**Somith Das** **2019W2_ELEC_221_201**

**Assignment Problem_Set_10 due 04/08/2020 at 11:59pm PDT**

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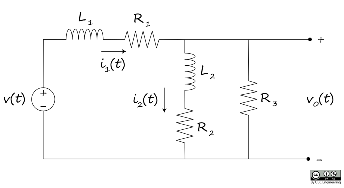
NB: Enter matrices with nested square brackets (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

In the circuit below, the input of the system is the voltage of the voltage source, $v(t)$, and the output is the voltage across the resistor, $v_o(t)$. Assume $R_1 = 3\ \Omega$, $R_2 = 9\ \Omega$, $R_3 = 5\ \Omega$, $L_1 = 8\ H$, and $L_2 = 4\ H$.




**a)** Find the state-space representation of the system and enter each of the $[A,B,C,D]$ matrices below using the state $x(t) = \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$

$A =$ _________

$B =$ _________

$C =$ _________

$D =$ _________

**b)** Find the observability matrix of this system

$M_o =$ _________

**c)** Is this system observable? [?/Yes/No]

*Part **c** will only be marked correct if part **b** is correct.*

*Correct Answers:*

- `[[-1,0.625],[1.25,-3.5]]`
- `[[0.125],[0]]`
- `[[5,-5]]`
- `[[0]]`
- `[[5,-5],[-11.25,20.625]]`
- `Yes`

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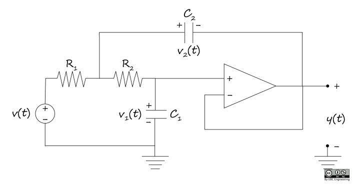
NB: Enter matrices with nested square brackets (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

The figure below shows an ideal op-amp circuit. The input to the system is from the voltage source, $v(t)$, and the output of the system is $y(t)$ as indicated on the figure. Find the equivalent state-space model of the system using the state vector $x(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$ and enter the $[A,B,C,D]$ matrices below. In the system, $R_1 = 6\ \Omega$, $R_2 = 6\ \Omega$, $C_1 = 4\ F$, and $C_2 = 3\ F$.




$A =$ _________

$B =$ _________

$C =$ _________

$D =$ _________

*Correct Answers:*

- `[[0,0.0416667],[-0.0555556,-0.111111]]`
- `[[0],[0.0555556]]`
- `[[1,0]]`
- `[[0]]`

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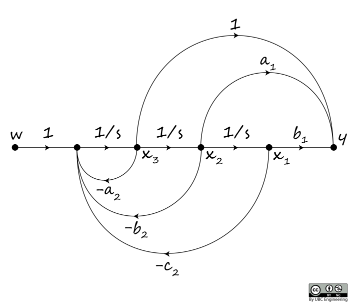
NB: Enter matrices with nested square brackets (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

The signal-flow graph of a system is given in the figure below. In the system, $a_1 = 3$, $b_1 = 6$, $a_2 = 5$, $b_2 = 2$, and $c_2 = 8$.




**a)** Find the state-space representation of the system in controller canonical form (use the form with ones on the superdiagonal of A). Enter each of the $[A,B,C,D]$ matrices below.

$A =$ ________

$B =$ ________

$C =$ ________

$D =$ ________

**b)** Find the corresponding transfer function, $H(s)$, of the system.

$H(s) =$ ________

**c)** Find the state-space representation of the system in observer canonical form (use the form with ones on the subdiagonal of A). Enter each of the $[A', B', C', D']$ matrices below.

$A' =$ ________

$B' =$ ________

$C' =$ ________

$D' =$ ________

*Correct Answers:*

- `[[0,1,0],[0,0,1],[-8,-2,-5]]`
- `[[0],[0],[1]]`
- `[[6,3,1]]`
- `[[0]]`
- `(s^2+3*s+6)/(s^3+5*s^2+2*s+8)`
- `[[0,0,-8],[1,0,-2],[0,1,-5]]`
- `[[6],[3],[1]]`
- `[[0,0,1]]`
- `[[0]]`

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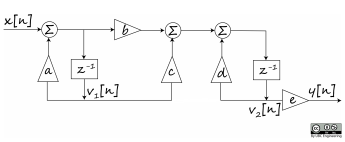
JY Note Apr 6, 2020: Please ignore part (b) which has an error in the answer.

NB: Enter your matrices with nested square brackets (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

The block-diagram representation of a discrete-time system is shown in the figure below. In the system, $a = 9$, $b = 4$, $c = 9$, $d = 4$, and $e = 6$.




