DATA FUSION: PROBABILISTIC AND DS BASED

ESTIMATION FUSION

Fusion methods (actual fusion of data, derived data or information, and inferences) can be based on:

(1) probabilistic and statistical models such as Bayesian reasoning, evidence theory, robust statistics, and recursive operators;
(2) least–square (LS) and mean square methods such as KF, optimization, regularization, and uncertainty ellipsoids; or
(3) other heuristic methods such as ANNs, fuzzy logic, approximate reasoning, and computer vision.

This is called the positional concept; however, it is valid for other state variables of the object being tracked, such as velocity, acceleration, jerk (change in accelerations, and even surge, if need be), and bearing (angle and orientation data).

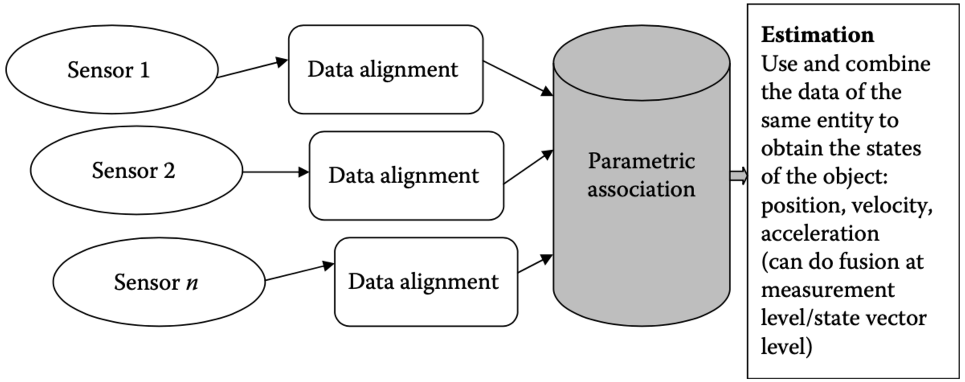


FIGURE 2.8
Basic sensor data-fusion process and taxonomy: implicitly involves certain aspects from the JDL–DFP model.

The parametric association involves the measure of association and the association strategies. The following tools can be used for measuring the data association:

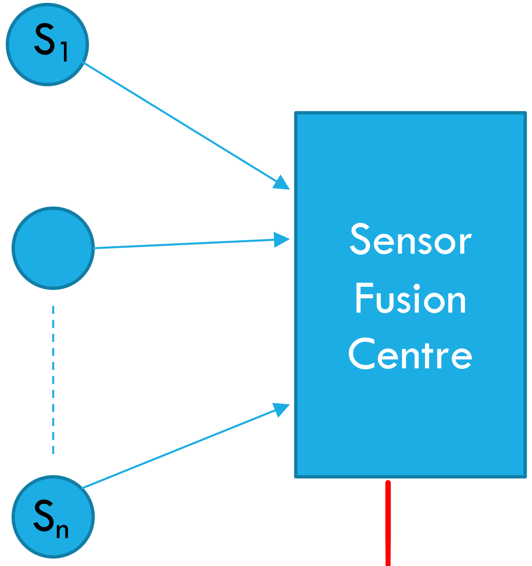
(1) correlation coefficients to correlate shapes, scatter, and elevation;
(2) distance measures, such as the Euclidean norm and the Mahalanobis distance; and
(3) probabilistic similarities. The association strategies involve
    - gating techniques (e.g., elliptical or rectangular gates and kinematic models); and
    - assignment strategy (nearest neighbor, probability data association, and so on).

THE ESTIMATION FUSION PROCESS

- $Consider\ a\ WSN\ with\ n\ sensors\ estimating\ the\ value\ of\ an\ object$ x
- Each sensor S_i obtains an observation Z_i relevant to the estimate of x
- R_i is the covariance matrix of the observations noise

$\hat{x}_i$ is the estimate of x according to S_i

- $P_i = \mathrm{cov}(\tilde{x}_i)$ is the covariance of the estimation error $\tilde{x}_i = x - \hat{x}_i$
- At the fusion center we have: $\hat{x}$ and $P = \mathrm{cov}(\tilde{x})$



Let us assume the following:

$$z_i = h_i x + \eta_i \tag{2.1}$$

Here, z_i is the measurement of the ith sensor, and η_i is the measurement noise. Next, a local estimate is viewed as an observation of the estimate [15–19], as shown below:

$$\hat{x}_i = x + (\hat{x}_i - x) = x + (-\tilde{x}_i) \tag{2.2}$$

Equation 2.2 is similar to an observation or measurement equation, where the "new observation" $\hat{x}_i$ is actually the estimate of x and the additive term is regarded as an error (noise). The above model is referred to as the data model for standard distributed fusion. Also consider the following equation:

$$\begin{aligned} \hat{y}_i = g_i(z_i) = a_i + B_i z_i &= B_i(h_i x + \eta_i) + a_i \\ &= B_i h_i x + (B_i \eta_i + a_i) = B_i h_i x + \hat{\eta}_i \end{aligned} \tag{2.3}$$

In Equation 2.3, z_i is processed linearly and sent to the fusion center. In this case, a_i and B_i are known. This model is referred to as the linearly-processed data model for distributed fusion.

