

# Assignment 2

Department of Electrical and Computer Engineering  
University of Waterloo  
ECE659 Intelligent Sensors and Sensor Networks

Submitted by  
Sai Teja G (20825009)  
Shadman Raihan (20858688)  
Somesh Kumar Gupta (20817245)

Q.1.

$$d_{A_{12}} = 40\text{m}, d_{A_{13}} = 110\text{m}, d_{23} = 35\text{m}$$

for finding the position of unknown node we first need to find the correction from each anchor nodes.

∴ Correction computation for Anchor node A<sub>1</sub>,

$$= \frac{40 + 110}{2 + 5}$$

$$= \frac{150}{7}$$

$$= 21.43 \text{ m}$$

Correction computation for Anchor node A<sub>2</sub>.

$$= \frac{40 + 35}{2 + 3}$$

$$= \frac{75}{5}$$

$$= 15 \text{ m}$$

Correction computation for Anchor node A<sub>3</sub>,

$$= \frac{110 + 35}{5 + 3}$$

$$= \frac{145}{8}$$

$$= 18.13 \text{ m}$$

Now the minimum number of hop is 2 from  
Anchor node  $A_2$   $\therefore$  the correction step is 15m.

$\therefore$  Distance of Unknown node from  $A_1$  is  
 $2 \times 15 = 30 \text{ m}$ .

Distance of unknown node from  $A_2$  is  
 $2 \times 15 = 30 \text{ m}$ .

Distance of unknown node from  $A_3$  is  
 $3 \times 15 = 45 \text{ m}$ .

Q.2. As Node A can hear the beacons located at (4,2) and (2,5) and Node B can hear beacons located at (2,5) and (3,7) we can find the distance b/w beacon points and the given position for the sensor. As the range is 2m, the distance should be less than equal to 2m.

- for position (3, 3.5) for Node A.

a) the distance from (4,2)

$$d = \sqrt{(4-3)^2 + (2-3.5)^2}$$

$$\begin{aligned} &= \sqrt{1 + (-1.5)^2} \\ &= \sqrt{3.25} \\ &= 1.803 \end{aligned}$$

b) the distance from (2,5)

$$\begin{aligned} d &= \sqrt{(2-3)^2 + (5-3.5)^2} \\ &= \sqrt{(-1)^2 + (1.5)^2} \\ &= \sqrt{1 + 2.25} \\ &= 1.803 \end{aligned}$$

- fore position  $(3, 4.5)$  for Node A

- the distance from  $(4, 2)$  is

$$d = \sqrt{(4-3)^2 + (2-4.5)^2}$$

$$= \sqrt{1^2 + (-2.5)^2}$$

$$= \sqrt{1+6.25}$$

$$= 2.693$$

- the distance from  $(2, 5)$

$$d = \sqrt{(2-3)^2 + (5-4.5)^2}$$

$$d = \sqrt{(-1)^2 + (0.5)^2}$$

$$d = \sqrt{1+0.25}$$

$$d = 1.118$$

Now the position of Node A is  $(3, 3.5)$  as it can hear beacons at positions  $(4, 2)$  and  $(2, 5)$  and the distance from both this points to  $(3, 3.5)$  is less than 2.  $\therefore$  the position is  $(3, 3.5)$  as the other point has a distance  $> 2$  from one of the beacon shown above.

- for position (2, 6) from Node B.

- a) the distance from (2, 5)

$$d = \sqrt{(2-2)^2 + (5-6)^2}$$

$$d = \sqrt{(-1)^2}$$

$$d = 1$$

- b) the distance from (3, 7)

$$d = \sqrt{(3-2)^2 + (7-6)^2}$$

$$d = \sqrt{1^2 + 1^2}$$

$$d = \sqrt{2}$$

$$d = 1.414$$

- for position (4, 5) from Node B to position (3, 7)

- $d = \sqrt{(4-3)^2 + (5-7)^2}$

$$= \sqrt{(-1)^2 + (-2)^2}$$

$$= \sqrt{1+4}$$

$$= \sqrt{5}$$

$$d = 2.236$$

b) the distance from  $(2,5)$  is

$$d = \sqrt{(2-4)^2 + (5-5)^2}$$

$$d = \sqrt{(-2)^2}$$

$$d = 2$$

$\therefore$  As from the distance above it is clear that position for Node B is  $(2,6)$  as the distance from both the beacons is less than 2 whence this is not the case with position  $(4,5)$ . hence position of B is  $(2,6)$ .

