Derivatives

Sum: $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Product: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

Quotient: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,du/dx - u\,dv/dx}{v^2}$

Power: $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$

Chain: $\frac{d}{dx}z(y(x)) = \frac{dz}{dy}\frac{dy}{dx}$

Inverse: $\frac{dx}{dy} = \frac{1}{dy/dx}$

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\csc x = -\csc x \cot x$

$\frac{d}{dx}e^{cx} = ce^{cx}$

$\frac{d}{dx}b^x = b^x \ln b$

$\frac{d}{dx}\ln x = \frac{1}{x}$

$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$

$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

$\int a^x\,dx = \frac{1}{\ln a}\cdot a^x$

Limits and Continuity

$\frac{\sin x}{x} \to 1 \quad \frac{1-\cos x}{x} \to 0 \quad \frac{1-\cos x}{x^2} \to \frac{1}{2}$

$a_n \to 0 : |a_n| < \epsilon$ for all $n > N$

$a_n \to L : |a_n - L| < \epsilon$ for all $n > N$

$f(x) \to L : |f(x) - L| < \epsilon$ for $0 < |x-a| < \delta$

$f(x) \to f(a)$: Continuous at a if $L = f(a)$

$\frac{f(x)-f(a)}{x-a} \to f'(a)$: Derivative at a

$\frac{f(x)-f(a)}{x-a} = f'(c)$: Mean Value Theorem

$\frac{f(x+\Delta x)-f(x)}{\Delta x} \to f'(x)$: Derivative at x

$\frac{f(x+\Delta x)-f(x-\Delta x)}{2\,\Delta x} \to f'(x)$: Centered

$\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$ l'Hôpital's Rule for $\frac{0}{0}$

Maximum and Minimum

Critical: $f'(x) = 0$ or no f' or endpoint

Minimum $\quad f'(x) = 0$ and $f''(x) > 0$

Maximum $\quad f'(x) = 0$ and $f''(x) < 0$

Inflection point $\quad f''(x) = 0$

Newton's Method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Iteration $x_{n+1} = F(x_n)$ attracted to

fixed point $x^* = F(x^*)$ if $|F'(x^*)| < 1$

Stationary in 2D: $\partial f/\partial x = 0$, $\partial f/\partial y = 0$

Minimum $\quad f_{xx} > 0 \quad f_{xx}f_{yy} > f_{xy}^2$

Maximum $\quad f_{xx} < 0 \quad f_{xx}f_{yy} > f_{xy}^2$

Saddle point $\quad f_{xx}f_{yy} < f_{xy}^2$

Newton in 2D $\left\{ \begin{array}{l} g + g_x\,\Delta x + g_y\,\Delta y = 0 \\ h + h_x\Delta x + h_y\Delta y = 0 \end{array} \right.$

Algebra

$\frac{x/a}{y/b} = \frac{bx}{ay} \qquad x^{-n} = \frac{1}{x^n} \qquad \sqrt[n]{x} = x^{1/n}$

$(x^2)(x^3) = x^5 \quad (x^2)^3 = x^6 \quad x^2/x^3 = x^{-1}$

$ax^2 + bx + c = 0$ has roots $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x^2 + 2Bx + C = 0$ has roots $x = -B \pm \sqrt{B^2 - C}$

Completing square $ax^2 + bx + c = a(x + \frac{b}{2a})^2 + c - \frac{b^2}{4a}$

Partial fractions $\frac{cx+d}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$

Mistakes $\quad \frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c} \quad \sqrt{x^2+a^2} \neq x + a$

Fundamental Theorem of Calculus

$\frac{d}{dx}\int_a^x v(t)dt = v(x) \quad \int_a^b \frac{df}{dx}dx = f(b) - f(a)$

$\frac{d}{dx}\int_{a(x)}^{b(x)} v(t)dt = v(b(x))\frac{db}{dx} - v(a(x))\frac{da}{dx}$

$\int_0^b y(x)dx = \lim_{\Delta x \to 0} \Delta x[y(\Delta x) + y(2\Delta x) + \cdots + y(b)]$

