LOCALIZATION

- Assuming distances to three points with known location are exactly given
- Solve system of equations (Pythagoras!)
  - (x_i,y_i) : coordinates of ***anchor point*** i, r_i distance to anchor i
  - (x_u, y_u) : unknown coordinates of node

$$(x_i - x_u)^2 + (y_i - y_u)^2 = r_i^2 \text{ for } i = 1, \ldots, 3$$

  - Subtracting eq. 3 from 1 & 2:

$$(x_1 - x_u)^2 - (x_3 - x_u)^2 + (y_1 - y_u)^2 - (y_3 - y_u)^2 = r_1^2 - r_3^2$$
$$(x_2 - x_u)^2 - (x_2 - x_u)^2 + (y_2 - y_u)^2 - (y_2 - y_u)^2 = r_2^2 - r_3^2.$$

  - Rearranging terms gives a linear equation in (x_u, y_u)!

$$2(x_3 - x_1)x_u + 2(y_3 - y_1)y_u = (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2)$$
$$2(x_3 - x_2)x_u + 2(y_3 - y_2)y_u = (r_2^2 - r_2^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2)$$

- Rewriting as a matrix equation:

$$2\begin{bmatrix} x_3 - x_1 & y_3 - y_1 \\ x_3 - x_2 & y_3 - y_2 \end{bmatrix}\begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2) \\ (r_2^2 - r_2^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2) \end{bmatrix}$$

- Example: $(x_1, y_1) = (2,1)$, $(x_2, y_2) = (5,4)$, $(x_3, y_3) = (8,2)$, $r_1 = 10^{0.5}$, $r_2 = 2$, $r_3 = 3$

$$2\begin{bmatrix} 6 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} 64 \\ 22 \end{bmatrix}$$

! $(x_u, y_u) = (5,2)$

- What if only distance estimation $r_i^0 = r_i + \varepsilon_i$ available?
- Use multiple anchors, overdetermined system of equations

$$2\begin{bmatrix} x_n - x_1 & y_n - y_1 \\ \vdots & \vdots \\ x_n - x_{n-1} & y_n - y_{n-1} \end{bmatrix} \begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} (r_1^2 - r_n^2) - (x_1^2 - x_n^2) - (y_1^2 - y_n^2) \\ \vdots \\ (r_{n-1}^2 - r_n^2) - (x_{n-1}^2 - x_n^2) - (y_{n-1}^2 - y_n^2) \end{bmatrix}$$

- Use (x_u, y_u) that minimize mean square error, i.e, $\|\mathbf{Ax} - \mathbf{b}\|_2$

GPS LOCALIZATION

GLOBAL POSITIONING SYSTEM (GPS)

Simplest localization method.

Allow all nodes to have GPS receivers, to obtain position data from GPS satellites.

That's generally a good idea, but:

- It doesn't work indoors.
- It doesn't work outdoors with obstacles to line-of-sight from the satellites (e.g., in the woods).
- Locations are only approximate---to within about $10-20$ feet.
- Somewhat expensive---OK for laptops, but may be prohibitive for cheaper devices like sensor nodes (but getting cheaper).
- Fairly large, use a lot of power (but getting smaller).

Want a technology that is deployable everywhere, accurate, and fairly low cost.

THE HISTORY OF GPS

- Feasibility studies begun in 1960's.
- Pentagon appropriates funding in 1973.
- First satellite launched in 1978.
- System declared fully operational in April, 1995.

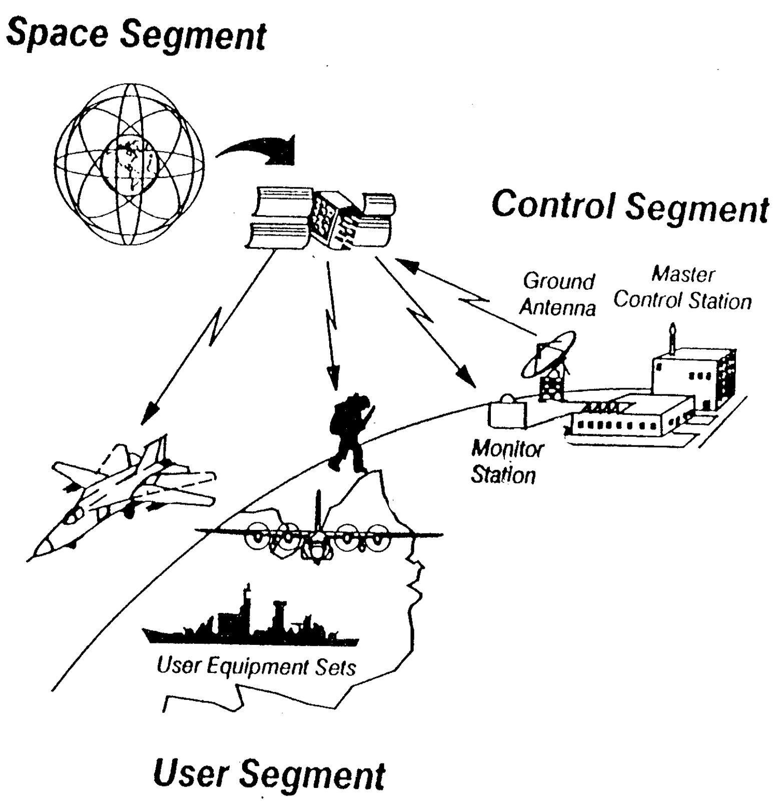
HOW GPS WORKS?

GPS CONFIGURATION

Space Segment
Satellites provide navigation signal and relay orbital and clock data.

Control Segment
Ground control tracks satellites and measures and uploads ephemeris and clock data.

User Segment
User tracks satellite signals, downloads data, and computes position, velocity, and time.



CONTROL SEGMENT

Master Control Station

Ground Antenna

Monitor Station

CONTROL SEGMENT

The Master Control facility is located at Schriever Air Force Base (formerly Falcon AFB) in Colorado.

Originally Schriever AFB and four other stations monitored and controlled satellite positions.

During August and September 2005, six more monitor stations of the NGA (National Geospatial-Intelligence Agency) were added to the grid.

Now, every satellite can be seen from at least two monitor stations.

The monitoring stations compute precise orbital data (ephemeris) and (space vehicle) SV clock corrections for each satellite and update each satellite.

CONTROL SEGMENT--CONT.

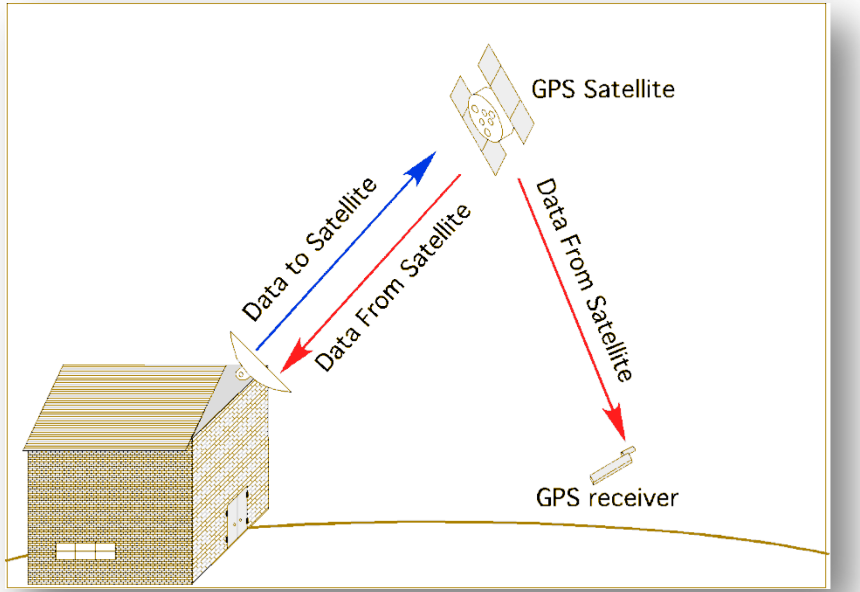
- **The Master Control station uploads ephemeris and clock data to the SVs.**
- **The SVs then send subsets of the orbital ephemeris data to GPS receivers over radio signals.**

SPACE SEGMENT

- The Air force insures that at least 24 satellites are operational at all times.
- There are six orbital planes (with nominally four space vehicles (SVs) in each), equally spaced (60 degrees apart), and inclined at about fifty-five degrees with respect to the equatorial plane.
  - The satellite orbits are controlled so that at least six should be available, unobstructed location, at all times.
  - Each satellite circles the earth twice a day.
  - Each satellite broadcasts a unique signal that tells the receiver its location and the exact time.

USER SEGMENT

- The primary use of GPS is navigation.
- Navigation receivers are made for aircraft, ships, ground vehicles, surveying, and for hand carrying by individuals.
- The accuracy of a receiver depends on the number of channels, compatibility with other navigational systems (WAAS, GLONAS, etc.) and design of the receiver (cost).



- **Most civilian hand held units have an accuracy of 10 meters.**
- **Survey quality GPS units may be as good as one centimeter.**

USER SEGMENT--CONT.

The GPS User Segment consists of all GPS receivers.

