Elec 341 Assignment #1

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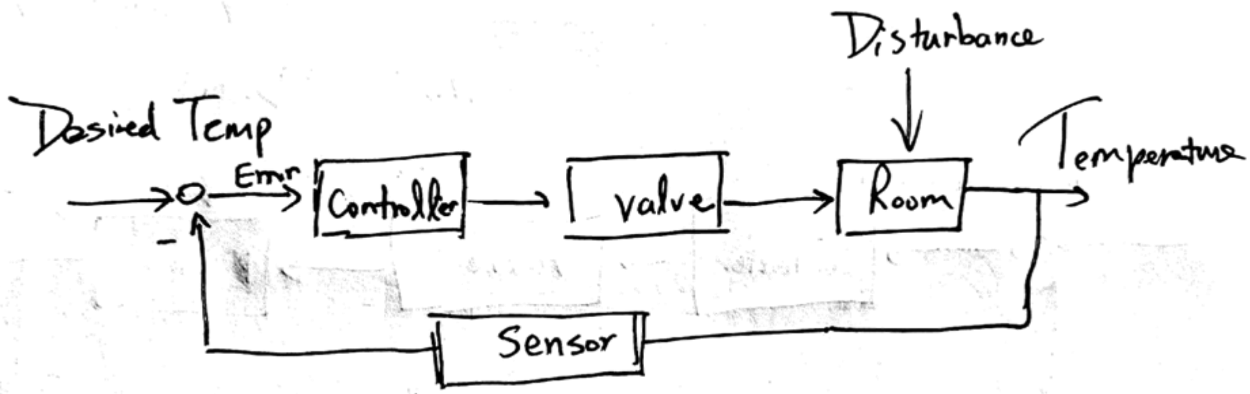
Assignment 1 (ELEC 341 L1_Introduction)

Problem 1:

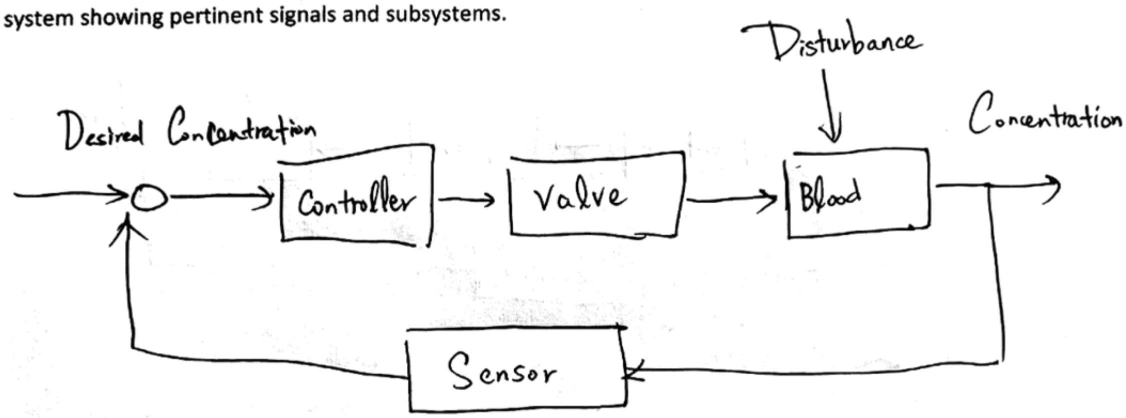
A temperature control system operates by sensing the difference between the thermostat setting and the actual temperature and then opening a fuel valve an amount proportional to this difference. Draw a functional closed-loop block diagram identifying the input and output transducers, the controller, and the plant. Further, identify the input and output signals of all subsystems.



Assignment 1 (ELEC 341 L1_Introduction)

Problem 2:

During a medical operation an anesthesiologist controls the depth of unconsciousness by controlling the concentration of isoflurane in a vaporized mixture with oxygen and nitrous oxide. The depth of anesthesia is measured by the patient's blood pressure. The anesthesiologist also regulates ventilation, fluid balance, and the administration of other drugs. In order to free the anesthesiologist to devote more time to the latter tasks, and in the interest of the patient's safety, we wish to automate the depth of anesthesia by automating the control of isoflurane concentration. Draw a functional block diagram of the system showing pertinent signals and subsystems.



Assignment 2 (ELEC 341 L2_LaplaceTransform)

Problem 1:

Find the inverse Laplace transform of the following function using tabulated Laplace transform pairs:

$F_1(s) = 1/(s+3)^2$

$$F_1(s) = \frac{1}{(s+3)^2} = \frac{1}{s^2+6s+9}$$

$$\downarrow$$

$$= e^{-3t}\, t\, u(t)$$

TABLE 1 Laplace transform table

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

TABLE 2 Laplace transform theorems

Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)]$	$= F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem ✓
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.