Circle, Line, and Plane

$x = r\cos\omega t, y = r\sin\omega t,$ speed ωr

$y = mx + b$ or $y - y_0 = m(x - x_0)$

Plane $ax + by + cz = d$ or

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

Normal vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$

Distance to (0,0,0): $|d|/\sqrt{a^2+b^2+c^2}$

Line $(x,y,z) = (x_0,y_0,z_0) + t(v_1,v_2,v_3)$

No parameter: $\frac{x-x_0}{v_1} = \frac{y-y_0}{v_2} = \frac{z-z_0}{v_3}$

Projection: $\mathbf{p} = \frac{\mathbf{b}\cdot\mathbf{a}}{\mathbf{a}\cdot\mathbf{a}}\mathbf{a}, |\mathbf{p}| = |\mathbf{b}|\cos\theta$

Sums and Infinite Series

$1 + x + \cdots + x^{n-1} = \frac{1-x^n}{1-x}$

$1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + x^n = (1+x)^n$

$1 + 2 + \cdots + n = \frac{1}{2}n(n+1) \approx \frac{n^2}{2}$

$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{n^3}{3}$

$1 + \frac{1}{2} + \cdots + \frac{1}{n} \approx \ln n \to \infty$ (harmonic)

$1 - \frac{1}{2} + \frac{1}{3} - \cdots = \ln 2$ (alternating)

$1 - \frac{1}{3} + \frac{1}{5} - \cdots = \frac{\pi}{4} \quad \sum \frac{1}{n^2} = \frac{\pi^2}{6} \quad \sum \frac{1}{n^4} = \frac{\pi^4}{90}$

$\frac{1}{1-x} = 1 + x + x^2 + \cdots$ (geometric: $|x| < 1$)

$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \cdots = \frac{d}{dx}\left(\frac{1}{1-x}\right)$

$\frac{1}{1+x} = 1 - x + x^2 - \cdots$ (geometric for $-x$)

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots = \int \frac{dx}{1+x}$

$\sin x = x - x^3/6 + x^5/120 - \cdots$ (all x)

$\cos x = 1 - x^2/2 + x^4/24 - \cdots$ (all x)

$e^x = 1 + x + \frac{x^2}{2!} + \cdots (e = 1 + 1 + \frac{1}{2!} + \cdots)$

$e^{ix} = \cos x + i \sin x$ (Euler's formula)

$\cosh x = \frac{1}{2}(e^x + e^{-x}) = 1 + \frac{x^2}{2!} + \cdots$

$\sinh x = \frac{1}{2}(e^x - e^{-x}) = x + \frac{x^3}{3!} + \cdots$

$(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$

$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + \cdots$ (Taylor)

$f(x,y) = f + xf_x + yf_y + \frac{x^2}{2!}f_{xx} + xyf_{xy} + \cdots$

Polar and Spherical

$x = r\cos\theta$ and $y = r\sin\theta$

$r = \sqrt{x^2 + y^2}$ and $\tan\theta = y/x$

$x + iy = r(\cos\theta + i \sin\theta) = re^{i\theta}$

Area $\int \frac{1}{2}r^2 d\theta$ Length $\int \sqrt{r_\theta^2 + r^2}\,d\theta$

$x = \rho\sin\phi\cos\theta, y = \rho\sin\phi\sin\theta, z = \rho\cos\phi$

Area $dA = dx\,dy = r\,dr\,d\theta = J\,du\,dv$

Volume $r\,dr\,d\theta\,dz = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

Stretching factor $J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$

Area - Volume - Length - Mass - Moment

Circle πr^2 Ellipse πab Wedge of circle $r^2\theta/2$

Cylinder side $2\pi rh$ Volume $\pi r^2 h$ Shell $dV = 2\pi rh\,dr$

Sphere surface $4\pi r^2$ Volume $\frac{4}{3}\pi r^3$ Shell $dV = 4\pi r^2 dr$

Cone or pyramid Volume $\frac{1}{3}$ (base area) (height)

