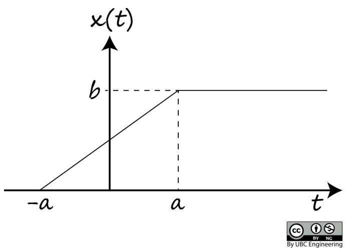
Somith Das **2019W2_ELEC_221_201**

Assignment Problem_Set_5 due 02/25/2020 at 11:59pm PST

Consider the signal $x(t)$ given in the figure below. Assume $a = \frac{1}{3}$ and $b = 2.66666666666667$.



a) Find a closed form expression for the Fourier transform $X(\omega)$ of the signal $x(t)$.

Hint: Use the integration and differentiation properties, as well as the Fourier transform pair of a rectangular pulse.

$X(\omega) =$ ______

b) Find the Fourier transform $G(\omega)$ of the signal $g(t) = x(t) - \frac{4}{3}$.

$G(\omega) =$ ______

In your answers, enter $D(t)$ instead of $\delta(t)$, and "w" for ω.

Correct Answers:

- `8*sin(w/3)/(j*w^2)+8*pi*D(w)/3`
- `8*sin(w/3)/(j*w^2)`

The frequency response of a causal continuous-time LTI system is given by $H(\omega) = \frac{j\omega+9}{32-\omega^2+12j\omega}$. The input signal to the system is $x(t)$ and the output is $y(t)$.

a) Find the differential equation that describes this system. ________

In your answers, enter "zpp" for $\frac{d^2z(t)}{dt^2}$, "zp" for $\frac{dz(t)}{dt}$, and "z" for $z(t)$, for any function $z(t)$.

b) Find the impulse response, $h(t)$, of the system.

$h(t) =$ ________

c) Find the output, $y(t)$, of the system when the input is $x(t) = e^{-9t}u(t) - te^{-9t}u(t)$.

$y(t) =$ ________

Correct Answers:

- `ypp+12*yp+32*y = xp+9*x`
- `1.25*e^(-4*t)*u(t)+(-0.25)*e^(-8*t)*u(t)`
- `(-0.2)*e^(-9*t)*u(t)-(-0.2)*e^(-4*t)*u(t)`

The transfer function of a filter is $H(s) = \dfrac{\sqrt{21}s}{s^2+(2\sqrt{5})s+6}$.

a) Find the poles, s_1 and s_2, and the zero, s_0, of the system.

$s_{1,2} =$ ____ $s_0 =$ ____

Enter the poles as a list, seperated by commas

b) Find the magnetude and phase (in degrees) of the frequency response at each of the frequencies given in the table below:

ω	$\lvert H(\omega)\rvert$	$\angle H(\omega)$ (degrees)
0	_____	_____
1	_____	_____
∞	_____	_____

c) What type of filter is this?

- ?
- High-pass
- Low-pass
- Band-pass
- Band-rejection

d) Find the impulse response, $h(t)$, of the filter.

$h(t) =$ ________

e) This filter is connected in series with a sinusoidal signal generator which generates biased sinusoids $x(t) = B + A\cos(\omega t)$ as the input to the filter. Find the frequency (or frequencies) ω_0 where the filter's output is $y(t) = A\cos(\omega_0 t + \theta)$, i.e, the amplitude of the sinusoid remains the same. If there are multiple frequencies where this occurs, enter the positive frequencies only as a list, separated by commas in decreasing order.

$\omega_0 =$ ____ rad/s

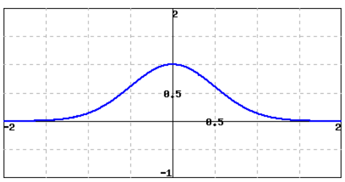
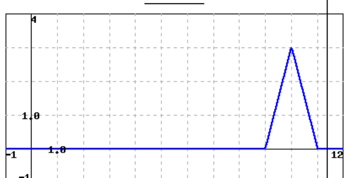
Correct Answers:

- `-2.23607-i, -2.23607+i`
- `0`
- `0`
- `0`
- `0.68313`

- `48.1897`
- `0`
- `0`
- `Band-pass`
- `4.58258*e^(-2.23607*t)*[cos(t)-2.23607*sin(t)]*u(t)`
- `3, 2`

For each of the signals given in the table below, first find the Fourier transform. Simplify your answer as much as possible and enter it as an expression consisting of a single term. Then, graph the Fourier transform, and from the figures 1 to 4 given at the bottom, find the one that best matches the Fourier transform that you have graphed.

Hint: For (A), one approach to find the FT is to apply the properties of Duality and of differentiation (both in time and frequency).

