Fairness-Aware Approach for Multi-Depot Vehicle Routing Problem

: A Case Study of Yuseong-gu, Daejeon

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Code Repository https://github.com/somin417/CusVRP

1. INTRODUCTION

1.1 Research Background

In modern urban logistics systems, the primary objective has traditionally been operational efficiency, characterized strictly by the minimization of total travel distances and the reduction of fleet operating times. While these cost-centric approaches have successfully optimized the service provider's resources, they often result in a critical side effect known as "service unfairness." Residents of specific buildings, often those situated at the tail end of optimal routes, consistently experience late service.

This issue is particularly exacerbated in high-density residential areas, such as large apartment complexes. When a complex comprising hundreds of households is systematically assigned a late delivery slot to minimize the driver's route deviation, the cumulative "social cost" of waiting is disproportionately higher than that of a single household.

This systematic bias generates significant social cost, which we define as the waiting time multiplied by the number of affected households ($n_i \times t_i$). For instance, a 500-household apartment complex facing a two-hour delay translates to an aggregate 1,000 household-hours of social cost, a figure vastly disproportionate to the cost incurred by smaller stops. Mitigating this specific, quantifiable service unfairness is the central problem this study addresses.

Addressing this critical gap, this study proposes a fairness-aware routing framework for last-mile delivery. Specifically, we address the Multi-Depot Vehicle Routing Problem (MDVRP) targeting the distribution from local Distribution Centers (DCs) to residential complexes within Daejeon, with a focus on Yuseong-gu. Our primary objective shifts from pure cost minimization to the equitable distribution of household-weighted waiting times.

1.2 Research Objectives

This study aims to establish a fairness-oriented vehicle routing framework that explicitly integrates social equity considerations into conventional efficiency-driven logistics planning. The research pursues this aim through a set of clearly defined primary and secondary objectives:

1. **Primary Objective: Minimization of Maximum Weighted Waiting Time** The central objective is to minimize the Maximum Weighted Waiting Time across all delivery points. This approach adopts a "Min-Max" fairness principle to alleviate the burden on the most

disadvantaged service recipients. We introduce the concept of "Weighted Waiting Time," defined as the waiting time multiplied by the number of households at a delivery point ($n_i \times t_i$).

In dense urban delivery systems, minimizing only the average waiting time inadequately represents distributional disparities. A one-hour delay at a large apartment complex results in a substantially greater cumulative impact than the same delay at a single detached dwelling. Assessing delays in terms of the number of affected individuals, rather than treating all stops as equivalent units, is therefore essential for accurately evaluating the social cost of routing decisions.

2. **Secondary Objectives: Efficiency and Uniformity** Although fairness is the central priority, the proposed routing system must remain operationally feasible. To this end, the study incorporates two secondary objectives. The first concerns operational efficiency, requiring that the total routing cost, measured in travel distance and time, remains within a reasonable deviation from conventional efficiency-optimized solutions. The second concerns distributional equity, expressed through minimizing the variance of Weighted Waiting Times across delivery points. This objective promotes consistency in service quality and prevents excessive concentration of delays in specific districts.

With these objectives in place, the problem is addressed by formulating a multi-objective mathematical optimization model for the Multi-Depot Capacitated Vehicle Routing Problem (MDCVRP). Given the NP-hard nature of this problem and the complexity added by the fairness constraints, we employ an Adaptive Large Neighborhood Search (ALNS) metaheuristic. This algorithm allows for the effective exploration of the solution space to find a high-quality balance between fairness and efficiency.

1.3 Research Contributions

While conventional studies on the Vehicle Routing Problem (VRP) have predominantly concentrated on operational efficiency—specifically the minimization of total travel distance, time, and fleet costs—this study differentiates itself through the following key contributions:

1. **Paradigm Shift to Fairness-First Optimization** Unlike traditional models that treat fairness merely as a secondary constraint or a minor penalty term, this study establishes Fairness as the primary objective function. We challenge the standard "efficiency-at-all-costs" approach by demonstrating that a logistics system can be designed to prioritize social equity while maintaining acceptable operational standards.
2. **Quantifying Social Impact via Household Weighting** Existing fairness studies often treat every delivery node equally, regardless of the population density. This study introduces a novel metric: household-weighted waiting time. By explicitly integrating the number of households into the optimization model, we shift the focus from "node-based fairness" to "population-based fairness." This ensures that the waiting time of a large apartment complex is prioritized over that of a single household, thereby minimizing the aggregate social cost of delay.
3. **Pragmatic Balance via Multi-Objective Approach** We adopt a multi-objective optimization framework to navigate the trade-off between conflicting goals (efficiency vs. fairness). Instead of pursuing a theoretical maximum of fairness that might bankrupt a logistics provider, our model seeks a realistic Pareto-optimal solution. This approach aims to demonstrate that significant improvements in social equity can be achieved with only a marginal increase in operational costs, providing a practical guideline for policymakers and logistics operators.

2. MATHEMATICAL MODEL

2.1 Problem Definition and Operational Challenges

Before presenting the formal mathematical formulation, we define the specific operational environment and the inherent challenges of the target Last-Mile Delivery problem. The core complexity arises from the heterogeneity of customer demand and the strict constraints of logistics operations, which can be categorized into three key aspects:

1. **Heterogeneous Demand Density:** Each residential building possesses a distinct number of households (n_i), ranging from small villa complexes (approx. 50 households) to large-scale apartment complexes (up to 500 households).

2. **Capacity Constraints:** Due to strict vehicle capacity limits, a single vehicle cannot service all customers simultaneously. Consequently, routes must be constructed sequentially, inevitably creating a sequence of waiting times.
3. **Efficiency-Equity Conflict:** When the routing objective solely focuses on minimizing distance (the traditional approach), the algorithm tends to prioritize clustered, easy-to-reach stops near the DC to reduce travel cost. As a result, large apartment complexes located in peripheral or less accessible areas are consistently deferred to the end of the route.

Based on these challenges, we formulate the problem as a Multi-Depot Capacitated Vehicle Routing Problem with a Fairness Objective (MDCVRP-Fairness).

2.2 Mathematical Formulation

In this subsection, we present the mathematical formulation of the Multi-Depot Capacitated Vehicle Routing Problem with Fairness (MDCVRP-Fairness). We consider a complete arc-weighted directed graph $(\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the set of vertices (depots and customer buildings) and $\mathcal{A}$ is the set of arcs (i, j) with $i \neq j$ representing feasible travel between locations. Each arc $(i, j) \in \mathcal{A}$ is associated with a non-negative travel cost (or time) $c_{ij} \geq 0$.

The vertex set is partitioned into the set of depots $\mathcal{D} \subset \mathcal{N}$ and the set of customer buildings $\mathcal{N}_c = \mathcal{N} \setminus \mathcal{D}$. We are given a fleet of vehicles $\mathcal{V}$, where each vehicle $v \in \mathcal{V}_k \subseteq \mathcal{V}$ is assigned to a specific depot $k \in \mathcal{D}$. Every vehicle must start at its associated depot, visit a subset of customers, and return to the same depot. Each customer building $i \in \mathcal{N}_c$ has a demand q_i, a number of households n_i, and a service time s_i. All vehicles share a common capacity Q.

To represent the routing decisions, we use a 3-index binary variable x_{ijv}, which equals 1 if vehicle v travels directly from node i to node j, and 0 otherwise. For fairness and timing, we introduce a continuous variable $t_i \geq 0$ denoting the service start time (waiting time) at customer i, a load variable $u_i \geq 0$ representing the cumulative load of the vehicle just after servicing customer i, a variable $W_{max} \geq 0$ indicating the maximum household-weighted waiting time, and $\bar{t} \geq 0$ for the household-weighted average waiting time. For the MAD variant, an auxiliary variable $\delta_i \geq 0$ is used to linearize absolute deviations from $\bar{t}$. A sufficiently large constant M is used in the time-based big-M constraints.

Table 1 Elements of the MDCVRP-Fairness formulation

Sets	
Symbol	**Description**
$\mathcal{D}$	Set of depots (Distribution Centers).
$\mathcal{N}_c$	Set of customer buildings.
$\mathcal{N}$	Set of all vertices, $\mathcal{D} \cup \mathcal{N}_c$.
$\mathcal{V}$	Set of vehicles.
$\mathcal{V}_k$	Set of vehicles assigned to depot k $\in \mathcal{D}$.
Decision variables	
Symbol	**Description**
x_{ijv}	1 if vehicle v $\in \mathcal{V}$ travels from node i $\in \mathcal{N}$ to node j $\in \mathcal{N}$; 0 otherwise.
t_i	Service start time (waiting time) at customer i $\in \mathcal{N}_c$.
u_i	Cumulative vehicle load after serving customer i $\in \mathcal{N}_c$.
W_{max}	Maximum household-weighted waiting time over all customers.
$\bar{t}$	Household-weighted average waiting time over all customers.
δ_i	Absolute deviation ($= \lvert t_i - \bar{t} \rvert$).
Parameters	
Symbol	**Description**
c_{ij}	Travel cost (or time) from node i to node j.
q_i	Demand at customer i.
n_i	Number of households at customer i.
s_i	Service time required at customer i.
Q	Vehicle capacity.
M	Big-M constant for time consistency constraints.