[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Assignment 2 (ELEC 341 L2_LaplaceTransform)

Problem 2:

Find the final value of $f(t)$ for the given $F(s)$ without calculating explicitly $f(t)$

$$F(s) = \frac{2s+51}{47s^2+67s}$$

using F.V.T. $\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$

$$\lim_{s \to 0} sF(s) = \frac{2s+51}{47s^2+67s} \cdot s = \frac{2s+51}{\cancel{47s^2+67s}} \cancel{s}$$

$$\cancel{s}(47s+67)$$

$$= \frac{\cancel{2s}+51}{47\cancel{s}+67} = \boxed{\frac{51}{67}}$$

Assignment 3

Friday, May 24, 2019 10:03 AM

Problem 1:

(1a) $\dot{x} + 7x = 5\cos 2t$

$sX(s) + 7X(s) = \dfrac{5s}{s^2+2^2}$

$X(s)(s+7) = \dfrac{5s}{s^2+2^2}$

$X(s) = \dfrac{5s}{(s^2+2^2)(s+7)}$

$X(s) = \dfrac{A}{s+7} + \dfrac{Bs+C}{s^2+2^2}$

$(s+7)(X(s)) \overset{s=-7}{=} A = \dfrac{5s}{s^2+4} = -\dfrac{35}{53}$

then $A(s^2+2^2) + Bs(s+7) + C(s+7) = 5s$

s^2: $A + B = 0$

$\rightarrow B = -A = \dfrac{35}{53}$

then,

s^0: $4A + 7C = 0$

$C = \frac{4}{7}A =$

$X(s) = \dfrac{A}{s+7} + \dfrac{Bs+C}{s^2+2^2}$

$\downarrow \mathcal{L}^{-1}$

$x(t) = Ae^{-7t}u(t) + Bu(t)\cos 2t + \frac{C}{2}u(t)\sin 2t$

$$\boxed{X(t) = \frac{-35}{53}e^{-7t}u(t) + \frac{35}{53}u(t)\cos 2t + \frac{10}{53}u(t)\sin 2t}$$

(1)(B) $\ddot{x} + 6\dot{x} + 8x = 5\sin 3t$

$\mathcal{L} \downarrow$ 6. $\sin \omega t u(t)$ $\dfrac{\omega}{s^2+\omega^2}$

$s^2X(s) + 6sX(s) + 8X(s) = \dfrac{15}{s^2+3^2}$

$X(s^2+6s+8) = X(s+2)(s+4)$

$X(s) = \dfrac{15}{(s+2)(s+4)(s^2+3^2)}$

$X(s) = \dfrac{A}{s+2} + \dfrac{B}{s+4} + \dfrac{Cs+D}{s^2+3^2}$

A: $X(s+2) \overset{s=-2}{=} \dfrac{15}{2(4+9)} = \dfrac{15}{2\cdot 13} = \dfrac{15}{26}$

$A: \; X(s+2) = \frac{}{2(4+9)} = \frac{}{2\cdot 13} = 26$

$B: \; X(s+4) \overset{s=-4}{=} \frac{15}{-2(16+9)} = \frac{15}{-2\cdot 25} = \frac{3}{-2\cdot 5} = \frac{-3}{10}$

$X(1) = \frac{15}{3\cdot 5\cdot 10} = \frac{A}{3} + \frac{B}{5} + \frac{C+D}{10}$

$1 = \frac{10}{3}A + 2B + C + D$

$X(0) = \frac{15}{2\cdot 4\cdot 9} = \frac{A}{2} + \frac{B}{4} + \frac{D}{9}$

$\frac{15}{8} = \frac{A}{2} + \frac{B}{4} + \frac{D}{9}$

$\frac{15}{8} - \frac{15}{2\cdot 26} + \frac{3}{10\cdot 4} = \frac{D}{9}$

$D = \frac{472}{65}$

$1 = \frac{10}{3}\left(\frac{15}{26}\right) + 2\left(\frac{-3}{10}\right) + C + \frac{472}{65}$

$\rightarrow C = -\frac{443}{65}$

$X(s) = A\frac{1}{s+2} + B\frac{1}{s+4} + \frac{Cs+D}{s^2+3^2}$

$\downarrow \mathcal{L}^{-1}$

$x(t) = Ae^{-2t}u(t) + Be^{-4t}u(t) + C\cos 3t + \frac{D}{3}\sin 3t$

$x(t) = \frac{75}{130}e^{-2t}u(t) - \frac{39}{130}e^{-4t}u(t) - 36\cos 3t - 2\sin 3t$

(1C) $\ddot{x} + 8\dot{x} + 25x = 10u(t)$

$\downarrow \mathcal{L}$

$X(s)(s^2+8s+25) = \frac{10}{s}$

$X(s) = \frac{10}{s(s^2+8s+25)} = \frac{A}{s} + \frac{Bs+C}{s^2+8s+25}$

$sX_s \overset{s=0}{=} \frac{10}{25} = A = \frac{2}{5}$

$10 = A(s^2+8s+25) + Bs^2 + Cs$

$s^1: \; 8A + C = 0$

$C = -8A = \frac{-16}{5}$

$s^2: \; 0 = A + B$

$B = -A = \frac{-2}{5}$

$s^2 + 8s + 25 = (s+4)^2 + 9$

$X(s) = \frac{2}{5}\frac{1}{s} + \frac{-\frac{2}{5}s}{(s+4)^2+9} + \frac{-\frac{16}{5}}{(s+4)^2+9}$

