

Assignment 4

Due Date: After the class on November 12.

Please prove the running time and the correctness of your algorithm.

1. (5 points) Given a binary tree, give an $O(n)$ time algorithm to find the height of the tree – height of a tree is the maximum depth of any node in the tree.
2. You are given an a matrix M of size $n \times n$ containing distinct integers such that entries in each row are in increasing order (from left to right) and entries in each column are in increasing order (from top to bottom). Given a integer x , your job is to find if x exists in some cell of M or not.
 - (a) (1 point) Design an algorithm with a running time of $O(n \log n)$ for the above problem.
 - (b) (10 points) Design an algorithm with a running time of $O(n)$ for the above problem.

-10	-5	-1	1	8
-8	-3	5	6	14
-4	4	10	11	19
0	9	23	26	38
2	15	30	33	40

Figure 1: A matrix M of size 5×5 . If $x = 26$, then you algorithm should output *yes*, else if $x = 17$, your algorithm should output *no*.

3. (10 points) You are given an an array A of size n . You have to design an algorithm that will do the following update on the array:
ADD(i, j, a): For each $k \in [i, j]$, $A[k] = A[k] + a$
QUERY(i): Return the number at index i of the array A
All the updates should take $O(\log n)$ time.