2.2.1 Objective functions

The MDCVRP-Fairness seeks to balance three objectives: (i) fairness for the worst-served customer, (ii) operational efficiency, and (iii) uniformity of service quality across customers. We first define three scalar measures:

1. Fairness component (Max household-weighted waiting time)

$$Z_1 = W_{max} \tag{1}$$

$$\text{where } n_i\, t_i \leq W_{max}, \forall i \in \mathcal{N}_c \tag{2}$$

2. Efficiency component (Total routing cost)

$$Z_2 = \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} c_{ij} x_{ijv} \tag{3}$$

$$\text{where } \bar{t} \cdot \textstyle\sum_{j \in \mathcal{N}_c} n_j = \sum_{j \in \mathcal{N}_c} n_j t_j \tag{4}$$

3. Uniformity component (two alternative definitions)

   (a) Variance mode

$$Z_3^{var} = \sum_{i \in \mathcal{N}_c} n_i\, (t_i - \bar{t})^2 \tag{5}$$

   (b) MAD mode

$$Z_3^{MAD} = \sum_{i \in \mathcal{N}_c} n_i\, \delta_i \tag{6}$$

$$\text{where } \delta_i \geq t_i - \bar{t},\ \ \delta_i \geq \bar{t} - t_i, \forall i \in \mathcal{N}_c \tag{7}$$

In the empirical analysis, we examine two variants of the model that differ only in the definition of Z_3: one using the variance form Z_3^{var} and the other using the MAD form Z_3^{MAD}. To combine these three components into a single scalar objective, we adopt a normalized weighted sum:

$$\min Z = \alpha \cdot \frac{Z_1}{Z_1^*} + \beta \cdot \frac{Z_2}{Z_2^*} + \gamma \cdot \frac{Z_3}{Z_3^*} \tag{8}$$

where $\alpha, \beta, \gamma \geq 0$ with $\alpha + \beta + \gamma = 1$, and $Z_1^*,\ Z_2^*,\ Z_3^*$ denote baseline values obtained from the VROOM solution S_0, which serve as normalization constants. The term Z_3 denotes either Z_3^{var} or Z_3^{MAD}, depending on the chosen fairness mode.

2.2.2 Constraints

The routing and timing decisions are constrained as follows:

$$\sum_{v \in \mathcal{V}} \sum_{j \in \mathcal{N}} x_{ijv} = 1, \qquad \forall i \in \mathcal{N}_c \tag{9}$$

$$\sum_{i \in \mathcal{N}} x_{ijv} = \sum_{k \in \mathcal{N}} x_{jkv}, \qquad \forall j \in \mathcal{N}_c, \forall v \in \mathcal{V} \tag{10}$$

$$\sum_{j\in\mathcal{N}_c} x_{kjv} = \sum_{i\in\mathcal{N}_c} x_{ikv} \leq 1, \qquad \forall k \in \mathcal{D}, \forall v \in \mathcal{V}_k \tag{11}$$

$$u_j \geq q_j \cdot \sum_{v\in\mathcal{V}_k} x_{kjv}, \qquad \forall k \in \mathcal{D}, \forall j \in \mathcal{N}_c \tag{12}$$

$$u_j \geq u_i + q_j - Q\left(1 - \sum_{v\in\mathcal{V}} x_{ijv}\right), \qquad \forall i,j \in \mathcal{N}_c \tag{13}$$

$$q_i \leq u_i \leq Q, \qquad \forall i \in \mathcal{N}_c \tag{14}$$

$$t_j \geq c_{kj} - M\left(1 - \sum_{v\in\mathcal{V}_k} x_{kjv}\right), \qquad \forall k \in \mathcal{D}, \forall j \in \mathcal{N}_c \tag{15}$$

$$t_j \geq t_i + s_i + c_{ij} - M\left(1 - \sum_{v\in\mathcal{V}} x_{ijv}\right), \qquad \forall i,j \in \mathcal{N}_c \tag{16}$$

$$t_i \geq 0, u_i \geq 0, \delta_i \geq 0, \qquad \forall i \in \mathcal{N}_c \tag{17}$$

$$W_{max} \geq 0, \bar{t} \geq 0, \tag{18}$$

$$x_{ijv} \in \{0,1\}, \qquad \forall i,j \in \mathcal{N}, \forall v \in \mathcal{V} \tag{19}$$

2.2.3 Interpretation of constraints

In line with the explanatory style of the previous sections, the role of each constraint set can be summarized as follows. Constraint (9) ensures that every customer building is visited exactly once by exactly one vehicle. Constraint (10) enforces flow conservation at customer nodes, requiring that, for each vehicle, the number of arcs entering a node equals the number of arcs leaving it. Constraint (11) guarantees that each vehicle departs from and returns to its assigned depot at most once, thereby defining one route per vehicle. Constraints (12)-(14) implement the capacity restrictions using a Miller–Tucker–Zemlin–type load variable u_i: they accumulate the vehicle load along the route, prevent violations of the vehicle capacity Q, and eliminate customer-only subtours.

Constraints (15) and (16) impose temporal consistency. They ensure that service start times respect travel times and service durations both from depots to customers and between customers. Constraint (4), together with (1), (2), and either (5) or (6)-(7), defines the household-weighted average waiting time, the maximum household-weighted waiting time, and the dispersion of waiting times (variance or MAD). Finally, constraints (17)-(19) specify the domains of the decision variables, enforcing non-negativity for time, load, and fairness variables and binary values for routing decisions.

Taken together, this formulation defines the exact MDCVRP-Fairness problem addressed in this study. Due to the NP-hard nature of the problem and the additional complexity introduced by fairness objectives, the model is not solved as a single monolithic mixed-integer program. Instead, we adopt a hierarchical solution strategy in which feasibility and efficiency are first ensured using a state-of-the-art VRP solver, and the fairness objectives defined in the formulation are subsequently optimized via metaheuristic search.

3. METHODOLOGY

This section presents the solution methodology for the MDCVRP-Fairness formulated in Section 2. Given the NP-hard nature of the problem and the additional complexity introduced by fairness objectives, directly solving the full formulation as a monolithic mixed-integer program is computationally impractical for realistic problem sizes. We therefore adopt a hierarchical solution framework that separates feasibility and efficiency from fairness optimization.

The proposed framework proceeds in three stages. First, a cost-efficient and feasible baseline solution is generated using a state-of-the-art VRP solver. Second, a simple greedy heuristic is introduced as a diagnostic benchmark to illustrate the limitations of local fairness repair. Finally, a fairness optimization head based on Adaptive Large Neighborhood Search (ALNS) is applied to systematically improve the fairness objectives defined in Section 2. This optimization head is further enhanced by an adaptive operator selection mechanism based on Contextual Thompson Sampling (CTS), which dynamically adjusts the search strategy according to the evolving state of the solution, while explicitly controlling cost degradation.

3.1 Hierarchical Solution Framework

The proposed framework decouples feasibility and efficiency from fairness optimization. Hard operational constraints—such as vehicle capacity, route continuity, and road-network travel times—are handled in the baseline stage, while fairness-related objectives are optimized through post-processing. This separation allows fairness considerations to be incorporated without redesigning or replacing the underlying routing engine.

Each stage of the framework serves a distinct role and is described below.

3.1.1 Tier 1: Baseline Generation (VROOM Solver)

The initial stage utilizes the Vehicle Routing Open-Source Optimization Machine (VROOM) to generate an efficiency-centric baseline. By assigning customers to the nearest Distribution Center (DC) based on road network proximity via OSRM, the system solves independent VRP instances to minimize total travel costs. This tier provides a mathematically valid initial solution, a cost-efficient solution (S_0) that minimizes total travel time (Z_2) and establishes the normalization constants ($Z_1^* = Z_1(S_0)$, $Z_2^* = Z_2(S_0)$, and $Z_3^* = Z_3(S_0)$) required for subsequent multi-objective evaluation.

However, this cost-centric solution is inherently blind to fairness. All customer nodes are treated as equivalent, regardless of the number of households served. As a result, high-demand residential complexes located at the periphery of the service area are frequently scheduled late in the route, leading to extremely large household-weighted waiting times Z_1.

