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How to Guess and Prove? and Master's Theorem

* Guess and Prove.

Works

Q $T(n) = T(\frac{n}{2}) + n$

$T(1) = 1$

assume $T(n) \leq cn$

Base case:

$n = 1$

$T(1) \leq c$

using induction, assume that lemma holds for $\frac{n}{2}$

$T(n) = T(\frac{n}{2}) + n$

$T(n) \leq c\frac{n}{2} + n$

$\leq cn(\frac{1}{2} + \frac{1}{c})$

$\Downarrow$

$\frac{1}{2} + \frac{1}{c} \leq 1$

$c \geq 2$

Not works

Q $T(n) = 4T(\frac{n}{2}) + n$

$T(1) = 1$

assume $T(n) \leq cn$

Base case

Induction.

$T(n) = 4 \cdot c\frac{n}{2} + n$

$= n(2c+1)$

$\Downarrow$

$2c+1 < c$

× wrong assumption

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Masters Theorem

Let $T(n)$ be a monotonically inc. f^n that satisfies

$$T(n) = aT\left(\frac{n}{b}\right) + n^d$$

$$T(1) = c$$

$a \geq 1 \quad b \geq 2 \quad c \geq 0 \quad d \geq 1$

then

$$T(n) = \begin{cases} O(n^d) & a < b^d \\ O(n^d \log n) & a = b^d \\ O(n^{\log_b a}) & a > b^d \end{cases}$$