Problem Set

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Part I: Functions, Domain, Range, and Transformations (30 Problems)

PC1: [S]Find the domain of the function $f(x) = \frac{\sqrt{x-3}}{x-5}$.

PC2: [S]Given $f(x) = 3x^2 - 1$ and $g(x) = 2x + 5$. Find the value of $(f \circ g)(-1)$.

PC3: [S]State the domain and range of the function $h(x) = -2|x + 1| + 3$.

PC4: [S]Describe the sequence of transformations required to obtain the graph of $g(x) = -\sqrt{x-4}$ from the parent function $f(x) = \sqrt{x}$.

PC5: [S]Determine algebraically whether $f(x) = x^3 - x$ is an even function, an odd function, or neither.

PC6: [S]Find the inverse function $f^{-1}(x)$ for $f(x) = 4x - 7$.

PC7: [S]Determine the interval(s) where the function $m(x) = x^2 - 6x + 5$ is increasing.

PC8: [S]Given the graph of $y = f(x)$, find the function's average rate of change on the interval $[-4, 0]$, where $f(-4) = 10$ and $f(0) = 2$.

PC9: [S]Sketch the graph of the piecewise function: $h(x) = \begin{cases} x+2 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$.

PC10: [S]If $f(x) = \frac{1}{x+2}$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$.

PC11: [S]Find the value of k such that the function $g(x) = x^2 - k$ is increasing on the interval $[0, \infty)$.

PC12: [S]Use the definition of a function to explain why the equation $x^2 + y^2 = 9$ (a circle) does not represent y as a function of x.

PC13: [I]Find the domain of the composite function $(g \circ f)(x)$ given $f(x) = \frac{1}{x+2}$ and $g(x) = \sqrt{x+1}$.

PC14: [I]Find the inverse function $g^{-1}(x)$ for the one-to-one function $g(x) = \frac{x-1}{x+3}$. State the domain of $g(x)$ and the range of $g^{-1}(x)$.

PC15: [I]Determine the intervals of increasing, decreasing, and constant behavior for the function $m(x) = |x^2 - 4|$.

PC16: [I]Graph the transformation $y = 3f(2x) - 1$ starting from a generic graph of $y = f(x)$ that passes through the point $(4, 6)$. State the new coordinates of this point.

PC17: [I]A rectangular field is to be enclosed with 400 feet of fencing. Express the area A of the field as a function of the length L of one side. Determine the domain of this function in context.

PC18: [I]The height of a falling object is given by $h(t) = -16t^2 + 64t + 80$. Find the maximum height reached by the object using properties of the quadratic function.

PC19: [I]Given $f(x) = \frac{x-1}{2x+3}$. Find the inverse function $f^{-1}(x)$ and verify the composition $(f \circ f^{-1})(x) = x$.

PC20: [I]A function $f(x)$ is defined on $[0, 5]$ with $f(0) = 3$ and $f(5) = 1$. If f is one-to-one, what is the domain and range of the function $y = f(x - 2) + 1$?

PC21: [I]Solve the inequality: $|2x - 3| \leq x + 6$.

PC22: [I]Show that the composition of two odd functions is an odd function, and the composition of two even functions is an even function.

PC23: [C]Consider the function $f(x) = \sqrt{\frac{x^2-4}{x+1}}$. Determine the domain of $f(x)$ using a sign chart analysis.

PC24: [C]Find the domain and range of the function $f(x) = \frac{e^x - e^{-x}}{2}$ (the hyperbolic sine function, $\sinh x$). Justify the range.

PC25: [C]The graph of $f(x) = \frac{x^2}{x^2+1}$ passes through the point $(k, 1/2)$. Find all possible values of k and state the range of $f(x)$ formally.

PC26: [C]A function $g(x)$ is known to have $g(x) = g(x+2)$ and $g(x) = g(-x)$. If $g(1) = 5$ and $g(1.5) = 2$, find $g(4.5)$. Explain the properties used.

PC27: [C]A function $f(x)$ is strictly increasing on $(-\infty, \infty)$. Prove that $f(x)$ must be one-to-one and thus have an inverse function.