Despite this limitation, the baseline solution plays two essential roles in the proposed framework. First, it provides a valid and efficient initialization for subsequent optimization. Second, the baseline objective values serve as normalization constants for the composite objective function defined in Section 2.

3.1.2 Tier 2: Benchmark Heuristic (Local Search)

To illustrate the limitations of simple fairness repair strategies, we implement a greedy local search heuristic, referred to as FairnessLocalSearch, which serves as a comparative benchmark by applying an iterative improvement algorithm that targets the most problematic node, the "worst-wait stop", defined as the customer node exhibiting the highest weighted waiting time ($n_i \cdot t_i$), within the baseline routes. By utilizing relocate and swap operators on a greedy basis, this stage attempts rapid, localized fairness improvements.

While this approach can reduce the worst waiting-time case in the short term, it suffers from two fundamental limitations. First, it optimizes only the single worst node and does not account for the full multi-objective trade-off among fairness, efficiency, and uniformity. Second, due to its greedy acceptance rule, the heuristic is prone to premature convergence and cannot escape local optima.

This benchmark is introduced to motivate the need for a more robust metaheuristic that evaluates candidate solutions using the full objective structure defined in Section 2.

3.1.3 Tier 3: Fairness Optimization Head (ALNS)

The final stage of our hierarchical framework is the Adaptive Large Neighborhood Search (ALNS) metaheuristic to systematically optimize fairness objectives while preserving operational feasibility.

In its basic form, ALNS operates through iterative destroy-and-repair cycles. The destruction phase employs Worst-Wait Removal, which removes the k customers with the largest household-weighted waiting times, directly targeting the primary fairness objective Z_1. The repair phase uses Regret-2 Insertion, which prioritizes customers whose second-best insertion position is significantly worse than the best one, thereby reducing myopic placements.

Candidate solutions are evaluated using the normalized composite objective function, as defined earlier in Section 2. And to ensure that the pursuit of social equity does not compromise economic viability, a strict cost budget constraint is imposed. This constraint limits the maximum allowable increase in routing cost relative to the baseline solution and guarantees that all accepted solutions remain operationally feasible. The specific parameter settings used in the experiments are described in Section 5.

Algorithm 1 presented below provides a concise pseudo-code representation of the baseline ALNS procedure employed in this study.

Algorithm 1 Basic Fairness-Aware ALNS

Input: Initial Solution S_0, Weights α, β, γ, Cost Budget ε

Output: Best found solution S_{best}

1: **procedure** BASIC-FAIRNESS-ALNS ($S_0, \alpha, \beta, \gamma, \varepsilon$)

2: $S_{best} \leftarrow S_0$, $S_{curr} \leftarrow S_0$

3: Calculate baselines Z_1^*, Z_2^*, Z_3^* from S_0

4: **for** $iter = 1$ **to** $max\_iterations$ **do**

5: $k \leftarrow$ **Random**$(2, \min(5, |N|10))$

6: $S_{temp} \leftarrow$ **DESTROY-WORST-WAIT**(S_{curr}, k)

$S_{new} \leftarrow$ **REPAIR-REGRET-2**(S_{temp})
Calculate $Z_1(S_{new}), Z_2(S_{new}), Z_3(S_{new})$
$Z(S_{new}) \leftarrow \alpha\left(\frac{Z_1}{Z_1^*}\right) + \beta\left(\frac{Z_2}{Z_2^*}\right) + \gamma\left(\frac{Z_3}{Z_3^*}\right)$
    if $Z_2(S_{new}) \leq (1+\varepsilon) \cdot Z_2^*$ **then**
        if $Z_2(S_{new}) < Z_{best}$ **then**
            $S_{best} \leftarrow S_{new}$
        end if
        if $Z(S_{new}) \leq Z(S_{curr})$ **then**
            $S_{curr} \leftarrow S_{new}$
        end if
    end if
end for
return S_{best}
end procedure

3.2 Adaptive Operator Selection via Contextual Thompson Sampling (CTS)

While the fixed-operator (the basic) ALNS described in Section 3.1.3 is effective at directly reducing extreme unfairness, a static search strategy may be suboptimal across different stages of the optimization process. In particular, early iterations often require broader exploration to resolve structural imbalances, whereas later iterations benefit from targeted exploitation under increasingly tight cost budgets.

To address this limitation, we extend the ALNS framework using Contextual Thompson Sampling (CTS), modeling the selection of destroy–repair operators as a contextual multi-armed bandit (CMAB) problem. In this formulation, each arm corresponds to a specific combination of destruction and repair operators, and the selection is conditioned on the current state of the solution.

3.2.1 Operator Arms

The CTS framework considers a discrete set of destroy–repair operator pairs, referred to as arms. These arms are constructed by combining three destruction strategies (worst-wait removal, cluster-based removal, and random removal) with two repair strategies (regret-2 insertion and best insertion). The full set of operator pairs is summarized in Table 2.

Table 2 Definitions of CMAB Components in the Proposed Framework

Component	Symbol	Description
Arms	$a \in \mathcal{A}$	The six combined destruction-repair operator pairs (*worst_k* + *regret2, random_k* + *best_insert, cluster_k* + *regret2, cluster_k* + *best_insert, random_k* + *regret2, worst_k* + *best_insert*).
Context	$\boldsymbol{x}(S)$	An 8-dimensional vector representing the solution's current state.
Reward	r	The observed improvement in the composite objective function Z (or a penalty if the cost budget is violated).

This design allows the search to alternate between fairness-oriented moves that aggressively target high household-weighted waiting times and diversification-oriented moves that promote broader exploration of the solution space.

3.2.2 Context Representation

The operator selection decision is conditioned on an 8-dimensional context vector that captures key characteristics of the current solution state. The context includes normalized objective values, remaining cost budget slack, workload imbalance across depots, the severity of the waiting-time tail, and the progress of the search. These features jointly describe both the fairness and structural properties of the current solution.

The complete set of context features and their interpretations are summarized in Table 3. By explicitly encoding this information, the CTS mechanism is able to adapt its behavior to the evolving optimization landscape.

Table 3 Features of the Context Vector $\boldsymbol{x}(\boldsymbol{S})$

	Feature	Calculation	Interpretation
x_0	**Fairness Level**	Z_1/Z_1^*	Normalized Max Weighted Waiting Time.
x_1	**Efficiency Level**	Z_2/Z_2^*	Normalized Total Routing Cost.
x_2	**Uniformity Level**	Z_3/Z_3^*	Normalized Variance of Waiting Time.
x_3	**Cost Slack**	$\max\left(0\,,\frac{\text{budget} - \text{cost}}{Z_2^*}\right)$	Remaining margin before budget violation.
x_4	**DC Imbalance**	$\frac{\sigma(\text{dur})}{\mu(\text{dur})}$	Workload imbalance across Distribution Centers.
x_5	**Tail Ratio**	$\frac{\text{top 10\% mean}}{\text{overall mean}}$	Severity of the fairness "long tail" problem.
x_6	**Boundary Ratio**	(Fraction of boundary stops)	Potential for re-assigning customers to different DCs.
x_7	**Progress**	$iter\ /\ max\_iter$	Current stage of the search (Exploration vs. Exploitation).

3.2.3 CTS-Enhanced ALNS Procedure

For each operator arm, CTS maintains a Bayesian belief over its expected reward, defined as the improvement in the composite objective function subject to cost feasibility. At each iteration, a parameter vector is sampled from the posterior distribution of each arm, and the arm with the highest predicted reward under the current context is selected. After executing the chosen destroy–repair pair, the belief distributions are updated based on the observed outcome.

The resulting CTS-enhanced ALNS procedure is summarized in Algorithm 2. Unlike the fixed-operator baseline (Algorithm 1), where the same destroy–repair pair is applied throughout the search, CTS dynamically selects operator pairs in response to the current solution state. This adaptive mechanism naturally balances exploration and exploitation over time, enabling more effective fairness repair while respecting operational cost constraints.