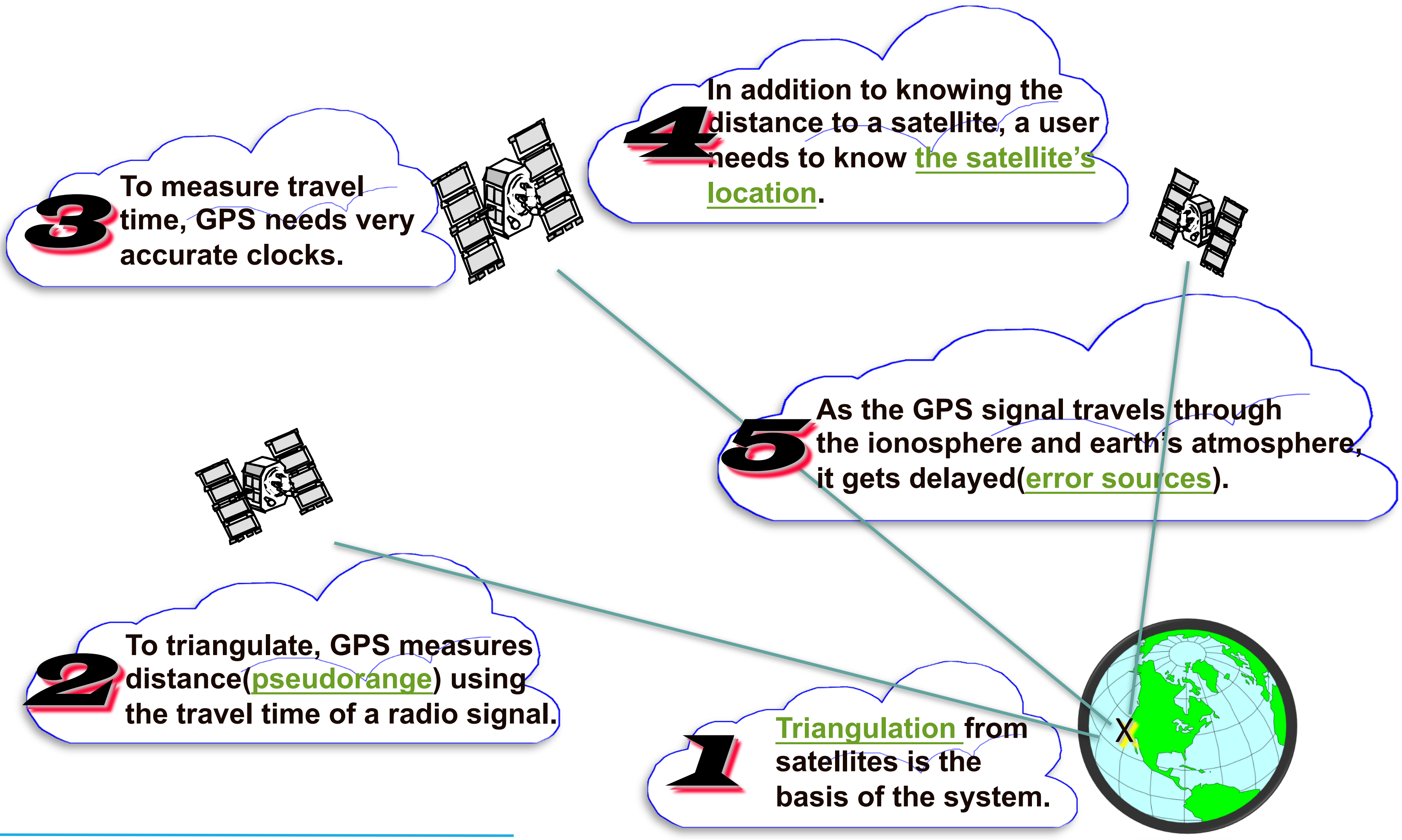
- Surveying
- Recreation
- Navigation

GPS receivers convert satellite signals into position, velocity, and time estimates.

Four satellites are required to compute the four dimensions of X, Y, Z (position) and Time.

- Time and frequency dissemination, based on the precise clocks on board the SVs and controlled by the monitor stations, is another use for GPS.
- Astronomical observatories, telecommunications facilities, and laboratory standards can be set to precise time signals or controlled to accurate frequencies by special purpose GPS receivers.
- The GPS signals are available to everyone, and there is no limit to the number and types of applications that use them.

HOW GPS WORKS ?



PRINCIPLES

The GPS system operates on the principles of trilateration, determining positions from distance measurements.

This can be explained using the velocity equation.

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}}$$

- **Rearranging the equation for distance:**

$$\text{Distance} = \text{Velocity x Time}$$

- **If the system knows the velocity of a signal and the time it takes for the signal to travel from the sender to the receiver, the distance between the sender and the receiver can be determined.**

TRILATERATION EXAMPLE

The signals from the GPS satellites travel at the speed of light--186,000 feet/second.

How far apart are the sender and the receiver if the signal travel time was 0.23 seconds?

Distance (ft) = Velocity (ft/sec) x Time (sec)

$$= 186{,}000 \ \frac{\text{ft}}{\text{sec}} \times 0.23 \text{ sec} = 42{,}780 \text{ ft}$$

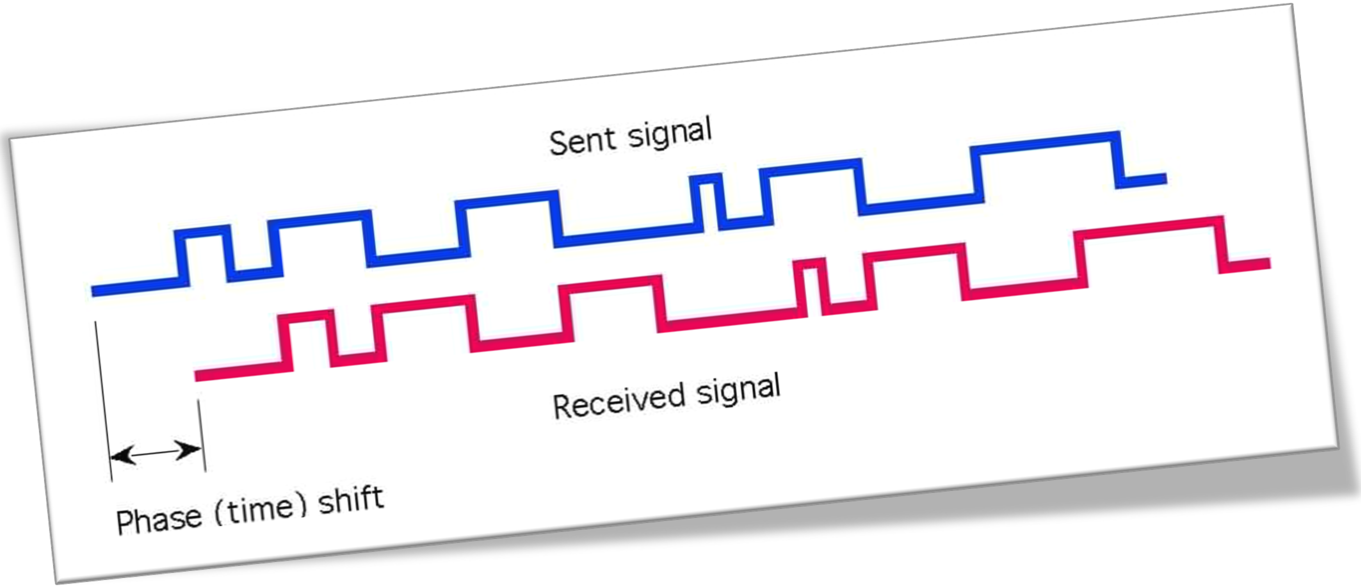
- **We know that trilateration requires three distances.**
- **In the GPS the satellites are at known positions and the receiver calculates its position by knowing the travel time for the signals from at least four satellites.**

SATELLITE SIGNALS

Each satellite has its own unique signal.

It continuously broadcasts its signal and also sends out a time stamp every time it starts.

The receiver has a copy of each satellite signal and determines the distance by recording the time between when the satellite says it starts its signal and when the signal reaches the receiver.

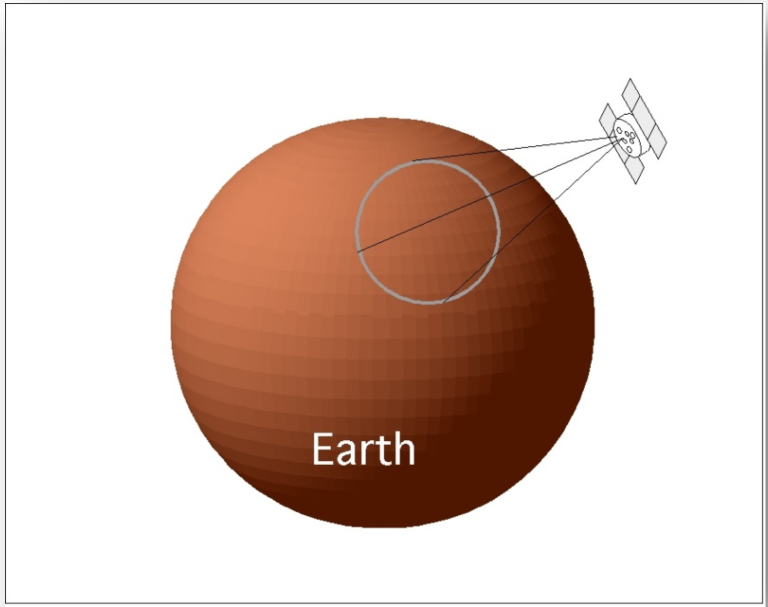


GPS TRILATERATION--CONT.

