Q.1.

$d_{A_{12}} = 40m$, $d_{A_{13}} = 110m$, $d_{23} = 35m$

for finding the position of unknown node we first need to find the correction from each anchor nodes.

∴ Correction computation for Anchor node A_1.

$$= \frac{40 + 110}{2 + 5}$$

$$= \frac{150}{7}$$

$$= 21.43 \text{ m}$$

Correction computation for Anchor node A_2.

$$= \frac{40 + 35}{2 + 3}$$

$$= \frac{75}{5}$$

$$= 15 \text{ m}$$

Correction computation for Anchor node A_3

$$= \frac{110 + 35}{5 + 3}$$

$$= \frac{145}{8}$$

$= 18.13\,m$

Now the minimum number of hop is 2 from Anchor node A_2 $\therefore$ The correction used is 15 m.

$\therefore$ Distance of Unknown node from A1 is

$2 \times 15 = 30\,m$.

Distance of unknown node from A_2 is

$2 \times 15 = 30\,m$.

Distance of unknown node from A_3 is

$3 \times 15 = 45\,m$.

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Q.2. As Node A can hear the becons located at (4,2) and (2,5) and Node B can hear beacons Located at (2,5) and (3,7) we can find the distance b/w beacon points and the given position for the sensor. As the range is 2m the distance should be Less than equall to 2m.

∴

- for position (3, 3.5) for Node A

a) the distance from (4,2) is

$$d = \sqrt{(4-3)^2 + (2-3.5)^2}$$

$$= \sqrt{1 + (-1.5)^2}$$

$$= \sqrt{3.25}$$

$$= 1.803$$

b) the distance from (2,5) is

$$d = \sqrt{(2-3)^2 + (5-3.5)^2}$$

$$= \sqrt{(-1)^2 + (1.5)^2}$$

$$= \sqrt{1 + 2.25}$$

$$= 1.803$$

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• for position $(3, 4.5)$ for Node A

a) the distance from $(4,2)$ is

$$d = \sqrt{(4-3)^2 + (2-4.5)^2}$$

$$= \sqrt{(1)^2 + (-2.5)^2}$$

$$= \sqrt{1 + 6.25}$$

$$= 2.693$$

b). the distance from $(2,5)$

$$d = \sqrt{(2-3)^2 + (5-4.5)^2}$$

$$d = \sqrt{(-1)^2 + (0.5)^2}$$

$$d = \sqrt{1 + .25}$$

$$d = 1.118$$

Now the position of Node A is $(3, 3.5)$ as it can hear beacon at positions $(4,2)$ and $(2,5)$ and the distance from both this points to $(3, 3.5)$ is less than 2. ∴ the position is $(3, 3.5)$ as the other point has a distance >2 for one of the beacon shown above.

- for position (2,6) for Node B.

a) the distance from (2,5) is

$$d = \sqrt{(2-2)^2 + (5-6)^2}$$

$$d = \sqrt{(-1)^2}$$

$$d = 1$$

b) the distance from (3,7) is

$$d = \sqrt{(3-2)^2 + (7-6)^2}$$

$$d = \sqrt{1^2 + 1^2}$$

$$d = \sqrt{2}$$

$$d = 1.414$$

- for position (4,5) for Node B to position (3,7)

a) $d = \sqrt{(4-3)^2 + (5-7)^2}$

$= \sqrt{(1)^2 + (-2)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$

$d = 2.236$

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b) the distance from (2,5) is

$$d = \sqrt{(2-4)^2 + (5-5)^2}$$

$$d = \sqrt{(-2)^2}$$

$$d = 2$$

$\therefore$ As from the distance above it is cleare that position for Node B is (2,6) as the distance from both the beacons is less than 2 where this is Not the case with position (4,5). Hence position of B is (2,6).

* In Addition this can also be varified be Node centroid theory.

- the centroid for the two beacon for Node A is

$$= \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$= \frac{4+2}{2}, \frac{5+2}{2}$$

$$= (3, 3.5)$$

$\therefore$ the position of Node A is (3,3.5)

• the centroid for two beacons for Node B is

$$= \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$= \frac{2+3}{2}, \frac{12}{2}$$

$$= (2.5, 6)$$

∴ the position for B has to be close to (2.5, 6) which is (2, 6) in this case hence the answers of position are verified.