$x(t) = \frac{12}{15}u(t) - \frac{6}{15}\cos 3t\, e^{-4t} - \frac{8}{15}e^{-4t}\sin 3t$

Assignment 3 (ELEC 341 L3_ODESolution)

Problem 2:

For each of the following transfer functions, write the corresponding differential equation:

a. $\dfrac{X(s)}{F(s)} = \dfrac{7}{s^2 + 5s + 10}$

b. $\dfrac{X(s)}{F(s)} = \dfrac{15}{(s + 10)(s + 11)}$

c. $\dfrac{X(s)}{F(s)} = \dfrac{s + 3}{s^3 + 11s^2 + 12s + 18}$

(a). $X(s)\,(s^2+5s+10) = 7F(s)$

$X(s)\left(\frac{1}{7}s^2+\frac{5}{7}s+\frac{10}{7}\right) = F(s)$

$\frac{1}{7}\frac{d^2X}{dt^2} + \frac{5}{7}\frac{dX}{dt} + \frac{10}{7}X = f(t)$

(b) $15F(s) = (s^2+11s+10s+110)X(s)$

$\left(\frac{1}{15}s^2 + \frac{11}{15}s + \cancel{\frac{10s}{15}} + \frac{110}{15}\right)X(s) = F(s)$, with $\frac{21}{15}$ and $\frac{22}{3}$

$$\boxed{\frac{1}{15}\frac{d^2X}{dt^2} + \frac{21}{15}\frac{dX}{dt} + \frac{22}{3}X = f(t)}$$

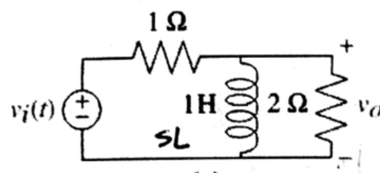
(c) $X(s)\,(s^3+11s^2+12s+18) = (s+3)F(s)$

$$\boxed{\frac{d^3X}{dt^3} + 11\frac{d^2X}{dt^2} + 12\frac{dX}{dt} + 18X = \frac{dF}{dt} + 3f}$$

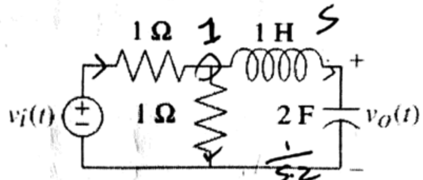
Assignment 4 (ELEC 341 L4_Model_ElecMech)

Problem 1:

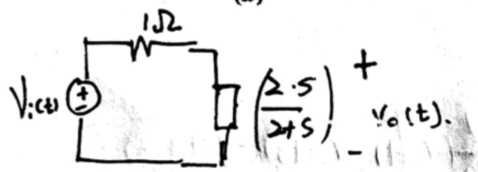
Find the transfer function, $G(s) = V_o(s)/V_i(s)$ for each network shown in the following figure:



(a)



(b)



$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{\text{Impedance for output}}{\text{total Impedance}}$$

$$= \frac{\left(\frac{2s}{2+s}\right)\cdot(2+s)}{\left[1+\frac{2s}{(2+s)}\right](2+s)}$$

$$= \frac{2s}{(2+s)+2s} = \boxed{\frac{2s}{2+3s}}$$

$$\begin{cases} \frac{V_i(t)-V_1}{1} = \frac{V_1}{1} + \frac{V_1-V_o}{s} \\ \frac{V_1-V_o}{s} = V_o\cdot 2s \end{cases}$$

$$V_1 - V_o = V_o\cdot 2s^2$$

$$V_1 = V_o\cdot 2s^2 + V_o$$

$$= V_o(1+2s^2)$$

$$V_o(1+2s^2) - V_o$$

$$\frac{V_i - V_o(1+2s^2)}{1} = \frac{V_o(1+2s^2)}{1} + \frac{V_o\cdot 2s^2}{s}$$

