

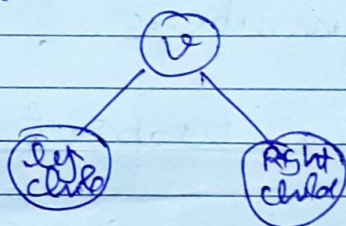
Heaps 1

① Complete Binary Tree

1. Each internal node has at most 2 children
2. All the levels except the last one are full.
3. Nodes Leaves appear from left to right at last levels.

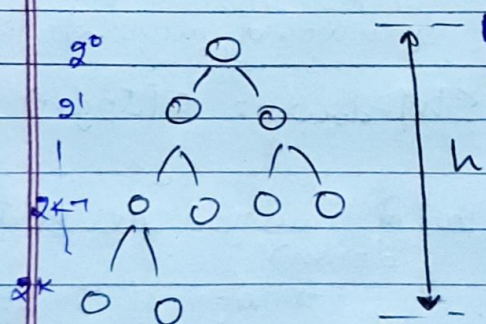
② Heap

Heap is a complete Binary Tree in which each internal nodes v satisfies.



$v < \text{left child and right child}$

③



$$2^0 + 2^1 + 2^2 + \dots + 2^k = n$$

Height of heap is $O(\log n)$

$n = \text{no. of nodes}$

- ④ If an array represents a heap, for zero based indexing

children of $i = 2i+1, 2i+2$

parent of $i = \begin{cases} \lfloor i/2 \rfloor & i = \text{odd} \\ \lfloor (i-1)/2 \rfloor & i = \text{even} \end{cases}$

- ⑤ Given a min heap, how can you use it to sort numbers.

- ⑥ Running time of heap sort = $O(n \log n)$
 each time heap sort is implemented and shift down is called, the height is decreased by 1, at max you would have to call heap sort n times.

Running time of Shift down is $O(\log n)$

\propto (no. of times you call shift down) \rightarrow height of the tree

- ⑦ Build heap algorithm.
 \rightarrow correctness
 \rightarrow running time

- ⑧ Given a heap, add or delete.