SPECIAL CASE

If $B_i = I$ and $\hat{\eta}_i = \eta_i$, then we have $\hat{y}_i = h_i x + \eta_i$, which is a centralized fusion model of Equation 2.1. If $B_i h_i = I$ and $\hat{\eta}_i = -\tilde{x}_i$, then we have $\hat{y}_i = x + (-\tilde{x}_i)$, which is Equation 2.2 in the standard distributed-fusion model. Next, the unified model of the data available to the fusion center is defined as

$$y_i = H_i x + v_i \tag{2.4}$$

In the batch-processing mode, it is represented as follows:

$$y^n = Hx + v^n \tag{2.5}$$

The above model is valid for centralized, distributed, and hybrid DF architectures. From Equations 2.4 and 2.5 in the above model, we get the following definitions [15–19]:

$$y_i = \begin{Bmatrix} z_i -> CL \\ x_i -> SD \\ \hat{y}_i -> DL \end{Bmatrix}; \quad v_i = \begin{Bmatrix} \eta_i \quad -> CL \\ -\tilde{x}_i -> SD \\ \hat{\eta}_i \quad -> DL \end{Bmatrix}$$

$$H_i = \begin{Bmatrix} h_i \quad -> CL \\ I \quad -> SD \\ B_i h_i -> DL \end{Bmatrix}; \quad y^n = \begin{Bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{Bmatrix}; \quad H = \begin{Bmatrix} H_1 \\ . \\ . \\ . \\ H_n \end{Bmatrix}; \quad v^n = \begin{Bmatrix} v_1 \\ . \\ . \\ . \\ v_n \end{Bmatrix} \tag{2.6}$$

Here, CL⇒centralized with linear observations, SD⇒standard distributed, and DL⇒distributed with linearly processed data. We further have:

CORRELATIONS

Let $C = \text{cov}(v^n) = \begin{Bmatrix} R & -> CL \\ .\mathfrak{R} & -> SD \\ \widehat{R} & -> DL \end{Bmatrix}$ be the general covariance matrix, with $R = \text{cov}(\eta_1, \ldots, \eta_n)$ (measurement-noise covariance matrix); $\mathfrak{R} = \text{cov}(\tilde{x}_1, \ldots, \tilde{x}_n)$ (the joint error covariance of the local estimates); and $\widehat{R} = \text{cov}(\widehat{\eta}_1, \ldots, \widehat{\eta}_n)$ (the covariance of equivalent observation noise). The noise components $(v_1, \ldots, v_n)$ of the UM could be correlated, and then "C" is not necessarily a block-diagonal matrix. R is usually assumed to be block-diagonal; however, this is not always true, because the measurement noises of a discrete-time multisensor system obtained by sampling the continuous time system would be correlated. Moreover, these components would be correlated if x is observed in a common noisy environment and if the sensors are in the same platform. In addition, measurement-noise statistics would depend on the distance (x, y, z) of the target from the measurement suite. The matrix $\mathfrak{R}$ is seldom block-diagonal, and the noise v and the estimate x could be correlated as follows:

$$\text{Cov}(x, v^n) = A \quad \text{and} \quad \text{cov}(x, v_i) = A_i$$

Thus, $A_i = \text{cov}(x, v_i) = \text{cov}\{(\tilde{x}_i + \hat{x}_i - \bar{x}), (-\tilde{x}_i)\}$

From Equation 2.1, we have $x = \hat{x} + \tilde{x}_i$, and after expansion and simplification, we have $\mathrm{cov}(x, v_i) = -P_i$, according to the orthogonality principle.

It follows that Equations 2.4 and 2.5 are the unified data model for the three linear types of fusion architectures described earlier; however, the measurement noises would be correlated with each other and with x. An alternative distributed fusion model is provided next.