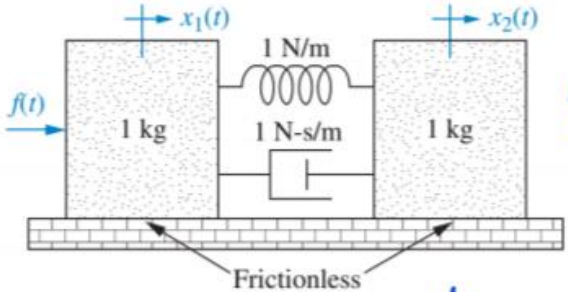
$$V_i = V_o(1+2s^2+2s) + V_o(1+2s^2)$$

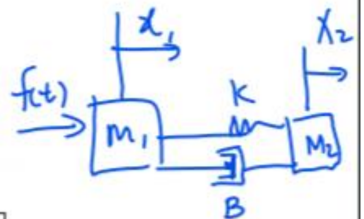
$$V_i = V_o(2+4s^2+2s)$$

$$\boxed{\frac{V_o}{V_i} = \frac{1}{4s^2+2s+2}}$$

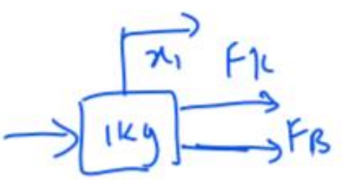
Problem 2:

Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical network shown in figure:





$$\begin{cases} M_1 x_1''(t) = -B\left(x_1'(t) - x_2'(t)\right) - K\left(x_1(t) - x_2(t)\right) - f(t) \\ M_2 x_2''(t) = -B\left(x_2'(t) - x_1'(t)\right) - K\left(x_2(t) - x_1(t)\right) \end{cases}$$



$$\ddot{x}_1(t) = f(t) - K(x_1 - x_2) - B(\dot{x}_1 - \dot{x}_2)$$

$$\Rightarrow s^2 X_1(s) + \left[(X_1(s) - X_2(s)\right] \times (1+s) = F(s)$$

$$\mathcal{L}\left\{ \ddot{x}_2(t) = -\left[K(x_1 - x_2) - B(\dot{x}_1 - \dot{x}_2)\right] \right\}$$

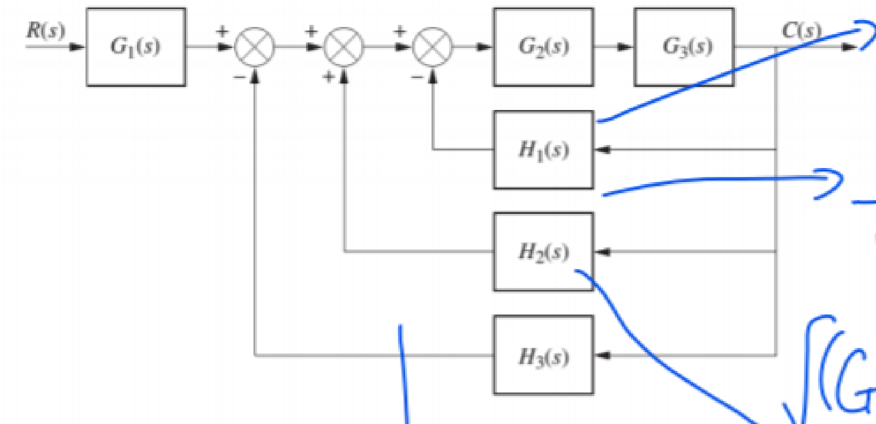
$$s^2 X_2(s) + X_2(s) + sX_2(s) = X_1(s) + sX_1(s)$$

$$X_2(s) = \frac{X_1(s)(1+s)}{s^2+s+1}$$

$$\therefore \boxed{\frac{X_2}{F_1} = \frac{1+s}{s^4+2s^3+2s^2}}$$

Problem 3:

Reduce the block diagram shown in figure to a single transfer function:



$$G_2(s)\,G_3(s)$$

$$\frac{G_2G_3}{1+G_2G_3H_1}$$

$$\frac{\left(\frac{G_2G_3}{1+G_2G_3H_1}\right)}{1+H_2\left(\frac{G_2G_3}{1+G_2G_3H_1}\right)}$$

$$\frac{\frac{\left(\frac{G_2G_3}{1+G_2G_3H_1}\right)}{\left(1+H_2\frac{G_2G_3}{1+G_2G_3H_1}\right)}}{1+H_3\frac{\left(\frac{G_2G_3}{1+G_2G_3H_1}\right)}{1+H_2\left(\frac{G_2G_3}{1+G_2G_3H_1}\right)}}$$

$$\therefore \boxed{\frac{G_1G_2G_3}{1+G_2G_3H_1+H_2G_2G_3+H_3G_2G_3}}$$

Assignment 4 (ELEC 341 L4_Model_ElecMech)

Problem 4:

Calculate the transfer function of the system represented in the state space model as

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \overset{A}{\begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overset{B}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} [u]$$

$$Y = \underset{C}{[0 \quad 1]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x}_1 = -x_1 - x_2 + u$$

$$\dot{x}_2 = x_1$$

$$Y = x_2$$

$$T(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

$$sI - A$$

$$= \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} s-1 & 1 \\ -1 & s \end{bmatrix}^{-1}$$

$$= \frac{1}{s(s-1)+1} \begin{bmatrix} s & -1 \\ 1 & s-1 \end{bmatrix}$$

$$\frac{1}{s(s-1)+1} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s & -1 \\ 1 & s-1 \end{bmatrix}$$

$$= \frac{1}{s(s-1)+1} \underset{1\times 2}{\begin{bmatrix} 1 & s-1 \end{bmatrix}} \overset{2\times 1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

