Q2)

A) $\varepsilon_1 = 6 \quad x < 0 \qquad \varepsilon_{r2} = 4 \quad x \geq 0$

$E_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \qquad E_2 = ?$ $\qquad E_{1T} = E_{2T}$ & $D_{1N} = D_{2N}$

$\varepsilon_1 E_{1N} = \varepsilon_2 E_{2N}$

$n = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad t = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$E_{1N} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \qquad E_{1T} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = E_{2T}$

$E_{2N} = \frac{\varepsilon_1}{\varepsilon_2} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -6/4 \\ 0 \end{bmatrix} \qquad \boxed{E_2 = \begin{bmatrix} 2 \\ -3/2 \\ 1 \end{bmatrix}}$

B)

$2x + 6y + 4z = 1 \qquad \varepsilon_{r1} = 6 \quad \varepsilon_{r2} = 1$

$\varepsilon_{r2} = 1$

$E_1 = 2\hat{a}_x = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \qquad \varepsilon_{r1} = 6$

$E_1 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

$\vec{n} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} \qquad \hat{n} = \frac{1}{\sqrt{56}} \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$

$E_{1N} = \frac{1}{\sqrt{56}} \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} \qquad E_{2N} = \frac{\varepsilon_1}{\varepsilon_2} E_{1N}$

$E_{1T} = E_1 - E_{1N} = \begin{bmatrix} 1.465477 \\ 0 \\ 0 \end{bmatrix} = E_{2T}$

$$\boxed{E_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}}$$

c)

$\varepsilon_{r_2} = 3$ $\varepsilon_{r_1} = 1$

$\uparrow$ cylinder $\uparrow$ sphere $r = 6cm$ @ origin

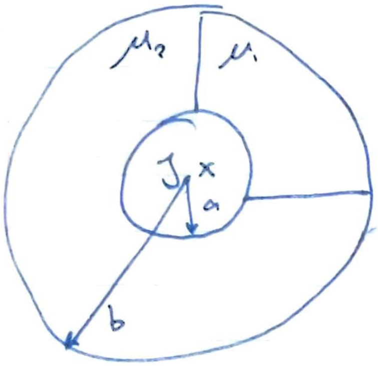
$Q = 1 \times 10^{-15}\ C$ @ $\Rightarrow (r, \theta, \phi) = (r/2, 0°, 0°)$

$\underbrace{P = (r/2, 60°, 0°)}_{\text{still within region 1}}$

$\vec{D}$ @ P $E = \dfrac{q}{R^2 4\pi\varepsilon_0} = \dfrac{Q}{\pi\varepsilon_0 r^2}$

$D_1 = \varepsilon_{r1}\varepsilon_0 \vec{E} = \dfrac{Q}{\pi r^2} = 8.842 \times 10^{-14}$ C/m^2

Q3



A) $J = \begin{bmatrix} 0 \\ 0 \\ 9 \end{bmatrix}$ $\mu_{r1} = \mu_{r2} = 9$

$H \ (0 < \rho < b)$

$$H = \frac{\iint_S J \cdot n\,dS}{2\pi\rho} \quad \star\ a < \rho < b$$

$$= \frac{\int_0^{2\pi}\int_0^{a} 9\rho\, d\rho\, d\phi}{2\pi\rho}$$

$$H = \frac{9a^2}{2\rho}\,\hat{a}_\phi \quad (a < \rho < b)$$

$0 < \rho < a$

$$H = \frac{\int_0^{2\pi}\int_0^{\rho} 9\rho\, d\rho\, d\phi}{2\pi\rho}$$

$$= \frac{2\pi \frac{9}{2}\rho^2}{2\pi\rho}$$

$$H = 4.5\rho\,\hat{a}_\phi \quad 0 < \rho < a$$

B) $J = 4 + \frac{4}{a}\rho\,\hat{a}_z \quad 0 < \rho < a$

$$H = \frac{\int_0^{2\pi}\int_0^{\rho}\left(4 + \frac{4}{a}\rho\right)\rho\, d\rho\, d\phi}{2\pi\rho}\,\hat{a}_\phi$$

$$H = \frac{2\rho^2 + \frac{4}{3a}\rho^3}{\rho}\,\hat{a}_\phi$$

$$H = 2\rho + \frac{4}{3a}\rho^2\,\hat{a}_\phi$$

C)

$\vec{J} = 4 + \frac{4\rho}{a}\,\hat{a_z}$ $\mu_{r1} = 9 \quad \mu_{r2} = 3\mu_{r1}$