PC28: [C]Consider the function $f(x) = \frac{x}{x-1}$. Find the n-th composition $f^{(n)}(x) = (f \circ f \circ \cdots \circ f)(x)$ and use induction (conceptually) to verify your pattern for $n = 3$.

PC29: [C]Show that the function $f(x) = x^3 + x$ is one-to-one without graphing, and then find the value of the inverse function at $x = 10$, i.e., $f^{-1}(10)$.

PC30: [C]Given $f(x) = \sqrt{x+1}$. Find a non-identity function $g(x)$ such that $(f \circ g)(x) = \sqrt{|x^2 - 4|}$. Specify the transformation $g(x)$ performs.

Part II: Polynomial and Rational Functions (35 Problems)

PC31: [S]For the polynomial $P(x) = -2x^3 + 6x - 4$, determine the end behavior using the Leading Coefficient Test.

PC32: [S]Find all real zeros of the polynomial $P(x) = x^4 - 2x^2 + 1$ and state their multiplicity. Does the graph cross or touch the x-axis at each zero?

PC33: [S]Find all horizontal and vertical asymptotes for $R(x) = \frac{5x^3-3x+1}{2x^3+x^2-6}$.

PC34: [S]Find the vertex and the y-intercept of the quadratic function $f(x) = 2x^2 - 8x + 6$.

PC35: [S]List all possible rational zeros for the polynomial $P(x) = 2x^3 - 5x^2 + x + 6$ using the Rational Zeros Theorem.

PC36: [S]Divide $P(x) = x^3 - 4x^2 + 2x + 5$ by $(x - 2)$ using synthetic division. State the quotient and the remainder.

PC37: [S]Find a polynomial of degree 3 with zeros at $x = 0$, $x = 1$, and $x = -3$.

PC38: [S]For the rational function $R(x) = \frac{x+2}{x^2-4}$, simplify $R(x)$ and identify the coordinates of the hole in the graph.

PC39: [S]If a polynomial has real coefficients and $3i$ is a zero, what is another zero that must exist? (Conjugate Pairs Theorem)

PC40: [S]Write the standard form of a parabola with vertex at $(-1, 4)$ and a vertical axis of symmetry.

PC41: [S]State the domain of the rational function $R(x) = \frac{x^2-9}{x^2+9}$.

PC42: [S]Solve the polynomial inequality: $x(x-1)(x+2) > 0$.

PC43: [S]Determine the maximum number of real zeros and the maximum number of turning points for a polynomial of degree 5.

PC44: [S]Find the oblique (slant) asymptote for the rational function $R(x) = \frac{x^2-4x+1}{x-1}$.

PC45: [S]Sketch the general shape of a polynomial function of odd degree with a positive leading coefficient.

PC46: [I]Construct a polynomial $P(x)$ of degree 4 with leading coefficient $a_4 = 1$, having zeros at $x = 2$ (multiplicity 2) and $x = 3i$. Write the polynomial in standard form.

PC47: [I]Find the equation of the slant asymptote for $R(x) = \frac{2x^3-x^2+3}{x^2+1}$.

PC48: [I]Use Descartes' Rule of Signs to determine the possible number of positive and negative real zeros for $P(x) = x^4 - 6x^3 + x^2 - 1$.

PC49: [I]Find the coordinates of the hole and the equations of all asymptotes for $R(x) = \frac{x^2-2x-8}{x^2-16}$.

PC50: [I]Given that $x = 2$ is a zero of $P(x) = x^3 - 7x + 6$. Use synthetic division and factoring to find the remaining zeros.

PC51: [I]Determine the standard form of the quadratic function that has a vertex at $(1, -5)$ and passes through the point $(3, 3)$.

PC52: [I]Solve the rational inequality: $\frac{x-1}{x+3} \leq 0$.

PC53: [I]A rational function $R(x)$ has a vertical asymptote at $x = 1$, a horizontal asymptote at $y = 2$, and a zero at $x = 3$. Construct a possible equation for $R(x)$.

PC54: [I]Find the oblique asymptote and sketch the end behavior of the function $f(x) = \frac{x^3-8}{x^2}$.