	Signal, $x(t)$	**FT of signal,** $X(\omega)$	**Figure Number**
A 	e^{-2t^2}	______	[?/1/2/3/4]
B			[?/1/2/3/4]

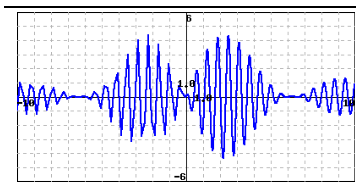


Figure 1

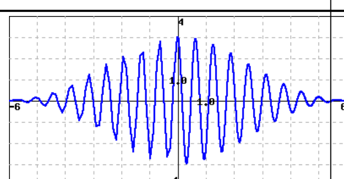


Figure 2

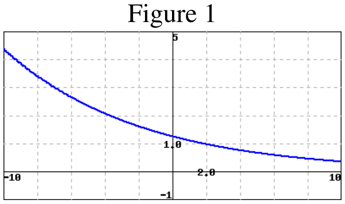


Figure 3

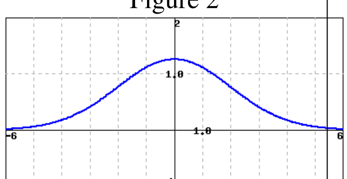


Figure 4

Correct Answers:

- `sqrt(pi/2)*e^(-w^2/8)`
- `4`
- `3*e^[(-10i)*w]*([sin(w/2)]/(w/2))^2`
- `2`

A system is described by the differential equation $\frac{d}{dt}y(t) + 7y(t) = 4\frac{d}{dt}x(t) + x(t)$, where $x(t)$ is the input and $y(t)$ is the output of the system.

In your answers below, enter $D(t)$ instead of $\delta(t)$, and "w" for ω.

a) Find the frequency response of the system, $H(\omega)$.$H(\omega) =$ ________

b) Find the impulse response of the system, $h(t)$.$h(t) =$ ________

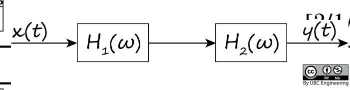
c) Find the output signal $y(t)$ that corresponds to the input signal $x(t) = e^{-6t}u(t)$. Solve the problem in the time as well as the frequency domain.$y(t) =$ ________

Correct Answers:

- `(1+4*i*w)/(7+i*w)`
- `4*D(t)+(-27)*e^(-7*t)*u(t)`
- `27*e^(-7*t)*u(t)+(-23)*e^(-6*t)*u(t)`

JY Note Feb 29, 2020: Corrected with impulses as described during Feb 28 tutorial.

Two filters with frequency responses $H_1(\omega) = jw$ and $H_2(\omega) = e^{-3jw}$ for $-\infty < \omega < \infty$, are cascaded together so that the output of the first filter is fed as the input to the second, as shown in the figure below.



Suppose that the input to this cascaded system is the signal $x(t) = cos\left(\frac{\pi t}{4}\right)[u(t+3) - u(t-3)]$.

a) Find the output, $y(t)$, of this cascaded system.$y(t) =$ ________

Hint: think about what each filter does and do your calculations in the time domain.

b) We now reverse the cascading order of the two filters. Find the output, $y'(t)$, of this new cascaded system.$y'(t) =$ ________

c) Does the output depend on the order of cascading? [?/Yes/No]

*Part **c** will only be marked correct if parts **a** and **b** are correct.*

Correct Answers:

- `-pi/4*sin(pi/4*(t-3))*[u(t+0)-u(t-6)]-D(t+0)+D(t-6)`
- `-pi/4*sin(pi/4*(t-3))*[u(t+0)-u(t-6)]-D(t+0)+D(t-6)`
- `No`

Consider an ideal low-pass filter with frequency response:

$$H(\omega) = \begin{cases} 5 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

The input to this filter is $x(t) = e^{-9t}u(t)$. Find the value of ω_c such that this filter passes exactly one-half of the normalized energy of the input signal.

$\omega_c =$___ rad/s

Correct Answers:

- 0.282836

NB: In this Webwork problem, take $sinc(t) = sin(t)/t$ (in contrast, in Signal Processing literature, $sinc(t) = sin(\pi t)/(\pi t)$.

Find the Fourier transform $X_1(\omega)$, $X_2(\omega)$, and $X_3(\omega)$ of the signals $x_1(t)$, $x_2(t)$, and $x_3(t)$, using the Fourier transform pair $x(t) = u(t+1) - u(t-1) \longleftrightarrow X(\omega) = 2sinc(\omega)$. Then select the Fourier transform property you used for each signal, from the corresponding drop-down menu.

In your answers, enter "w" for omega.

a) $x_1(t) = -3u(t+2) + 5u(t) - 2u(t-2) X_1(\omega) =$ ________

- ?
- Modulation
- Time-shift
- Compression in time
- Duality
- Expansion in time
- Frequency-shift

b) $x_2(t) = cos(5\pi t)[u(t+1) - u(t-1)] X_2(\omega) =$ ________

- ?
- Modulation
- Time-shift
- Compression in time
- Duality
- Expansion in time
- Frequency-shift

c) $x_3(t) = 3[u(t+\frac{1}{3}) - u(t-\frac{1}{3})] X_3(\omega) =$ ________

- ?
- Modulation
- Time-shift
- Compression in time
- Duality
- Expansion in time
- Frequency-shift

Answers to the drop-downs will only be marked correct if their corresponding FT's are correct.

Correct Answers:

- `2*sinc(w)*[-3*e^(i*w)+2*e^(-i*w)]`
- `Time-shift`
- `sinc(w+pi*5)+sinc(w-pi*5)`
- `Modulation`
- `2*sinc(w/3)`
- `Compression in time`