Z-Xform Tables from L&V

Discrete-time signal $\forall n \in \mathbb{Z}$	Z transform $\forall z \in RoC(x)$	$Roc(x) \subset \mathbb{C}$	Reference
$x(n) = \delta(n-M)$	$\hat{X}(z) = z^{-M}$	$\mathbb{C}$	Example 12.12
$x(n) = u(n)$	$\hat{X}(z) = \frac{z}{z-1}$	$\{z \mid \lvert z\rvert > 1\}$	Example 12.7
$x(n) = a^n u(n)$	$\hat{X}(z) = \frac{z}{z-a}$	$\{z \mid \lvert z\rvert > \lvert a\rvert\}$	Example 13.3
$x(n) = a^n u(-n)$	$\hat{X}(z) = \frac{1}{1-a^{-1}z}$	$\{z \mid \lvert z\rvert < \lvert a\rvert\}$	Exercise 1 in Chapter 12
$x(n) = \cos(\omega_0 n)u(n)$	$\hat{X}(z) = \frac{z^2 - z\cos(\omega_0)}{z^2 - 2z\cos(\omega_0)+1}$	$\{z \mid \lvert z\rvert > 1\}$	Example 13.3
$x(n) = \sin(\omega_0 n)u(n)$	$\hat{X}(z) = \frac{z\sin(\omega_0)}{z^2 - 2z\cos(\omega_0)+1},$	$\{z \mid \lvert z\rvert > 1\}$	Exercise 1
$x(n) = \frac{1}{(N-1)!}(n-1)\cdots(n-N+1)\, a^{n-N}u(n-N)$	$\hat{X}(z) = \frac{1}{(z-a)^N}$	$\{z \mid \lvert z\rvert > \lvert a\rvert\}$	(13.13)
$x(n) = \frac{(-1)^N}{(N-1)!}(N-1-n)\cdots(1-n)\, a^{n-N}u(-n)$	$\hat{X}(z) = \frac{1}{(z-a)^N}$	$\{z \mid \lvert z\rvert < \lvert a\rvert\}$	(13.14)

Table 13.1: Z transforms of key signals. The signal u is the unit step (12.13), δ is the Kronecker delta, a is any complex constant, ω_0 is any real constant, M is any integer constant, and $N > 0$ is any integer constant.

Time domain $\forall n \in \mathbb{Z}$	Frequency domain $\forall z \in RoC$	RoC	Name (reference)
$w(n) = ax(n) + by(n)$	$\hat{W}(z) = a\hat{X}(z) + b\hat{Y}(z)$	$RoC(w) \supset RoC(x) \cap RoC(y)$	**Linearity** (Section 13.1.1)
$y(n) = x(n-N)$	$\hat{Y}(z) = z^{-N}\hat{X}(z)$	$RoC(y) = RoC(x)$	**Delay** (Section 13.1.2)
$y(n) = (x*h)(n)$	$\hat{Y}(z) = \hat{X}(z)\hat{H}(z)$	$RoC(y) \supset RoC(x) \cap RoC(h)$	**Convolution** (Section 13.1.3)
$y(n) = x^*(n)$	$\hat{Y}(z) = [\hat{X}(z^*)]^*$	$RoC(y) = RoC(x)$	**Conjugation** (Section 13.1.4)
$y(n) = x(-n)$	$\hat{Y}(z) = \hat{X}(z^{-1})$	$RoC(y) = \{z \mid z^{-1} \in RoC(x)\}$	**Time reversal** (Section 13.1.5)
$y(n) = nx(n)$	$\hat{Y}(z) = -z\frac{d}{dz}\hat{X}(z)$	$RoC(y) = RoC(x)$	**Scaling by n** (page 597)
$y(n) = a^{-n}x(n)$	$\hat{Y}(z) = \hat{X}(az)$	$RoC(y) = \{z \mid az \in RoC(x)\}$	**Exponential scaling** (Section 13.1.6)

Table 13.2: Properties of the Z transform. In this table, a, b are complex constants, and N is an integer constant.