Length of curve $\int ds = \int \sqrt{1 + (dy/dx)^2}\,dx$

Area between curves $\int (v(x) - w(x))dx$

Surface area of revolution $\int 2\pi r\,ds (r = x$ or $r = y)$

Volume of revolution: Slices $\int \pi y^2 dx$ Shells $\int 2\pi xh\,dx$

Area of surface $z(x,y) : \iint \sqrt{1 + z_x^2 + z_y^2}\,dx\,dy$

Mass $M = \iint \rho\,dA$ Moment $M_y = \iint \rho x\,dA$

$\overline{x} = M_y/M, \overline{y} = M_x/M$ Moment of Inertia $I_y = \iint \rho x^2 dA$

Work $W = \int_a^b F(x)dx = V(b) - V(a)$ Force $F = dV/dx$

Partial Derivatives of $z = f(x,y)$

Tangent plane $z - z_0 = (\frac{\partial f}{\partial x})(x - x_0) + (\frac{\partial f}{\partial y})(y - y_0)$

Approximation $\Delta z \approx (\frac{\partial f}{\partial x})\Delta x + (\frac{\partial f}{\partial y})\Delta y$

Normal $\mathbf{N} = (f_x, f_y, -1)$ or (F_x, F_y, F_z)

Gradient $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$

Directional derivative: $D_{\mathbf{u}}f = \nabla f \cdot \mathbf{u} = f_x u_1 + f_y u_2$

Chain rule: $\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

Vector field $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$

Work $\int \mathbf{F} \cdot d\mathbf{R}$ Flux $\int M\,dy - N dx$

Divergence of $\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

Curl of $\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ M & N & P \end{vmatrix}$

Conservative $\mathbf{F} = \nabla f =$ gradient of f if curl $\mathbf{F} = \mathbf{O}$

Green's Theorem $\oint M\,dx + N\,dy = \iint (\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y})\,dx\,dy$

Divergence Theorem $\iint \mathbf{F} \cdot \mathbf{n}\,dS = \iiint \text{div}\,\mathbf{F}\,dV$

Stokes' Theorem $\oint \mathbf{F} \cdot d\mathbf{R} = \iint (\text{curl}\,\mathbf{F}) \cdot \mathbf{n}\,dS$

An additional table of integrals is included just after the index.

Exponentials and Logarithms

$y = b^x \leftrightarrow x = \log_b y \quad y = e^x \leftrightarrow x = \ln y$

$e = \lim(1+\frac{1}{n})^n = \sum_{n=0}^{\infty} \frac{1}{n!} = 2.71828\cdots$

$e^x = \lim(1+\frac{x}{n})^n = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\ln y = \int_1^y \frac{dx}{x} \qquad \ln 1 = 0 \quad \ln e = 1$

$\ln xy = \ln x + \ln y \quad \ln x^n = n \ln x$

$\log_a y = (\log_a b)(\log_b y) \quad \log_a b = 1/\log_b a$

$e^{x+y} = e^x e^y \quad b^x = e^{x \ln b} \quad e^{\ln y} = y$

Equations and Their Solutions

$y' = cy$	$y_0 e^{ct}$
$y' = cy + s$	$y_0 e^{ct} + \frac{s}{c}(e^{ct} - 1)$
$y' = cy - by^2$	$\frac{c}{b+de^{-ct}} \quad d = \frac{c-by_0}{y_0}$
$y'' = -\lambda^2 y$	$\cos \lambda t$ and $\sin \lambda t$
$my'' + dy' + ky = 0$	$e^{\lambda_1 t}$ and $e^{\lambda_2 t}$ or $te^{\lambda_1 t}$
$y_{n+1} = ay_n$	$a^n y_0$
$y_{n+1} = ay_n + s$	$a^n y_0 + s\frac{a^n-1}{a-1}$

Vectors and Determinants

$\mathbf{A} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$

$|\mathbf{A}|^2 = \mathbf{A}\cdot\mathbf{A} = a_1^2 + a_2^2 + a_3^2$ (length squared)

$\mathbf{A}\cdot\mathbf{B} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{A}||\mathbf{B}|\cos\theta$

$|\mathbf{A}\cdot\mathbf{B}| \le |\mathbf{A}||\mathbf{B}|$ (Schwarz inequality: $|\cos\theta| \le 1$)

$|\mathbf{A}+\mathbf{B}| \le |\mathbf{A}| + |\mathbf{B}|$ (triangle inequality)