Algorithm 2 CTS-Enhanced Fairness ALNS

Input: Initial Solution S_0, Weights α, β, γ, Cost Budget ε, Fairness Measure $M \in \{MAD, Var\}$

Output: Best found solution S_{best}

1: **procedure** CTS-FAIRNESS-ALNS ($S_0, \alpha, \beta, \gamma, \varepsilon$)

2: **Initialize** CTS parameters μ_a, Σ_a for all Arms $a \in \mathcal{A}$

3: $S_{best} \leftarrow S_0$, $S_{curr} \leftarrow S_0$

4: **for** $iter = 1$ **to** $max\_iterations$ **do**

Construct Context vector x_t from S_{curr}
for each Arm $a \in \mathcal{A}$ **do**
Sample $\tilde{\theta}_a \sim \mathcal{N}(\mu_a, \Sigma_\mathrm{a})$
Predict Reward $\tilde{r}_a \leftarrow x_t^T \tilde{\theta}_a$
end for
Select Arm $a^* \leftarrow argmax_a \tilde{r}_a(Action)$
$S_{temp} \leftarrow$ **DESTROY**($S_{curr}, a^*.dest$)
$S_{new} \leftarrow$ **REPAIR**($S_{temp}, a^*.rep$)
Calculate $Z_1(S_{new})$ using measure M and Weights
Calculate Reward r (Improzement in Z or Penalty)
Update $\mu_{a^*}, \Sigma_{\mathrm{a}^*}$ using Bayes Rule with (r, x_t)
if $Z_2(S_{new}) \leq (1+\varepsilon) \cdot Z_2^*$ **then** *(Check Budget)*
if $Z_2(S_{new}) < Z_{best}$ **then** $S_{best} \leftarrow S_{new}$
if Accept($Z(S_{curr}, S_{new})$ **then**
if $Z(S_{new}) \leq Z(S_{curr})$ **then** $S_{curr} \leftarrow S_{new}$
end for
return S_{best}
end procedure

3.3 Multi-Objective Trade-off Evaluation

The proposed CTS-ALNS framework optimizes the solution for a specific set of weights (α, β, γ). However, the definition of an "optimal" balance between fairness and efficiency is subjective and varies depending on logistics policy. Therefore, strictly fixing these parameters a priori would limit the practical applicability of the model.

To provide a comprehensive decision-making framework, we adopt a parametric variation strategy. Instead of seeking a single optimal point, we systematically vary the weight of the primary fairness objective (α) relative to the efficiency objective (β), while keeping the variance weight (γ) as a stabilizing factor. Through this, we examine how improvements in household-weighted waiting time translate into changes in routing cost and dispersion.

This parametric evaluation provides a transparent characterization of the trade-off structure and directly informs the comparative analysis presented in Section 5.

4. RESEARCH SCOPE AND DATA DESCRIPTION

4.1 Study Area

To ensure empirical relevance, we anchor our study in a concrete, geographically bounded delivery scenario within Daejeon, South Korea. Specifically, the district of Yuseong-gu was selected as the testbed due to its distinct urban topology. This region features a unique coexistence of high-density apartment clusters in the urban core and low-density residential areas in the periphery. Such structural heterogeneity provides an ideal environment to demonstrate the necessity and efficacy of our proposed fairness-aware routing algorithm, particularly regarding its ability to balance service levels between dense and dispersed demand zones.

4.2 Data Acquisition and Preprocessing

Spatial data was derived from the GIS Building Integrated Information dataset for Daejeon. The raw data, originally in .shp format, was converted to .gpkg to ensure stability and processing efficiency using QGIS. The preprocessing workflow involved the following steps:

1. Spatial Filtering: From the full dataset, we extracted only buildings located within the administrative boundary of Yuseong-gu.
2. Attribute Filtering: We filtered for residential building types, specifically selecting 'Apartment/Multi-unit Dwellings'(공동주택) and 'Detached Houses' (단독주택) based on the building use classification code (A9).
3. Data Cleaning: We utilized the 'Number of Above-ground Floors' attribute (A26) to determine building height. Records with a value of zero (A26=0) were removed to eliminate erroneous or missing data.
4. Geometry Conversion: Building polygons were converted into representative point geometries using the Centroid tool in QGIS. These points were assigned latitude and longitude coordinates for subsequent spatial and statistical analysis.

As a result of this process, a total of 12,406 building units were retained for analysis, preserving only the essential attributes: Address (A4), Building Use (A9), and Number of Floors (A26).

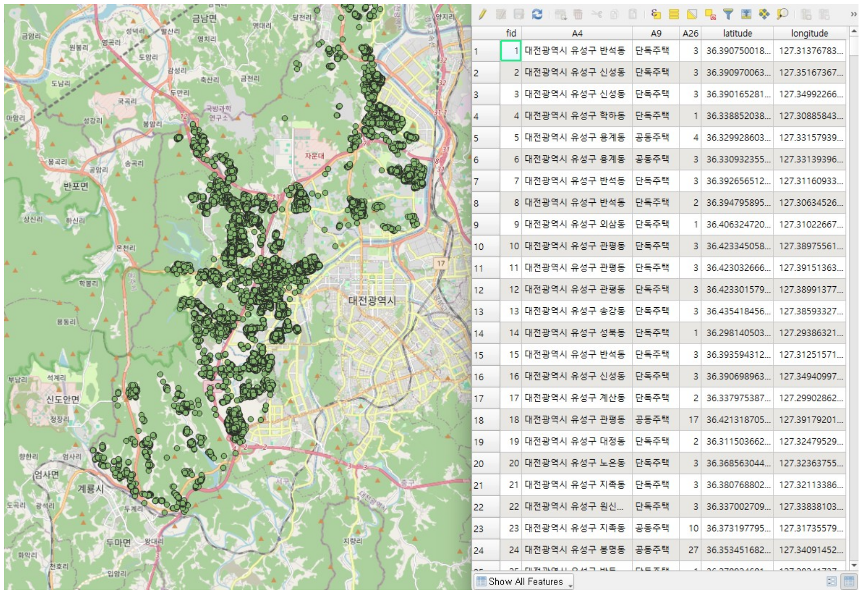


Figure 1 All the buildings in Yuseong-gu, Daejeon

4.3 Simulation Environment

The simulation models a realistic last-mile delivery operation, defined by the parameters summarized in Table 4. To accurately reflect the urban density of Yuseong-gu, we selected 50 representative delivery nodes (e.g., apartment complex entrances, villa clusters) from the processed dataset. These nodes collectively serve an aggregated demand of approximately 200 to 400 households.

Table 4 Summary of Simulation Parameters and Scenario Settings

Parameter	Value	Description
Study Area	Yuseong-gu, Daejeon	High-density urban and low-density periphery.
Distribution Centers (DCs)	3	Multi-Depot structure, mimicking major logistics hubs.
Delivery Points (N_c)	50	A mix of apartment complexes and villa districts.
Total Households (D_{total})	200~400	Aggregated demand across all locations (200–400 range).
Fleet Size	Fixed per DC (e.g., 3 vehicles)	All vehicles are assumed to be homogeneous with defined capacity constraints.

To replicate a realistic multi-depot environment, the locations of the three Distribution Centers (DCs) were fixed based on actual logistics hubs operating in the region (Logen, Hanjin, and CJ Logistics). Their specific coordinates are detailed in Table 5.

Table 5 Geographical Coordinates of Distribution Centers (DCs)

Distribution Center	Location (Lat, Lon)
Logen	(36.3800587, 127.3777765)
Hanjin	(36.3711833, 127.4050933)
CJ	(36.449416, 127.4070349)

5. RESULTS

This section reports the performance of the proposed fairness-aware routing framework under progressively more realistic experimental settings. The results are organized to highlight three key aspects: (i) the limitations of simple fairness heuristics, (ii) the effectiveness of fairness-aware ALNS under different fairness formulations, and (iii) the additional benefits of adaptive operator selection via Contextual Thompson Sampling (CTS).

Two types of outputs are presented. First, small-scale sanity-check runs (10–20 delivery points) are included to visually verify how simple heuristics and ALNS modify the waiting-time tail. These sanity-check figures are used only for interpretation and are not used for quantitative claims. Second, all tables report the main experiment results (50 delivery points), which are used for the actual comparison across algorithms.

5.1 Baseline vs. FairnessLocalSearch: Limits of Greedy Fairness Repiar

We first compare the baseline VROOM solution with a simple greedy fairness heuristic, FairnessLocalSearch, to examine whether local adjustments are sufficient to mitigate extreme unfairness.

Figure 2 presents a sanity-check comparison using a small number of delivery points. As expected, FairnessLocalSearch is able to reduce the most extreme household-weighted waiting times by relocating the worst-served nodes earlier in the route. However, this improvement is highly localized and does not account for global efficiency or dispersion effects.