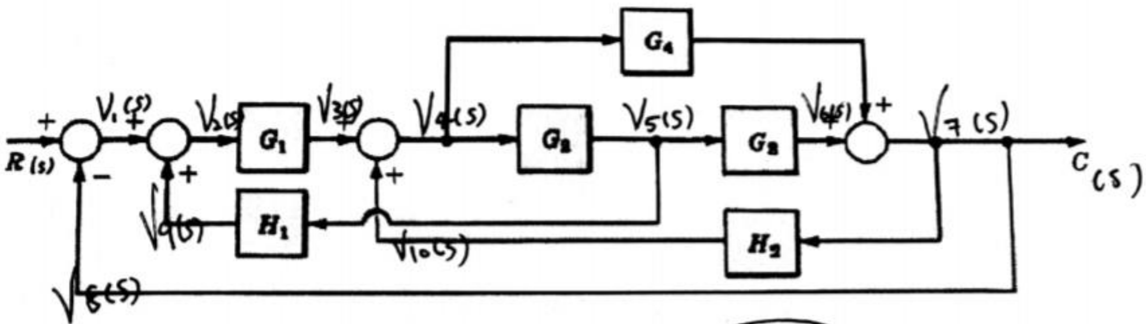
$$= \frac{1}{s(s-1)+1} \cdot 1$$

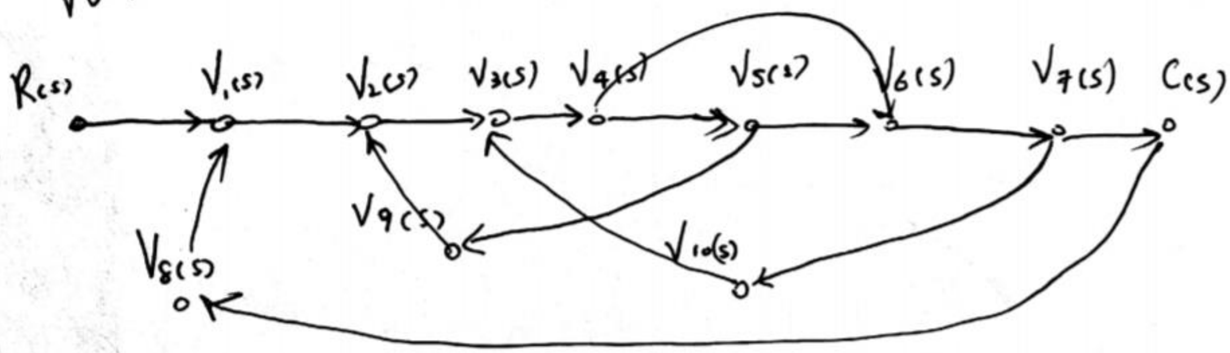
$$= \boxed{\frac{1}{s^2 - s + 1}}$$

Assignment 4 (ELEC 341 L4_Model_ElecMech)

Problem 5:

Find the signal flow graph for the following block diagram:





Assignment 5 (ELEC 341 L5_Model_DCMotor)

Problem 1:

Find the linearize form of the following system using the equilibrium point of $v_0 = 30$. That is, re-write the ODE in terms of perturbation variables:

$$m\dot{v}(t) + K_f \cdot v(t) + K_a . v^2(t) = F_m(t)$$

Use the following numerical values for the constant parameters in your calculations:
$m = 1000$; $K_f = 0.1$; $K_a = 1$

$V_0 = 30$

$m\dot{v}(t) + K_f v(t) + K_a v^2(t) = F_m(t)$

$m = 1000, \; K_f = 0.1, \; K_a = 1$

$1000\,\dot{v}(t) + 0.1\,v(t) + v^2(t) = F_m(t)$

$f(\dot{v}, v, F) = f(\dot{v}_0, v_0, F_0) + \left.\frac{\partial f}{\partial \dot{v}}\right|_{op} \delta\dot{v} + \left.\frac{\partial f}{\partial v}\right|_{op} \delta v + \left.\frac{\partial f}{\partial F}\right|_{op} \delta F$

$= 1000\,\dot{v}(t) + 0.1\,v(t) + v^2(t) - F_m(t).$

$\frac{\partial f}{\partial \dot{v}} = 1000$

$\frac{\partial f}{\partial v} = 0.1 + 2\cancelto{30}{V_0} = 60.1$

$\frac{\partial f}{\partial F} = -1$

$\cancelto{0}{f(\dot{v}, v, F)} = \cancelto{0}{f(\dot{v}_0, v_0, F_0)} + 1000\,\delta\dot{v} + 60.1\,\delta v - \delta F$

$1000\,\delta\dot{v} + 60.1\,\delta v = \delta F$

Assignment 6 (ELEC 341 L6_Stability)

Problem 1:

Tell how many roots of the following polynomial are in the right half-plane, in the left half-plane, and on the $j\omega$-axis:

$$P(s) = s^5 + 3s^4 + 5s^3 + 4s^2 + s + 3$$ 5 in total.