Let $\hat{x}_i$ be represented as follows:

$$\hat{x}_i = \bar{x}_i + K_i(z_i - h_i\bar{x}_i) = [I - K_i h]\bar{x}_i + K_i z_i \tag{2.7}$$

using the linear measurement model of Equation 2.1 for z_i. Then, we have y_i as the linearly processed measurement form [15–19], as shown below:

$$\hat{x}_i = [I - K_i h]\bar{x}_i + K_i(h_i x + \eta_i) \tag{2.8}$$

$$y_i = \hat{x}_i - [I - K_i h]\bar{x}_i = K_i h_i x + K_i \eta_i = B_i h_i x + v_i \tag{2.9}$$

The covariance of v_i is $C = \mathrm{cov}(v) = [C_{ij}] = [K_i R_{ij} K_j'] = KRK'$; $K = diag[K_1 \ldots K_n]$. We can see that C would be block-diagonal if R is also block-diagonal. We need to send data y_i to and gain K_i to the fusion center. This architecture is referred to as simple nonstandard distributed fusion.

BEST LINEAR UNBIASED ESTIMATION FUSION: COMPLETE PRIOR KNOWLEDGE

Let us consider the UM of Equation 2.5. The best linear unbiased estimation (BLUE) fusion rule (FR) with complete prior knowledge is given when the prior mean $E\{x\} = \overline{x}$ and the prior covariance $P_0 = \text{cov}(x)$ (and A) are specified:

$\hat{x}^B = \arg\min E\{(x - \hat{x})(x - \hat{x})' \mid Y \mid\}$; the minimum is taken over $\hat{x} = a + By^n$.

The details are as follows [15–19]:

$$
\begin{aligned}
&\hat{x} = \overline{x} + K(y^n - H\overline{x}) \\
&P = \text{cov}(x + \hat{x} \mid Y \mid) = P_0 - KSK' \\
&S = HP_0H' + C + HA + A'H' \\
&U = I - KH \\
&P = UP_0U' + KCK' - UAK' - (UAK')' \\
&\hat{x} = \overline{x} + K(y^n - H\overline{x})
\end{aligned}
\tag{2.10}
$$

BEST LINEAR UNBIASED ESTIMATION FUSION: WITHOUT PRIOR KNOWLEDGE

If there is no prior knowledge about the estimate (i.e., $E\{x\}$ is not known) or the prior covariance matrix either is not known or does not exist, then the BLUE FR for the model Equation 2.5 is represented as follows [15–19]:

$$\begin{aligned}
\hat{x} &= Ky^n = \tilde{K}y^n \\
P &= KCK' = \tilde{K}C\tilde{K}' \\
\tilde{K} &= H^+(I - C\{TCT\}^+) \\
K &= \tilde{K} = H^+(I - CT^{1/2}(T^{1/2\prime}CT^{1/2})^{-1}T^{1/2\prime})
\end{aligned} \tag{2.11}$$

with $(H, C^{1/2})$ having full-row rank; $T^{1/2}$ is the full-rank square root of $T = I - HH^+$ *and* $C^{1/2}$ *is the* SQRT(C). If C is nonsingular, then $\tilde{K} = PH'C^{-1}$.

BEST LINEAR UNBIASED ESTIMATION: INCOMPLETE KNOWLEDGE

If prior information about some components of $E\{x\}$ is not available, then it is proper to assume that P_0 does not exist (i.e., P_0^{-1} is singular). We can then set certain elements (or the eigenvalues) of P_0 as infinity. We can assume that $\bar{x} = E\{x\}$, $Cov(x, v^n) = A$, and a positive semidefinite symmetric but singular matrix P_0^{-1} is given. Then, for the UM of Equation 2.5, the optimal BLUE FR is generated by the following equations [15–19]:

$$\begin{aligned} \hat{x} &= VK[(V_1'\bar{x})', y^{n'}] \\ P &= VK\tilde{C}K'V' \\ K &= \tilde{H}^{+}(I - \tilde{C}\{T\tilde{C}T\}^{+}) \end{aligned} \tag{2.12}$$

Here, $\tilde{H} = \begin{bmatrix} [I_{rxr}, 0] \\ HV \end{bmatrix}$; $\tilde{C} = \begin{bmatrix} \Lambda_1 & -V_1'A \\ -(V_1'A)' & C \end{bmatrix}$; $V = [V_1, V_2]$

The matrix V diagonalizes P_0^{-1} as $P_0^{-1} = V\,\mathrm{diag}(\Lambda_1^{-1}, 0, \ldots, 0)V'$, with $\Lambda_1 = \mathrm{diag}(\lambda_1, \ldots, \lambda_r) > 0$; $r = \mathrm{rank}(P_0^{-1})$.

OPTIMAL-WEIGHTED LEST SQUARES FUSION

For the UM of Equation 2.5, we have the following weighted least-squares (WLS) fusion equations [15–19]:

$$\begin{aligned} \hat{x} &= Ky^n \\ P &= KCK' = [H'C^{-1}H]^+ \\ K &= PH'C^{-1} \end{aligned} \tag{2.13}$$

OPTIMAL GENERALIZED WEIGHTED SQUARES FUSION

For similar conditions as in the BLUE fusion rule with complete prior knowledge, we have the following WLS fusion rule formulae.

