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Semah 1910206

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Midsem exam

Q2.

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + n \quad \forall n > 5$$

$$T(n) = 1 \quad \forall n \leq 5$$

$$T(n) = \left[T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right) + \frac{n}{2} \right] + \left[T\left(\frac{n}{8}\right) + T\left(\frac{n}{16}\right) + \frac{n}{4} \right] + \dots$$

=

$$T\left(\frac{n}{4}\right) + 2T\left(\frac{n}{8}\right) + T\left(\frac{n}{16}\right) + \left[\frac{n}{2} + \frac{n}{4} + \dots \right]$$

↓
(A)↓
(B)↓
(C)

4+1+2

 $\left(\frac{2n}{4} \right)$

(A) =

$$T\left(\frac{n}{8}\right) + T\left(\frac{n}{16}\right) + \frac{n}{4}$$

(B) =

$$T\left(\frac{n}{16}\right) + T\left(\frac{n}{32}\right) + \frac{n}{8}$$

(C) =

$$T\left(\frac{n}{32}\right) + T\left(\frac{n}{64}\right) + \frac{n}{16}$$

=

$$\left(T\left(\frac{n}{16}\right) + T\left(\frac{n}{8}\right) + \frac{n}{4} \right)$$

$$+ 2 \left(T\left(\frac{n}{16}\right) + T\left(\frac{n}{32}\right) + \frac{n}{8} \right)$$

$$+ \left(T\left(\frac{n}{32}\right) + T\left(\frac{n}{64}\right) + \frac{n}{16} \right)$$

$$T(n) \leq 2 \cdot T\left(\frac{n}{2}\right) + n \quad n \geq 5$$

$$T(1), T(2), T(3), T(4), T(5) = 1$$

$$T(n) \leq 2 \left(T\left(\frac{n}{2}\right) + n \right)$$

$$T(n) \leq c n$$

$$T\left(\frac{n}{2}\right) \leq c \frac{n}{2}$$

$$2 \cdot c \frac{n}{2} + n$$

$$T(n) \leq cn + n$$

$$n(c+1)$$

$$cn < c \quad \times$$

wrong assumption.

$$T(n) \leq 2T\left(\frac{n}{2}\right) + n, \quad n \leq 5$$

$$T(n) = 1, \quad n \leq 5$$

 $n \geq 5$

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$T(n) = 2\left(2T\left(\frac{n}{4}\right) + \frac{n}{2}\right) + n$$

$$= 2^2 T\left(\frac{n}{4}\right) + 2 \cdot \frac{n}{2} + \frac{2 \cdot n}{2}$$

$$= 2^k T\left(\frac{n}{2^k}\right) + 2 \cdot \left(\frac{n}{2^k} + \dots + \frac{n}{2}\right)$$

$$\frac{n}{2^k} \leq 5$$

$$\frac{n}{5} \leq 2^k$$

$$\frac{n}{5} \leq 2^k$$

$$\log_2(n/5) \leq k$$

$$k \geq \log_2(n/5)$$

$$2^{\log(n/5)} T\left(\frac{n}{2^k}\right) + 2 \left(\frac{n}{2^k} + \dots + \frac{n}{2} \right)$$

$$GP: \frac{n}{2} + \frac{n}{2^2} + \dots + \frac{n}{2^k}$$

$$\frac{n \left(\frac{1}{2} \right) \left(\left(\frac{1}{2} \right)^k - 1 \right)}{\left(\frac{1}{2} \right) - 1}$$

$$\frac{n \left(\frac{1}{2} \right) \left(1 - \left(\frac{1}{2} \right)^k \right)}{1 - \frac{1}{2}}$$

$$n \left(1 - \left(\frac{1}{2} \right)^k \right)$$

$$T(n) = \frac{5 \cdot n}{5} + 2n \left(1 - \left(\frac{1}{2} \right)^{\log_2 \frac{n}{5}} \right)$$

1.

we can create an array of size $n+1$ where in $A[i] = i$, i is the no. of elements already in the data str.

For adding x , ~~we~~ we can simply do
 $A[x] = x$;

For reporting the minimum
we can just print
 $A[0]$ [0 based indexing]

3. Take input parameter k.

for (int i = 0 to n-1) {

 tweak_quicksort(A, 0, i, k)
 print A[k-1]

}

3.

where tweak_quicksort is

void tweak_quicksort(A, low, high, k)
if (low >= high)
 return

 pivot = position(arr, low, high)

 if (pivot == k-1)
 return;

3.

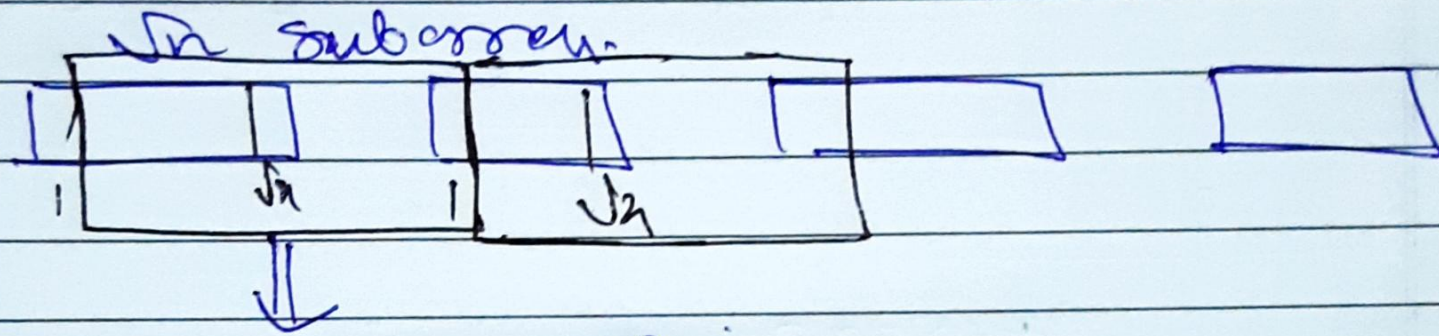
→ compare
position
function
used in class

 tweak_qs(~~arr~~ A, low, pivot-1, k);
 tweak_qs(A, pivot+1, high, k)

3.

④ First make \sqrt{n} calls on (\sqrt{n} no. of subarray, dividing the whole array into equal parts)

then include the first element of every subarray



Call on this

$$(\sqrt{n}) + (\sqrt{n}-1) + (\sqrt{n}-2) \dots (\sqrt{n}-k) \text{ as }.$$

$$(k \leq \sqrt{n})$$