Problem Set

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Part I: Antiderivatives, Definite Integrals, and the FTC (35 Problems)

IC1: [S]Evaluate the indefinite integral: $\int \left(4x^3 - \frac{2}{\sqrt{x}} + \frac{5}{x}\right) dx$.

IC2: [S]Find the general antiderivative of $f(x) = \sec^2(x) - e^{-x}$.

IC3: [S]Evaluate the definite integral: $\int_1^e \frac{1}{x} dx$.

IC4: [S]Find $F(x)$ given $F'(x) = 6x^2 - 4x + 1$ and $F(1) = 5$. (Initial Value Problem).

IC5: [S]Evaluate the integral: $\int \left(\frac{1}{1+x^2} - \sin(3x)\right) dx$.

IC6: [S]Use the Fundamental Theorem of Calculus (FTC II) to evaluate: $\int_0^{\pi/4} \cos(2x) dx$.

IC7: [S]A particle's velocity is $v(t) = 2t + 3$ m/s. Find the displacement of the particle from $t = 1$ to $t = 4$ seconds.

IC8: [S]Evaluate $\int \frac{(t^2-1)^2}{t^2} dt$ by first expanding and simplifying the integrand.

IC9: [S]Find the specific antiderivative $f(x)$ if $f''(x) = 12x$ and $f(0) = 0$, $f'(0) = 3$.

IC10: [S]If $\int_2^5 f(x)dx = 7$ and $\int_5^2 g(x)dx = -3$, find $\int_2^5 [2f(x) + g(x)]dx$.

IC11: [S]State the geometric significance of the constant of integration, C, in $\int f(x)dx = F(x) + C$.

IC12: [S]Approximate $\int_0^4 x^2 dx$ using a Right Riemann Sum with $n = 2$ rectangles.

IC13: [S]True or False: If $f(x)$ is continuous on $[a, b]$, then $\int_a^b f(x)dx = \int_b^a f(x)dx$. Justify your answer.

IC14: [S]Evaluate $\int_{-1}^1 \sin(x)dx$ and explain the result in terms of function symmetry.

IC15: [S]Given $F(x) = \int_0^x (t^2 - 4)dt$. Find $F'(x)$.

IC16: [I])Find the total area bounded by $f(x) = x^2 - 4x$ and the x-axis on the interval $[0, 5]$.

IC17: [I])Evaluate $\int_1^8 \frac{(1+\sqrt[3]{x})^2}{\sqrt{x}} dx$.

IC18: [I])Given $\int_0^9 f(x)dx = 4$, find the value of $\int_0^3 xf(x^2)dx$.

IC19: [I])Use the limit definition of the definite integral (Riemann Sum) to evaluate $\int_0^2 (3x - x^2)dx$. (You may use summation formulas).

IC20: [I])A rocket accelerates at a rate $a(t) = \frac{1}{\sqrt{t+1}}$ m/s^2. If the initial velocity is $v(0) = 10$ m/s, find the velocity after 3 seconds.

IC21: [I])Given that $F(x)$ is an antiderivative of $f(x) = e^{x^2}$. Evaluate $\int_1^3 xe^{x^2} dx$ in terms of $F(x)$ and then evaluate the integral explicitly.

IC22: [I])If $\int_0^x f(t)dt = \cos(x^2) - 1$, find the function $f(x)$.

IC23: [I])Explain why $\int_0^1 \frac{1}{x} dx$ cannot be evaluated using the FTC II.

IC24: [I])Find the value of k that satisfies $\int_1^k \frac{1}{x^2} dx = \frac{1}{2}$, where $k > 1$.

IC25: [I])Approximate $\int_0^2 e^{-x^2} dx$ using a Midpoint Riemann Sum with $n = 4$.

IC26: [I])A function $f(x)$ is odd and continuous on $[-a, a]$. Explain why $\int_{-a}^{a} f(x)dx = 0$.

IC27: [I])Find the critical points of $G(x) = \int_0^x (t^3 - 4t)dt$.

IC28: [C])Determine the values of a and b such that the function $F(x) = \int_a^x f(t)dt$ has a local maximum at $x = 4$ and a local minimum at $x = 1$, given $f(x) = (x-1)(x-4)$.