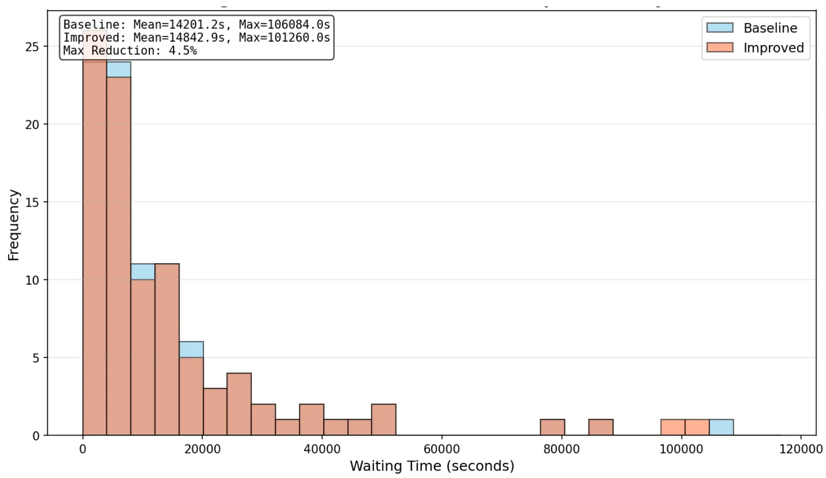


Figure 2 [Baseline vs. FairnessLocalSearch]
Weighted Waiting Time Distribution

The main experimental results are summarized in Table 6. Despite its targeted intervention, FairnessLocalSearch fails to improve the composite objective Z and, in some cases, leads to higher routing cost and increased dispersion. This outcome highlights the fundamental limitation of greedy fairness repair: alleviating a single bottleneck does not guarantee an overall improvement in fairness–efficiency trade-offs.

Table 6 Baseline vs. FairnessLocalSearch

Metric	Baseline	Local Search	Change (%)
Z	1.000	1.021	-2.06%
$\mathbf{Z_1}$	112236.000	112236.000	+0.00%
$\mathbf{Z_2}$	12051.000	12680.300	-5.22%
$\mathbf{Z_3}$	3269672141.167	3350553652.718	-2.47%

These findings motivate the need for a global search strategy that evaluates candidate solutions using the full objective structure defined in Section 2.

5.2 Baseline vs. Basic Fairness-Aware ALNS (MAD)

We next evaluate Basic Fairness-Aware ALNS introduced in Section 3.1.3, which stematically applies destroy-repair operations under a cost budget constraint. Figure 3 illustrates the effect of ALNS in a sanity-check setting. By repeatedly removing the worst-served customers and reinserting them using a regret-based strategy, ALNS reshapes the tail of the household-weighted waiting-time distribution more consistently than local heuristics.

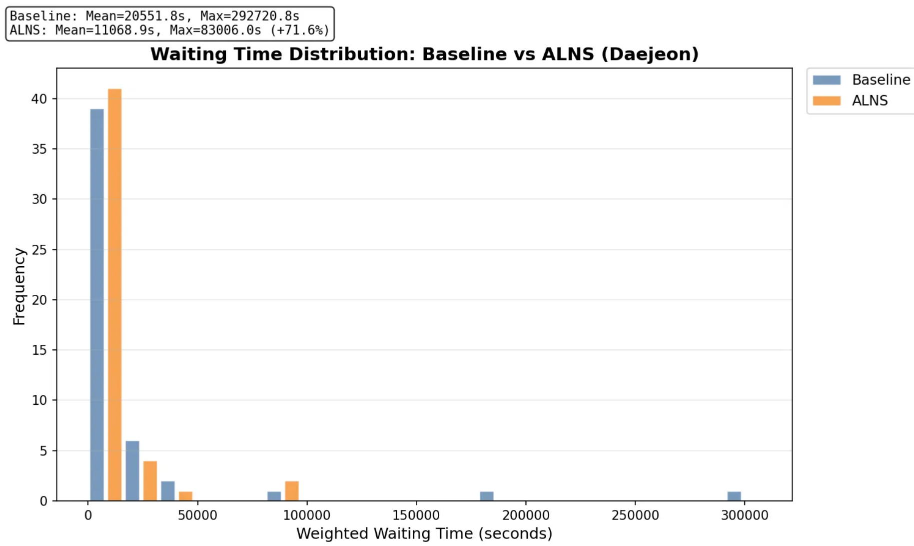


Figure 3 [Baseline vs. ALNS]
Weighted Waiting Time Distribution (Sanity-check, 10–20 locations)

The quantitative comparison on the main experimental setting is reported in Table 7. Compared to the baseline solution, ALNS (MAD) achieves a substantial reduction in the maximum household-weighted waiting time Z_1, while maintaining routing cost increases within the prescribed budget. The composite objective Z improves accordingly, reflecting a balanced gain in fairness without excessive efficiency loss.

Table 7 Baseline vs. ALNS (MAD)

Metric	Baseline	ALNS	Change (%)
Z	1.000	0.638	+36.23%
$\mathbf{Z_1}$	112236.000	25256.000	+77.50%
$\mathbf{Z_2}$	12051.000	15707.100	-30.34%
$\mathbf{Z_3}$	734696.016	492837.393	+32.92%

These results demonstrate that fairness-aware ALNS is significantly more effective than greedy local search for mitigating extreme service inequities.

5.3 Variance vs. MAD in ALNS: Choice of Uniformity Measure

To examine the impact of the uniformity componentZ_3, we compare two ALNS variants that differ only in how dispersion around the household-weighted mean waiting time is penalized: variance and mean absolute deviation (MAD).

The results are summarized in Table 8, with the corresponding waiting-time distributions shown in Figure 4. Both formulations achieve comparable reductions in the primary fairness metric Z_1. However, the variance-based formulation exhibits a stronger reduction in dispersion at the cost of a larger increase in routing cost, whereas the MAD-based formulation yields more moderate dispersion improvements while better preserving efficiency. Especially, figure 4 shows that both formulations reduce the long-tail pattern in which a small number of high-demand buildings experience disproportionately late service. However, under a practical cost budget perspective, the MAD formulation provides more stable behavior: it still mitigates extreme outcomes in Z_1 while avoiding the larger cost increase often observed in the variance setting.

Table 8 Baseline vs. Variance vs. MAD

Metric	Baseline	Variance Change (%)	MAD Change (%)
Z	1.000	+37.4%	+36.23%
$\mathbf{Z_1}$	112236.000	+76.8%	+77.50%
$\mathbf{Z_2}$	12051.000	-41.4%	-30.34%
$\mathbf{Z_3}$	(different base)	+57.3%	+32.92%

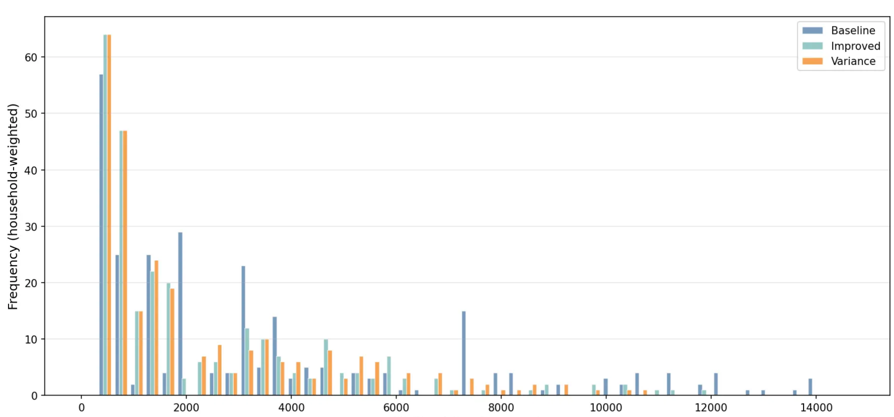


Figure 4 [Baseline vs. ALNS (Variance) vs. ALNS (MAD)]
Waiting Time Distribution (household-weighted frequency)

Given that the primary objective of this study is to improve fairness under realistic operational constraints, the MAD formulation provides a more stable and practically relevant balance. Accordingly, ALNS (MAD) is adopted as the reference variant in subsequent analyses.

5.4 Distribution-Level Effects of Fairness Optimization

Finally, we present the distribution-level comparison under the main experiment setting (50 delivery points). Figure 5 compares the household-weighted waiting-time distributions of the baseline solution, FairnessLocalSearch, and ALNS (MAD) under the main experimental setting. The baseline solution exhibits a pronounced long-tail pattern, in which a small number of high-demand buildings experience disproportionately late service.