When the receiver knows its distance from only one satellite, its location could be anywhere on the earths surface that is an equal distance from the satellite.

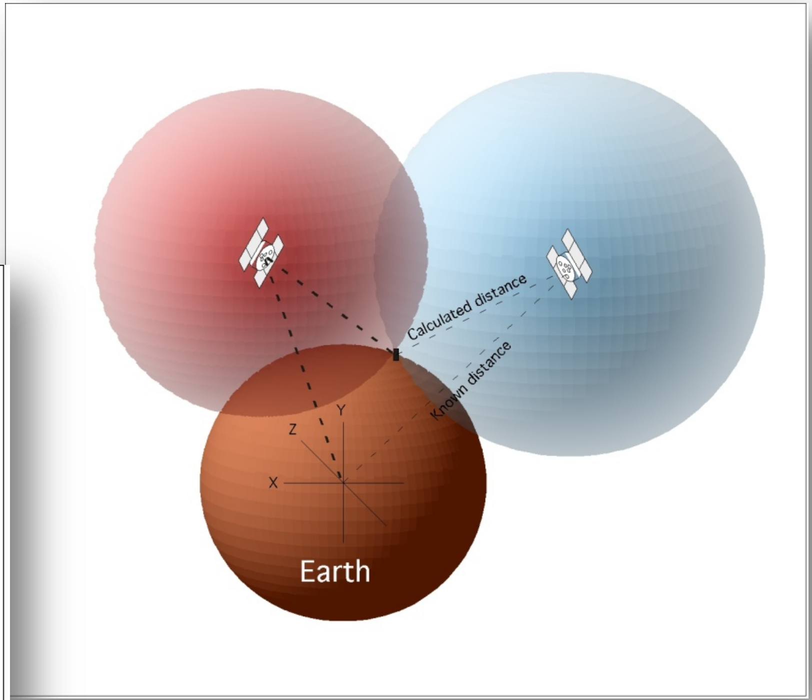
All the receiver can determine is that it is some where on the perimeter of a circle that is an equal distance from the satellite.

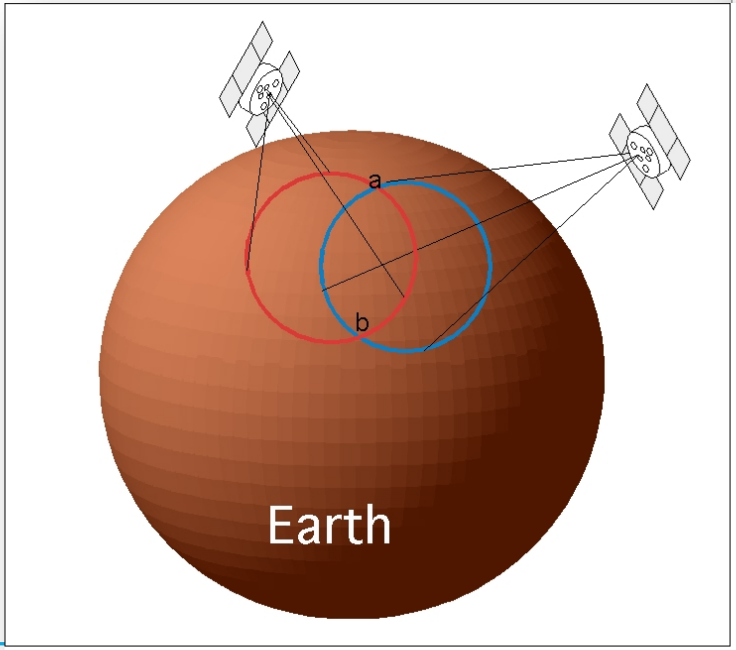
The receiver must have additional information.



GPS TRILATERATION--CONT.

With signals from two satellites, the receiver can narrow down its location to just two points on the earths surface.





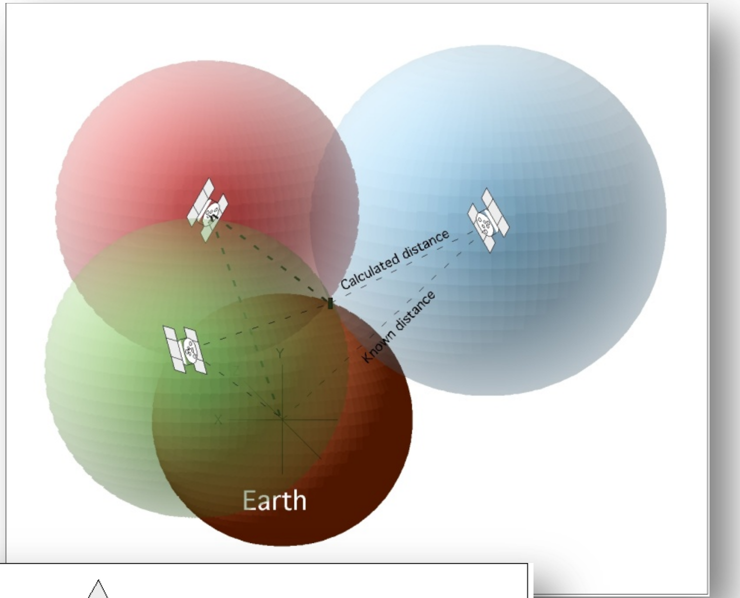
GPS TRILATERATION--CONT.

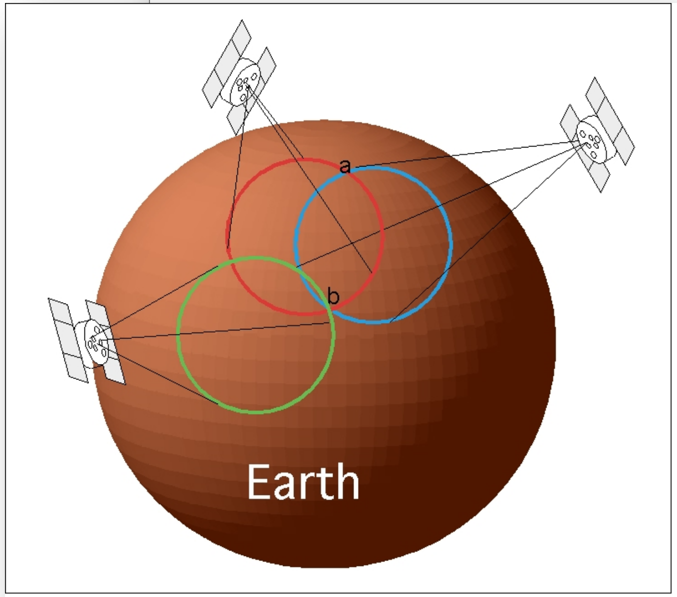
Knowing its distance from three satellites, the receiver can determine its location because there is only two possible combinations and one of them is out in space.

In this example, the receiver is located at b.

Most receivers actually require four to insure the receiver has full information on time, and satellite positions.

The more satellite positions that are used, the greater the potential accuracy of the position location.





FACTORS INFLUENCING POSITION ACCURACY

The number of satellites (channels) the receiver can track.

- The number of channels a receiver has is part of it's design.
- The higher the number of channels---the greater the potential accuracy.
- The higher the number of channels---the greater the cost.

The number of satellites that are available at the time.

- Because of the way the satellites orbit, the same number are not available at all times.
- When planning precise GPS measurements it is important to check for satellite availability for the location and time of measurement.
- If a larger number of channels are required $(6-10)$, and at the time of measurement the number available was less than that, the data will be less accurate.

FACTORS INFLUENCING POSITION ACCURACY--CONT.

The system errors that are occurring during the time the receiver is operating.

- The GPS system has several errors that have the potential to reduce the accuracy.
- To achieve high levels of precision, differential GPS must be used.

Differential GPS uses one unit at a known location and a rover.