IC29: [C])Evaluate the definite integral $\int_0^1 \arcsin(x)dx$ by interpreting it as an area and using the area of the inverse function. (Hint: Requires a geometric argument, not IBP).

IC30: [C])A continuous function $f(x)$ satisfies the relationship $f(x) = 5 + \int_0^x tf(t)dt$. Find an explicit formula for $f(x)$. (Requires combining FTC I and solving a differential equation).

IC31: [C])Given $G(x) = \int_{x^2}^{x^3} \frac{t^2}{1+t^2}dt$. Find $G'(1)$. (Requires FTC I with variable limits).

IC32: [C])Prove the Linearity Property of the definite integral: $\int_a^b [cf(x)+dg(x)]dx = c\int_a^b f(x)dx + d\int_a^b g(x)dx$, given the properties of summation.

IC33: [C])If $f(x)$ is continuous and $\int_0^9 f(\sqrt{x})dx = 12$, find $\int_0^3 xf(x)dx$.

IC34: [C])Show that if $f(x)$ is continuous and periodic with period T, then $\int_a^{a+T} f(x)dx$ is independent of a.

IC35: [C])Consider the integral function $F(x) = \int_1^x \frac{e^t}{t}dt$. Find the interval(s) where $F(x)$ is concave up.

Part II: Technique I: u-Substitution and Integration by Parts (35 Problems)

IC36: [S]Evaluate $\int x\cos(x^2)dx$ using u-substitution.

IC37: [S]Evaluate $\int \frac{e^x}{e^x+1}dx$.

IC38: [S]Evaluate $\int x^2 \ln(x)dx$ using integration by parts.

IC39: [S]Evaluate the definite integral $\int_0^1 xe^{x^2}dx$.

IC40: [S]Evaluate $\int \tan(x)dx$.

IC41: [S]Find the value of k that makes the following substitution correct: $\int \frac{1}{x\ln x}dx = \int k\frac{1}{u}du$.

IC42: [S]Evaluate $\int e^{3x}dx$.

IC43: [S]Evaluate $\int \arcsin(x)dx$. (Hint: Use IBP with $dv = dx$).

IC44: [S]Evaluate $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}dx$.

IC45: [S]Find the error in the following: $\int \frac{1}{x^2+4}dx = \frac{1}{2}\arctan(x/2) + C$.

IC46: [I])Evaluate $\int x^3\cos(x^2)dx$. (Requires u-sub followed by IBP).

IC47: [I])Evaluate $\int e^x \sin(x)dx$. (Requires recursive IBP).

IC48: [I])Evaluate the definite integral $\int_1^e \frac{\ln x}{x^2}dx$.

IC49: [I])Evaluate $\int \frac{1}{\sqrt{e^{2x}-1}}dx$. (Requires substitution, possibly $u = \sqrt{e^{2x}-1}$).

IC50: [I])Evaluate $\int \sec^3 x dx$. (Requires IBP and a substitution/identity).

IC51: [I])Solve the general antiderivative: $\int \frac{x \arctan x}{\sqrt{1+x^2}} dx$.

IC52: [I])Use the substitution $u = \sqrt{x+1}$ to evaluate $\int x\sqrt{x+1} dx$.

IC53: [I])Evaluate the definite integral $\int_0^{\pi/2} \cos x \cdot e^{\sin x} dx$.

IC54: [I])Find a reduction formula for $I_n = \int x^n e^x dx$.

IC55: [I])Evaluate $\int \frac{x}{\sqrt{4-x^4}} dx$. (Requires u-sub to simplify to an inverse trig form).

IC56: [I])Show that $\int x^n \ln x dx = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + C$ for $n \neq -1$.

IC57: [I])Evaluate $\int \cos(\ln x) dx$. (Requires recursive IBP).

IC58: [I])Find the area under the curve $y = x\sqrt{x-1}$ from $x = 1$ to $x = 5$.

IC59: [I])Evaluate $\int \frac{\sin x}{1+\cos^2 x} dx$.

IC60: [I])Use IBP to derive the formula $\int \ln x dx = x \ln x - x + C$.