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3

1. $(x_A - x_D)^2 + (y_A - y_D)^2 = r_{AD}^2$ — ①

$(x_B - x_D)^2 + (y_A - y_D)^2 = r_{BD}^2$ — ②

$(x_C - x_D)^2 + (y_C - y_D)^2 = r_{CD}^2$ — ③

Substituting ③ from ① and ②, we get

$$2(x_c - x_A)x_D + 2(y_C - y_A)y_D = (r_{AB}^2 - r_{CD}^2) - (x_A^2 - x_c^2) - (y_A^2 - y_C^2)$$

$$2(x_c - x_B)x_D + 2(y_C - y_B)y_D = (r_{BD}^2 - r_{CD}^2) - (x_B^2 - x_c^2) - (y_B^2 - y_C^2)$$

$$2\begin{bmatrix} x_c - x_A & y_c - y_A \\ x_c - x_B & y_c - y_B \end{bmatrix}\begin{bmatrix} x_D \\ y_D \end{bmatrix} = \begin{bmatrix} (r_{AD}^2 - r_{CD}^2) - (x_A^2 - x_c^2) - (y_A - y_c)^2 \\ (r_{BD}^2 - r_{CD}^2) - (x_B^2 - x_c^2) - (y_B - y_c^2) \end{bmatrix}$$

$$2\begin{bmatrix} -2 & 10 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x_D \\ y_D \end{bmatrix} = \begin{bmatrix} 15 \\ 8 \end{bmatrix}$$

$x_D = 1.3125$

$y_D = 1.0125$

$\therefore (x_D, y_D) = (1.3125, 1.0125)$

- Node E.

$$\Rightarrow 2\begin{bmatrix} x_D - x_A & y_A - y_A \\ x_D - x_C & y_D - y_C \end{bmatrix}\begin{bmatrix} x_E \\ y_E \end{bmatrix} = \begin{bmatrix} 21.5 \\ -82.3 \end{bmatrix}$$

$$\Rightarrow \quad x_E = 2.36$$

$$y_E = 5.67$$

$\therefore E(2.36, 5.67)$

- Node G

Similar to A, E, J we find G by

$$2\begin{bmatrix} x_J - x_A & y_J - y_A \\ x_J - x_E & y_J - y_A \end{bmatrix}\begin{bmatrix} x_G \\ y_G \end{bmatrix} = \begin{bmatrix} (r_{AG}^2 - r_{JG}^2) - (x_A^2 - x_J^2) - (y_A^2 - y_J^2) \\ (r_{EG}^2 - r_{JG}^2) - (x_E^2 - x_J^2) - (y_E^2 - y_J^2) \end{bmatrix}$$

$$2\begin{bmatrix} 6 & 8 \\ 7.64 & 0.33 \end{bmatrix}\begin{bmatrix} x_G \\ y_G \end{bmatrix} = \begin{bmatrix} 75.75 \\ 58.012 \end{bmatrix}$$

$(x_G, y_G) = G(3.71, 1.95)$

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- Node F.

$$\Rightarrow 2\begin{bmatrix} x_J - x_A & y_J - y_A \\ x_J - x_G & y_J - y_G \end{bmatrix}\begin{bmatrix} x_F \\ y_F \end{bmatrix} = \begin{bmatrix} (r_{AF}^2 - r_{JF}^2) - (x_A^2 - x_J^2) - (y_A^2 - y_J^2) \\ (r_{AF}^2 - r_{JF}^2) - (x_G^2 - x_J^2) - (y_G^2 - y_J^2) \end{bmatrix}$$

$$\Rightarrow 2\begin{bmatrix} 6 & 8 \\ 6.29 & 4.05 \end{bmatrix}\begin{bmatrix} x_F \\ y_F \end{bmatrix} = \begin{bmatrix} 156 \\ 118.4 \end{bmatrix}$$

$x_F = 6.06$

$y_F = 5.2$

$F\,(6.06, 5.2)$

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Q.3 The distance between the Anchore node can be found and is shown below.

- Distance between A and B.

