0(-2y_i) - r_i^2 = -x_i^2 - y_i^2$$

- Subtract the last equation from the previous ones to get rid of quadratic terms.

$$2\textcolor{red}{x}_0(x_k - x_i) + 2\textcolor{red}{y}_0(y_k - y_i) = r_i^2 - r_k^2 - x_i^2 - y_i^2 + x_k^2 + y_k^2$$

- Note that this is linear.

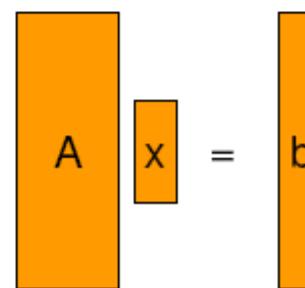
# LINEARIZATION AND MMLSE

- In general, we have an over-constrained linear system

$$Ax = b$$

$$b = \begin{bmatrix} r_1^2 - r_k^2 - x_1^2 - y_1^2 + x_k^2 + y_k^2 \\ r_2^2 - r_k^2 - x_2^2 - y_2^2 + x_k^2 + y_k^2 \\ \vdots \\ r_{k-1}^2 - r_k^2 - x_{k-1}^2 - y_{k-1}^2 + x_k^2 + y_k^2 \end{bmatrix} \quad A = \begin{bmatrix} 2(x_k - x_1) & 2(y_k - y_1) \\ 2(x_k - x_2) & 2(y_k - y_2) \\ \vdots & \vdots \\ 2(x_k - x_{k-1}) & 2(y_k - y_{k-1}) \end{bmatrix}$$

$$x = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$


$$\begin{matrix} A & | & x \\ & & = \\ & & b \end{matrix}$$

## SOLVING THE MMLSE

The linearized equations in matrix form become

$$Ax = b$$

Now we can use the least squares equation to compute an estimation.

$$x = (A^T A)^{-1} A^T b$$

**Beacon nodes must not lie on the same line**