**a)** Find the state-space representation of the system and enter each of the $[A, B, C, D]$ matrices below.

$A =$ ________

$B =$ ________

$C =$ ________

$D =$ ________

**b)** Determine the impulse response of the system for $n \geq 1$.

$h[n] =$ ________

*Correct Answers:*

- `[[9,0],[45,4]]`
- `[[1],[4]]`
- `[[0,6]]`
- `[[0]]`
- `[(-162)*9^(n-1)+(-42)*4^(n-1)]/5`

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The state and output equations of a continuous-time system are given by:

$$\frac{d}{dt}x_1(t) + 4x_1(t) = u(t)\frac{d}{dt}x_2(t) + x_1(t) + x_2(t) = 5u(t)y(t) = x_2(t)$$

$x_1(t)$ and $x_2(t)$ are the two state variables, $y(t)$ is the output and $u(t)$ is the unit-step input to the system. The initial conditions are: $x_1(0) = 4$, $x_2(0) = 0$. Find $x_1(t)$ and $x_2(t)$ for $t > 0$.

$x_1(t) =$ ________

$x_2(t) =$ ________

*Correct Answers:*

- `0.25+15*e^(-4*t)/4`
- `4.75+18*e^(-t)/(-3)+1.25*e^(-4*t)`

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JY Note Mar 24, 2020: There is a known issue with the eigenvalues for part (d) and potentially later parts that is being reviewed. You may wish to delay entering answers for this question until this is resolved.

NB: Enter your matrices with nested square brackets (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

Consider a discrete-time system whose input, $x[n]$ and output $y[n]$ are related by the difference equation below.

$$y[n] - 972y[n-2] - 971y[n-3] = x[n-1] - 971x[n-2] - 972x[n-3]$$

**a)** Compute the impulse response, $h[n]$ of the system for $0 \leq n \leq 7$ and enter it in the table below.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $h[n]$ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ |

**b)** Find the controllable canonical realization of this system, $[A_c, B_c, C_c, D_c]$, and enter the controllability matrix, $M_c$, and the observability matrix, $M_o$ below.

$M_c =$ _________

$M_o =$ _________

**c)** Is the system controllable? [?/Yes/No] Is the system observable? [?/Yes/No]

**d)** Find and the eigenvalues of $A_c$ and enter them in the table below. Then select if the system is stable, marginally stable or unstable.

| Eigenvalue of $A_c$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | Stability |
|---|---|---|---|---|
| | ___ | ___ | ___ | • ?<br>• Marginally stable<br>• Asymptotically stable<br>• Unstable |

**e)** For the controllable realization of the system that you have found, find an input sequence to drive the state from $v[0] = [000]^T$ to $v[3] = [754]^T$ and then back to $v[6] = [000]^T$. Enter your answer as an ordered list, separated by commas (e.g. $x(0)$, $x(1)$, $x(2)$, $x(3)$, $x(4)$, $x(5)$).

$x(0) \cdots x(5) =$ _________

**f)** Find the observable canonical realization of this system, $[A_o, B_o, C_o, D_o]$, and enter the controllability matrix, $M_c'$, and the observability matrix, $M_o'$ below.

$M_c' =$ _________

$M_o' =$ _________

**g)** Is the system controllable? [?/Yes/No] Is the system observable? [?/Yes/No]

**h)** For the observable canonical realization of the system that you have found, determine the initial state $v[0]$ given that you observe, with $x[0] = x[1] = x[2] = 9$, $y[0] = 4$, $y[1] = 3$ and $y[2] = 2$. Enter the vector $v[0]$ below:

$v[0] =$ _________

*Part **c** will only be marked correct if the answer to part **b** is correct. Part **g** will only be marked correct if the answer to part **f** is correct.*

*Correct Answers:*

- 0
- 1
- -971
- 0
- -942841
- -942841
- -9.16441E+08
- -1.83194E+09
- [[0,0,1],[0,1,0],[1,0,972]]
- [[-972,-971,1],[971,0,-971],[-942841,-942841,0]]
- Yes
- No
- -18
- -18
- 2.99691
- Unstable
- 7, 5, -6800, -11657, -8743, -3884
- [[-972,971,-942841],[-971,0,-942841],[1,-971,0]]
- [[0,0,1],[0,1,0],[1,0,972]]
- No
- Yes
- [[4844],[-6],[4]]

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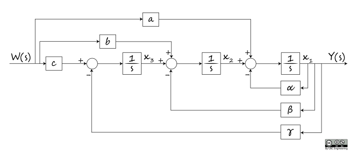
JY Note Apr 4, 2020: Changed entry method for C matrices to be consistent with other parts.

NB: Enter matrices with nested square brackets (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

The figure below shows the block-diagram for the observer canonical form representation of an LTI system. Assume that $x_1(t)$, $x_2(t)$ and $x_3(t)$ are the the three state variables, $w(t)$ is the input and $y(t)$ is the output. In the system, $a = 4$, $b = 9$, $c = 7$, $\alpha = 7$, $\beta = 6$, and $\gamma = 3$.