- In addition this can also be verified by Node Centroid theory.

- The centroid for the two beacon for Node A is

$$= \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$$

$$= \frac{4+2}{2}, \frac{5+2}{2}$$

$$= (3, 3.5)$$

$\therefore$  the position of Node A is  $(3, 3.5)$

- the centroid for two beacons for Node B is

$$= \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$= \frac{2+3}{2}, \frac{12}{2}$$

$$= (2.5, 6)$$

$\therefore$  the position for B has to be close to  $(2.5, 6)$  which is  $(2, 6)$  in this case  
hence the answers or position are verified.

3

$$1. (x_A - x_D)^2 + (y_A - y_D)^2 = r_{AD}^2 \quad - \textcircled{1}$$

$$(x_B - x_D)^2 + (y_B - y_D)^2 = r_{BD}^2 \quad - \textcircled{2}$$

$$(x_C - x_D)^2 + (y_C - y_D)^2 = r_{CD}^2 \quad - \textcircled{3}$$

Substituting  $\textcircled{3}$  from  $\textcircled{1}$  and  $\textcircled{2}$ , we get

$$2(x_C - x_A)x_D + 2(y_C - y_A)y_D = (r_{AB}^2 - r_{CD}^2) - (x_A^2 - x_C^2) \\ - (y_A^2 - y_C^2)$$

$$2(x_C - x_B)x_D + 2(y_C - y_B)y_D = (r_{BD}^2 - r_{CD}^2) - (x_B^2 - x_C^2) \\ - (y_B^2 - y_C^2)$$

$$2 \begin{bmatrix} x_C - x_A & y_C - y_A \\ x_C - x_B & y_C - y_B \end{bmatrix} \begin{bmatrix} x_D \\ y_D \end{bmatrix} = \begin{bmatrix} (r_{AB}^2 - r_{CD}^2) - (x_A^2 - x_C^2) - (y_A^2 - y_C^2) \\ (r_{BD}^2 - r_{CD}^2) - (x_B^2 - x_C^2) - (y_B^2 - y_C^2) \end{bmatrix}$$

$$2 \begin{bmatrix} -2 & 10 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x_D \\ y_D \end{bmatrix} = \begin{bmatrix} 15 \\ 8 \end{bmatrix}$$

$$x_D = 1.3125$$

$$y_D = 1.0125$$

$$\therefore (x_D, y_D) = (1.3125, 1.0125)$$

• Node E

$$\Rightarrow 2 \begin{bmatrix} x_D - x_A - y_D - y_A \\ x_D - x_E - y_D - y_E \end{bmatrix} \begin{bmatrix} x_E \\ y_E \end{bmatrix} = \begin{bmatrix} 21.5 \\ -82.5 \end{bmatrix}$$

$$\Rightarrow x_E = 2.36$$

$$y_E = 5.67$$

$$\therefore E(2.36, 5.67)$$

• Node G

Similar to A, E, J we find G by

$$2 \begin{bmatrix} x_J - x_A & y_J - y_A \\ x_J - x_E & y_J - y_E \end{bmatrix} \begin{bmatrix} x_G \\ y_G \end{bmatrix} = \begin{bmatrix} (x_{AG}^2 - x_{JG}^2) - (x_A^2 - x_J^2) - (y_A^2 - y_J^2) \\ (x_{EG}^2 - x_{JG}^2) - (x_E^2 - x_J^2) - (y_E^2 - y_J^2) \end{bmatrix}$$

$$2 \begin{bmatrix} 6 & 8 \\ 7.64 & 0.33 \end{bmatrix} \begin{bmatrix} x_G \\ y_G \end{bmatrix} = \begin{bmatrix} 75.75 \\ 58.012 \end{bmatrix}$$

$$(x_G, y_G) = G(3.71, 1.95)$$

• Node F .