$|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}||\mathbf{B}||\sin\theta|$ (cross product)

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{array}{l} \mathbf{i}(a_2b_3 - a_3b_2) \\ +\mathbf{j}(a_3b_1 - a_1b_3) \\ +\mathbf{k}(a_1b_2 - a_2b_1) \end{array}$$

Right hand rule $\mathbf{i}\times\mathbf{j} = \mathbf{k}$, $\mathbf{j}\times\mathbf{k} = \mathbf{i}$, $\mathbf{k}\times\mathbf{i} = \mathbf{j}$

Parallelogram area $= |a_1b_2 - a_2b_1| = |\text{Det}|$

Triangle area $= \frac{1}{2}|a_1b_2 - a_2b_1| = \frac{1}{2}|\text{Det}|$

Box volume $= |\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})| = |\text{Determinant}|$

Matrices and Inverses

$Ax =$ combination of columns $= b$

Solution $x = A^{-1}b$ if $A^{-1}A = I$

Least squares $A^TA\bar{x} = A^Tb$

$Ax = \lambda x$ (λ is an eigenvalue)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$(AB)^{-1} = B^{-1}A^{-1}, (AB)^T = B^TA^T$

$$\begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{bmatrix}^{-1} = \frac{1}{D}\begin{bmatrix} \mathbf{b}\times\mathbf{c} \\ \mathbf{c}\times\mathbf{a} \\ \mathbf{a}\times\mathbf{b} \end{bmatrix}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{array}{l} +\,a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 \\ -\,a_1b_3c_2 - a_2b_1c_3 - a_3b_2c_1 \end{array}$$

	SI Units	Symbols
length	meter	m
mass	kilogram	kg
time	second	s
current	ampere	A
frequency	hertz	Hz $\sim$ 1/s
force	newton	N $\sim$ kg·m/s^2
pressure	pascal	Pa $\sim$ N/m^2
energy, work	joule	J $\sim$ N·m
power	watt	W $\sim$ J/s
charge	coulomb	C $\sim$ A·s
temperature	kelvin	K

From	To	Multiply by
degrees	radians	.01745
calories	joules	4.1868
BTU	joules	1055.1
foot-pounds	joules	1.3558
feet	meters	.3048
miles	km	1.609
feet/sec	km/hr	1.0973
pounds	kg	.45359
ounces	kg	.02835
gallons	liters	3.785
horsepower	watts	745.7

Speed of light $c = 2.9979 \times 10^8$ m/s

Gravity $G = 6.6720 \times 10^{-11}$ Nm^2/kg^2

Radius at Equator $R = 6378$ km $= 3964$ miles

Acceleration $g = 9.8067$ m/s^2 $= 32.174$ ft/s^2

Trigonometric Identities

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$ (divide by $\cos^2 x$)

$1 + \cot^2 x = \csc^2 x$ (divide by $\sin^2 x$)

$\sin 2x = 2 \sin x \cos x$ (double angle)

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\sin(s \pm t) = \sin s \cos t \pm \cos s \sin t$ (Addition

$\cos(s \pm t) = \cos s \cos t \mp \sin s \sin t$ formulas)

$\tan(s+t) = (\tan s + \tan t)/(1 - \tan s \tan t)$

$c^2 = a^2 + b^2 - 2ab\cos\theta$ (Law of cosines)

$a/\sin A = b/\sin B = c/\sin C$ (Law of sines)

$a\cos\theta + b\sin\theta = \sqrt{a^2+b^2}\cos(\theta - \tan^{-1}\frac{b}{a})$

$\cos(-x) = \cos x$ and $\sin(-x) = -\sin x$

$\sin(\frac{\pi}{2} \pm x) = \cos x$ and $\cos(\frac{\pi}{2} \pm x) = \mp \sin x$

$\sin(\pi \pm x) = \mp \sin x$ and $\cos(\pi \pm x) = -\cos x$

Trigonometric Integrals

$\int \sin^2 x\, dx = \frac{x - \sin x \cos x}{2} = \int \frac{1-\cos 2x}{2} dx = \frac{x}{2} - \frac{\sin 2x}{4}$

