Q.5.

1. the posperiore can be calculated as below.

$$P(\mu | D_x, D_y) = N(\mu | m_n, \lambda_N^{-1})$$

$$m_n = \lambda_x n_x \bar{x} + \lambda_y n_y \bar{y}$$

$$\lambda_n = \lambda_0 + n_x \lambda_x + n_y \lambda_y$$

here $\bar{x} = \frac{1}{n_x} \sum_{k=1}^{N} x_N$ and $\bar{y} = \frac{1}{n_y} \sum_{k=1}^{N} y_N$ are the Sample average. We put the expressions fore the mean and the Variance / precision in terms of known variables. x, y, n_x, n_y, v_x, v_y. In addition, the prior is non-informative Gaussian so $\lambda_0 = 0$

$$m_n = \frac{\lambda_x n_x \bar{x} + \lambda_y n_y \bar{y}}{n_x \lambda_x + n_y \lambda_y}$$

$$= \frac{\frac{1}{v_x} n_x \bar{x} + \frac{1}{v_y} n_y \bar{y}}{\frac{n_x}{v_x} + \frac{n_y}{v_y}}$$