Use Routh-Hurwitz criterion.

$$\begin{array}{c|ccc} s^5 & 1 & 5 & 1 \\ s^4 & 3 & 4 & 3 \\ s^3 & \frac{11}{3} & 0 & \\ s^2 & 4 & 3 & \\ s^1 & -\frac{11}{4} & 0 & \\ s^0 & 3 & & \end{array}$$

$$\frac{3\times5-1\times4}{3} = \frac{15-4}{3} = \frac{11}{3}$$

$$\frac{1\times3-1\times3}{3} = 0$$

$$\frac{4\times\frac{11}{3}-0}{\frac{11}{3}} = 4.$$

$$\frac{4\times0-\frac{11}{3}\times3}{4} = \frac{-11}{4}.$$

2 roots on the RHP.

3 roots on the LHP

0 root on the $j\omega$-axis

Assignment 7 (ELEC 341 L7_RouthHurwitzEx)

Problem 1:

Tell how many roots of the following polynomial are in the right half-plane, in the left half-plane, and on the $j\omega$-axis:

$$P(s) = s^5 + 6s^3 + 5s^2 + 8s + 20$$

Use Routh-Hurwitz criterion.

s^5	1	6	8
s^4	$\cancel{0}$ ε	5	20
s^3	a	b	
s^2	$\cancel{c}$ 5	$\cancel{d}$ 20	
s^1	$\cancel{e}$ $\cancel{0}$ 10		
s^0	$\cancel{f}$ 20		

$$a = \frac{6\varepsilon - 5}{\varepsilon} \text{ as } \varepsilon \to 0 = \frac{-5}{\varepsilon}$$

$$b = \frac{8\varepsilon - 20}{\varepsilon} \text{ as } \varepsilon \to 0 = \frac{-20}{\varepsilon}$$

$$c = \frac{5 \cdot a - b\varepsilon}{a} = \frac{5\cdot\left(\frac{-5}{\varepsilon}\right) - \frac{-20}{\cancel{\varepsilon}}\cdot\cancel{\varepsilon}}{\left(\frac{-5}{\varepsilon}\right)}$$

$$= \frac{\frac{-25}{\varepsilon} + 20}{\frac{-5}{\varepsilon}} \overset{\lim_{\varepsilon\to 0}}{=} \frac{-25 + 20\varepsilon}{-5} = 5$$

$$\therefore \boxed{c = 5}$$

$$e = \frac{5b - 20a}{5} = \frac{5\cdot\left(\frac{-20}{\varepsilon}\right) - 20\left(\frac{-5}{\varepsilon}\right)}{5}$$

$$= \frac{-20}{\varepsilon} + \frac{20}{\varepsilon} = 0$$

$$P(s) = (5s^2 + 20s^0) = 5s^2 + 20$$

$$P'(s) = 10s$$

2 sign changes in the first column ⇒ 2 Roots on RHP

2 roots on the $j\omega$-axis

$$5s^2 + 20 = 0$$

$$s^2 + 5 = 0$$

$$s = \pm\sqrt{5}\,j$$

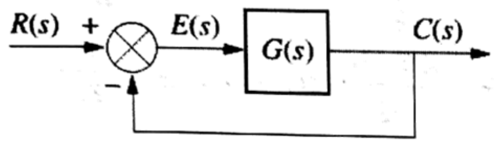
1 root on the LHP

Problem 2:

In the system of Figure , let

$$G(s) = \frac{K(s+2)}{s(s-1)(s+3)}$$

Find the range of K for closed-loop stability.



$$\frac{2(K-3)-2K}{2} = -3.$$

FIGURE

Use Routh-Hurwitz criterion.

$$\begin{array}{c|cc} s^3 & 1 & K-3 \\ s^2 & 2 & 2K \\ s^1 & -3 & \\ s^0 & 2K & \end{array}$$

$$G(s) = \frac{K(s+2)}{s(s-1)(s+3)}.$$

$$\therefore 1 + G(s) = 0$$

$$1 + \frac{K(s+2)}{s(s-1)(s+3)} = 0$$

$$s(s-1)(s+3) + K(s+2) = 0$$

$$s(s^2+3s-s-3) + Ks + 2K = 0$$

$$s^3 + 2s^2 - 3s + Ks + 2K = 0$$

$$s^3 + 2s^2 + (K-3)s + 2K = 0.$$

$\because$ Always have sign change

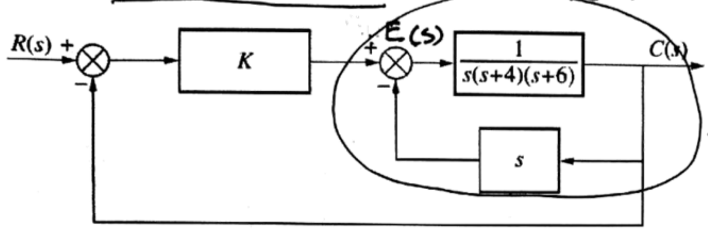
$\therefore$ system is always unstable no matter how we change K.