PC55: [I]Prove that if a polynomial $P(x)$ has integer coefficients, any rational zero p/q must satisfy that p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n.

PC56: [I]Show that the graph of $f(x) = x^4 + 3x^2 + 1$ has no real zeros.

PC57: [I]Find all complex zeros (real and non-real) of $P(x) = x^4 - 16$.

PC58: [C]Construct the equation of a rational function $R(x)$ that satisfies all the following conditions: (i) Vertical asymptotes at $x = \pm 2$, (ii) Slant asymptote $y = 3x$, (iii) x-intercept at $x = 1$.

PC59: [C]Determine the equation of the quadratic function $f(x) = ax^2 + bx + c$ whose graph passes through the three points $(1, -2)$, $(2, 3)$, and $(-1, 0)$.

PC60: [C]A polynomial $P(x)$ has degree 5, a leading coefficient of -1, and the following zeros: 2 (multiplicity 3), and $4i$. Write $P(x)$ in factored form with real coefficients.

PC61: [C]Use the Intermediate Value Theorem to show that the polynomial $P(x) = x^3 - 4x - 2$ has a real zero between $x = 2$ and $x = 3$.

PC62: [C]Prove that a rational function $R(x)$ cannot cross its vertical asymptote, but it can cross its horizontal asymptote. Give an example of a function that crosses its HA.

PC63: [C]Solve the inequality: $\frac{x^2+3x-4}{x^2-1} \geq 0$.

PC64: [C]Given the definition of a polynomial of degree n, prove that a non-zero polynomial of degree n can have at most n roots.

PC65: [C]Find all values of k such that the rational function $R(x) = \frac{(k-1)x^2+2x-1}{2x^2+5x+3}$ has a horizontal asymptote at $y = 3$.

Part III: Exponential and Logarithmic Functions (35 Problems)

PC66: [S]Describe the transformations of $f(x) = e^x$ to obtain $h(x) = 5e^{-x} - 4$. State the domain, range, and the equation of the Horizontal Asymptote (HA).

PC67: [S]Find the domain and the Vertical Asymptote (VA) of the function $k(x) = \log(2x + 6) + 1$.

PC68: [S]Condense the following expression into a single logarithm: $3 \log x + \frac{1}{2} \log y - \log z$.

PC69: [S]Expand the logarithmic expression: $\log_3 \left(\frac{9x^4}{y\sqrt{z}} \right)$.

PC70: [S]Solve for x: $3^{2x-1} = 27$.

PC71: [S]Solve for x: $\log_4(x + 5) = 2$.

PC72: [S]Given a principal of \$1000 compounded annually at a rate of 5%. Write the formula for the balance $A(t)$ after t years.

PC73: [S]Convert the equation $y = e^x$ into its equivalent logarithmic form.

PC74: [S]State the domain, range, and VA for the parent logarithmic function $f(x) = \ln(x)$.

PC75: [S]If $e^x = 5$, what is x in terms of natural logarithm?

PC76: [S]Use a calculator to approximate $e^{2.5}$ and $\ln(10)$ to four decimal places.

PC77: [S]Determine if $f(x) = (0.7)^x$ represents exponential growth or decay.

PC78: [S]Simplify the expression: $e^{2 \ln x}$.

PC79: [S]Find the value of $\log_5(1)$ and $\log_2(2^7)$.

PC80: [S]A house appreciates by 3% annually. If its initial value is \$200,000, write an equation for its value $V(t)$ after t years.

PC81: [I]Solve the exponential equation $4^{2x} - 4^x - 12 = 0$. (Hint: Use substitution $u = 4^x$).

PC82: [I]Solve for x: $\log_2(x) + \log_2(x - 2) = 3$. Check for extraneous solutions.

PC83: [I]A sum of \$5000 is invested at 6% annual interest rate, compounded continuously. How many years will it take for the investment to triple? (Answer in terms of ln).

PC84: [I]Use the change-of-base formula to evaluate $\log_7(20)$ and round to four decimal places.