$$d_{AB} = \sqrt{(4-(-1))^2 + (-2-(3))^2}$$

$$= \sqrt{(5)^2 + (-5)^2}$$

$$= \sqrt{25+25}$$

$$= \sqrt{50}$$

$$d_{AB} = 7.07$$

- Distance between A and C is

$$d_{AC} = \sqrt{(4-2)^2 + (-2-8)^2}$$

$$= \sqrt{(2)^2 + (-10)^2}$$

$$= \sqrt{4+100}$$

$$= \sqrt{104}$$

$$= 10.198$$

- Distance between A and J is

$$d_{AJ} = \sqrt{(4-10)^2 + (-2-6)^2}$$

$$= \sqrt{(-6)^2 + (-8)^2}$$

$$= \sqrt{36+64}$$

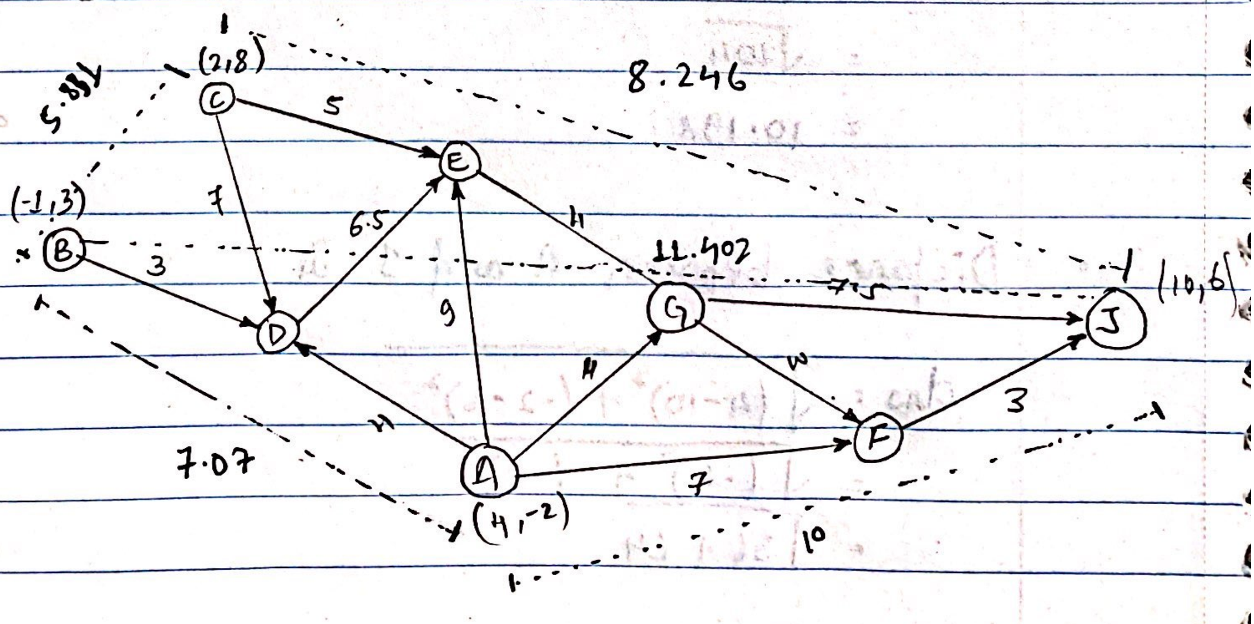
$d_{AJ} = 10$

- Distance between B and C is

$$d_{BC} = \sqrt{(-1-2)^2 + (3-8)^2}$$
$$= \sqrt{(-3)^2 + (-5)^2}$$
$$= \sqrt{9 + 25}$$
$$= \sqrt{34}$$
$$d_{BC} = 5.831$$

- Distance between C and J is

$$d_{CJ} = \sqrt{(10-2)^2 + (6-8)^2}$$
$$= \sqrt{(8)^2 + (-2)^2}$$
$$= \sqrt{64 + 4}$$
$$= \sqrt{68}$$
$$d_{CJ} = 8.246$$



- Distance between node B and J is

$$d_{BJ} = \sqrt{(11)^2 + (3)^2}$$

$$= \sqrt{(121) + 9}$$

$$d_{BJ} = \sqrt{130}$$

$$d_{BJ} = 11.402$$

Q.4.

1. $\overline{RSSI} = -10(4)\log(d) + (-50)$

$= -10(4)\log(0)$

$\overline{RSSI}_1 = -50$

for $d = 70$.

$\overline{RSSI}_{70} = -10(4)\log(70) - 50$

$= -10(4)\log(70) - 50$

$= -40(1.845) - 50$

$= -123.8$ dBm

$\overline{RSSI}_{140} = -10(4)\log(140) - 50$

$= -10(4)(2.146) - 50$

$= -10(8.584) - 50$

$= -135.84$ dBm.

Q.5.