$$\Rightarrow 2 \begin{bmatrix} x_j - x_A & y_j - y_A \\ x_j - x_G & y_j - y_G \end{bmatrix} \begin{bmatrix} x_F \\ y_F \end{bmatrix} = \begin{bmatrix} (x_{AF}^2 - x_{GF}^2) - (x_A^2 - x_j^2) - (y_A^2 - y_j^2) \\ (x_{AF}^2 - x_{GF}^2) - (x_G^2 - x_j^2) - (y_G^2 - y_j^2) \end{bmatrix}$$

$$\Rightarrow 2 \begin{bmatrix} 6 & 8 \\ 6.29 & 4.05 \end{bmatrix} \begin{bmatrix} x_F \\ y_F \end{bmatrix} = \begin{bmatrix} 156 \\ 118.4 \end{bmatrix}$$

$$x_F = 6.06 \quad F(6.06, 5.2)$$

$$y_F = 5.2$$

$$\begin{bmatrix} (x_P - x_A) & (y_P - y_A) & (x_S - x_A) \\ (x_P - x_G) & (y_P - y_G) & (x_S - x_G) \end{bmatrix} = \begin{bmatrix} 3 & 4 & 3 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_R & y_R & x_T \\ x_R & y_R & x_T \end{bmatrix}$$

$$\begin{bmatrix} 26.80 \\ 16.80 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 3 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 8.00 & 10.00 \\ 8.00 & 10.00 \end{bmatrix}$$

$$(x_R, y_R) = (8.0, 10.0) \quad (x_T, y_T) = (10.0, 10.0)$$

$x_R = 8.0$

$y_R = 10.0$

$x_T = 10.0$

Q.3 The distance between the Anchor node can be found and is shown below.

- Distance between A and B.