PC85: [I]Determine the time t required for an object to cool from 100°C to 70°C in a room kept at 20°C, given Newton's Law of Cooling $T(t) = T_s + (T_0 - T_s)e^{-kt}$ and $k = 0.05$.

PC86: [I]Solve the equation: $\ln(x + 1) - \ln(x) = 2$.

PC87: [I]Graph the function $y = \log_2(x - 1) + 3$. State the domain, range, and VA.

PC88: [I]Find the inverse function $f^{-1}(x)$ for $f(x) = 2^x + 5$.

PC89: [I]Condense the expression $2 \ln(x) - \ln(x^2 - 1) + \ln(x + 1)$ into a single logarithm.

PC90: [I]A bacterial culture grows from 100 cells to 500 cells in 5 hours. Assuming exponential growth $N(t) = N_0 e^{kt}$, find the growth constant k.

PC91: [I]Solve the inequality: $\left(\frac{1}{2}\right)^x < 4$.

PC92: [I]Explain why $\log_b(M) = \frac{\ln M}{\ln b}$ is mathematically equivalent to the definition of a logarithm.

PC93: [C]Solve the equation $x^2e^x - 5xe^x + 6e^x = 0$.

PC94: [C]The half-life of Carbon-14 is 5730 years. Derive the value of the decay constant k for the formula $A(t) = A_0e^{-kt}$.

PC95: [C]Consider the function $f(x) = \frac{1}{1+e^{-x}}$ (the logistic function). Find the inverse function $f^{-1}(x)$. State the domain and range of $f^{-1}(x)$.

PC96: [C]Prove the product rule of logarithms: $\log_b(MN) = \log_b M + \log_b N$, using the definition of a logarithm $x = b^{\log_b x}$.

PC97: [C]Solve the system of equations: $2^x \cdot 4^y = 1$ and $\log_2(x + y) = 0$.

PC98: [C]Find all values of x for which the function $f(x) = \log_x(4 - x^2)$ is defined. (Requires analyzing base and argument restrictions).

PC99: [C]A sound level L (in decibels) is given by $L = 10 \log\left(\frac{I}{I_0}\right)$. If the intensity I of a siren is 1000 times greater than the intensity of a normal conversation I_c, by how many decibels is the siren louder?

PC100: [C]Determine the domain of the function $f(x) = \ln(x - \sqrt{x + 2})$ using algebraic techniques.

Part IV: Angles, Ratios, Graphing, and Inverse Functions (30 Problems)

PC101: [S]Convert 210° to radians and $\frac{5\pi}{6}$ radians to degrees.

PC102: [S]An angle θ in standard position has its terminal side passing through the point $(-5, 12)$. Find the value of $\sin\theta$ and $\tan\theta$.

PC103: [S]Find the exact value of $\sec(\frac{\pi}{3})$.

PC104: [S]Find the amplitude, period, and phase shift for the function $y = 4\sin(2x - \pi)$.

PC105: [S]State the domain and range of the function $y = \arcsin(x)$.

PC106: [S]Find the arc length s subtended by a central angle of 45° in a circle with radius $r = 8$ cm. (Use the formula $s = r\theta$ with θ in radians).

PC107: [S]State the equations of the vertical asymptotes for $y = \tan(x)$ on the interval $[-\pi, 2\pi]$.

PC108: [S]Determine the quadrant in which the angle θ lies if $\sec\theta < 0$ and $\cot\theta > 0$.

PC109: [S]Evaluate the hyperbolic cosine function $\cosh(x) = \frac{e^x+e^{-x}}{2}$ at $x = 0$.

PC110: [S]Write $\tan\theta$ in terms of $\sin\theta$ and $\cos\theta$.

PC111: [S]If $\cos\theta = 0.5$, find the values of θ in the interval $[0, 2\pi)$.

PC112: [S]Find the exact value of $\arccos\left(-\frac{\sqrt{3}}{2}\right)$.

PC113: [I]The height of a tidal wave is modeled by $h(t) = 8\cos(\frac{\pi}{6}t) + 10$ meters, where t is the time in hours. Find the maximum and minimum height of the tide and the time interval between consecutive high tides.