1. the posterior can be calculated as below.

$$P(\mu|D_x,D_y) = \mathcal{N}(\mu|m_n, \lambda_N^{-1})$$

$$m_n = \lambda_x n_x \bar{x} + \lambda_y n_y \bar{y}$$

$$\lambda_n = \lambda_0 + n_x\lambda_x + n_y\lambda_y$$

here $\bar{x} = \frac{1}{n_x}\sum_{k=1}^{N} x_N$ and $\bar{y} = \frac{1}{n_y}\sum_{k=1}^{N} y_N$ are the sample average. we put the expression for the mean and the variance/precision in terms of known variables. x, y, n_x, n_y, v_x, v_y. In addition, the prior is non-informative Guassian so $\lambda_0 = 0$.

$$m_n = \frac{\lambda_x n_x \bar{x} + \lambda_y n_y \bar{y}}{n_x\lambda_x + n_y\lambda_y}$$

$$= \frac{\frac{1}{v_x} n_x \bar{x} + \frac{1}{v_y} n_y \bar{y}}{\frac{n_x}{v_x} + \frac{n_y}{v_y}}$$

$$= \frac{V_y n_x \bar{x} + V_x n_y \bar{y}}{V_y n_x + V_x n_y}$$

$\therefore$

$$\lambda_n = n_x \lambda_x + n_y \lambda_y$$

$$\Rightarrow \frac{1}{V_n} = \frac{n_x}{V_x} + \frac{n_y}{V_y}$$

$$V_n = \frac{V_x V_y}{n_x V_y + n_y V_x}$$

$\therefore$ this is the variance.

2. The sensor fusion using the two sensor data is performed with the variances. The posterior mean is the weighted average of the prior mean and the sample average of the two sensors.

In addition, the posterior precision is the sum of prior precision with the weighted precision of the sensors and as it is clear the variance is the inverse of the precision. Therefore from the above equation we can say that the posterior variance decreases with the number of measurements taken n_1, n_2 and it is lower than the original variances V_1, V_2.

Q.6.

As the formula for getting the new data is as given

$$P(x=F|y_1^1,y_1^2) = \frac{P(x=F|y_1^1)\,P(x=F|y_1^2)\,P(x=F|y_0^1,y_0^2)}{P(F|y_0^1)\,P(F|y_0^2)} \quad -①$$

∴ from ① we can get the value of $P(x=F|y_0^1,y_0^2)$ as

$$P(x=F|y_0^1,y_0^2) = \frac{P(x=F|y_1^1,y_1^2)\times P(F|y_0^1)\,P(F|y_0^2)}{P(x=F|y_1^1)\,P(x=F|y_1^2)} \quad -②$$

using eq ② we can get the values for $x=F$, $x=M$ and $x=A$.

Now

- For $x=F$

$$P(x=F|y_0^1,y_0^2) = \frac{P(x=F|y_1^1,y_1^2)\times P(F|y_0^1)\,P(F|y_0^2)}{P(x=F|y_1^1)\,P(x=F|y_1^2)}$$

$$= \frac{0.88\times 0.4\times 0.6}{0.7\times 0.8}$$

$$P(x=F|y_0^1,y_0^2) = 0.3771$$

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- For $x = M$

$$P(x=M \mid y_0^1, y_0^2) = \frac{P(x=M \mid y_i^1, y_i^2)\, P(F \mid y_0^1)\, P(F \mid y_0^2)}{P(x=F \mid y_i^1)\, P(x=F \mid y_i^2)}$$

$$= \frac{0.11 \times 0.4 \times 0.3}{0.29 \times 0.15}$$

$$= 0.3034$$

- For $x = A$

$$P(x=A \mid y_0^1, y_0^2) = \frac{P(x=A \mid y_i^1, y_i^2)\, P(A \mid y_0^1)\, P(A \mid y_0^2)}{P(x=A \mid y_i^1)\, P(x=A \mid y_i^2)}$$

* Note :→ As in question the value of $P(x=A \mid y_i^1, y_i^2) = 0.2$ which exceeds the total value of more than 1 we have assumed it to be 0.02

Now using $P(x=A \mid y_i^1, y_i^2) = 0.02$ we get.

$$= \frac{0.02 \times 0.2 \times 0.1}{0.01 \times 0.05}$$

$$= 0.8$$

Now adding the probability to check value w.r.t 1.

$= P(x=F \mid y_0^1, y_0^2) + P(x=M \mid y_0^1, y_0^2) + P(x=A \mid y_0^1, y_0^2)$

$= 0.3771 + 0.3034 + 0.8$

$= 1.4805$

As the value is greater than 1 we will normalize it.