$\int \cos^2 x\, dx = \frac{x + \sin x \cos x}{2} = \int \frac{1+\cos 2x}{2} dx = \frac{x}{2} + \frac{\sin 2x}{4}$

$\int \tan^2 x\, dx = \tan x - x$

$\int \cot^2 x\, dx = -\cot x - x$

$\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx$

$\int \cos^n x\, dx = +\frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx$

$\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx$

$\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx$

$\int \tan x\, dx = -\ln|\cos x|$

$\int \cot x\, dx = \ln|\sin x|$

$\int \sec x\, dx = \ln|\sec x + \tan x|$

$\int \csc x\, dx = \ln|\csc x - \cot x| = -\ln|\csc x + \cot x|$

$\int \sec^3 x\, dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x|$

$\int \sin px \sin qx\, dx = \frac{\sin(p-q)x}{2(p-q)} - \frac{\sin(p+q)x}{2(p+q)}$

$\int \cos px \cos qx\, dx = \frac{\sin(p-q)x}{2(p-q)} + \frac{\sin(p+q)x}{2(p+q)}$

$\int \sin px \cos qx\, dx = -\frac{\cos(p-q)x}{2(p-q)} - \frac{\cos(p+q)x}{2(p+q)}$

Additional integrals follow the index

Integration by Parts

$\int \ln x\, dx = x \ln x - x$

$\int x^n \ln x\, dx = \frac{x^{n+1}\ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2}$

$\int x^n e^x dx = x^n e^x - n\int x^{n-1} e^x dx$

$\int e^{cx} \sin kx\, dx = \frac{e^{cx}}{c^2+k^2}(c \sin kx - k\cos kx)$

$\int e^{cx} \cos kx\, dx = \frac{e^{cx}}{c^2+k^2}(c \cos kx + k\sin kx)$

$\int x \sin x\, dx = \sin x - x\cos x$

$\int x \cos x\, dx = \cos x + x \sin x$

$\int x^n \sin x\, dx = -x^n \cos x + n\int x^{n-1}\cos x\, dx$

$\int x^n \cos x\, dx = +x^n \sin x - n\int x^{n-1}\sin x\, dx$

$\int \sin^{-1} x\, dx = x \sin^{-1} x + \sqrt{1-x^2}$

$\int \tan^{-1} x\, dx = x \tan^{-1} x - \frac{1}{2}\ln(1+x^2)$

Integrals with x^2 and a^2 and $D = b^2 - 4ac$

$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| = \frac{1}{a}\tanh^{-1}\frac{x}{a}$

$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln|x + \sqrt{x^2+a^2}|$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$

$\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln|x+\sqrt{x^2+a^2}|$

$\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\cos^{-1}\frac{a}{x}$

$\int \frac{dx}{x\sqrt{x^2+a^2}} = \frac{1}{a}\ln\left|\frac{\sqrt{x^2+a^2}-a}{x}\right|$

$$\int \frac{dx}{ax^2+bx+c} = \frac{1}{\sqrt{D}}\ln\left|\frac{2ax+b-\sqrt{D}}{2ax+b+\sqrt{D}}\right|,\ D>0$$
$$= \frac{2}{\sqrt{-D}}\tan^{-1}\frac{2ax+b}{\sqrt{-D}},\ D<0$$
$$= \frac{-2}{2ax+b},\ D=0$$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}|$$
$$= \frac{1}{\sqrt{-a}}\sin^{-1}\frac{-2ax-b}{\sqrt{D}}, a<0$$

Definite Integrals

$\int_0^\infty x^n e^{-x} dx = n! = \Gamma(n+1)$

$\int_0^\infty e^{-a^2x^2} dx = \sqrt{\pi}/2a$

$\int_0^1 x^m (1-x)^n dx = \frac{m!n!}{(m+n+1)!}$

$\int_0^\infty \frac{\sin^2 x}{x^2}\, dx = \int_0^\infty \frac{\sin x}{x}\, dx = \frac{\pi}{2}$

$\int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx =$

$\frac{1}{2}\frac{3}{4}\cdots\frac{n-1}{n}\left(\frac{\pi}{2}\right)$ (n even) or $\frac{2}{3}\frac{4}{5}\cdots\frac{n-1}{n}$ (n odd > 1)