Assignment 7 (ELEC 341 L7_RouthHurwitzEx)

Problem 3:

For the system shown in Figure, find the value of gain, K, that will make the system oscillate. Also, find the frequency of oscillation. ?

unstable



FIGURE

$$\rightarrow \frac{C(s)}{E(s)} = \frac{\frac{1}{s(s+4)(s+6)}}{1+\frac{1}{(s+4)(s+6)}}$$

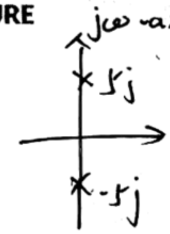
$$\frac{(s+4)(s+6)+1}{(s+4)(s+6)}$$

$$\frac{1}{s\cancel{(s+4)}\cancel{(s+6)}} \cdot \frac{\cancel{(s+4)}\cancel{(s+6)}}{(s+4)(s+6)+1}$$

$$= \frac{1}{s[(s+4)(s+6)+1]}$$

Use Routh-Hurwitz criterion.

jω-axis



$$1 + \frac{K}{s[(s+4)(s+6)+1]} = 0$$

$$s(s+4)(s+6)+s+K=0$$

$$s(s^2+6s+4s+24)+s+K=0$$

$$s^3+10s^2+25s+K=0$$

$$\begin{array}{c|cc} s^3 & 1 & 25 \\ s^2 & 10 & \cancel{K} \rightarrow 250 \\ s^1 & \cancel{\frac{250-K}{10}} \rightarrow 0 \rightarrow 20. & \\ s^0 & K & \end{array}$$

$$\frac{250-K}{10}$$

oscillation → on the jω axis
↓
At least 1 of Roz

$250-K=0$

$K=250$

$P(s)=10s^2+250$

$P'(s)=20s$

$10s^2+250=0$

$10s^2=-250$

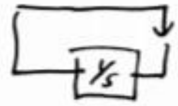
$s^2=-25$

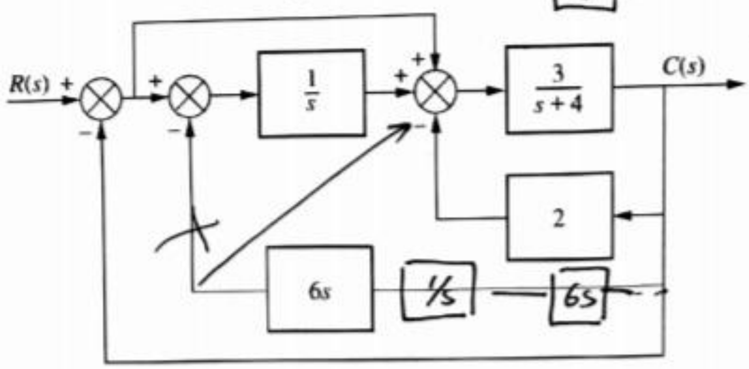
$s=\pm 5j \Rightarrow$ frequency of oscillation is 5

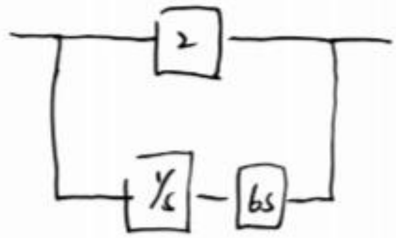
Assignment 8 (ELEC 341 L8_SSError)

Problem 1:

For the system shown in Figure, what steady-state error can be expected for the following test inputs: $10u(t)$, $10tu(t)$, $10t^2u(t)$.

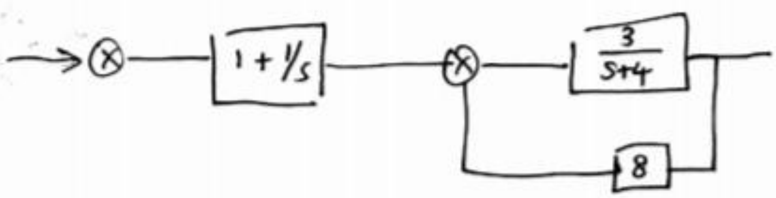
$\frac{1}{s}+1 =$  Paraller





$2 + \frac{1}{s} \cdot 6s = 6 + 2 = 8.$

FIGURE



$$\frac{\frac{3}{s+4}}{1+\frac{8\cdot 3}{s+4}} = \frac{\frac{3}{s+4}}{1+\frac{24}{s+4}} = \frac{3}{s+28}$$

$$L(s) = \left(1+\frac{1}{s}\right)\left(\frac{3}{s+28}\right)$$

$$= \frac{3}{s+28} + \frac{3}{s(s+28)}$$

e_{ss} — step. $K_p = \lim_{s\to 0} L(s) = \infty \rightarrow e_{ss} = \frac{10}{1+\infty} = 0$