$H \quad a < \rho < b$

H_1 is normal to H_2

$$H = \frac{\int_0^{2\pi}\int_0^{a}\left(4\rho + \frac{4}{a}\rho^2\right) d\rho\, d\phi}{2\pi\rho}$$

$" \frac{4}{2}a^2 + \frac{4}{3}a^3 "$

$$= \frac{\left(2 + \frac{4}{3}\right)a^2}{2\pi\rho}\,\hat{a}_\phi \quad \therefore \quad H_2 = \frac{10a^2}{3\rho}$$

$\cancel{H_1 = H_2}$ $B_{1N} = B_{2N} \therefore \mu_{r1} H_1 = \mu_{r2} H_2$

$$H_1 = \frac{\cancel{\mu_{r2}}^{\,3}}{\cancel{\mu_{r1}}} H_2 = \frac{10a^2}{\rho}$$

D)

$L = \frac{N\Phi}{I}\Big|_{N=1}$

$\Phi = \iint_S B \cdot \hat{n}\, dS$ Flux in = Flux out

$$= \int_a^b \int_0^z \mu_n \mu_0 H_1\, dz\, d\rho$$

$$\frac{L}{z} = \mu_{r1}\mu_0 \ln(b/a)$$

unit length

$$= \frac{\mu_{r1}\mu_0 a^2 \ln(b/a) \cdot 10}{3}\, z$$

$$I = \frac{10}{3}a^2$$

Q4)

a) $\begin{bmatrix} 2y \\ xz \\ -x+3 \end{bmatrix} = \vec{F}$ $\quad \nabla \cdot F = 0+0+0=0$

$$\int_R \nabla \cdot F \, dv = 0$$

cap $x^2+y^2=16$

$y=(16-x^2)^{1/2}$

$\hat{n} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$x\,[0,4]$

$$\iint -x+3 \; dx\,dy$$

$$4\int_0^4 \int_0^{(16-x^2)^{1/2}} (-x+3)\,dy\,dx = (16.36578)4$$

$$\iint_M \vec{F} \cdot \vec{n}\,ds = \iiint_R \nabla \cdot F\,dv - \iint_{cap} \vec{F} \cdot n\,ds$$

$$\boxed{= -65.463}$$

b) $F = \begin{bmatrix} -y \\ x \\ 5z \end{bmatrix}$ $\nabla \cdot F = 5$ cap @ bottom & top @ $z = \pm 6$

$$\iint_M F \cdot \hat{n}\, dS = \iiint_R (\nabla \cdot F)\, dS - 2\iint_{\substack{\text{Cap} \\ \text{Surface}}}^{x^2+y^2=36} F \cdot \hat{n}\, dS$$

$$\int_R \nabla \cdot F\, dS = \int_{-6}^{6} \int_{0}^{2\pi} \int_{0}^{6} 5\rho\, d\rho\, d\phi\, dz = 2160\pi$$

$$2\iint_{\text{cap surface}} F \cdot \hat{n}\, ds = 8\int_0^6 \int_0^{(36-x^2)^{1/2}} 5\cancel{z}^{\,6}\, \cancel{dx}\, dy\, dz = 2160\pi$$

$$\boxed{\iint_M F \cdot \hat{n}\, dS = 0}$$

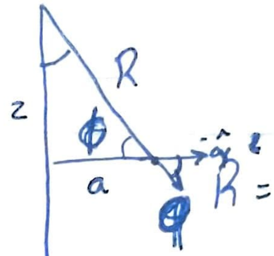
COS

Half pipe

$x^2 + y^2 = a^2 \qquad y \geq 0 \qquad \rho_s = \rho_0 \ \text{nC/m}^2$

$dQ = \rho_s \, dz \, d\theta$

$\dfrac{dQ}{4\pi\varepsilon_0 R^2}$

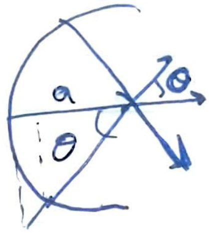


$R = a^2 + z^2$

$E = \dfrac{\rho_s \, dz \, d\theta}{4\pi\varepsilon_0 (a^2+z^2)} \cos\theta \dfrac{a}{(a^2+z^2)^{1/2}}$