IC61: [C])Evaluate $\int \frac{1}{x\sqrt{1-\ln^2 x}} dx$.

IC62: [C])Evaluate the definite integral $\int_0^{\pi} x \sin(x) dx$ and use the result to evaluate $\int_0^{\pi} x^2 \sin x dx$.

IC63: [C])Evaluate $\int \sqrt{1-e^{2x}} dx$. (Requires substitution and then a trigonometric substitution).

IC64: [C])Derive a reduction formula for $I_n = \int \tan^n x dx$.

IC65: [C])Evaluate the definite integral $\int_0^1 \frac{\arctan x}{x^2} dx$. (Requires IBP and analysis of the boundary at $x = 0$).

IC66: [C])Prove the integration by parts formula $\int u dv = uv - \int v du$ using the product rule for differentiation.

IC67: [C])Evaluate the definite integral $\int_{-1}^{1} \frac{x^2 \sin x}{1+x^6} dx$ using a symmetry argument.

IC68: [C])Find the integral $\int \frac{x^2}{1+x^2} \arctan x dx$.

IC69: [C])Find $f(x)$ if $\int f(x)e^x dx = xe^x + C$.

IC70: [C])Evaluate the definite integral $\int_0^1 \frac{\arcsin x}{\sqrt{1-x^2}} dx$.

Part III: Technique II: Trig Integrals, Trig Sub, and Partial Fractions (35 Problems)

IC71: [S]Evaluate $\int \sin^3 x \cos x dx$.

IC72: [S]Evaluate $\int \cos^2 x dx$ using the half-angle identity.

IC73: [S]Evaluate $\int \frac{1}{x^2+4} dx$.

IC74: [S]Evaluate $\int \frac{1}{x^2-4} dx$ using partial fraction decomposition.

IC75: [S]Evaluate $\int \tan^2 x dx$.

IC76: [S]Write out the form of the partial fraction decomposition for $\frac{3x+1}{(x-1)^2(x^2+1)}$. (Do not solve for constants).

IC77: [S]State the appropriate trigonometric substitution for the integral $\int \sqrt{9-x^2}dx$.

IC78: [S]Evaluate $\int \frac{1}{\sqrt{4-x^2}}dx$.

IC79: [S]Evaluate $\int \frac{\sec^2 x}{\tan x}dx$.

IC80: [S]Solve $\int \frac{x}{x^2-4}dx$.

IC81: [I])Evaluate $\int \sin^2 x \cos^3 x dx$.

IC82: [I])Evaluate $\int \frac{x^3+4x^2-x}{x^2+4}dx$. (Requires long division first).

IC83: [I])Evaluate $\int \frac{x^2}{\sqrt{16-x^2}}dx$ using trigonometric substitution.

IC84: [I])Evaluate $\int \sec^4 x \tan x dx$.

IC85: [I])Evaluate $\int \frac{1}{x^3+x^2-2x}dx$ using partial fractions.

IC86: [I])Evaluate $\int \frac{dx}{\sqrt{x^2+2x+5}}$ by completing the square and using trig substitution.

IC87: [I])Evaluate $\int \frac{x+1}{x^2(x^2+1)}dx$ using partial fractions.

IC88: [I])Evaluate $\int \tan^3 x \sec^4 x dx$.

IC89: [I])Evaluate $\int \frac{dx}{x^2\sqrt{x^2-9}}$.

IC90: [I])Evaluate $\int \frac{1}{1+\sin x}dx$ by multiplying by the conjugate.

IC91: [I])Evaluate $\int \frac{1}{2\sin x+\cos x}dx$ using the Weierstrass substitution ($u = \tan(x/2)$).

IC92: [I])Find the area bounded by $y = \frac{1}{x^2+3x+2}$ and the x-axis from $x = 0$ to $x = 2$.

IC93: [I])Show that the substitution $x = a\sin\theta$ in $\int \sqrt{a^2-x^2}dx$ yields $\frac{a^2}{2}(\theta + \sin\theta\cos\theta) + C$.

IC94: [I])Evaluate $\int \frac{x^3}{(x+1)^2}dx$.