— Ramp: $K_v = \lim_{s\to 0} sL(s) = \frac{3}{28} \rightarrow e_{ss} = \frac{10}{3/28} = \frac{280}{3}$

— Parabolic: $K_a = \lim_{s\to 0} s^2 L(s) = 0 \longrightarrow e_{ss} = \frac{10}{0} = \infty$

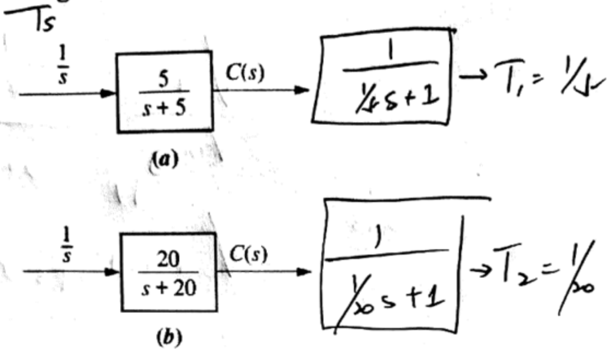
Assignment 9 (ELEC 341 L9_StepResponse)

Problem 1:

Find the output response, $c(t)$, for each of the systems shown in Figure. Also find the time constant, rise time, and settling time for each case.

τ T_r T_s

$$\frac{K}{Ts+1}$$



FIGURE

Dc. Gain for

(a) $= G(0) = 1$

(b) $= G(0) = 1$

Use the difference between 10% to 90% of y_{ss} as the rise time and also use a 2% settling time.

Settling time for (a) $4T_1 = 4 \times \frac{1}{5} = \frac{4}{5}$ sec

(b) $4T_2 = 4 \times \frac{1}{20} = \frac{1}{5}$ sec

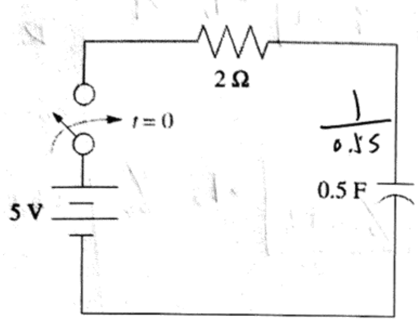
Rise time $(2.2T)$ for (a) $2.2 \times T_1 = 2.2 \times \frac{1}{5} = \underline{0.44 \text{ sec}}$

(b) $2.2 \times T_2 = 2.2 \times \frac{1}{20} = \underline{0.11 \text{ sec}}$

Problem 2:

Find the capacitor voltage in the network shown in Figure if the switch closes at $t = 0$. Assume zero initial conditions. Also find the time constant, rise time, and settling time for the capacitor voltage.

$$\frac{V_{out}}{\cancel{V_{in}}\ 5V} = \frac{\frac{1}{0.5s}}{2+\frac{1}{0.5s}}$$



FIGURE

$$V_{out}\cdot\left(2+\frac{1}{0.5s}\right) = \frac{5}{0.5s}$$

$$V_{out} = \mathcal{L}^{-1}\left\{\left(\frac{5}{0.5s}\right)\Big/\left(2+\frac{1}{0.5s}\right)\right\}$$

Use the difference between 10% to 90% of y_{ss} as the rise time and also use a 2% settling time. Use Matlab to verify and plot your output response.

$$V_{out} = 5e^{-t}$$

$$\boxed{\frac{1}{s+1}} \rightarrow \text{transfer function}$$

$$\frac{K}{Ts+1}$$

T = time constant = 1 sec

$T_r = 2.2T = 2.2$ sec

$T_s = 4\times T = 4$ sec

Problem 3:

For the second-order system that follows, find ζ, ω_n, T_s, T_p, T_r, and $\%OS$.

$$T(s) = \frac{16}{s^2 + 3s + 16}$$

$$\begin{cases} 16 = \omega_n^2 \\ 2\zeta\omega_n = 3 \end{cases} \Rightarrow \boxed{\omega_n = 4}$$

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$8\zeta = 3$$

$$\boxed{\zeta = 3/8}$$

$$2\%\ T_s = \frac{4}{\zeta\omega_n} = \frac{4}{3/8 \times 4} = \boxed{\frac{8}{3}\ \text{sec}}$$

$$5\%\ T_s = \frac{3}{\zeta\omega_n} = \boxed{2\ \text{sec}}$$

$$T_{peak} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{4\sqrt{1-(3/8)^2}} = \boxed{0.84722\ \text{sec}}$$

Rising time

$$\omega_n T_r = 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1$$

$$T_r = 0.35594\ \text{sec}$$

$$P.O\% = 100\, e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 28.0596\%$$