PC114: [I]Find the exact value of the composite function $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$.

PC115: [I]Graph one full period of the function $y = \sec(x)$ and determine its domain and range.

PC116: [I]Find the equation of the sine function that has an amplitude of 3, a period of π, and a phase shift of $\frac{\pi}{4}$ to the right.

PC117: [I]A wheel with a 10-inch radius is rotating at 3 revolutions per minute. Find the linear speed of a point on its rim in inches per minute.

PC118: [I]Given that $\tan\theta = -4/3$ and θ is in Quadrant IV. Find the exact value of $\sin\theta$ and $\cos\theta$.

PC119: [I]Find the exact value of $\cosh(\ln 3)$.

PC120: [I]Find the period, phase shift, and the equations of two consecutive vertical asymptotes for the function $y = \cot(3x - \frac{\pi}{2})$.

PC121: [I]Solve for x: $3\cos x - 1 = 0$ for $0 \le x < 2\pi$. (Provide calculator approximations to 3 decimal places).

PC122: [I]Explain why $\arcsin(\sin(3\pi/4)) \neq 3\pi/4$. Find the correct value.

PC123: [C]Evaluate the expression $\tan\left(\arcsin\left(\frac{x}{\sqrt{x^2+1}}\right)\right)$ by drawing a reference triangle.

PC124: [C]Find the domain and range of the function $f(x) = 2\arccos(3x - 1) - \pi$.

PC125: [C]Derive the identity $\csc^2 x = 1+\cot^2 x$ from the fundamental Pythagorean identity $\sin^2 x+\cos^2 x = 1$.

PC126: [C]A point $P(x, y)$ moves along a circle with radius $r = 1$ such that its x-coordinate is given by $x(t) = \cos(\omega t)$ and $y(t) = \sin(\omega t)$. Prove that the period of the motion is $T = 2\pi/\omega$.

PC127: [C]Find the values of x for which $\sinh(x) = \frac{3}{4}$. (Requires solving an exponential equation).

PC128: [C]Determine the equation of the graph shown, assuming it is a sine function, given a minimum at $(\frac{\pi}{6}, -2)$ and a maximum at $(\frac{\pi}{3}, 4)$.

PC129: [C]Define the concept of a **co-terminal angle**. Show that for a given angle θ, the set of all co-terminal angles is represented by $\theta + 2\pi n$, where $n \in \mathbb{Z}$.

PC130: [C]Given $f(x) = \arcsin(x)$. Use the definition of the inverse function to derive the range of $f(x)$ from the domain of $\sin(x)$.

Part V: Analytic Trigonometry and Equations (30 Problems)

PC131: [S]Verify the identity: $\tan x + \cot x = \sec x \csc x$.

PC132: [S]Find the exact value of $\cos(105°)$ using the sum formula $\cos(A + B) = \cos A\cos B - \sin A\sin B$. (Use $105° = 60° + 45°$).

PC133: [S]Given $\sin\theta = 4/5$ and θ is acute. Find the exact value of $\sin(2\theta)$ using the double-angle formula.

PC134: [S]If $\cos\theta = -1/8$ and θ is in the interval $90° < \theta < 180°$. Determine the quadrant of $\theta/2$.

PC135: [S]Solve the trigonometric equation: $2\sin x - \sqrt{2} = 0$ for $0 \le x < 2\pi$.

PC136: [S]Use a reciprocal identity to prove that $\frac{\sin^2 x-1}{\cos x} = -\cos x$.

PC137: [S]Write $\cos(4\theta)$ as a double-angle formula in terms of 2θ.

PC138: [S]Use the Pythagorean identity to find $\sin x$ if $\cos x = 3/5$ and x is in Quadrant IV.

PC139: [S]Find the exact value of $\tan(\frac{\pi}{8})$ using the half-angle formula for tangent.

PC140: [S]Determine if the following statement is a valid identity: $\sin(\frac{\pi}{2} - x) = \cos x$.

PC141: [S]Solve for x: $4\cos^2 x - 3 = 0$ for $0 \leq x < 2\pi$.