Let the model be written as:

$$y_o = x + v_o = \bar{x} \tag{2.14}$$

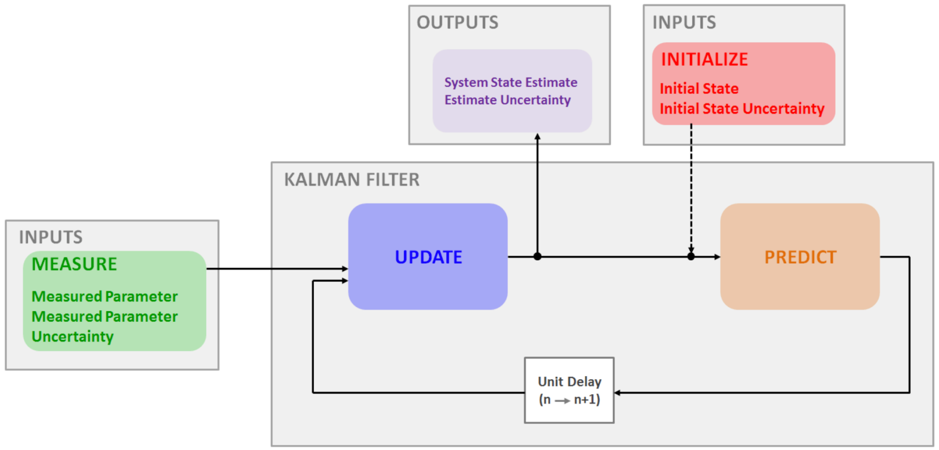
with $Cov(v_o) = \text{cov}(\bar{x} - x) = P_o$. In addition, let

$$\hat{y}^n = \begin{bmatrix} \bar{x} \\ y_n \end{bmatrix} \quad \hat{H} = \begin{bmatrix} I \\ H \end{bmatrix} \quad \hat{v}^n = \begin{bmatrix} v_o \\ v_n \end{bmatrix} \quad \hat{C} = \begin{bmatrix} P_o & -A \\ -A' & C \end{bmatrix} \quad \hat{y}^n = \hat{H}x + \hat{v}^n \tag{2.15}$$

Then, the estimator is represented as [15–19]:

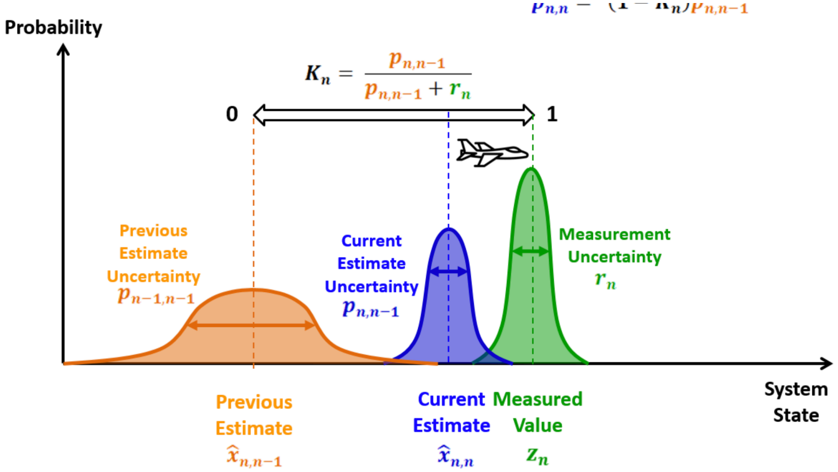
$$\begin{aligned} \hat{x} &= \arg\min(\hat{y}^n - \hat{H}\hat{x})'\hat{C}^{-1}(\hat{y}^n - \hat{H}\hat{x}) = K\hat{y}^n \\ P &= K\hat{C}K = [\hat{H}'\hat{C}^{-1}\hat{H}]^+ \\ K &= P\hat{H}'\hat{C}^{-1} \end{aligned} \tag{2.16}$$

KALMAN FILTER DATA FUSION



The three KF widely used methods to perform fusion at the kinematic level are:

- (1) fusion of the raw observational and measurement data (the data con- verted to engineering units), called centralized fusion;
- (2) fusion of the estimated state vectors or state-vector fusion; and
- (3) the hybrid approach, which allows fusion of raw data and the processed state vector, as desired.



Kalman filtering has evolved to become a very high-level state-of-the-art method for estimation of the states of dynamic systems . The main reason for its success is that it has a very intuitively appealing state-space formulation and a predictor-corrector estimation and recursive-filtering structure; furthermore, it can be easily implemented on digital computers and digital signal processing units. It is a numerical data processing algorithm, which has tremendous real-time and online application potential.