$-\hat{a}_y$ - Resultant $\vec{E}_{field}$

$\cos\phi = \dfrac{a}{(a^2+x^2)^{1/2}}$



$\hat{a}_r$

$E_{resultant} \ \hat{a}_y$

$\cos\theta$

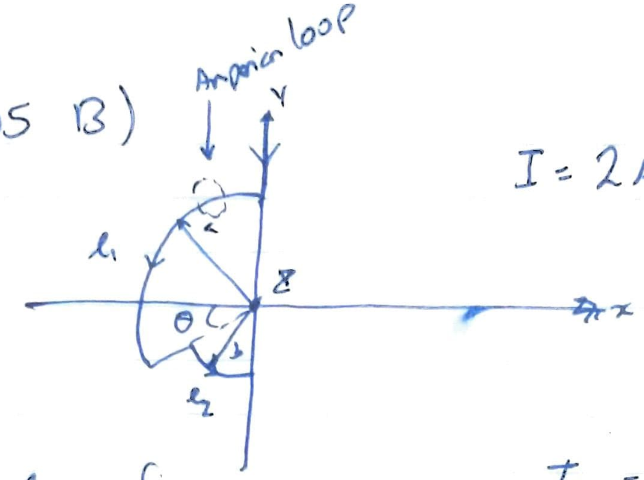
$E = \dfrac{\rho_s}{4\pi\varepsilon_0} \displaystyle\int_{-\infty}^{\infty}\int_{\pi/2}^{3\pi/2} \dfrac{a\cos\theta}{(a^2+z^2)^{3/2}} \, d\theta \, dz$

$= \dfrac{\rho_s}{4\pi\varepsilon_0} \displaystyle\int_{-\infty}^{\infty} \dfrac{a}{(a^2+z^2)^{3/2}} \underbrace{\left[\sin\theta\right]_{\pi/2}^{3\pi/2}}_{-2} dz$

$= \dfrac{-2\rho_s}{4\pi\varepsilon_0} \left[\dfrac{z}{a(a^2+z^2)^{1/2}}\right]_{-\infty}^{\infty} = \dfrac{-\rho_s}{2\pi\varepsilon_0 a} \cdot 2 = \dfrac{-\rho_s}{\pi\varepsilon_0 a} \hat{a}_y$

$\vec{E} = -31.4571 \ \hat{a}_y$

Q5 B)



$I = 2A$ $a = 10cm$

$b = 4cm$

$\theta = \pi/5$ rad

l_1: $\oint H \cdot dL = I_{enc}$ $\qquad I_{enc} = I\,d\theta$

$$H_1 = \int \frac{I\,d\theta}{2\pi a} = \int_{\pi/2}^{\pi/2+\theta} \frac{I\,d\theta}{2\pi a} = \frac{I\theta}{2\pi a} = 2\hat{a}_z$$

l_2:

$$H_2 = \int_{\pi/2+\theta}^{\pi} \frac{I\,d\theta}{2\pi b} = \frac{(\pi/2 - \theta)I}{2\pi b}\hat{a}_z = 7.5\,\hat{a}_z$$

$$H_T = H_1 + H_2 = 9.5\,\hat{a}_z$$

Q6)

A) $N = 159$ $\quad 7 \times 7cm$ $\quad z = 10cm$ plane

$I = 415mA$ $\quad$ CW $\quad B = \begin{bmatrix} 3 \\ 2 \\ 9 \end{bmatrix}$

$\tau = \vec{m} \times \vec{B}$ $\qquad m = NIS\,\hat{a_z}$

$$\tau = \left[\left| \begin{matrix} a_x & a_y & a_z \\ 0 & 0 & NIS \\ 3 & 2 & 9 \end{matrix} \right|\right] = \begin{bmatrix} -2NIS \\ 3NIS \\ 0 \end{bmatrix} = \begin{bmatrix} -0.646653 \\ 0.9699795 \\ 0 \end{bmatrix} Nm$$

B)

$K = 19\hat{a_x}$ $\qquad z = 0$ plane $\qquad w = 105$

$$H = \frac{1}{2} K \times \hat{a_z} = \frac{1}{2} \begin{bmatrix} 0 \\ -19 \\ 0 \end{bmatrix} \qquad \Phi = \underbrace{\iint B \cdot \hat{n}\, dS = 0 \quad \hat{n}\, dS = a_z\, dx\, dy}_{\text{Flux} = 0}$$