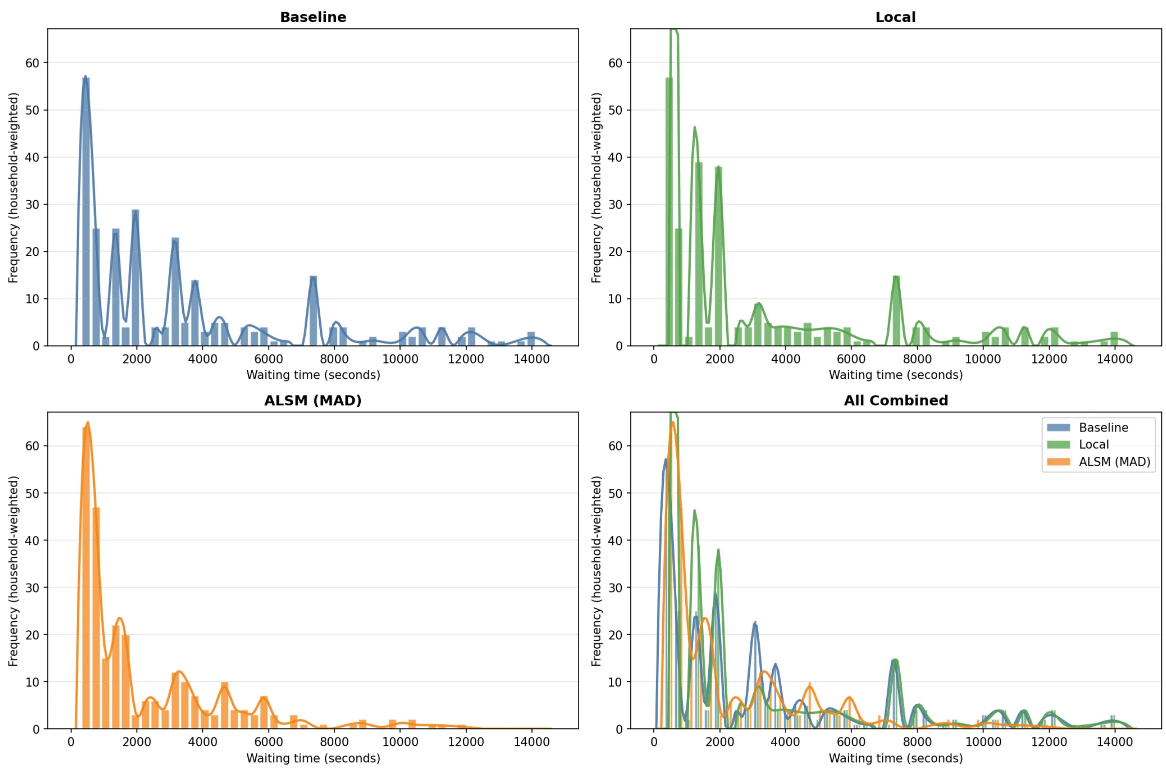
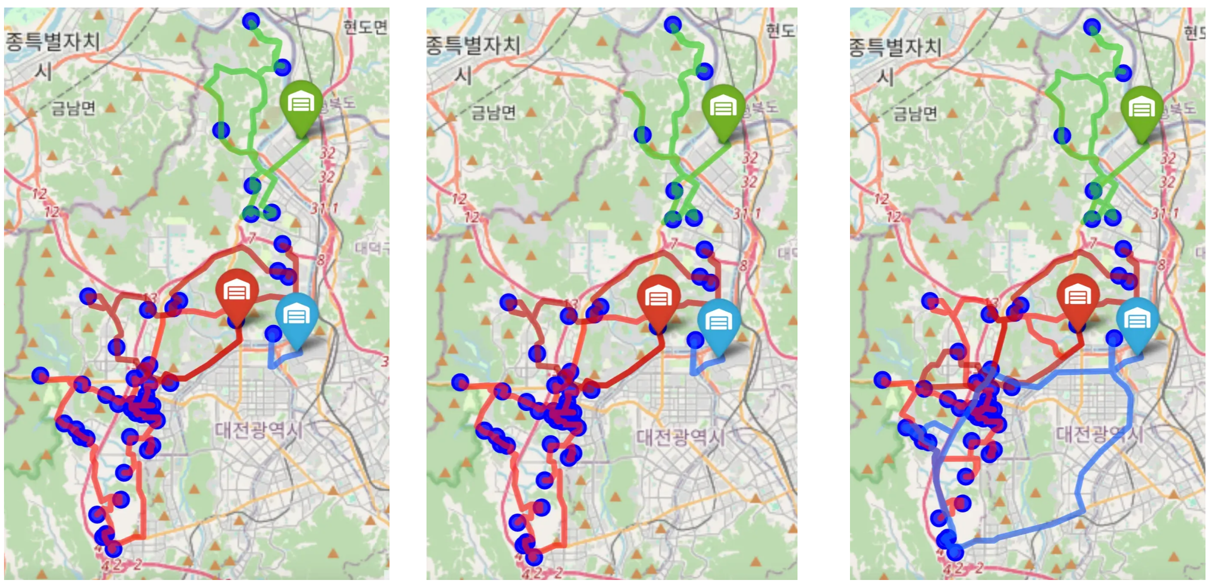


Figure 5 [Baseline vs FairnessLocalSearch vs ALNS(MAD)]
Household-weighted Waiting Time Distributions

While FairnessLocalSearch partially reduces some extreme cases, its effect is inconsistent across the distribution. In contrast, ALNS (MAD) consistently compresses the tail of the distribution, substantially reducing the severity of extreme delays without uniformly shifting waiting times across all customers.

5.4.1 Structural Mechanisms Behind Z_1 Reduction

The observed reduction in the maximum household-weighted waiting time can be attributed to two recurring structural mechanisms, illustrated in Figure 6. First, high-demand delivery points are systematically moved earlier within routes, preventing large apartment complexes from being consistently served last. Second, for customers located near depot boundaries, reassignment to alternative depots shortens effective service paths when feasible.



(a) (b) (c)

Figure 6 Illustration of route changes that reduce $\mathbf{Z_1}$: (a) Baseline, (b) FairnessLocalSearch, (c) ALNS (MAD)

These mechanisms primarily target the tail of the waiting-time distribution rather than uniformly redistributing delays, explaining the pronounced reduction in Z_1.

5.5 Adaptive Operator Selection: ALNS vs. CTS

While fixed-operator ALNS provides substantial fairness improvements, its static search strategy may not be optimal throughout the optimization process. We therefore compare ALNS with its CTS-enhanced variant introduced in Section 3.2. The quantitative comparison between the baseline solution and CTS is reported in Table 9. CTS achieves a larger improvement in the composite objective Z than fixed ALNS, primarily driven by further reductions in Z_1 and dispersion Z_3, while incurring a smaller increase in routing cost.

Table 9 Baseline (VROOM) vs CTS

Metric	Baseline	CTS	Change (%)
Z	1.000	0.615	+38.48
Z_1	112236.000	26049.600	+76.79
Z_2	12051.000	15029.500	-24.72
Z_3	734696.016	459101.090	+37.51

Figures 7 and 8 compare the waiting-time and household-weighted waiting-time distributions, respectively. CTS not only truncates the extreme tail more effectively than fixed ALNS but also avoids redistributing excessive delays into the middle of the distribution. This behavior is further illustrated in Figure 9, which directly contrasts ALNS and CTS under identical conditions.

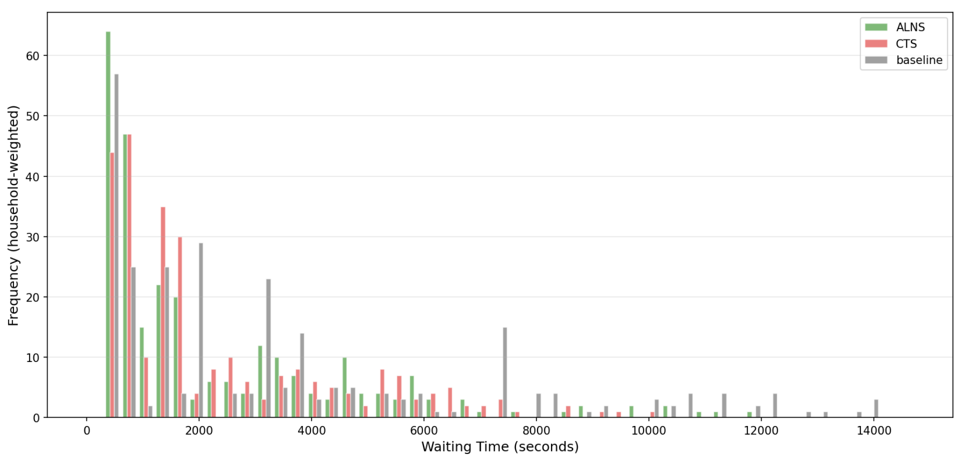


Figure 7 [Baseline vs. CTS vs. ALNS]
Waiting Time Distribution

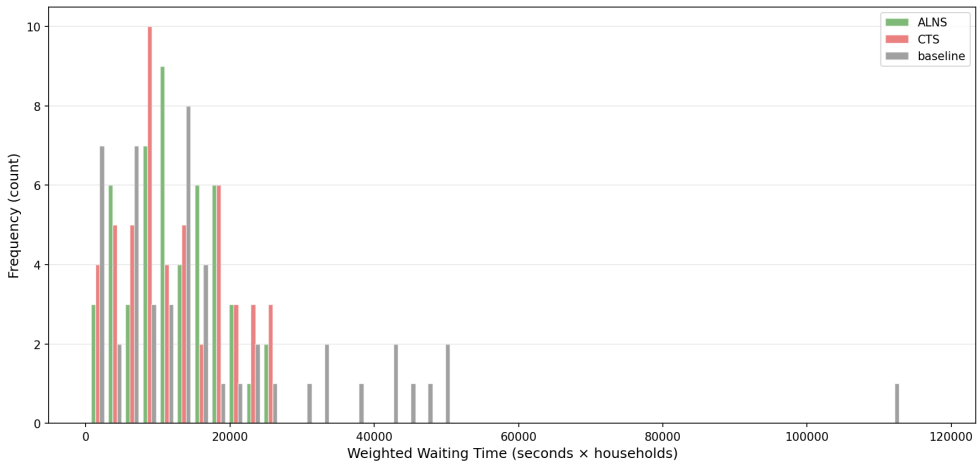


Figure 8 [Baseline vs. CTS vs. ALNS]
Household-weighted Waiting Time Distribution