$$d_{AB} = \sqrt{(4 - (-1))^2 + (-2 - (3))^2}$$

$$= \sqrt{(5)^2 + (-5)^2}$$

$$= \sqrt{25 + 25}$$

$$= \sqrt{50}$$

$$d_{AB} = 7.07$$

- Distance between A and C is

$$d_{AC} = \sqrt{(4 - 2)^2 + (-2 - 8)^2}$$

$$= \sqrt{(2)^2 + (-10)^2}$$

$$= \sqrt{4 + 100}$$

$$= \sqrt{104}$$

$$= 10.198$$

- Distance between A and J is

$$d_{AJ} = \sqrt{(4 - 10)^2 + (-2 - 6)^2}$$

$$= \sqrt{(-6)^2 + (-8)^2}$$

$$= \sqrt{36 + 64}$$

$$d_{AB} = 10$$

- Distance between B and C is

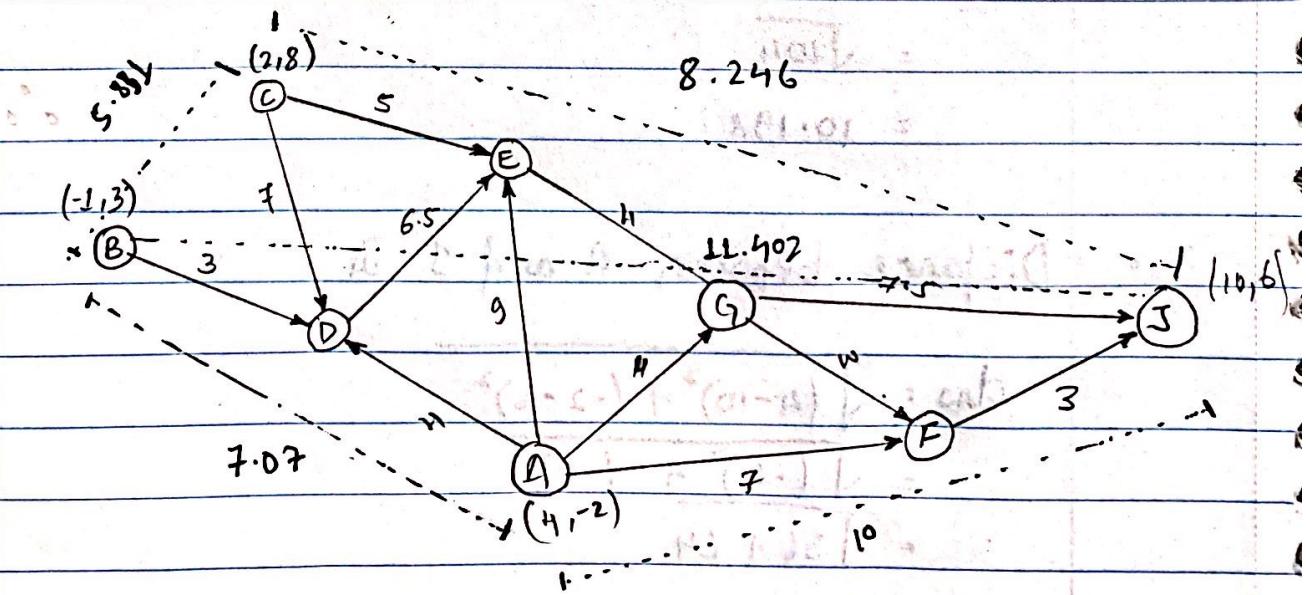
$$\begin{aligned} d_{BC} &= \sqrt{(-1-2)^2 + (3-8)^2} \\ &= \sqrt{(-3)^2 + (-5)^2} \\ &= \sqrt{9+25} \\ &= \sqrt{34} \end{aligned}$$

$$d_{BC} = 5.83$$

- Distance between C and J is

$$\begin{aligned} d_{CJ} &= \sqrt{(0-2)^2 + (6-8)^2} \\ &= \sqrt{(8)^2 + (-2)^2} \\ &= \sqrt{64+4} \\ &= \sqrt{68} \end{aligned}$$

$$d_{CJ} = 8.246$$



- Distance between node B and J is

$$d_{BJ} = \sqrt{(11)^2 + (3)^2}$$

$$= \sqrt{(121) + 9}$$

$$d_{BJ} = \sqrt{130}$$

$$d_{BJ} = 11.402$$

Q.4.

$$1. \overline{RSSI} = -10(4) \log(d) + (-50)$$

$$= -10(4) \log(10)$$

$$\overline{RSSI}_1 = -50$$

for  $d = 70$ .

$$\overline{RSSI}_{70} = -10(4) \log(70) - 50$$

$$= -10(4) \log(70) - 50$$

$$= -40(1.845) - 50$$

$$= -123.8 \text{ dBm}$$

$$\overline{RSSI}_{140} = -10(4) \log(140) - 50$$

$$= -10(4)(2.146) - 50$$

$$= -10(8.584) - 50$$

$$= -135.84 \text{ dBm.}$$

Q.S.

1. the posterior can be calculated as below.

$$P(\mu | D_x, D_y) = N(\mu | m_n, \lambda_n^{-1})$$

$$m_n = d_x n_x \bar{x} + d_y n_y \bar{y}$$

$$\lambda_n = d_0 + n_x d_x + n_y d_y$$

here  $\bar{x} = \frac{1}{n_x} \sum_{k=1}^N x_k$  and  $\bar{y} = \frac{1}{n_y} \sum_{k=1}^N y_k$  are the sample average. We put the expression for the mean and the variance / precision in terms of known variables.