PC142: [S]Simplify the expression: $\sin(x)\cot(x) + \cos(x)$.

PC143: [I]Verify the identity: $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 2\sec x$.

PC144: [I]Solve the general solution for the equation: $\cos(2x) = \sin(x)$. (Requires substitution using the double-angle formula).

PC145: [I]If $\cos A = -12/13$ with A in Q3, and $\tan B = 4/3$ with B in Q1, find the exact value of $\tan(A - B)$.

PC146: [I]Solve the equation: $\tan^2 x - 3\tan x + 2 = 0$ for $0 \leq x < 2\pi$.

PC147: [I]Use the half-angle formula to find the exact value of $\sin(15°)$.

PC148: [I]Verify the identity: $\frac{\sin(x+y)}{\sin x \cos y} = 1 + \cot x \tan y$.

PC149: [I]A student claims that $1 - 2\sin^2 x$ is equivalent to $\cos(2x)$. Prove or disprove this claim.

PC150: [I]Solve the equation: $\sin(3x) = -1$ for $0 \leq x < 2\pi$.

PC151: [I]Prove the product-to-sum identity: $\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$.

PC152: [I]Find the exact value of $\cos(2\theta)$ if $\tan\theta = -3$ and θ is in Quadrant II.

PC153: [C]Solve the equation $2\cos^2 x + \sin x - 1 = 0$ for the general solution.

PC154: [C]Prove the identity $\frac{\sin x}{1-\cos x} = \csc x + \cot x$. (Hint: Multiply by the conjugate of the denominator).

PC155: [C]Derive the half-angle formula for $\cos(\frac{\theta}{2})$ from the double-angle formula $\cos(2A) = 2\cos^2 A - 1$.

PC156: [C]Use De Moivre's Theorem (conceptual connection) to prove the triple-angle formula: $\cos(3x) = 4\cos^3 x - 3\cos x$.

PC157: [C]Find the exact value of $\sin\left(2\arccos\left(\frac{1}{3}\right)\right)$.

PC158: [C]Solve the system of trigonometric equations for $0 \leq x, y < 2\pi$: $\sin x + \sin y = 1$ and $\cos x + \cos y = 1$.

PC159: [C]Verify the identity: $\ln|\tan x| = \ln|\sin x| - \ln|\cos x|$ and discuss the domain restriction imposed by the ln function.

PC160: [C]Prove that the expression $\frac{\sin(x+h)-\sin x}{h}$ (the difference quotient for $\sin x$) is equal to $\cos x\left(\frac{\sin h}{h}\right) - \sin x\left(\frac{1-\cos h}{h}\right)$.

Part VI: Analytic Geometry, Sequences, and Series (40 Problems)

PC161: [S]Write the standard form of the equation of an ellipse with foci at $(\pm 4, 0)$ and vertices at $(\pm 5, 0)$.

PC162: [S]Identify the conic section represented by the general equation $4x^2 - 9y^2 - 16x - 18y - 29 = 0$ by calculating the discriminant $B^2 - 4AC$.

PC163: [S]Find the focus and directrix of the parabola $x^2 = 12y$.

PC164: [S]Write the standard equation of a circle with center $(-3, 5)$ and radius $r = 4$.

PC165: [S]Find the 10th term of the arithmetic sequence with first term $a_1 = 2$ and common difference $d = 5$.

PC166: [S]Find the sum of the first 8 terms of the geometric sequence $1, 2, 4, 8, \ldots$.

PC167: [S]Find the sum of the infinite geometric series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$.

PC168: [S]Expand $(2x - y)^4$ using the Binomial Theorem.

PC169: [S]Find the distance between the points $(1, -2)$ and $(4, 2)$ in the Cartesian plane.

PC170: [S]Find the center and radius of the circle given by $x^2 + y^2 - 6x + 4y - 3 = 0$ by completing the square.

PC171: [S]Identify the center, vertices, and asymptotes of the hyperbola $\frac{y^2}{4} - \frac{x^2}{1} = 1$.

PC172: [S]Find the $k = 3$ term (the third term) of the binomial expansion of $(x + y)^6$.

