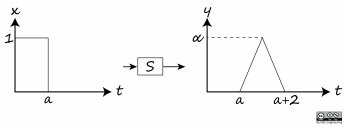
Somith Das **2019W2_ELEC_221_201**

Assignment Problem_Set_3 due 02/04/2020 at 11:59pm PST

For an LTI system, S, an input and its corresponding output signal are shown in the figure below. Assume $a = 8$ and $\alpha = -10$.



For each of the following input signals, determine the corresponding output signal:

a) $x_1(t) = u(t) - u(t-8) - u(t-2) + u(t-10)$**b)** $x_2(t) = 7.5u(t+8) - 15u(t) + 7.5u(t-8)$**c)** $x_3(t) = \delta(t) - \delta(t-8)$

$y_1(t) =$_____ $y_2(t) =$_____ $y_3(t) =$_____

Use $r(t)$ to represent the ramp function.

Hint: Solving this problem using Laplace Transforms can get messy. Instead, express each of the inputs ('x_i(t)') in terms of the original input 'x(t)' and use the LTI system property.

Correct Answers:

- `-10*[r(t-8)-2*r(t-8-1)+2*r(t-8-3)-r(t-8-4)]`
- `-10*(7.5*[r(t)-2*r(t-1)+r(t-2)]-7.5*[r(t-8)-2*r(t-8-1)+r(t-8-2)])`
- `-10*[u(t-8)-2*u(t-8-1)+u(t-8-2)]`

For a continuous-time LTI system S, suppose that the input-output relationship is given according to the differential equations $\frac{dy(t)}{dt} = x(t) - 7y(t)$ and the system is initially at rest.

a) Find the output of the system given that the input is described by $x(t) = e^{(-9+3j)t}u(t)$.

$y(t) =$_____

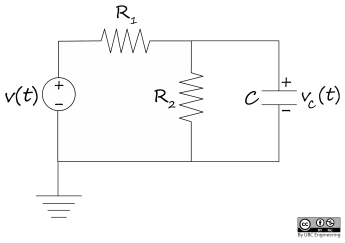
b) Is this a causal system? [?/Yes/No]

*Part **b** will only be marked correct if part **a** is correct.*

Correct Answers:

- `u(t)*[-e^(-7*t)+e^[(-9+3*j)*t]]/[(-2)+3*j]`
- `Yes`

In the circuit shown in the figure, the input is the voltage source, $v(t)$, and the output is the voltange $v_c(t)$ across the capacitor. Determine the transfer function, $H(s)$, the impulse response $h(t)$, and the step response $d(t)$, of this circuit. Assume $R_1 = 2$ $k\Omega$, $R_2 = 4$ $k\Omega$ and $C = 6$ mF.



$H(s) =$ _____

$h(t) =$ _____ volts

$d(t) =$ _____ volts

Correct Answers:

- `0.0833333/(s+0.125)`
- `0.0833333*e^(-0.125*t)*u(t)`
- `0.666667*[1-e^(-0.125*t)]*u(t)`

For each of the following impulse responses, determine the Laplace transform as well as the region of convergence, if the system is causal, anti-causal or two-sided, and if it is BIBO stable or not. Use "s" to represent σ.

Impulse Response	**Laplace Transform**	**Region of C**
	Causality	**BIBO**
$h_0(t) = 10e^{-7t}u(t) + 8e^{7t}u(t)$	_____	__
$h_1(t) = -8e^{-7t}u(-t) - 16e^{9t}u(-t)$	_____	__
$h_2(t) = 5e^{-4t}u(t) - 14e^{2t}u(-t)$	_____	__

**For regions of convergence, answer in interval notation e.g. (-INF,a), (a,b) or (b, INF).*
Answers to Causality and BIBO stability will only be marked correct if their corresponding Laplace transforms are correct.
Correct Answers:

- `10/(s+7)+8/(s-7)`
- `(7,infinity)`
- `Causal`
- `No`
- `8/(s+7)-16/(-s+9)`
- `(-infinity,-7)`

- Anti-Causal
- No
- 5/(s+4)-14/(-s+2)
- (-4,2)
- Two-Sided
- Yes

A system is represented by the ordinary differential equation

$$9\frac{d^2y(t)}{dt^2}+6\frac{dy(t)}{dt}+5y(t)=x(t)$$

where $x(t)$ is the input and $y(t)$ is the output.

a) Find the transfer function of the system, $H(s)=\frac{Y(s)}{X(s)}$.

$H(s)=$_____

b) Find the poles of the system. Separate your answers with commas.

$s=$_____

c) Is the system BIBO stable?

[?/Yes/No]

d) Suppose that the input $x(t)=u(t)$ and that the system is at rest. Find the steady state response of the system, $y_{ss}(t)$.

$y_{ss}(t)=$_____

*Part **c** will only be marked correct if part **b** is correct.*

Correct Answers:

- 1/(9*s^2+6*s+5)
- -0.333333+0.666667i, -0.333333-0.666667i
- Yes
- 0.2

In a continuous-time system, the laplace transform of the input $X(s)$ and the output $Y(s)$ are related by $Y(s)=\frac{(s-3)X(s)+6}{(s+2)^2+5}$.

a) If $x(t)=u(t)$, find the zero-state response of the system, $y_{zs}(t).y_{zs}(t)=$ _____

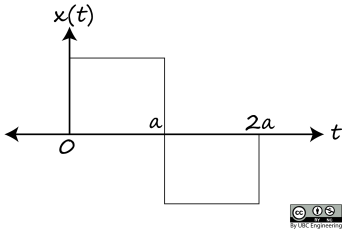
b) Find the zero-input response of the system, $y_{zi}(t).y_{zi}(t)=$ _____

c) Find the steady-state solution of the system, $y_{ss}(t).y_{ss}(t)=$ ___

Correct Answers:

- -0.333333*u(t)-(-0.333333)*e^(-2*t)*cos(2.23607*t)*u(t)+1.60607*e^(-2*t)*sin(2.23607*t)*u(t)/2.23607
- 6*e^(-2*t)*sin(2.23607*t)*u(t)/2.23607
- -0.333333

The input to an LTI system is shown in the graph below. Assume $a=6$.



Given that the Laplace transform of the output is $Y(s)=\frac{(s+6)(1-e^{-6s})^2}{s(s+2)^2}$,

a) Find the transfer function of the system and the region of convergence for $\sigma=Re(s).H(s)=$_____$RoC:$ ___

For regions of convergence, answer in interval notation e.g. (-INF,a), (a,b) or (b, INF).

b) Is the system BIBO stable for a causal input?[?/Yes/No]

c) Find the impulse response of the system.$h(t)=$___

*Part **b** will only be marked correct if the answers to part **a** are correct.*

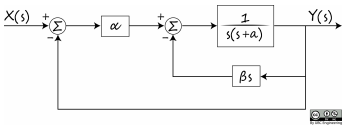
Correct Answers:

- (s+6)/[(s+2)^2]
- (-2,infinity)
- Yes
- e^(-2*t)*(1+4*t)*u(t)

For the feedback system shown in the figure below,

a) determine the overall transfer function, $H(s)$. Assume $a=5$, $\alpha=175$, $\beta=4$.

$H(s)=$ _____



b) For the input $u(t)$, is this system BIBO stable? [?/Yes/No]

*Part **b** will only be marked correct if the answer to part **a** is correct.*

Correct Answers:

- 175/(s^2+9*s+175)
- Yes

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