- The stationary unit compares its calculated GPS location with the actual location and computes the error.
- The rover data is adjusted for the error.
  - Real Time Kinematic (RTK)
  - Post processing

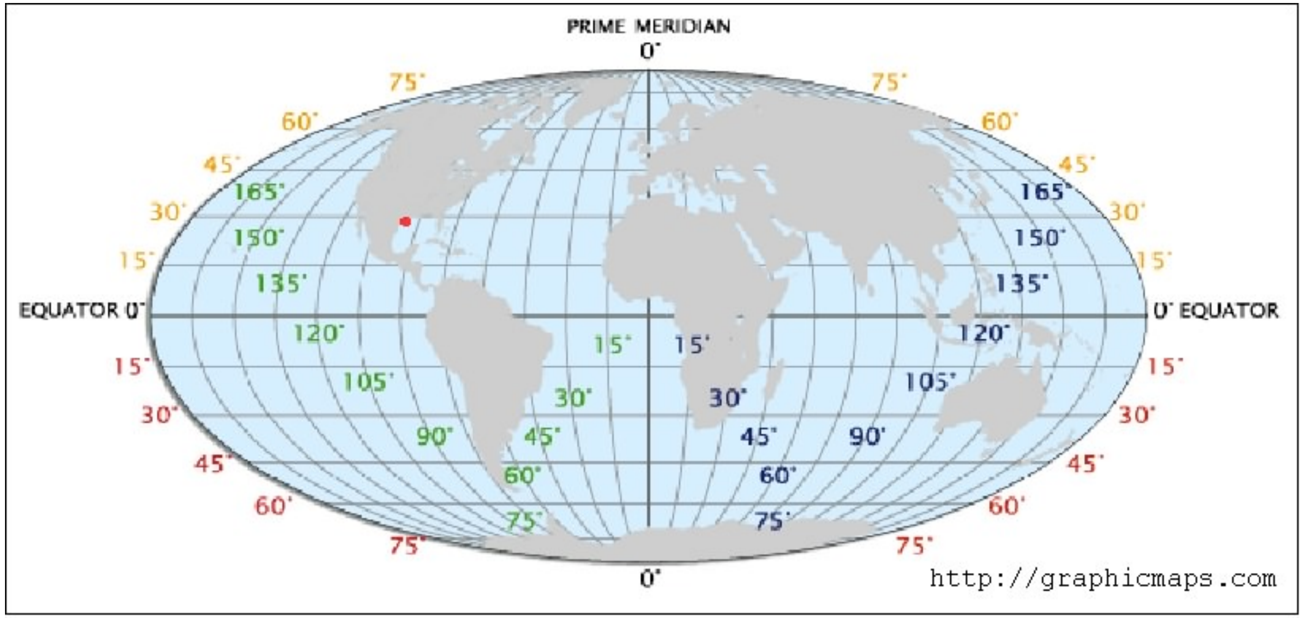
LOCATION

Once the GPS receiver has located its position it is usually displayed in one of two common formats:

- Latitude and longitude
- Universal transverse mercator (UTM).

LATITUDE AND LONGITUDE

Latitudes and longitudes are angles.

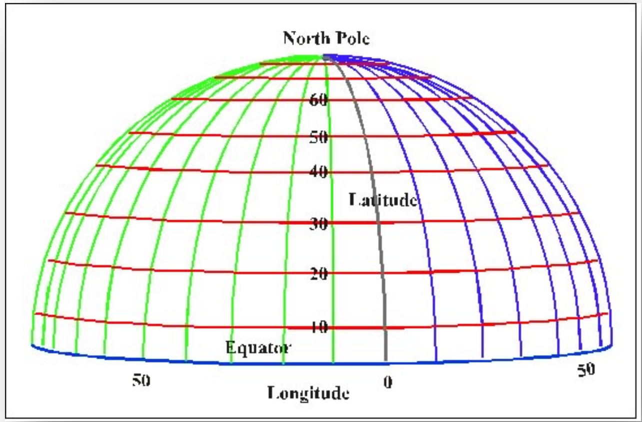


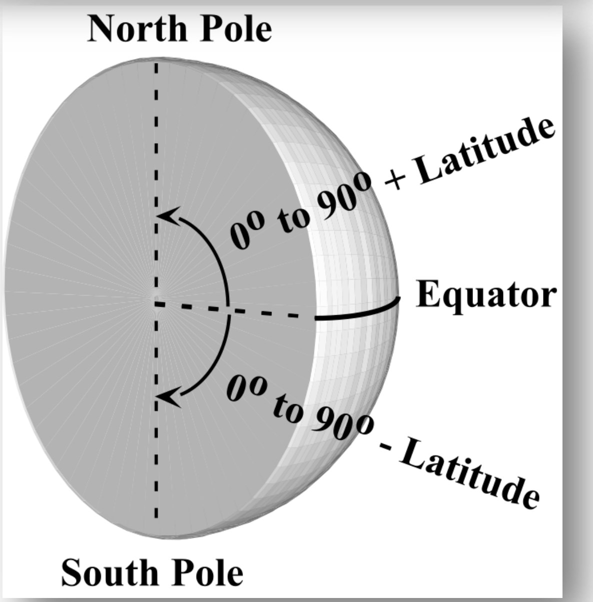
Both use the center of the earth as the vertex, and both utilize the equator, but they use a different zero reference.

LATITUDE

Latitude gives the location of a place on the Earth north or south of the Equator.

Latitude is an angular measurement in degrees (marked with °) ranging from $0°$ at the Equator to $90°$ at the poles ($90°$ N for the North Pole or $90°$ S for the South Pole)





The earth's circumference is approximately 24,859.82 miles around the poles.

$$\frac{\text{Miles}}{\text{Degree}} = \frac{24859.82 \text{ miles}}{360 \text{ degrees}} = 69.05 \text{ miles/degree}$$

Each degree of latitude ≈ 69 miles

LATITUDE--EQUATOR

- The Equator is an imaginary circle drawn around the planet at a distance halfway between the poles.

- The equator divides the planet into a Northern Hemisphere and a Southern Hemisphere.
- The latitude of the equator is, by definition, 0°.

LATITUDE--CONT.

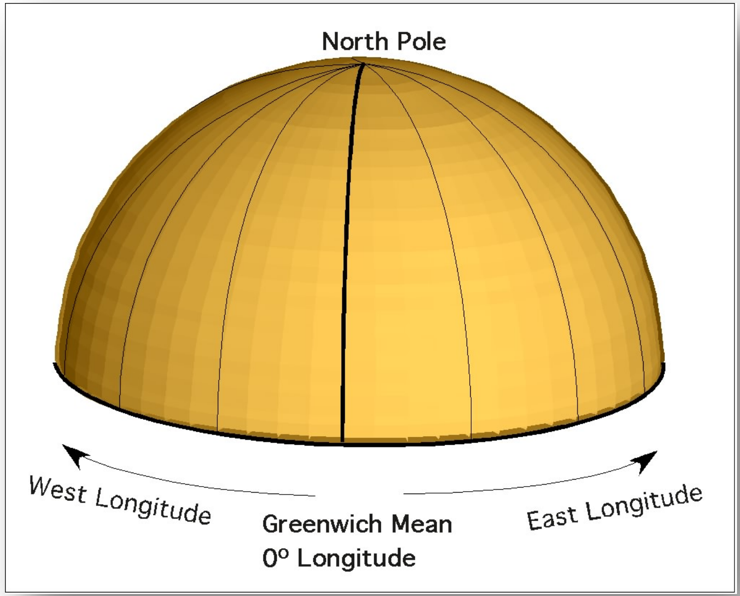
Four lines of latitude are named because of the role they play in the geometrical relationship with the Earth and the Sun.

- Arctic Circle — 66° 33′39″N
- Tropic of Cancer — 23° 26′22″N
- Tropic of Capricorn — 23° 26′22″S
- Antarctic Circle — 66° 33′39″S

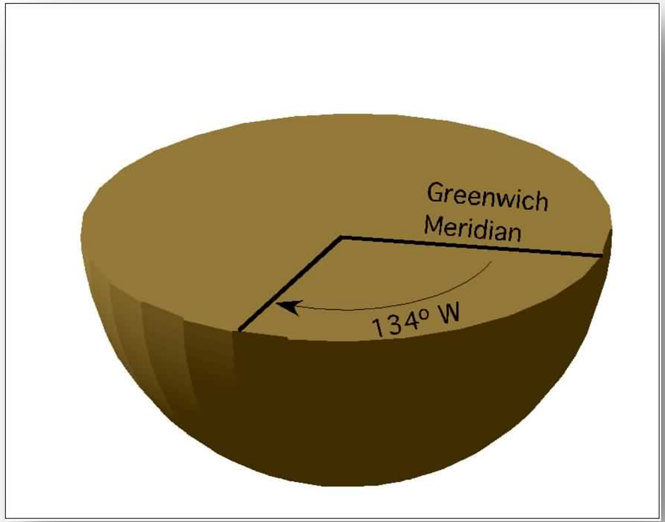
LONGITUDE

Longitude describes the location of a place on earth east or west of a north-south line called the Prime Meridian.

- Longitude is given as an angular measurement ranging from 0° at the Prime Meridian to +180° eastward and −180° westward.
- In 1884, the International Meridian Conference adopted the Greenwich meridian as the universal prime meridian or zero point of longitude.



LONGITUDE--CONT.



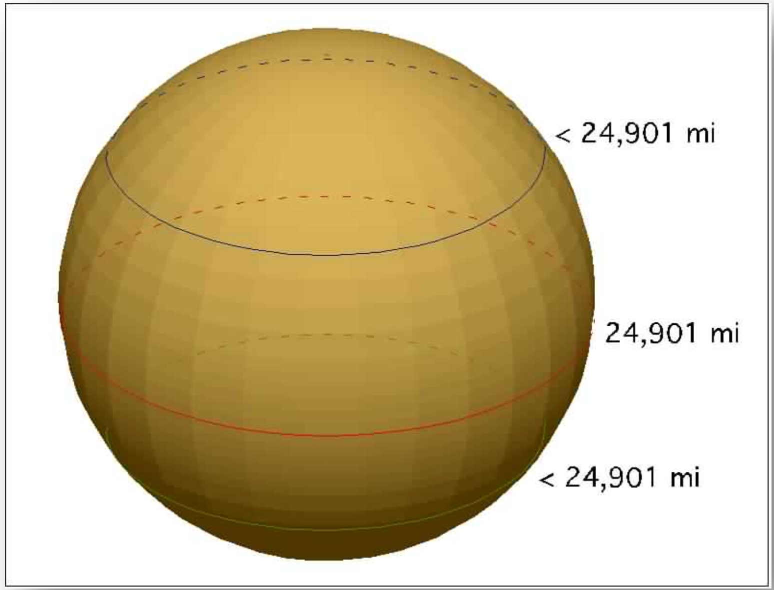
The circumference of the earth at the equator is approximately 24,901.55 miles.

$$\frac{\text{Miles}}{\text{Degree}} = \frac{24901.55 \text{ miles}}{360 \text{ degrees}} = 69.17 \ \text{Miles}/\text{Degree}$$

$$\text{Each degree of longitude} \approx 69 \text{ miles}$$

A longitude of 134° west would be 9,246 west of the prime meridian.