PC173: [S]Find the common ratio r of the geometric sequence where $a_2 = 12$ and $a_5 = 324$.

PC174: [S]Convert the rectangular coordinates $(3, 3)$ to polar coordinates (r, θ).

PC175: [S]Write the equation of a line perpendicular to $y = 3x - 5$ that passes through the point $(6, 1)$.

PC176: [I]Complete the square to write the equation of the parabola $y^2 + 2x - 4y + 6 = 0$ in standard form. State the vertex and the direction it opens.

PC177: [I]Find the equation of the hyperbola with foci at $(0, \pm 5)$ and asymptotes $y = \pm 2x$.

PC178: [I]A company's sales increase by 20% each month. If the sales are \$10,000 in the first month, what will the total accumulated sales be after 6 months?

PC179: [I]Prove that the sum of the first n terms of an arithmetic sequence is $S_n = \frac{n}{2}(a_1 + a_n)$.

PC180: [I]Find the term containing x^3 in the expansion of $(x - 2y)^7$.

PC181: [I]A ball is dropped from a height of 10 feet. On each bounce, it rises to 80% of its previous height. What is the total vertical distance the ball travels? (Requires two separate geometric series).

PC182: [I]Find the equation of the ellipse centered at the origin, passing through the point $(3, 2\sqrt{3})$, with a focus at $(0, 2)$.

PC183: [I]Convert the polar equation $r = 4\sin\theta$ to its rectangular form.

PC184: [I]Find the components of the vector v with magnitude $||v|| = 10$ and direction angle $\theta = 150°$.

PC185: [I]Find the equation of the tangent line to the circle $(x - 1)^2 + (y + 2)^2 = 25$ at the point $(5, 1)$.

PC186: [I]Find the general term a_n for the arithmetic sequence where $a_4 = 15$ and $a_9 = 35$.

PC187: [I]A series is defined by $\sum_{k=1}^{10}(2k - 1)$. Find the sum without listing all terms.

PC188: [I]Find the equation of the parabola with focus $(2, 3)$ and directrix $y = 1$.

PC189: [I]Analyze the graph of the polar equation $r = 1 + \cos\theta$ (a Cardioid). Determine its intercepts and sketch its general shape.

PC190: [I]Find the center, vertices, and foci of the conic section given by $25x^2 + 9y^2 + 100x - 36y - 89 = 0$.

PC191: [C]The definition of an ellipse is the set of all points where the sum of the distances from the two foci is constant. Prove that the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ satisfies this definition, where the foci are $(\pm c, 0)$ and $a^2 = b^2 + c^2$.

PC192: [C]Use the Principle of Mathematical Induction to prove the summation formula for the sum of the first n positive integers: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

PC193: [C]Prove the formula for the sum of a finite geometric series: $S_n = \frac{a_1(1-r^n)}{1-r}$, where $r \neq 1$.

PC194: [C]Find the equation of the plane (conceptual extension to 3D Analytic Geometry) that contains the three non-collinear points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

PC195: [C]Given the general second-degree equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Explain the role of the discriminant $B^2 - 4AC$ in classifying the conic section, especially when $B \neq 0$.

PC196: [C]A parabola has a vertex at $V(-2, 1)$ and a focus at $F(0, 1)$. Find the equation of the directrix and the equation of the parabola in standard form.

PC197: [C]Use the Binomial Theorem to approximate $(1.01)^5$ to five decimal places.

PC198: [C]For what values of k does the system of equations $x^2 + y^2 = 25$ and $y = 2x + k$ have exactly one solution? (Requires setting the discriminant of the resulting quadratic equation to zero).

PC199: [C]Determine the domain and range of the function defined parametrically by $x(t) = 3\cos t$ and $y(t) = 4\sin t$. Sketch the curve and write the rectangular equation.

PC200: [C]A sequence is defined recursively by $a_1 = 1, a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$ (Fibonacci sequence). Express the ratio of consecutive terms $\frac{a_{n+1}}{a_n}$ as $n \to \infty$ in terms of the Golden Ratio ϕ.