IC95: [I])Evaluate $\int \frac{1-\tan x}{1+\tan x}dx$.

IC96: [C])Evaluate $\int \frac{1}{(x^2+1)^2}dx$. (Requires a trigonometric substitution and then a half-angle identity).

IC97: [C])Evaluate $\int \frac{\sqrt{x^2-1}}{x}dx$ and use the result to find $\int \frac{\sec\theta\tan^2\theta}{\sec\theta}d\theta$.

IC98: [C])Evaluate $\int \frac{x^4}{x^4-1}dx$. (Requires long division and partial fractions for four linear factors).

IC99: [C])Find the integral $\int \frac{1}{x\sqrt{x+4}}dx$. (Requires substitution $u = \sqrt{x+4}$ or $u^2 = x + 4$).

IC100: [C])Derive the reduction formula for $\int \sin^n x dx$ for $n \geq 2$.

IC101: [C])Evaluate the definite integral $\int_0^1 \frac{1}{x^4+1}dx$. (Requires factoring the quartic denominator into two irreducible quadratics).

IC102: [C])Find $\int \frac{e^{2x}}{e^{4x}+4}dx$. (Requires u-sub followed by an inverse trig form).

IC103: [C])Show that $\int \csc x dx = \ln|\csc x - \cot x| + C$.

IC104: [C])Evaluate $\int \frac{\sqrt{1-x}}{1-\sqrt{x}}dx$. (Requires multiple substitutions).

IC105: [C])Solve the integral $\int \frac{3x+5}{x^3-2x^2-5x+6}dx$.

Part IV: Applications I: Area and Volume (40 Problems)

IC106: [S]Find the area of the region bounded by $y = x^2$ and $y = x$.

IC107: [S]Find the area of the region bounded by $x = y^2$ and the y-axis from $y = 1$ to $y = 3$.

IC108: [S]Find the volume of the solid formed by revolving the region bounded by $y = \sqrt{x}$, $x = 4$, and $y = 0$ about the x-axis (Disk method).

IC109: [S]Find the volume of the solid formed by revolving the region bounded by $y = x$ and $y = x^2$ about the x-axis (Washer method).

IC110: [S]Set up the integral for the volume of the solid formed by revolving the region bounded by $y = x^2$ and $y = x$ about the y-axis using the Shell method. (Do not evaluate).

IC111: [S]A solid has a base bounded by $y = x$, $y = -x$, and $x = 1$. Cross-sections perpendicular to the x-axis are squares. Set up the integral for the volume.

IC112: [S]Write the formula for the volume of a solid of revolution using the Washer method when revolving around the line $y = k$.

IC113: [S]Find the area of the region bounded by $y = e^x$, $y = 0$, $x = 0$, and $x = 1$.

IC114: [S]True or False: The Shell method integral for volume of revolution around the y-axis involves integration with respect to x.

IC115: [S]Find the area enclosed by the curves $y = \cos x$ and $y = \sin x$ from $x = 0$ to $x = \pi/2$.

IC116: [S]Find the volume of a sphere of radius R by revolving the curve $y = \sqrt{R^2 - x^2}$ about the x-axis.

IC117: [S]Set up the integral for the volume generated by revolving the region bounded by $y = \sin x$ and $y = \cos x$ from $x = 0$ to $x = \pi/4$ about the line $y = -1$.

IC118: [S]Find the radius of the washer at $x = 1$ when the region bounded by $y = x$ and $y = x^2$ is revolved about the y-axis.

IC119: [S]The base of a solid is a circle $x^2 + y^2 = 4$. Cross-sections perpendicular to the x-axis are semicircles. Find the area function $A(x)$ for the cross-section.

IC120: [S]Find the area of the region bounded by $x = y^2 - 4y$ and $x = 0$.

IC121: [I])Find the volume of the solid formed by revolving the region bounded by $y = x^2$ and $y = 4$ about the line $x = 2$ using the Shell method.

IC122: [I])Find the area between the curves $y = x^3 - 6x^2 + 8x$ and $y = 0$.

IC123: [I])The region bounded by $y = 3\sqrt{x}$, $x = 8$, and $y = 0$ is revolved about the line $x = 8$. Use the Disk method to find the volume.