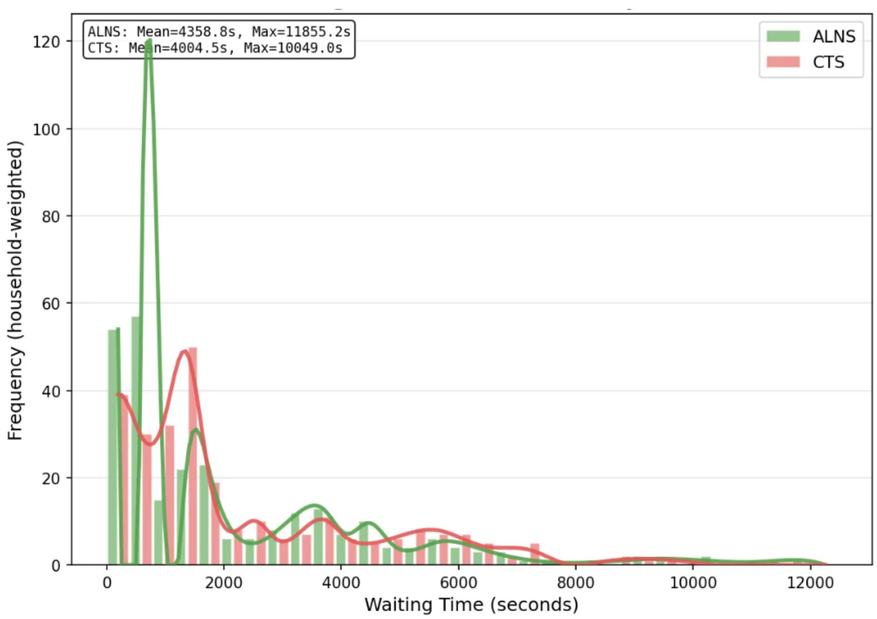
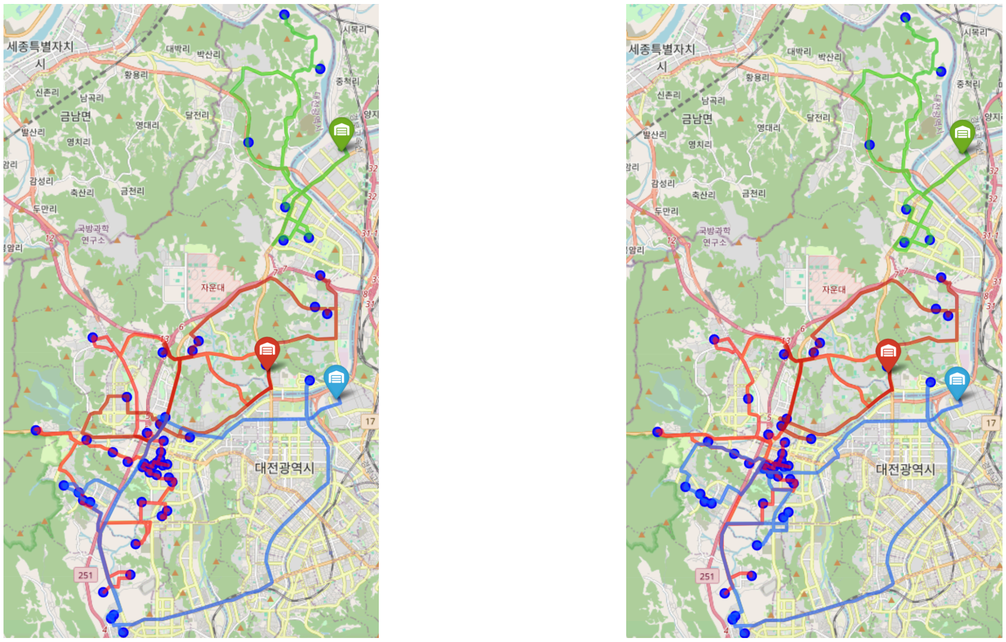


Figure 9 [ALNS vs. CTS]
Household-weighted Waiting Time Distributions

Finally, Figure 10 visualizes representative route structures generated by ALNS and CTS. CTS produces spatially tighter routes, consistent with its improved cost efficiency.



(a) (b)

Figure 10 Illustration of Routes: (a) ALNS, (b) CTS

5.6 Trade-off Analysis under Varying Fairness Preferences

To assess the policy sensitivity of the proposed framework, we evaluate CTS under three representative objective-weight configurations: fairness-focused, balanced, and uniformity-focused. The weights are set as ($\alpha = 0.60, \beta = 0.30, \gamma = 0.10$), ($\alpha = 0.35, \beta = 0.30, \gamma = 0.35$), ($\alpha = 0.10, \beta = 0.30, \gamma = 0.60$) respectively.

The results for each configuration are summarized in Tables 10–12, while a consolidated comparison is provided in Table 13 and visualized in Figure 11. When fairness is prioritized, CTS achieves the largest reduction in the maximum household-weighted waiting time, albeit with moderate cost increases. Balanced configurations retain most fairness gains while limiting efficiency loss, whereas uniformity-focused configurations primarily reduce dispersion but offer weaker protection for the worst-served customers.

Table 10 BASELINE vs CTS (0.6, 0.3, 0.1)

Metric	Baseline	CTS	Change (%)
Z	1.000	0.528	+47.23
Z_1	129022.000	25821.200	+79.99
Z_2	6570.000	7042.600	-7.19

Z_3	152887.100	131607.044	+13.92

Table 11 BASELINE vs CTS (0.35, 0.3, 0.35)

Metric	Baseline	CTS	Change (%)
Z	1.000	0.700	+30.04
Z_1	129022.000	28582.000	+77.85
Z_2	6570.000	6988.600	-6.37
Z_3	152887.100	132330.372	+13.45

Table 12 BASELINE vs CTS (0.1, 0.3, 0.6)

Metric	Baseline	CTS	Change (%)
Z	1.000	0.764	+23.62
Z_1	129022.000	78294.000	+39.32
Z_2	6570.000	7225.300	-9.97
Z_3	152887.100	95087.067	+37.81

Table 13 CTS Performance under Different Objective Weight Configurations

Configuration	α	β	γ	Z	Z_1	Z_2	Z_3
Baseline (VROOM)	–	–	–	1.0000	129,022.0	6,570.0	152,887.1
Fairness-focused	0.60	0.30	0.10	**0.5277**	**25,821.2**	7,042.6	131,607.0
Balanced	0.35	0.30	0.35	0.6996	28,582.0	**6,988.6**	132,330.4
Uniformity-focused	0.10	0.30	0.60	0.7638	78,294.0	7,225.3	**95,087.1**

