

Assignment 5

Due Date: After the class on Nov 26.

1. (10 points) The girth of a undirected graph is the length of the smallest cycle in it. Given a graph G , design an algorithm to find its girth. Prove its correctness and running time.
2. (10 points) Given an undirected unweighted graph G with two nodes s and t , show that if the distance between s and t is $> n/2$, then there exists a vertex v such that deleting v destroys all st path in G . Give an $O(m + n)$ algorithm to find such a vertex v .
3. (a) (5 points) In the **bottleneck-path problem**, you are given a graph G with edge weights, two vertices s and t , and a particular weight W ; your goal is to find a path from s to t in which every edge has at least weight W . Describe an efficient algorithm to solve this problem. Your algorithm should work even if the edge weights are negative and/or the particular weight W is negative.
(b) (5 points) In the **maximum-bottleneck-path** problem, you are given a graph G with edge weights, and two vertices s and t ; your goal is to find a path from s to t whose minimum edge weight is maximized. Describe an efficient algorithm to solve this problem that uses an efficient algorithm from Part (a) as a subroutine.