IC124: [I])Use the cylindrical shell method to find the volume of the solid generated by revolving the region bounded by $y = \ln x$, $x = e$, and the x-axis about the y-axis.

IC125: [I])Find the volume of the solid whose base is the region bounded by $y = x^2$ and $y = 4$, and whose cross-sections perpendicular to the y-axis are equilateral triangles.

IC126: [I])Find the volume of the solid generated by revolving the region bounded by $x = y^2$ and $x = y + 2$ about the line $y = 3$.

IC127: [I])Find the area of the loop of the curve $y^2 = x^2(x + 3)$.

IC128: [I])Find the total area bounded by the curve $y = \frac{x}{\sqrt{x^2-1}}$ and the x-axis from $x = 2$ to $x = 4$.

IC129: [I])Explain why the Shell method is generally preferred over the Washer method when revolving the region under $y = e^{-x^2}$ about the y-axis.

IC130: [I])Find the volume of the solid generated by revolving the region bounded by $y = \sec x$, $y = 1$, $x = -\pi/4$, and $x = \pi/4$ about the line $y = 1$.

IC131: [I])Determine the constant c such that the area of the region bounded by $y = x^2$ and $y = c$ is $\frac{32}{3}$.

IC132: [I])Find the volume of the solid created by revolving the region bounded by $y = \sin x^2$, $x = 0$, $x = \sqrt{\pi}$, and $y = 0$ about the y-axis using the Shell method. (Requires u-substitution).

IC133: [I])Set up and evaluate the integral for the area of the region bounded by $y = \frac{4x}{x^2+1}$ and the x-axis from $x = 0$ to $x = 3$.

IC134: [I])A region is bounded by $y = x^2$ and $y = 6 - x$. Set up the integral(s) for the volume when revolved about the line $x = 3$ using the Washer method.

IC135: [I])Find the volume of the solid generated by revolving the region bounded by $y = e^x$, $y = 1$, and $x = 2$ about the line $y = e^2$.

IC136: [C])A vase is designed by revolving the curve $y = x^4$ for $0 \leq x \leq 2$ about the y-axis. Find the volume of the vase and the amount of water needed to fill it to a height of $h = 10$.

IC137: [C])Find the area enclosed by the graphs of $y = |x^2 - 4|$ and $y = 5$.

IC138: [C])Find the volume of the solid generated by revolving the region bounded by the curves $y = \frac{1}{1+x^2}$ and $y = \frac{1}{2}$ about the x-axis.

IC139: [C])Show that the volume of a frustum of a right circular cone with height h and radii r_1 and r_2 is $V = \frac{1}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)$ by integral calculus.

IC140: [C])Find the volume of the solid whose base is an elliptical region $x^2/4 + y^2/9 = 1$, and whose cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

IC141: [C])Find the equation of a line $y = m$ that divides the area bounded by $y = x^2$ and $y = 4$ into two equal parts.

IC142: [C])Calculate the volume of the torus (doughnut shape) generated by revolving the circle $(x-R)^2+y^2 = r^2$ $(R > r)$ about the y-axis. (Requires Shell Method and trigonometric substitution).

IC143: [C])Set up the integral for the volume of the solid formed by revolving the region bounded by $y = \frac{1}{x}$, $y = x$, and $x = 2$ about the line $x = -1$.

IC144: [C])Find the area of the region bounded by $y = \frac{x}{\sqrt{x-1}}$ and $y = \frac{1}{\sqrt{x-1}}$ and the lines $x = 2$ and $x = 5$.

IC145: [C])Derive the general formula for the volume of a solid by known cross-sections $A(x)$ perpendicular to the x-axis, given the area $A(x)$ is a function of x.

Part V: Applications II: Arc Length, Avg Value, and Rates of Change (30 Problems)

IC146: [S]Find the average value of $f(x) = x^2 + 1$ on the interval $[0, 3]$.

IC147: [S]Write down the integral for the arc length of $y = \ln x$ from $x = 1$ to $x = e$. (Do not evaluate).

IC148: [S]A particle moves with velocity $v(t) = \cos(2t)$ m/s. Find the net displacement from $t = 0$ to $t = \pi$.

