
Econometrics for Causal Inference

URP Part 1: Causal Inference and Difference-in-Difference

Sungkyunkwan University
- Machine Learning and Econometrics -

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Econometrics and Causal Inference

- ▶ Mastering Econometrics
- ▶ Mostly Harmless Econometrics
- ▶ Empirical Strategy for Microeconomics

Econometrics

- ▶ Examine empirical relationship between variables

$$y = X\beta + \epsilon$$

1. $E[Y|X] = X\beta$: explain the average behavior of Y given X
 - Association between variables
 - Causal inference
2. $Y_{t+1} = E[Y|X_{t+1}]$: predict Y based on X

What is Causal Inference

- ▶ Forecasting and Causal Relationship
: Causal Inference vs Machine Learning
- ▶ **Pattern** between variables \longrightarrow predicting Y using X

$$Y = f(X)$$

What is Causal Inference

- ▶ Forecasting and Causal Relationship
: Causal Inference vs Machine Learning
- ▶ **Causal Association**
 - Association: variables “move together” (Correlation)
 - Ceteris Paribus: other conditions remaining the same
 - specific change (T) \longrightarrow consequence (Y)
 - $E[Y|X, T = 1] - E[Y|X, T = 0]$
- ▶ Causality = Association + Ceteris Paribus + Direction

Causal Inference and Regression

- ▶ Regression in the perspective of causal inference

$$y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \dots + \beta_k x_k + \epsilon$$

$$\beta_1 = \frac{\partial y}{\partial x_1}$$

- ▶ Estimation procedure
 1. x_1 to $x_2, \dots, x_k \rightarrow \text{Residual}_{x_1}$
 2. y to $x_2, \dots, x_k \rightarrow \text{Residual}_y$
 3. Residual_y to Residual_{x_1}
- ▶ If we perfectly make a regression model, then each coefficient can be interpreted as a causal relationship

Identification of Causal Relationship

- ▶ Causal association: specific change \longrightarrow consequence
 - Specific change: **Treatment**
 - Consequence: **Effect**
- ▶ Example: the causal effect of graduate school

Identification of Causal Relationship

- ▶ Causal association: specific change \longrightarrow consequence
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- ▶ Example: the causal effect of graduate school
 - Treatment: graduate school (G)
 - Effect: income, etc.... (y)

Identification of Causal Relationship

- ▶ How can we make "Ceteris Paribus"

Counterfactual!

- ▶ Example: the causal effect of graduate school
 - Graduate school person A: Y_{1A}
 - No graduate school person A: Y_{0A}
- ▶ Treatment effect = $Y_{1A} - Y_{0A}$
- ▶ Average treatment effect = $E[Y_{1i} - Y_{0i}]$

Identification of Causal Relationship

- ▶ However.... actually there's not "a counterfactual"
- ▶ Real: $E[Y_{1i} - Y_{0j}] = E[Y_{1i} - Y_{0j}] + E[Y_{0i} - Y_{0j}]$
- ▶ We need to make strategies for identifying 'causal relationship'
- ▶ **Difference-in-Difference** is the one of the most important strategy for identifying 'causal relationship'

Difference-in-Difference

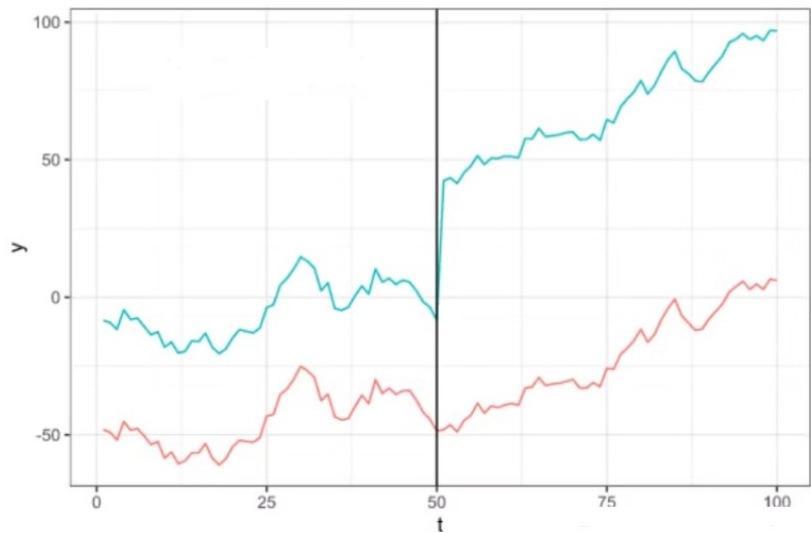


Figure: Difference-in-Difference

What is DiD?