LONGITUDE--CONT.



There is an important difference between latitude and longitude.

The circumference of the earth declines as the latitude increase away from the equator.

This means the miles per degree of longitude changes with the latitude.

This makes determining the distance between two points identified by longitude more difficult.

SELECTIVE AVAILABILITY (S/A)

- The Defense Department dithered the satellite time message, reducing position accuracy to some GPS users.
- S/A was designed to prevent America's enemies from using GPS against US and its allies.
- In May 2000 the Pentagon reduced S/A to zero meters error.
- S/A could be reactivated at any time by the Pentagon.

SOURCES OF GPS ERROR

Standard Positioning Service (SPS): Civilian Users

Source	Amount of Error
❑ Satellite clocks:	1.5 to 3.6 meters
❑ Orbital errors:	< 1 meter
❑ Ionosphere:	5.0 to 7.0 meters
❑ Troposphere:	0.5 to 0.7 meters
❑ Receiver noise:	0.3 to 1.5 meters
❑ Multipath:	0.6 to 1.2 meters
❑ Selective Availability	(see notes)
❑ User error:	Up to a kilometer or more

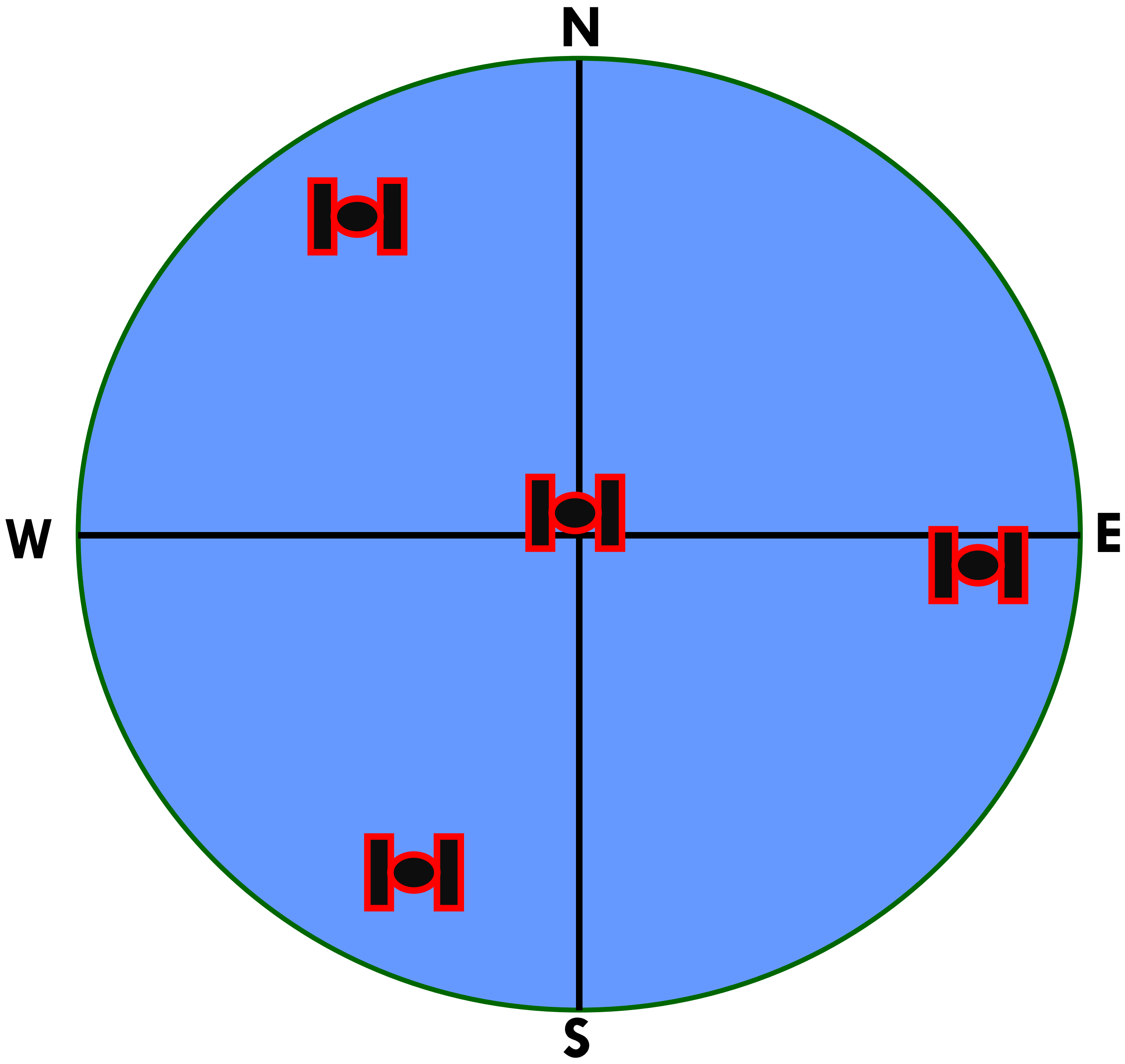
Vertical Error 51.4m (total), Horizontal error 41.1 m (total)

Errors are cumulative and increased by PDOP (Position Dilution of Precision).

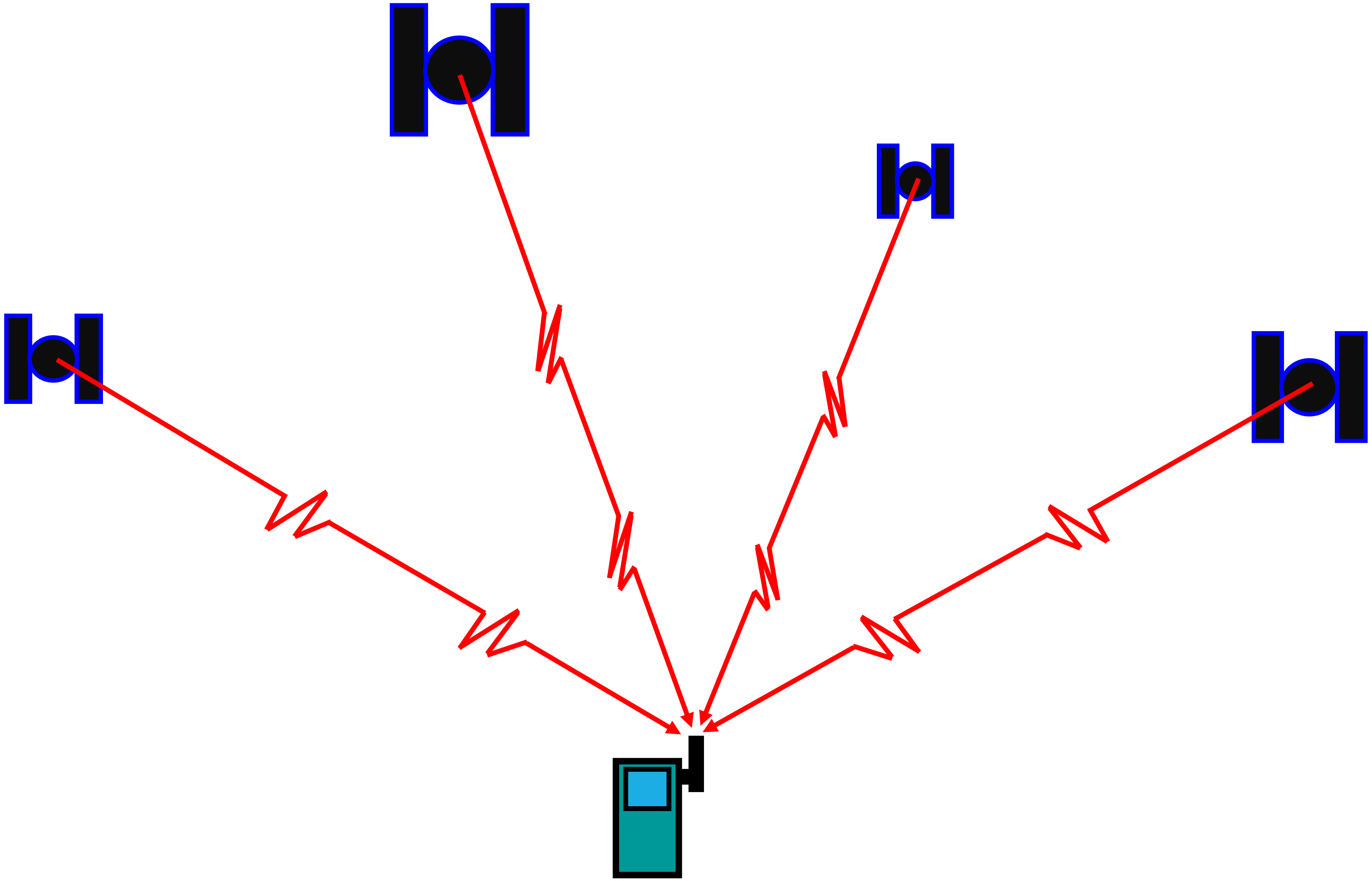
GPS SATELLITE GEOMETRY

- Satellite geometry can affect the quality of GPS signals and accuracy of receiver trilateration.
- Dilution of Precision (DOP) reflects each satellite's position relative to the other satellites being accessed by a receiver.
- There are five distinct kinds of DOP.
- Position Dilution of Precision (PDOP) is the DOP value used most commonly in GPS to determine the quality of a receiver's position.
- It's usually up to the GPS receiver to pick satellites which provide the best position triangulation.
- Some GPS receivers allow DOP to be manipulated by the user.