$\therefore$ Normalize values are:

- For $x = F$

$$P(x=F \mid y_0^1, y_0^2) = \frac{0.3771}{1.4805}$$

$$= 0.2547$$

- For $x = M$

$$P(x=M \mid y_0^1, y_0^2) = \frac{0.3034}{1.4805}$$

$$= 0.2049$$

- For $x = A$

$$P(x=A \mid y_0^1, y_0^2) = \frac{0.8}{1.4805}$$

$$= 0.5404$$

Now adding value to check closness to 1.

$= P(x=F|y_0^1,y_0^2) + P(x=M|y_0^1,y_0^2) + P(x=A|y_0^1,y_0^2)$

$= 0.2547 + 0.2049 + 0.5404$

$= 1.$

• Case : Note : As in question even with the value of 0.02 the total probability for the fused Data exceeds 1. we take a value of $P(x=A|y_1^1,y_1^2) = 0.01$

∴ the new data is.

• For $x=A$

$$P(x=A|y_0^1,y_0^2) = \frac{P(x=A|y_1^1,y_1^2) \cdot P(x=A|y_0^1)\, P(x=A|y_1^1)}{P(x=A|y_0^1)\, P(x=A|y_0^2)}$$

$$= \frac{0.01 \times 0.2 \times 0.1}{0.01 \times 0.05}$$

$$= 0.4$$

Now adding value to check if it is 1:

$= P(x=F|y_0^1,y_0^2) + P(x=M|y_0^1,y_0^2) + P(x=A|y_0^1,y_0^2)$

$= 0.3771 + 0.3034 + 0.4$

$= 1.0805$

$\therefore$ we normalize the values as it exceeds 1.

$\therefore$

- For $x = F$

$$P(x=F \mid y_0^1, y_0^2) = \frac{0.3771}{1.0805}$$

$$= 0.3490$$

- For $x = M$

$$P(x=M \mid y_0^1, y_0^2) = \frac{0.3034}{1.0805}$$

$$= 0.2808$$

- For $x = A$

$$P(x=A \mid y_0^1, y_0^2) = \frac{0.4}{1.0805}$$

$$= 0.3702$$

Now adding to check

$= P(x=F \mid y_0^1, y_0^2) + P(x=M \mid y_0^1, y_0^2) + P(x=A \mid y_0^1, y_0^2)$

$= 0.3490 + 0.2808 + 0.3702$

$= 1.$

Q7. The mass value for each is given therefore.

$m_1\{F\} = 0.3$ $\qquad$ $m_2\{F\} = 0.40$

$m_1\{M\} = 0.15$ $\qquad$ $m_2\{M\} = 0.10$

$m_1\{A\} = 0.03$ $\qquad$ $m_2\{A\} = 0.02$

$m_1\{Animal\} = 0.42$ $\qquad$ $m_2\{Animal\} = 0.45$

$m_1\{unknown\} = .10$ $\qquad$ $m_2\{Unknown\} = 0.03$

Sensor 2

			F 0.40	M 0.10	A 0.02	Animal 0.045	Unknown 0.03
Sensor 1	F	0.3	0.12	ϕ	ϕ	ϕ	0.009
	M	0.15	ϕ	0.015	ϕ	ϕ	0.0045
	A	0.03	ϕ	ϕ	0.006	ϕ	0.0009
	Animal	0.42	ϕ	ϕ	ϕ	0.189	0.0126
	Unknown	0.10	0.04	0.01	0.002	0.045	0.003

The conflict factor can be calculated as shown below.

$$K = (0.3 \times 0.10) + (0.3 \times 0.02) + (0.3 \times 0.45) + (0.15 \times 0.4) + (0.15 \times 0.02) + (0.15 \times 0.45) + (0.03 \times 0.4) + (0.03 \times 0.10) + (0.03 \times 0.45) + (0.42 \times 0.4) + (0.42 \times 0.10) + (0.42 \times 0.02)$$

$$K = 0.5484$$

Now the value

$1 - K = 0.4516$

Now the value after the combined data is.

$$m(F) = \frac{0.12 + 0.009 + 0.04}{0.4516}$$

$$= 0.374$$

$$m(A) = \frac{0.0006 + 0.0009 + 0.002}{0.4516}$$

$$= 0.0007$$

$$m(\text{Animal}) = \frac{0.189 + 0.0126 + 0.045}{0.4516}$$

$$= 0.546$$

$$m(\text{Unknown}) = \frac{0.003}{0.4516}$$

$$m(M) = \frac{0.015 + 0.0045 + 0.01}{0.4516}$$

$$= 0.065$$

Q.8.

The code and the graph for part b is shown below.