- ▶ Specific treatment happens at a point
- ▶ How can we analyze the effect of treatment?
- ▶ Counterfactual: untreated group similar to treated group
- ▶ Compare groups before and after treatment

Identification Strategy

$$Y_{it} = \beta_0 + \beta_1 \text{Treat}_i + \beta_2 \text{Post}_t + \tau \text{Treat}_i \times \text{Post}_t + \epsilon_{it}$$

	Post=0	Post=1
Treat=0	β_0	$\beta_0 + \beta_2$
Treat=1	$\beta_0 + \beta_1$	$\beta_0 + \beta_1 + \beta_2 + \tau$

$$\begin{aligned} \text{Effect} &= (E[Y | \text{Treat} = 1, \text{Post} = 1] - E[Y | \text{Treat} = 1, \text{Post} = 0]) \\ &\quad - (E[Y | \text{Treat} = 0, \text{Post} = 1] - E[Y | \text{Treat} = 0, \text{Post} = 0]) = \tau \end{aligned}$$

Difference-in-Difference

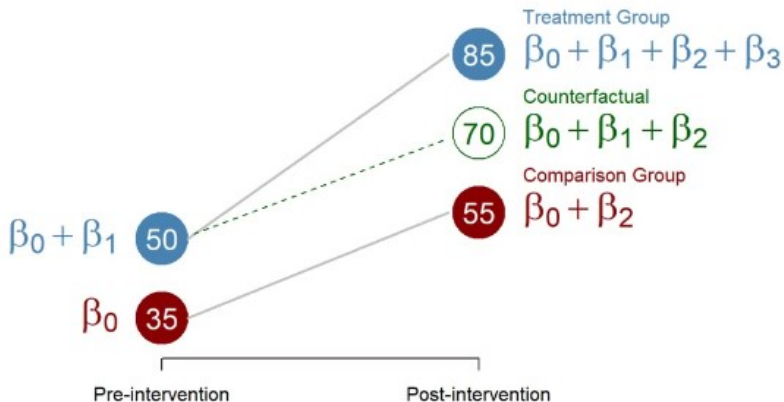


Figure: Difference-in-Difference

Mississippi Experiment: The Great Depression

- ▶ US Federal Reserve System is organized into 12 districts.
 - ▶ St. Louis Fed - 6th Districts
 - ▶ Atlanta Fed - 8th Districts
 - cut Mississippi state into halves
- ▶ The effect of Fed monetary policy
 - ▶ Treatment: Caldwell fails / banking crisis begins
 - ▶ St. Louis Fed(Treatment): active monetary policy
 - ▶ Atlanta Fed(Control): inactive monetary policy

Richardson and Troost (2009)

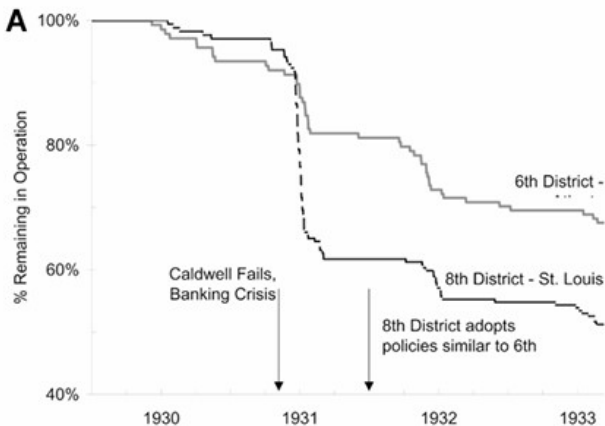


Figure: Richardson and Troost (2009)

Card and Kruger (1994)

- ▶ The effect of minimum wage using DiD
- ▶ What is the treatment in this research?
- ▶ Find the treatment group and control group
- ▶ Find main τ in this paper and replicate it in two ways
- ▶ What can be the potential problems in this research?
- ▶ Write one page report including above things

Next Time: Problems and Solution

$$\begin{aligned}E[Y_{1i} - Y_{0j}] &= E[Y_{1i} - Y_{0i}] + E[Y_{0i} - Y_{0j}] \\&= ATE + SelectionBias\end{aligned}$$

$$\begin{aligned}\tau^{\hat{D}D} &= (Y_{i,t=1}(1) - Y_{i,t=0}(0)) - (Y_{j,t=1}(0) - Y_{j,t=0}(0)) \\&= (Y_{i,t=1}(1) - Y_{i,t=0}(0)) - (Y_{j,t=1}(0) - Y_{j,t=0}(0)) \\&\quad + (Y_{i,t=1}(0) - (Y_{i,t=1}(0))) \\&= (Y_{i,t=1}(1) - Y_{i,t=1}(0)) + [(Y_{i,t=1}(0) - Y_{i,t=0}(0)) - (Y_{j,t=1}(0) - Y_{j,t=0}(0))] \\&\approx E[Y_{i,t=1}(1) - Y_{i,t=1}(0) | T_i = 1] + E[Y_{i,t=1}(1) - Y_{i,t=1}(0) | T_i = 1] \\&\quad - E[Y_{j,t=1}(1) - Y_{j,t=1}(0) | T_i = 0] \\&= \tau + CounterfactualTrend - UntreatedTrend\end{aligned}$$