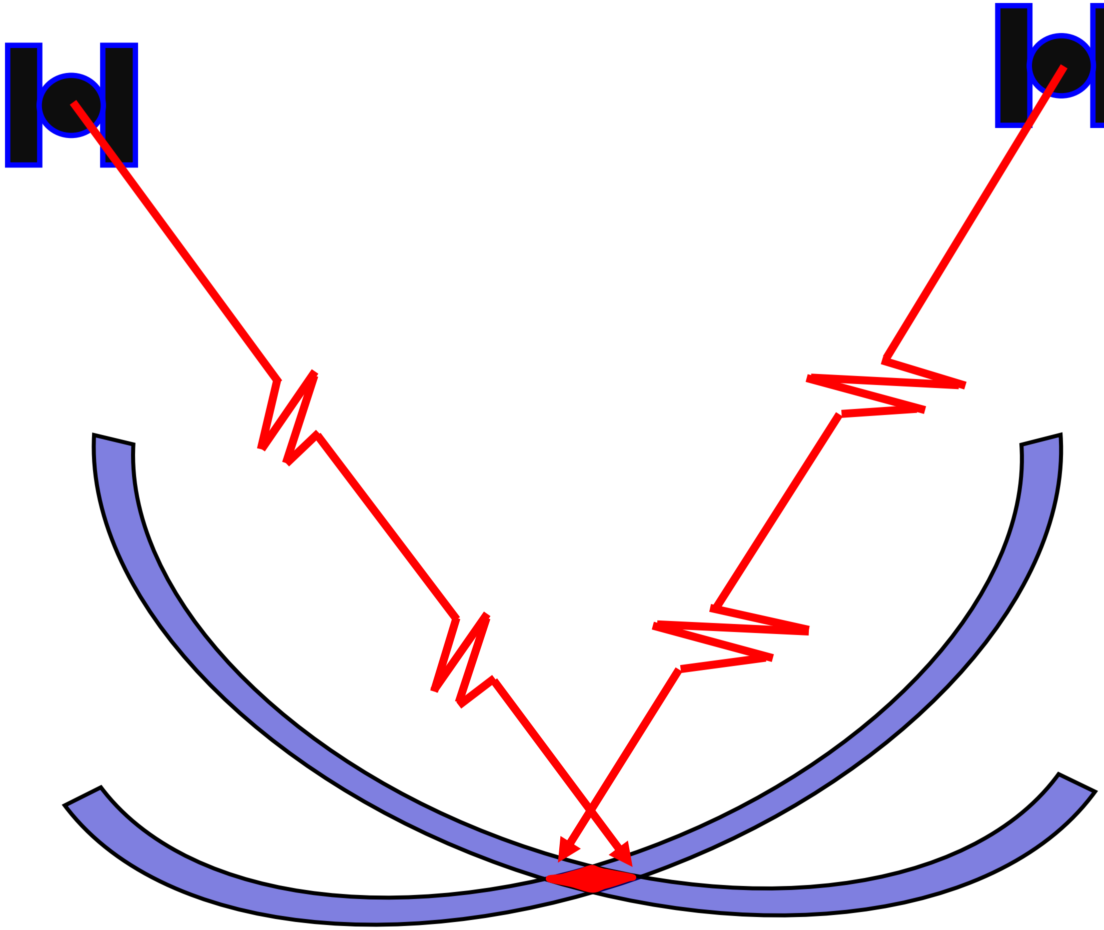
IDEAL SATELLITE GEOMETRY



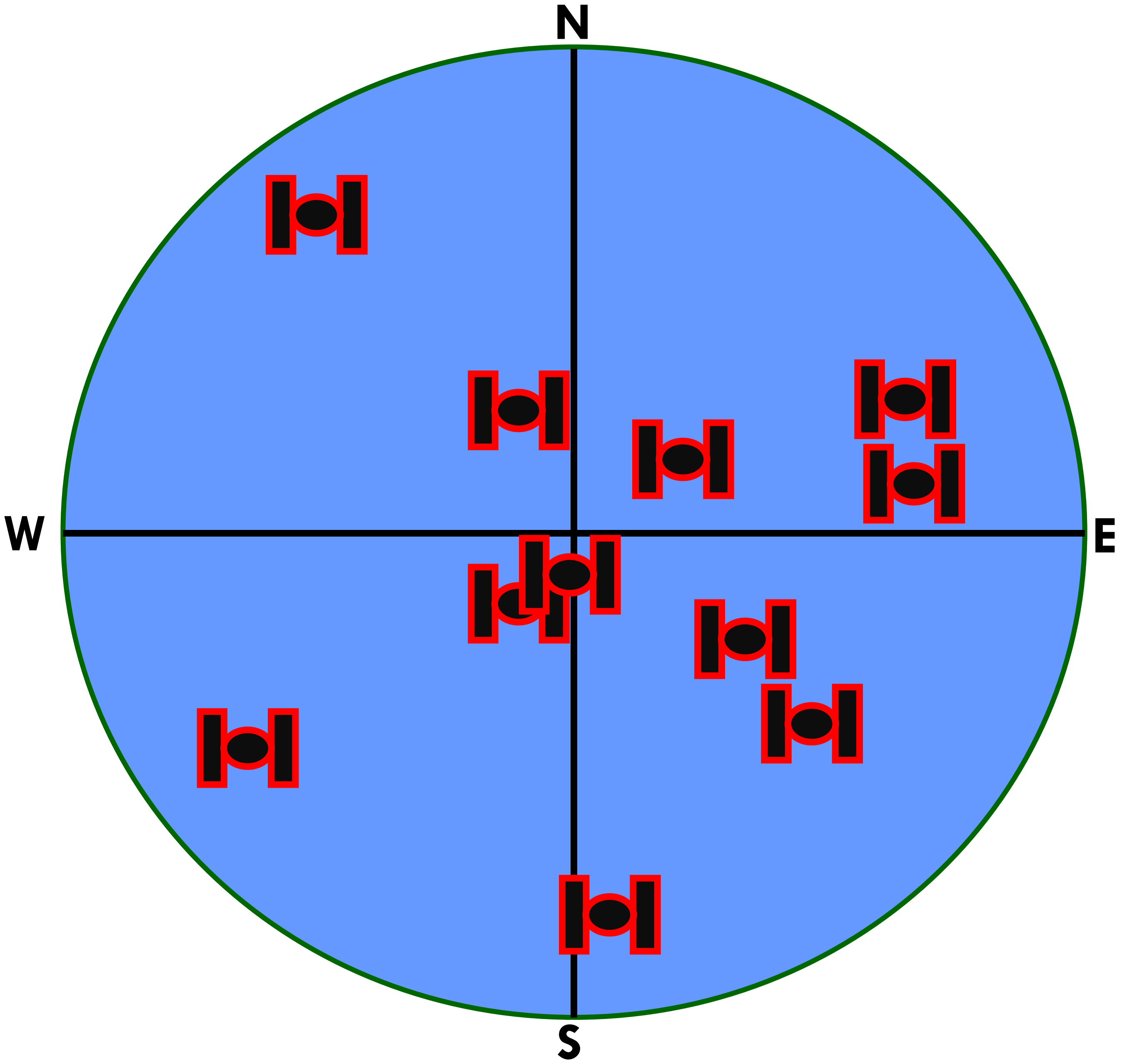
GOOD SATELLITE GEOMETRY



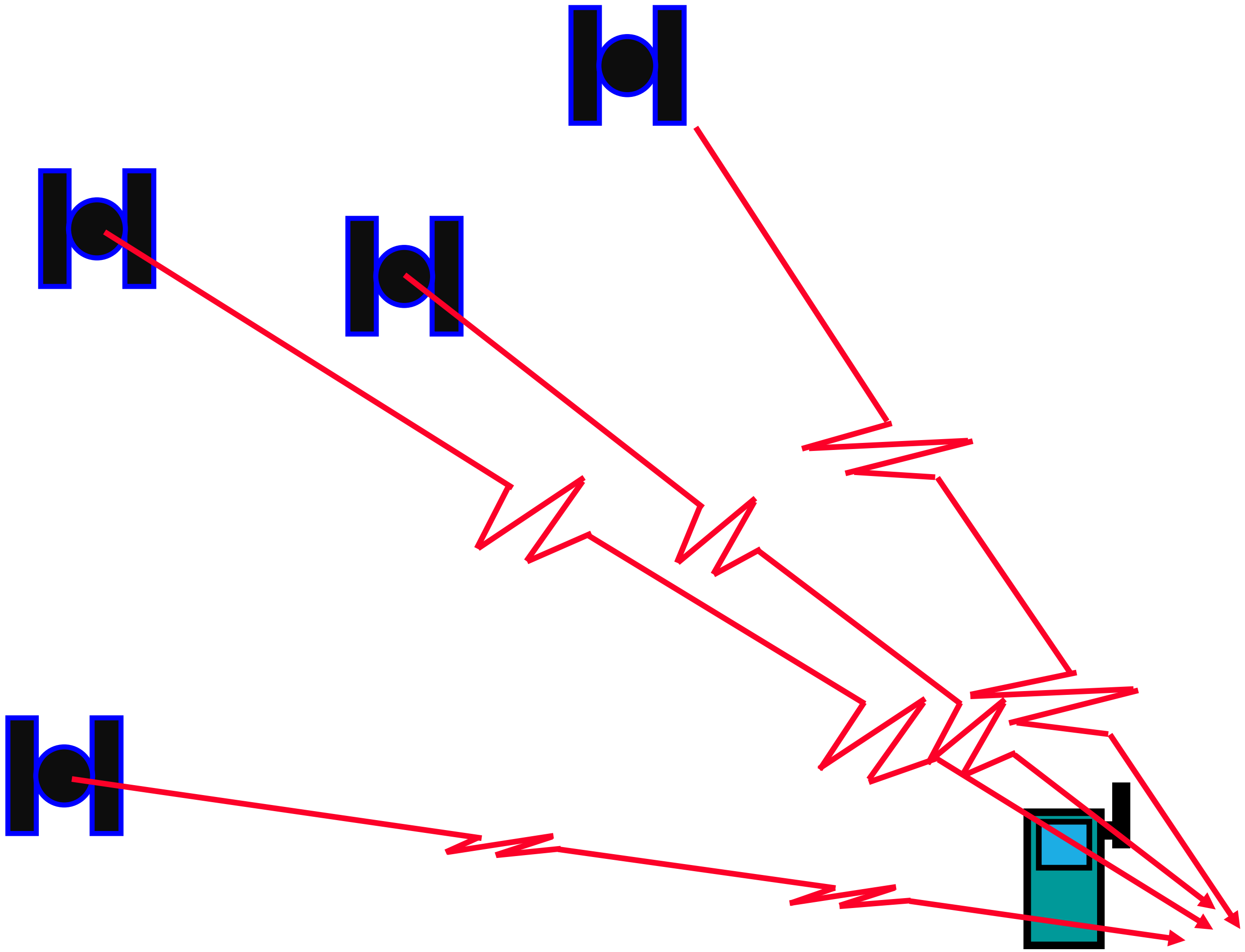
GOOD SATELLITE GEOMETRY



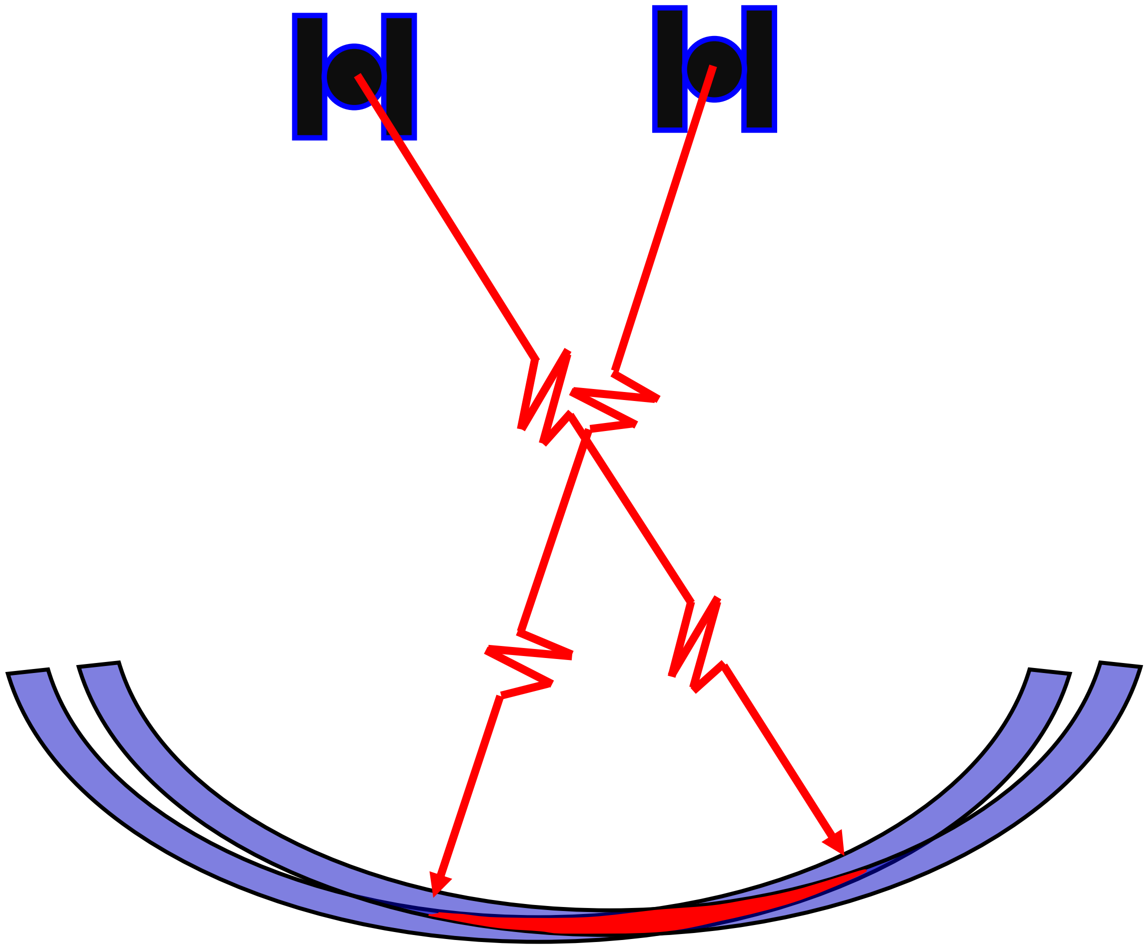
POOR SATELLITE GEOMETRY



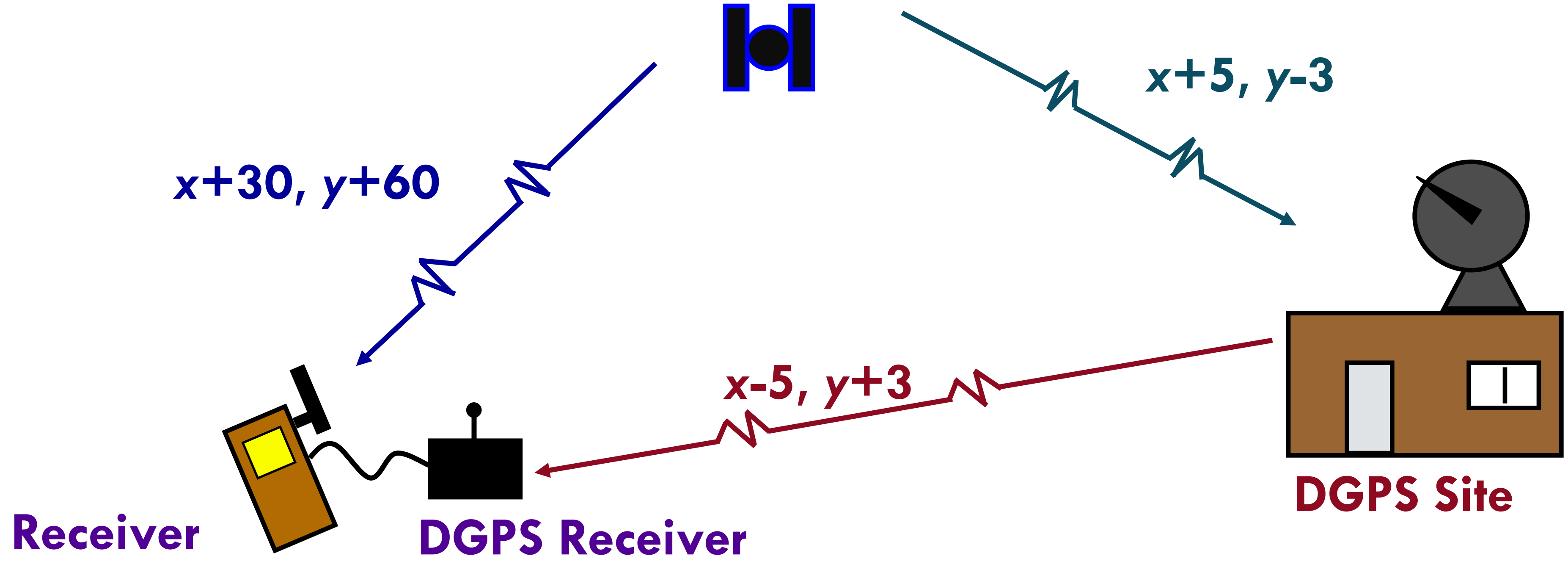
POOR SATELLITE GEOMETRY



POOR SATELLITE GEOMETRY



REAL TIME DIFFERENTIAL GPS



DGPS correction = $x+(30-5)$ and $y+(60+3)$

True coordinates = $x+25$, $y+63$

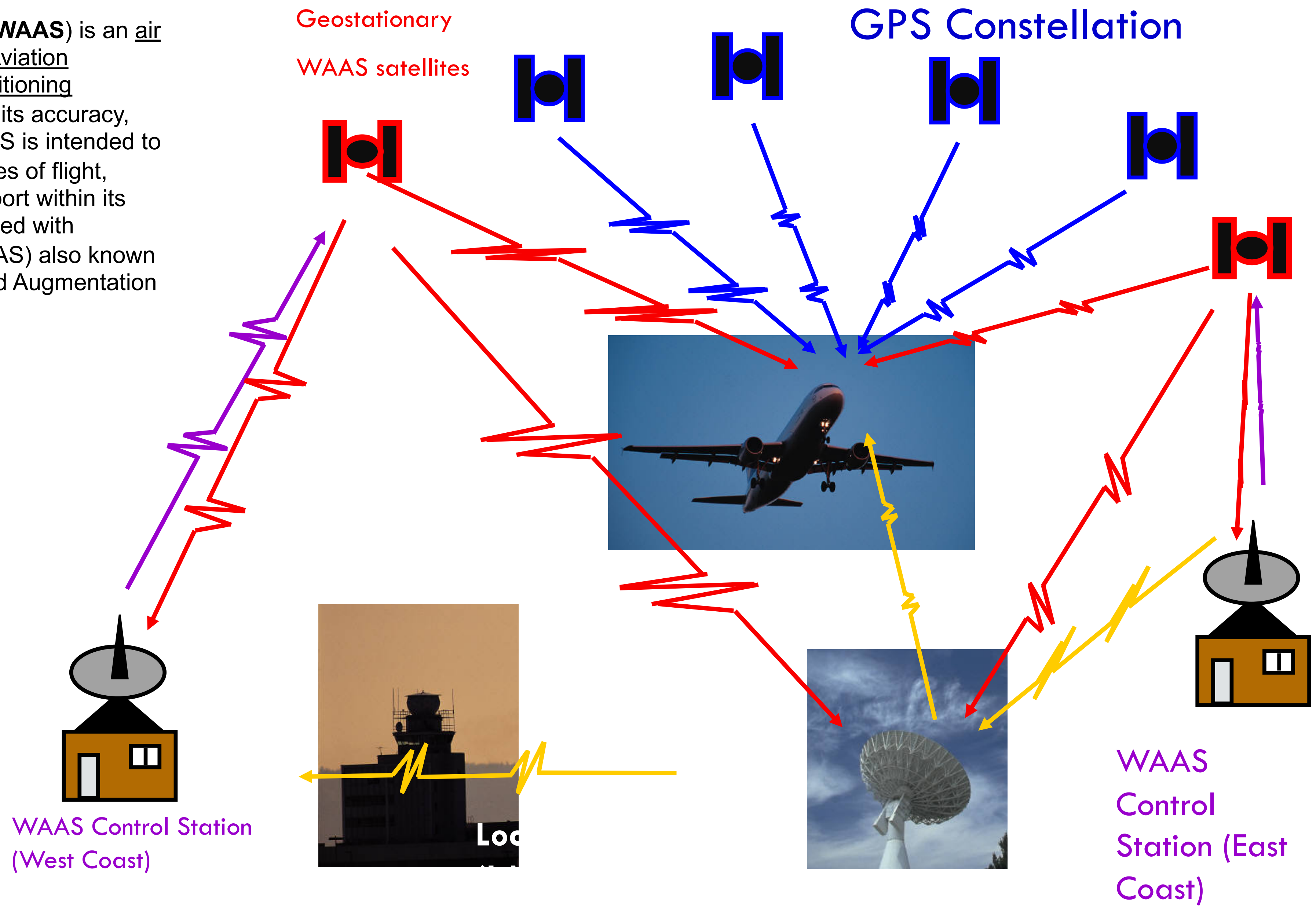
True coordinates = $x+0$, $y+0$

Correction = $x-5$, $y+3$

Wide Area Augmentation System

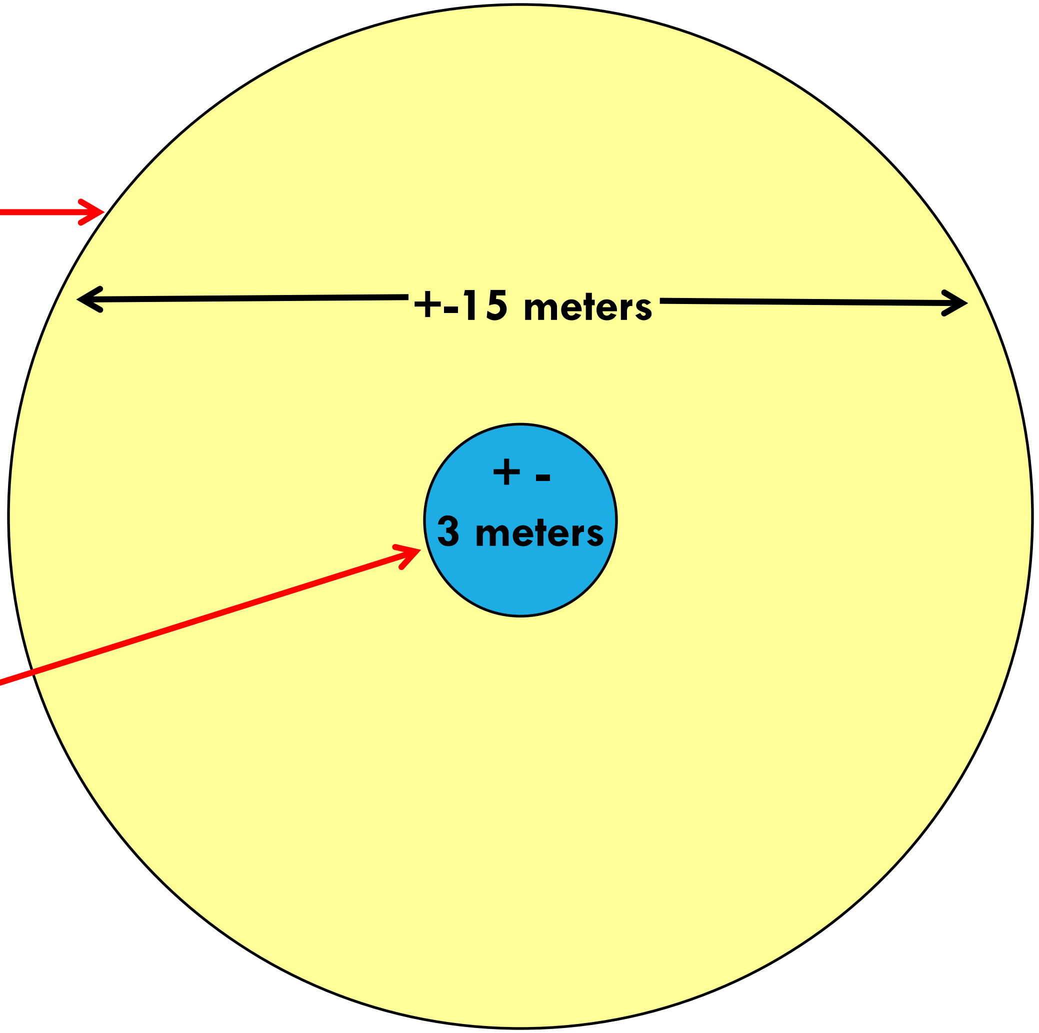
The **Wide Area Augmentation System** (**WAAS**) is an air navigation aid developed by the Federal Aviation Administration to augment the Global Positioning System (GPS), with the goal of improving its accuracy, integrity, and availability. Essentially, WAAS is intended to enable aircraft to rely on GPS for all phases of flight, including precision approaches to any airport within its coverage area.[1] It may be further enhanced with the Local Area Augmentation System (LAAS) also known by the preferred ICAO term Ground-Based Augmentation System (GBAS) in critical areas.

WAAS uses a network of ground-based reference stations, in North America and Hawaii, to measure small variations in the GPS satellites' signals in the western hemisphere. Measurements from the reference stations are routed to master stations, which queue the received Deviation Correction (DC) and send the correction messages to geostationary WAAS satellites in a timely manner (every 5 seconds or better). Those satellites broadcast the correction messages back to Earth, where WAAS-enabled GPS receivers use the corrections while computing their positions to improve accuracy.



How good is WAAS?

With Selective Availability set to zero, and under ideal conditions, a GPS receiver without WAAS can achieve fifteen meter accuracy most of the time.*



Under ideal conditions a WAAS equipped GPS receiver can achieve three meter accuracy 95% of the time.*

* Precision depends on good satellite geometry, open sky view, and no user induced errors.