DTFT Tables from L&V

Signal	DTFT	Reference
$\forall n \in \mathbb{Z}, \quad x(n) = \delta(n)$	$\forall \omega \in \mathbb{R}, \quad X(\omega) = 1$	Example 10.8
$\forall n \in \mathbb{Z},$ $x(n) = \delta(n-N)$	$\forall \omega \in \mathbb{R}, \quad X(\omega) = e^{-i\omega N}$	Example 10.8
$\forall n \in \mathbb{Z}, \quad x(n) = K$	$\forall \omega \in \mathbb{R},$ $X(\omega) = 2\pi K \sum_{k=-\infty}^{\infty} \delta(\omega - k2\pi)$	Section 10.7.5
$\forall n \in \mathbb{Z},$ $x(n) = a^n u(n), \quad \|a\| < 1$	$\forall \omega \in \mathbb{R},$ $X(\omega) = \frac{1}{1 - ae^{-i\omega}}$	Exercise 18
$\forall n \in \mathbb{Z},$ $x(n) = \begin{cases} 1 & \text{if } \|n\| \leq M \\ 0 & \text{otherwise} \end{cases}$	$\forall \omega \in \mathbb{R},$ $X(\omega) = \frac{\sin(\omega(M+0.5))}{\sin(\omega/2)}$	Exercise 8
$\forall n \in \mathbb{Z},$ $x(n) = \frac{\sin(Wn)}{\pi n}, \quad 0 < W < \pi$	$\forall \omega \in [-\pi, \pi],$ $X(\omega) = \begin{cases} 1 & \text{if } \|\omega\| \leq W \\ 0 & \text{otherwise} \end{cases}$	–

Table 10.3: Discrete time Fourier transforms of key signals. The function u is the unit step, given by (2.16).

Time domain	Frequency domain	Reference
$\forall n \in \mathbb{Z}, \quad x(n)$ is real	$\forall \omega \in \mathbb{R}, \quad X(\omega) = X^*(-\omega)$	Section 10.7.2
$\forall n \in \mathbb{Z}, \quad x(n) = x^*(-n)$	$\forall \omega \in \mathbb{R}, \quad X(\omega)$ is real	Section 10.7.2
$\forall n \in \mathbb{Z}, \quad y(n) = x(n-N)$	$\forall \omega \in \mathbb{R}, \quad Y(\omega) = e^{-i\omega N} X(\omega)$	Section 10.7.3
$\forall n \in \mathbb{Z}, \quad y(n) = e^{i\omega_1 n} x(n)$	$\forall \omega \in \mathbb{R}, \quad Y(\omega) = X(\omega - \omega_1)$	Section 10.7.6
$\forall n \in \mathbb{Z},$ $y(n) = \cos(\omega_1 n) x(n)$	$\forall \omega \in \mathbb{R},$ $Y(\omega) = (X(\omega - \omega_1) + X(\omega + \omega_1))/2$	Example 10.11
$\forall n \in \mathbb{Z},$ $y(n) = \sin(\omega_1 n) x(n)$	$\forall \omega \in \mathbb{R},$ $Y(\omega) = (X(\omega - \omega_1) - X(\omega + \omega_1))/2i$	Exercise 12
$\forall n \in \mathbb{Z},$ $x(n) = ax_1(n) + bx_2(n)$	$\forall \omega \in \mathbb{R},$ $X(\omega) = aX_1(\omega) + bX_2(\omega)$	Section 10.7.4
$\forall n \in \mathbb{Z}, \quad y(n) = (h * x)(n)$	$\forall \omega \in \mathbb{R}, \quad Y(\omega) = H(\omega) X(\omega)$	Section 10.7.1
$\forall n \in \mathbb{Z}, \quad y(n) = x(n) p(n)$	$\forall \omega \in \mathbb{R},$ $Y(\omega) = \frac{1}{2\pi} \int_0^{2\pi} X(\Omega) P(\omega - \Omega) d\Omega$	Box on page 459
$\forall n \in \mathbb{Z},$ $y(n) = \begin{cases} x(n/N) & n \text{ multiple of } N \\ 0 & \text{otherwise} \end{cases}$	$\forall \omega \in \mathbb{R},$ $Y(\omega) = X(N\Omega)$	Exercise 14

Table 10.8: Properties of the DTFT.