$x, y, n_x, n_y, v_x, v_y$ . In addition, the prior is non-informative Gaussian so  $d_0 = 0$ .

$$m_n = \frac{d_x n_x \bar{x} + d_y n_y \bar{y}}{n_x d_x + n_y d_y}$$

$$= \frac{\frac{1}{v_x} n_x \bar{x} + \frac{1}{v_y} n_y \bar{y}}{\frac{n_x}{v_x} + \frac{n_y}{v_y}}$$

$$\frac{n_x}{v_x} + \frac{n_y}{v_y}$$

$$= \frac{V_y n_x \bar{x} + V_x n_y \bar{y}}{V_y n_x + V_x n_y}$$

$$V_n = \frac{V_y n_x + V_x n_y}{n_x + n_y}$$

$$\lambda_n = n_x \lambda_x + n_y \lambda_y$$

$$\Rightarrow \frac{1}{V_n} = \frac{n_x}{V_x} + \frac{n_y}{V_y}$$

$$V_n = \frac{V_x V_y}{n_x V_y + n_y V_x}$$

so this is the variance.

2. The sensor fusion using the two sensor data is performed with the variances. The posterior mean is the weighted average of the prior mean and the sample average of the two sensors.

In addition, the posterior precision is the sum of prior precision with the weighted precision of the sensors and as it is clear the variance is the inverse of the precision. Therefore from the above equation we can say that the posterior variance decreases with the number of measurements taken  $n_1, n_2$  and it is lower than the original variances  $V_1, V_2$ .

Q.6.

As the formula for getting the new data is as given

$$P(x=F|y_1^1, y_1^2) = \frac{P(x=F|y_1^1) P(x=F|y_1^2)}{P(F|y_0^1) P(F|y_0^2)} - ①$$

∴ from ① we can get the value of  $P(x=F|y_0^1, y_0^2)$  as

$$P(x=F|y_0^1, y_0^2) = \frac{P(x=F|y_1^1, y_1^2) \times P(F|y_0^1) P(F|y_0^2)}{P(x=F|y_1^1) P(x=F|y_1^2)} - ②$$

using eq ② we can get the values for  $x=F, x=N$  and  $x=A$ .

Now

For  $x=F$

$$P(x=F|y_0^1, y_0^2) = \frac{P(x=F|y_1^1, y_1^2) \times P(F|y_0^1) P(F|y_0^2)}{P(x=F|y_1^1) P(x=F|y_1^2)}$$

$$= \frac{0.88 \times 0.4 \times 0.6}{0.7 \times 0.8}$$

$$P(x=F|y_0^1, y_0^2) = 0.371$$

- For  $x = M$

$$P(x=M | y_0^1, y_0^2) = \frac{P(x=M | y_1^1, y_1^2) P(F | y_0^1) P(F | y_0^2)}{P(x=F | y_1^1) P(x=F | y_1^2)}$$

$$\textcircled{1} - \quad = \frac{0.11 \times 0.4 \times 0.3}{0.29 \times 0.15}$$

- For  $x = A$

$$P(x=A | y_0^1, y_0^2) = \frac{P(x=A | y_1^1, y_1^2) P(A | y_0^1) P(A | y_0^2)}{P(x=A | y_1^1) P(x=A | y_1^2)}$$

+ Note: → As in question the value of  $P(x=A | y_1^1, y_1^2) = 0.2$   
 which exceeds the total value of more than 1  
 we have assumed it to be 0.02

$$\text{Now using } P(x=A | y_1^1, y_1^2) = 0.02 \text{ we get .}$$

$$= \frac{0.02 \times 0.2 \times 0.1}{0.018 \times 0.05}$$

$$= 0.8$$

Now adding the probability to check value  
 w.r.t 1.

$$\begin{aligned}
 &= P(x=F | y_0^1, y_0^2) + P(x=M | y_0^1, y_0^2) + P(x=A | y_0^1, y_0^2) \\
 &= 0.3771 + 0.3034 + 0.8 \\
 &= 1.4805
 \end{aligned}$$

as the value is greater than 1 we will normalize it.