IC149: [S]A function $f(x)$ has an average value of 5 on the interval $[2, 8]$. Find $\int_2^8 f(x)dx$.

IC150: [S]Find the area of the polar region enclosed by the curve $r(\theta) = 2 + 2\cos\theta$.

IC151: [S]Use the Net Change Theorem to find the final amount $Q(b)$ given an initial amount $Q(a)$ and a rate of change $Q'(t)$.

IC152: [S]Find the location of the absolute maximum of $F(x) = \int_0^x (t-1)(t-3)dt$ on $[0, 4]$.

IC153: [S]Find the average value of $f(x) = \sin^2 x$ on the interval $[0, \pi]$.

IC154: [S]Find the arc length of the curve $y = 2x + 1$ from $x = 0$ to $x = 5$ using the distance formula and verify with the integral formula.

IC155: [S]Find the total accumulation (positive change) of a quantity Q over $[0, 2]$ if $Q'(t) = t^2 - 2t$.

IC156: [I])Find the exact arc length of the curve $y = \frac{1}{6}x^3 + \frac{1}{2x}$ from $x = 1$ to $x = 2$. (Requires simplification of the integrand $\sqrt{1 + (y')^2}$).

IC157: [I])A city's population grows at a rate $P'(t) = 100e^{0.02t}$ people/year, where $t = 0$ is 2020. Find the total population growth between 2020 and 2030.

IC158: [I])Find the average value of $f(x) = x\sqrt{x^2 + 9}$ on the interval $[0, 4]$.

IC159: [I])Find the exact arc length of the polar curve $r(\theta) = 4\cos\theta$ for $0 \le \theta \le \pi$.

IC160: [I])A particle moves with velocity $v(t) = t^2 - 4t$ m/s. Find the total distance traveled from $t = 0$ to $t = 5$ seconds.

IC161: [I])The function $F(x) = \int_0^x \frac{\sin t}{t}dt$ is defined. Find the location of the first non-trivial local maximum for $F(x)$ for $x > 0$.

IC162: [I])Find the average value of the function $f(x) = \frac{x}{\sqrt{x+1}}$ on the interval $[0, 8]$.

IC163: [I])A water tank leaks at $L(t) = 4 + 2t$ gal/hr while being pumped at $P(t) = 16$ gal/hr. If the tank is full at $t = 0$, find the net change in water volume over $[0, 4]$.

IC164: [I])Find the length of the curve defined parametrically by $x(t) = \cos^3 t$ and $y(t) = \sin^3 t$ for $0 \le t \le \pi/2$. (Conceptual link to arc length of parametric curves).

IC165: [I])Find the value of c such that $f(c)$ is equal to the average value of $f(x) = \frac{1}{x}$ on $[1, e^2]$.

IC166: [C])Find the exact arc length of the curve $y = \int_1^x \sqrt{\sec^4 t - 1}dt$ from $x = \pi/6$ to $x = \pi/3$.

IC167: [C])Show that the Mean Value Theorem for Integrals implies that for a continuous function $f(x)$, the average value must be attained at some point c in the interval.

IC168: [C])Find the total distance traveled by a particle from $t = 0$ to $t = 2$ if its acceleration is $a(t) = 6t - 6$ and $v(0) = 9$.

IC169: [C])Given $F(x) = \int_0^x f(t)dt$. Prove that if $f(t)$ is continuous and concave up, then $F(x)$ is increasing and concave up.

IC170: [C])A curve is defined by $y = \int_0^x \sqrt{3t^4 + 2t^2}dt$. Find the arc length of this curve from $x = 0$ to $x = 2$.

IC171: [C])Find the area of the region bounded by the inner loop of the polar curve $r(\theta) = 1 + 2\cos\theta$. (Requires solving $r(\theta) = 0$ for limits).

IC172: [C])Use the concept of integration to prove that the centroid (center of mass) $\bar{x}$ of a continuous region between $f(x)$ and $g(x)$ is $\bar{x} = \frac{\int_a^b x[f(x)-g(x)]dx}{\int_a^b [f(x)-g(x)]dx}$.