**a)** Find the corresponding state-space representation of the system and enter the $[A, B, C, D]$ matrices below.

$A =$ _________

$B =$ _________

$C =$ _________

$D =$ _________

**b)** Write the transfer function for the system, $H(s) = \frac{Y(s)}{W(s)}$

$H(s) =$ _________

**c)** Use the transformation matrix $T = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ to obtain a different set of state-variables $v(t) = Tx(t)$. Enter the new set of $[\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}]$ below.

JY Note Apr 6, 2020: $A = T\tilde{A}T^{-1}$

$\tilde{A} =$ ________

$\tilde{B} =$ ________

$\tilde{C} =$ ________

$\tilde{D} =$ ________

**d)** Is the new representation using $[\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}]$ equivalent to the one you found in part **a**? [?/Yes/No]

*Part **d** will only be marked correct if the answers to part **c**are correct.*

*Correct Answers:*

- `[[-7,1,0],[-6,0,1],[-3,0,0]]`
- `[[4],[9],[7]]`
- `[[1,0,0]]`
- `[[0]]`
- `(4*s^2+9*s+7)/(s^3+7*s^2+6*s+3)`
- `[[0,-3,-11],[-0.2,0.2,0.4],[0.6,-3.6,-7.2]]`
- `[[9],[2],[1]]`
- `[[0,1,2]]`
- `[[0]]`
- `Yes`

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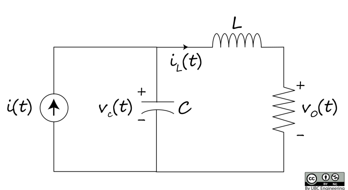
NB: Enter matrices with nested square brackets (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

Consider an RLC circuit shown in the figure below with$R = 70$ $\Omega$, $L = 7$ $H$, and $C = \frac{1}{147}$ $F$. The state of this system can be described by a set of state variables ($x_1$,$x_2$), where $x_1$ is the capacitor voltage , $v_c(t)$, and $x_2 s$ is the inductor current, $i_L(t)$. The input from is the current source, $i(t)$, and the output is the voltage across the resistor, $v_0(t)$.




**a)** Find the state-space representation of the system. Enter the corresponding $[A, B, C, D]$ matrices below.

$A =$ ________

$B =$ ________

$C =$ ________

$D =$ ________

**b)** Find the transfer function, $H(s)$, of this RLC circuit.

$H(s) =$ ________

**c)** Find the state-transition matrix, $\Phi(t)$.

$\Phi(t) =$ ________

**d)** Find the time-domain response, $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ of the circuit for zero input ($i(t) = 0$) with initial conditions $x_1(0) = 4$, $x_2(0) = 5$.

$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} =$ ________

*Correct Answers:*

- `[[0,-147],[0.142857,-10]]`
- `[[147],[0]]`
- `[[0,70]]`
- `[[0]]`
- `1470/(s^2+10*s+21)`
- `-0.25*[[3*e^(-7*t)-7*e^(-3*t),(-147)*[e^(-7*t)-e^(-3*t)]],`
- `-0.25*[[3*e^(-7*t)-7*e^(-3*t),(-147)*[e^(-7*t)-e^(-3*t)]],`

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JY Note Apr 6, 2020: For (b), ensure your first row is normalized to have all ones.

NB: Enter matrices with nested square brackets (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

For the matrix $A = \begin{bmatrix} 0 & 8 \\ -\frac{10}{8} & 7 \end{bmatrix}$,

**a)** Diagonalize the matrix (ie. find matrices $T$ and $D$ such that $A = TDT^{-1}$) and enter the diagonal matrix below.

$D =$ ________

**b)** Enter the $T$ matrix below.

$T =$ ________

*Part **b** will only be marked correct if part **a** is correct.*

**c)** Use your answer from part a to find $A^n$ and $A^{-1}$.

$A^n =$ ________

$A^{-1} =$ __________

**d)** Use your answer from part a to find $e^{At}$.

$e^{At} =$ __________

**e)** The Cayley-Hamilton Theorem (CHT) requires existence of coefficients $\alpha_0$ and $\alpha_1$ such that $f(A) = \alpha_0 I + \alpha_1 A$. Find $\alpha_0$ and $\alpha_1$ when $f(A) = A^n$.

$\alpha_0 =$ __________

$\alpha_1 =$ __________

**Important:** Try using the coefficients you found in part **d** to redo parts **b** and **c** and verify that you get the same answer using both approaches. Note that you should be able to use both CHT and diagonalization technique in an exam.

*Correct Answers:*

- `[[2,0],[0,5]] or [[5,0],[0,2]]`
- `[[1,1],[0.25,0.625]]`
- `[[1,1],[0.25,0.625]]*[[2^n,0],[0,5^n]]*[[1.66667,-2.66667]`
- `[[0.7,-0.8],[0.125,0]]`
- `[[1,1],[0.25,0.625]]*[[e^(2*t),0],[0,e^(5*t)]]*[[1.66667,-`
- `(5*2^n-2*5^n)/3`
- `(5^n-2^n)/3`