$\therefore$  Normalize values are

For  $x = F$

$$\begin{aligned}
 P(x=F | y_0^1, y_0^2) &= \frac{0.3771}{1.4805} \\
 &= 0.2547
 \end{aligned}$$

For  $x = M$

$$\begin{aligned}
 P(x=M | y_0^1, y_0^2) &= \frac{0.3034}{1.4805}
 \end{aligned}$$

$$= 0.2049$$

For  $x = A$

$$\begin{aligned}
 P(x=A | y_0^1, y_0^2) &= \frac{0.8}{1.4805} \\
 &= 0.5404
 \end{aligned}$$

Now adding value to check closeness to 1.

$$\begin{aligned}
 &= P(x=F|y_0^1, y_0^2) + P(x=M|y_0^1, y_0^2) + P(x=A|y_0^1, y_0^2) \\
 &= 0.2547 + 0.2049 + 0.5404 \\
 &= 1
 \end{aligned}$$

Case : Note : As in question even with the value of 0.02 the total probability for the fused Data exceed 1. we take a value of  $P(x=A|y_1^1, y_1^2) = 0.01$  so the new data is.

For  $x = A$

$$\begin{aligned}
 P(x=A|y_0^1, y_0^2) &= \frac{P(x=A|y_1^1, y_1^2) \cdot P(x=A|y_0^1) \cdot P(x=A|y_1^1)}{P(x=A|y_0^1) \cdot P(x=A|y_0^2)} \\
 &= \frac{0.01 \times 0.2 \times 0.1}{0.01 \times 0.05} \\
 &= 0.4
 \end{aligned}$$

Now adding value to check if it is 1.

$$\begin{aligned}
 &= P(x=F|y_0^1, y_0^2) + P(x=M|y_0^1, y_0^2) + P(x=A|y_0^1, y_0^2) \\
 &= 0.3771 + 0.3034 + 0.4 \\
 &= 1.0805
 \end{aligned}$$

$\therefore$  we normalize the values or if exceeds 1.  
 $\therefore$

- For  $x = F$

$$P(x=F | y_0^1, y_0^2) = \frac{0.3771}{1.0805}$$
$$= 0.3490$$

- For  $x = M$

$$P(x=M | y_0^1, y_0^2) = \frac{0.3034}{1.0805}$$
$$= 0.2808$$

- For  $x = A$

$$P(x=A | y_0^1, y_0^2) = \frac{0.4}{1.0805}$$
$$= 0.3702$$

Now adding to check

$$= P(x=F | y_0^1, y_0^2) + P(x=M | y_0^1, y_0^2) + P(x=A | y_0^1, y_0^2)$$
$$= 0.3490 + 0.2808 + 0.3702$$
$$= 1.$$

Q7. If the mass value for each is given therefore.

$$m_1 \{F\} = 0.3$$

$$m_1 \{M\} = 0.15$$

$$m_1 \{A\} = 0.03$$

$$m_1 \{\text{Animal}\} = 0.42$$

$$m_1 \{\text{Unknown}\} = 0.10$$

$$m_2 \{F\} = 0.40$$

$$m_2 \{M\} = 0.10$$

$$m_2 \{A\} = 0.02$$

$$m_2 \{\text{Animal}\} = 0.45$$

$$m_2 \{\text{Unknown}\} = 0.03$$

198-3

Sensor 2

		F	M	A	Animal	Unknown
	F	0.40	0.10	0.02	0.045	0.03
Sensor 1	M	0.3	0.12	φ	φ	0.009
A	0.15	φ	0.015	φ	φ	0.0045
Animal	0.03	φ	φ	0.006	φ	0.0009
Unknown	0.42	φ	φ	φ	0.189	0.0126
		0.10	0.04	0.01	0.002	0.045

The conflict factor can be calculated as shown below.