(i) $\mathcal{E}_{mf} = 0$

(ii) y direction $\quad w = 105$

$$\vec{B} = \frac{\mu_0}{2} \begin{bmatrix} 0 \\ -19 \\ 0 \end{bmatrix} \qquad S = 7 \times 7cm \quad \mathcal{E}_{mf} = -wBS =$$

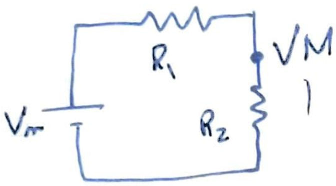
$$\mathcal{E}_{mf} = 6.259 \times 10^{-6}\ V$$

Q6 Pt C)

$S_1 = 6cm^2$ $l_1 = 10cm$ $|B_1|_{max} = 1.5T$

$S_2 = 1cm^2$ $l_2 = 8cm$

$R_1 = \frac{l_1}{\mu S_1}$ & $R_2 = \frac{l_2}{\mu S_2}$



$$\Phi_{in} = \Phi_{out} \therefore \Phi_1 = \Phi_2$$

$$R_1 + R_2 = V_m\left(\frac{1}{\Phi_1} + \frac{1}{\Phi_2}\right)$$

$$V_m = \frac{R_1 + R_2}{2/\Phi_1}$$

$$VM = V_m \cdot \frac{R_2}{R_1+R_2} = \frac{R_2}{2/\Phi_1} = \frac{\Phi_1 R_2}{2} = \frac{B_1 S_1 \cdot l_2/S_2}{2\mu}$$

$$VM = \frac{B_1 S_1 l_2}{2\mu S_2} = \frac{0.36}{\mu} = 286478.9\ V$$

$\mu = \mu_r \mu_0$

not given assume $\mu_r = 1$

Q7

a)

$M \quad x^2+y^2+z^2=36 \quad z \geq 0$ upwards

$$\vec{F} = \begin{bmatrix} 6y + z\cos(y) \\ -12x + z^2 e^{x} \\ \ln(z+1) \end{bmatrix} \qquad L(t)\begin{cases} x = 6\cos(t) \\ y = 6\sin(t) \\ z=0 \end{cases}$$

$$L(t) = \begin{bmatrix} 6\cos(t) \\ 6\sin t \\ 0 \end{bmatrix} \qquad t \in [0, 2\pi]$$

$$\vec{F} = \begin{bmatrix} 36\sin(t) \\ -72\cos(t) \\ 0 \end{bmatrix} \qquad dl = \begin{bmatrix} -6\sin t \\ 6\cos t \\ 0 \end{bmatrix} dt$$

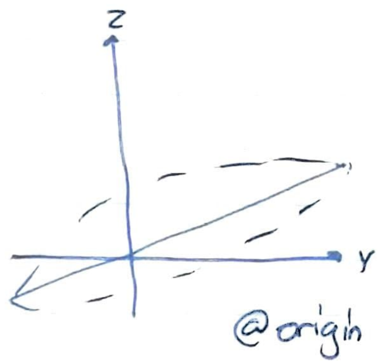
$$\int_0^{2\pi} -216\sin^2(t) - 432\cos(t)\, dt$$

$$= \left[-216\left(\frac{t}{2} - \frac{1}{4}\sin(2t)\right) - 432\left(\frac{t}{2} + \frac{1}{4}\sin(2t)\right)\right]_0^{2\pi}$$

$$-216\pi - 432\pi = -648\pi \Rightarrow = -2035.752$$

7b)
b)

$y = 4z$ circle in this plane



$r = 6$

$$\vec{F} = \begin{bmatrix} 3z + e^{(x^2)} \\ x + \cos 2^y \\ y + \ln(\ln(z + 1 4 1)) \end{bmatrix}$$

$$L(t) = \begin{bmatrix} 6\cos t \\ 6\sin t \\ 6/4 \sin t \end{bmatrix} \quad t(0, 2\pi)$$

$x = 6\cos t$
$y = 6\sin t$
$z = \frac{6}{4}\sin t$

$$F = \begin{bmatrix} \frac{18}{4}\sin t + e^{36\cos^2 t} \\ 6\cos(t) + \cos\left(2^{6\sin(t)}\right) \\ 6\sin(t) + \ln\left(\ln\left(\frac{6}{4}\sin t + 14\right)\right) \end{bmatrix}$$

$$dl = \begin{bmatrix} -6\sin t \\ 6\cos t \\ \frac{6}{4}\cos t \end{bmatrix}$$

$$\int_e F \cdot \hat{n}\, dl = -73.0325$$