IC173: [C])For a continuous function $f(x)$, if $\int_a^b f(x)dx = \int_a^c f(x)dx$, prove that $\int_b^c f(x)dx = 0$.

IC174: [C])Find the volume of a solid of revolution generated by revolving the region bounded by $y = xe^{-x}$ and $y = 0$ for $0 \leq x \leq 2$ about the y-axis using the Shell method.

IC175: [C])Find the exact arc length of the parabola $y = \frac{x^2}{2}$ from $x = 0$ to $x = 1$ (requires a trigonometric substitution).

Part VI: Advanced Topics: Improper Integrals and Special Functions (25 Problems)

IC176: [S]Evaluate the Type I improper integral: $\int_1^\infty \frac{1}{x^3}dx$.

IC177: [S]Determine if the p-integral $\int_1^\infty \frac{1}{\sqrt{x}}dx$ converges or diverges.

IC178: [S]Evaluate the Type II improper integral: $\int_0^1 \frac{1}{\sqrt{x}}dx$.

IC179: [S]Write the definition of the Gamma function, $\Gamma(z)$.

IC180: [S]State the functional property of the Gamma function: $\Gamma(z+1) = \dots$

IC181: [I])Determine the convergence or divergence of $\int_0^\infty xe^{-x}dx$. (Requires IBP).

IC182: [I])Evaluate the improper integral $\int_e^\infty \frac{1}{x(\ln x)^2}dx$.

IC183: [I])Evaluate the improper integral $\int_{-1}^1 \frac{1}{\sqrt[3]{x^2}}dx$. (Type II at $x = 0$).

IC184: [I])Use the Comparison Test to determine if $\int_1^\infty \frac{x^2}{x^4+\sin^2 x}dx$ converges.

IC185: [I])Evaluate $\int_0^\infty \frac{1}{x^2+4x+5}dx$. (Requires completing the square and limits).

IC186: [I])For which value(s) of p does the improper integral $\int_0^1 x^p \ln x dx$ converge?

IC187: [I])Evaluate $\Gamma(5/2)$ using the property $\Gamma(z+1) = z\Gamma(z)$ and the fact that $\Gamma(1/2) = \sqrt{\pi}$.

IC188: [I])Determine the value of k for which $\int_{-\infty}^k e^{2x}dx = \frac{1}{2}$.

IC189: [I])Show that the improper integral $\int_2^\infty \frac{1}{\ln x}dx$ diverges. (Requires comparison).

IC190: [I])Find the Beta function value $B(1/2, 1/2)$ and use the Beta-Gamma relationship to confirm the value of $\Gamma(1/2)$.

IC191: [C])Find the value of the Gaussian integral $\int_{-\infty}^\infty e^{-x^2}dx$. (State the known result, and conceptually discuss the method used for its proof - double integral/polar coordinates).

IC192: [C])Evaluate the improper integral $\int_1^\infty \frac{\arctan x}{x^2}dx$. (Requires IBP and limit evaluation).

IC193: [C])Show that the integral $\int_0^\infty \sin(x^2)dx$ converges (Fresnel Integral - conceptual comparison/Dirichlet test).

IC194: [C])Prove the convergence of the p-integral $\int_a^\infty \frac{1}{x^p}dx$ for $p > 1$ using the limit definition.

IC195: [C])Evaluate $\int_0^{\pi/2} \ln(\sin x)dx$ (A challenging definite integral requiring symmetry and substitution).

IC196: [C])Evaluate the Dirichlet integral $\int_0^\infty \frac{\sin x}{x} dx$. (State the known result and discuss its non-convergence by the Comparison Test).

IC197: [C])Determine all values of p for which the improper integral $\int_2^\infty \frac{1}{x(\ln x)^p} dx$ converges.

IC198: [C])Use the Beta function definition $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$ to evaluate $\int_0^1 \sqrt{t - t^2} dt$.

IC199: [C])Show that $\int_0^\infty \frac{x^2}{e^x} dx = \Gamma(3)$.

IC200: [C])Determine the convergence or divergence of $\int_0^\infty \frac{1}{\sqrt{x(x+1)(x+2)}} dx$ (Requires analysis of singularities at $x = 0$ and $x = \infty$).