$$\begin{aligned}
 K = & (0.3 \times 0.10) + (0.3 \times 0.02) + (0.3 \times 0.45) + (0.15 \times 0.4) + \\
 & (0.15 \times 0.02) + (0.15 \times 0.45) + (0.03 \times 0.4) + (0.03 \times 0.10) \\
 & + (0.03 \times 0.45) + (0.42 \times 0.4) + (0.42 \times 0.10) + (0.42 \times 0.02)
 \end{aligned}$$

$$K = 0.5484$$

Now the value

$$1-k = 0.4516$$

Now the value after the combined data is.

$$m(F) = \frac{0.12 + 0.009 + 0.04}{0.4516}$$
$$= 0.374$$

$$m(A) = \frac{0.0006 + 0.0009 + 0.002}{0.4516}$$

$$= 0.007$$

$$m(\text{Animal}) = \frac{0.189 + 0.0126 + 0.045}{0.4516}$$

$$= 0.546$$

$$m(\text{Unknown}) = \frac{0.003}{0.4516}$$

$$m(M) = \frac{0.015 + 0.00451 + 0.01}{0.4516}$$

$$= 0.065$$

Q.8.

Note 1

The code and the graph for part b) is shown below.

Code 1:

graph LR; A(( )) --> B(( )); B --> C(( )); C --> D(( )); D --> E(( )); E --> F(( )); F --> G(( )); G --> H(( )); H --> I(( )); I --> J(( )); J --> K(( )); K --> L(( )); L --> M(( )); M --> N(( )); N --> O(( )); O --> P(( )); P --> Q(( )); Q --> R(( )); R --> S(( )); S --> T(( )); T --> U(( )); U --> V(( )); V --> W(( )); W --> X(( )); X --> Y(( )); Y --> Z(( )); Z --> A;

Code 2:

Q2 = (a1)(f1)(f2)(a2)

Q2 = (f1)(f2)(a1)(a2)

max 3 + 6 + 1 = 10

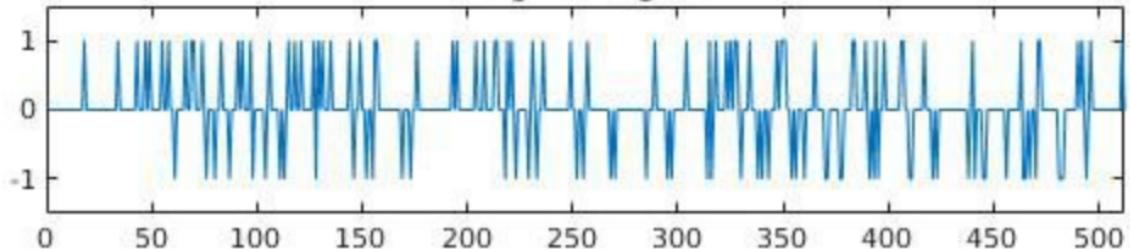
Q2 = (a1)(f1)(f2)(a2)

Q2 = (f1)(f2)(a1)(a2)

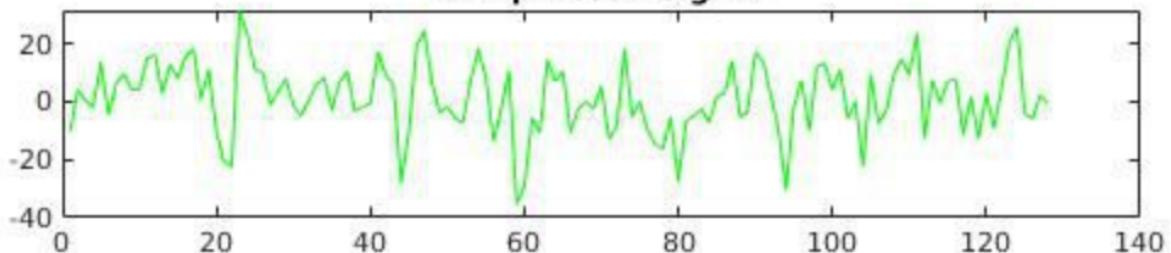
Q2 = (f2)(f1)(a1)(a2)

max 3 + 6 + 1 = 10

**Original Signal**



**Compressed Signal**



**Recovered Signal**