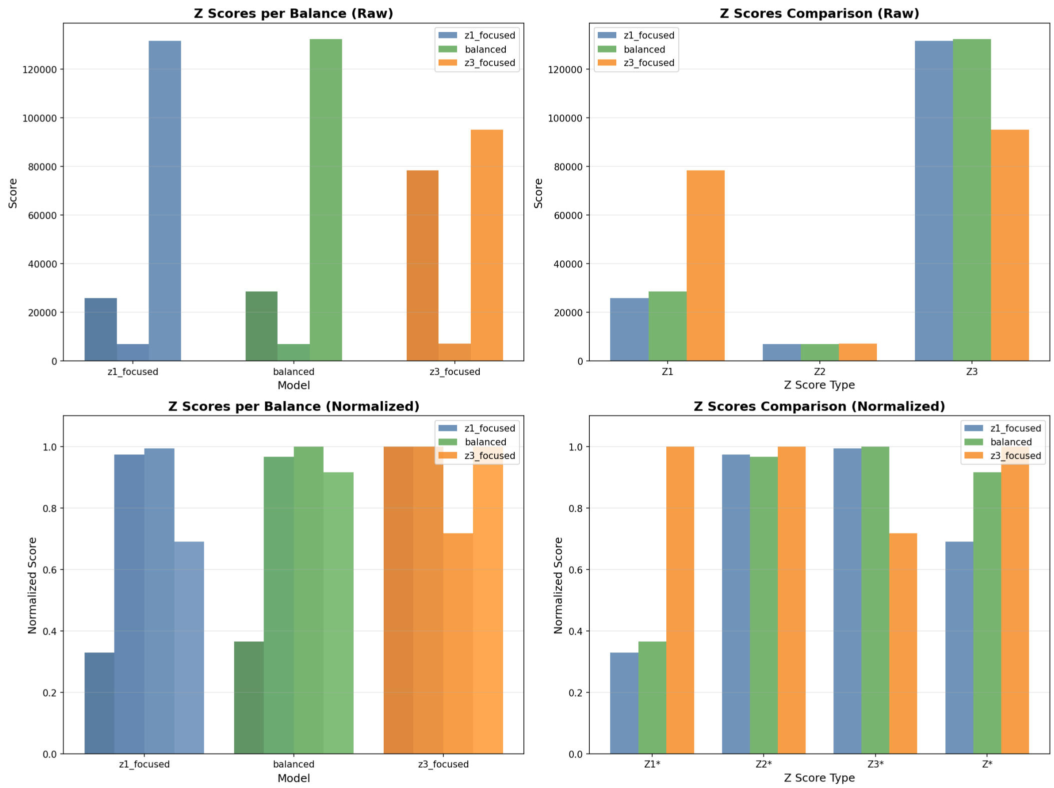


Figure 11 Objective Component Comparison under Different Fairness Preferences

Overall, these findings demonstrate that fairness-aware routing decisions should be interpreted as structured, multi-dimensional trade-offs rather than single-metric optimizations. The CTS framework facilitates this interpretation by exposing how different fairness preferences lead to qualitatively distinct routing outcomes, thereby enabling informed, policy-sensitive decision-making.

6. CONCLUSION

This study addresses a fairness issue in multi-depot vehicle routing by focusing on household-weighted waiting time, where delays at high-household locations impose larger social cost. Starting from a standard VRP baseline solution (generated by VROOM), we propose a fairness-aware post-optimization framework that explicitly targets long-tail unfairness while maintaining operational feasibility.

The core method is an ALNS-based fairness optimization layer that evaluates candidate solutions using a composite objective Z consisting of: (i) worst-case fairness Z_1 (maximum household-weighted waiting time), (ii) operational cost Z_2, and (iii) distributional uniformity Z_3 (dispersion of household-weighted waiting time). To clarify design choices and behavior, we first tested a simple greedy heuristic (FairnessLocalSearch) and then introduced ALNS with different uniformity definitions (Variance vs. MAD). Finally, we extended the framework with Contextual Thompson Sampling (CTS) to adaptively select destroy–repair operator pairs during the search, and we conducted a trade-off analysis under multiple objective-weight configurations to reflect different fairness preferences.

6.1 Key Findings and Implications

Across the results, the main takeaway is that fairness improvements are achievable without breaking operational constraints, but only when fairness is optimized directly rather than treated as a side-effect of cost minimization.

First, the experiments show that a greedy approach (FairnessLocalSearch) can reduce extreme cases in some runs, but it does not reliably improve the overall multi-objective quality because it reacts only to the current worst stop and does not manage the cost–fairness trade-off. This supports the need for a method like ALNS that evaluates solutions using the composite objective and can explore structural changes beyond local edits.

Second, the fairness-aware ALNS framework consistently mitigates long-tail unfairness in household-weighted waiting times. Qualitatively, the main distributions show that the baseline tends to produce a heavy upper tail—meaning a small set of locations receive disproportionately late service—while ALNS reshapes this tail toward a less unfair allocation. Mechanistically, the route map examples suggest two recurring structural patterns behind the improvement: pulling high-household stops earlier within routes, and occasionally reassigning boundary stops to different depots or routes when this reduces extreme household-weighted delays. This matters because it links the fairness metrics to observable routing decisions that practitioners can understand and audit.

Third, the comparison between dispersion penalties (Variance vs MAD) highlights a practical modeling point: both can improve uniformity, but they do so with different cost sensitivity. Variance tends to penalize large deviations more aggressively, which can reduce Z_3 strongly but may increase the risk of worsening operational cost Z_2. MAD provides a milder, more stable penalty and is therefore better aligned with a "fairness-first but operationally feasible" goal. This implies that the definition of uniformity is not cosmetic, but it changes how the algorithm spends cost to buy fairness.

Fourth, CTS improves the fairness-aware ALNS framework by removing the assumption that one operator strategy remains optimal throughout the search. The CTS-enhanced ALNS achieves strong fairness gains while keeping cost increases smaller than fixed-operator ALNS, and it also improves dispersion more consistently. The interpretation is that CTS adapts to the evolving structure of unfairness: it tends to use more exploratory operators early (helpful for depot/route restructuring) and more exploitative operators later (helpful for fine-tuning within tight cost slack). Practically, this means CTS is not just "another heuristic," but a mechanism to stabilize performance across search phases and reduce the risk of over-paying cost for marginal fairness gains.

Finally, the weight-sensitivity study demonstrates that fairness-aware routing is inherently a multi-dimensional policy choice. Fairness-focused weights deliver the strongest protection for worst-served households (large reductions in Z_1), balanced weights preserve most of that benefit with smaller cost increases, and uniformity-focused weights improve dispersion Z_3 but can leave the worst-case households less protected. The key implication is that a single composite score is not enough to explain outcomes; decision makers should interpret Z_1, Z_2, and Z_3 together and choose weights according to the service policy they want to enforce.

6.2 Practical Value and Limitations

Our approach is designed to be practical to integrate into an existing routing pipeline. The overall structure is modular: a standard VRP solver (VROOM) first produces a feasible baseline plan, and then a fairness optimization module refines it. This "post-processing" style design lowers the adoption barrier because it does not require replacing the full routing engine. Instead, the optimization head can be plugged into the current system and turned on when fairness control is needed (e.g., when complaints concentrate in specific regions or when service-level fairness becomes a KPI).

Another advantage is interpretability. The objective is decomposed into components that are easy to communicate: Z_1 captures the worst household-weighted waiting-time burden (the most disadvantaged location), Z_2 reflects operational cost, and Z_3 represents the dispersion of waiting times. In practice, this separation helps operators and decision makers understand why a solution is preferred and what trade-off is being made. In addition, the main analysis can be supported with simple visual evidence—distribution plots and route maps—which makes it easier to explain how the algorithm reduces long-tail unfairness and what structural changes (reordering or depot reassignment) cause the improvement.

However, this study has several limitations that should be acknowledged. First, the experiments are conducted under a static setting where all demands are known in advance. Real last-mile operations are typically dynamic, with new requests arriving over time and unexpected events (traffic, cancellations, vehicle issues) occurring during execution. As a result, the reported improvements may not directly transfer to a real-time environment without additional mechanisms for frequent re-optimization and stability control.

Second, fairness is modeled primarily through household-weighted waiting time. While household counts provide a clear and meaningful proxy for service impact, real-world fairness may involve additional dimensions such as vulnerable populations, accessibility constraints, priority customers, or contractual service levels. The current formulation does not capture these multi-dimensional fairness considerations.

Third, the evaluation scope is limited to a specific scenario and scale. Although the main experiment uses 50 delivery points and provides distribution-level evidence, broader validation across different cities, depot layouts, fleet sizes, and demand patterns is necessary to test generalizability. Finally, because the method is heuristic, the resulting solutions are not guaranteed to be globally optimal; performance can also depend on hyperparameters (e.g., iteration budget, destroy size range, penalty weights).

6.3 Future Implications

Several extensions can strengthen the practical relevance of this work. A primary direction is dynamic routing. A rolling-horizon framework could periodically re-optimize routes as new demands arrive, while adding constraints that limit how much the plan changes between updates to avoid driver confusion. In this setting, fairness improvement must be balanced not only with cost but also with route stability.

A second direction is to incorporate richer operational realism. This includes heterogeneous fleets (different vehicle capacities or service speeds), time-dependent travel times that reflect congestion, and additional constraints such as time windows, pickup-and-delivery, and priority classes. Since the proposed framework is modular, these constraints can be integrated either in the baseline solver stage or in the ALNS evaluation and feasibility checks.

Third, the fairness model can be expanded beyond household counts. Future studies can explore multi-factor fairness weights (e.g., demographic vulnerability or service criticality), and evaluate whether the improved fairness aligns with stakeholder expectations. Finally, large-scale empirical validation is needed. Testing on larger instances (hundreds to thousands of stops) and, ideally, real operational datasets would allow evaluation of scalability, runtime feasibility, and the impact on downstream outcomes such as complaint rates or perceived service quality.

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