WW8, Q3

$u[n] = u[n-1] + \delta[n]$
$= u[n-2] + \delta[n] + \delta[n-1]$

For the two discrete-time LTI systems described below, find the transfer function $H(z)$, if:

a) In system A, where an input-output signal pair is given by:

$$x[n] = \begin{cases} 3 & n = 0, 1 \\ 0 & otherwise \end{cases}$$

$$y[n] = \begin{cases} 8 & n = 0, 3 \\ 2 & n = 1, 4 \\ 0 & otherwise \end{cases}$$

$H(z) =$

b) In system B, where an input-output signal pair is given by:

$$x[n] = (-0.4)^n u[n]$$

$$y[n] = \begin{cases} 0 & n < 0 \\ 4(n+1) & n = 0, 1, 2 \\ \boxed{2(-0.4)^n \quad n \geq 3} \end{cases}$$

$H(z) =$

$g[n] = 2(-0.4)^n u[n-3]$

$G(z) = ?$

METHOD A:

$g[n] = 2(-0.4)^n u[n] - 2\delta[n] + 0.8\delta[n-1] - 0.32\delta[n-2]$

$\downarrow$

$G(z)$ EASY

METHOD B: $g[n] = 2(-0.4)^{n-3}(-0.4)^3 u[n-3] = K f[n-3]$

WHERE $f[n] = \alpha^n u[n] \rightarrow F(z)$ EASY

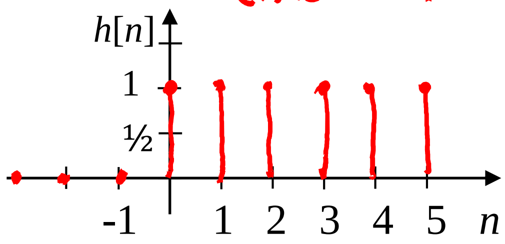
$\Big\}$ $G(z)$ EASY

$$h[n] \text{ for } H(z) = \frac{1}{1-z^{-1}} = \frac{z}{z-1}?$$

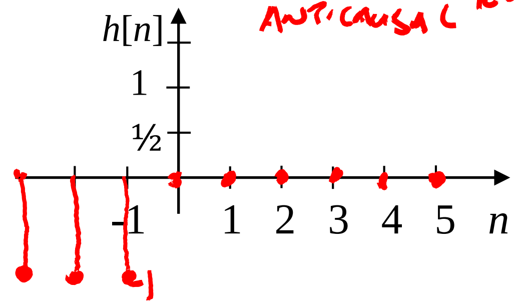
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-z^{-1}} \xRightarrow{\mathcal{Z}^{-1}} y[n] = y[n-1] + x[n]$$

IMPULSE RESPONSE IF $x[n] = \delta[n]$

CAUSAL ROC: $|z|>1$

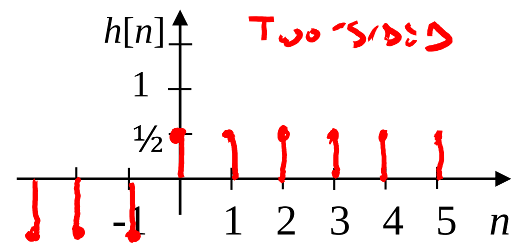


ANTICAUSAL ROC: $|z|<1$



TWO-SIDED

Z-XFORM DOESN'T EXIST



Revisit CT $h(t)$ for $H(s) = \frac{1}{s-a}$?

$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s-a} \xrightarrow{\mathcal{L}^{-1}} \dot{y}(t) = a\,y(t) + x(t)$

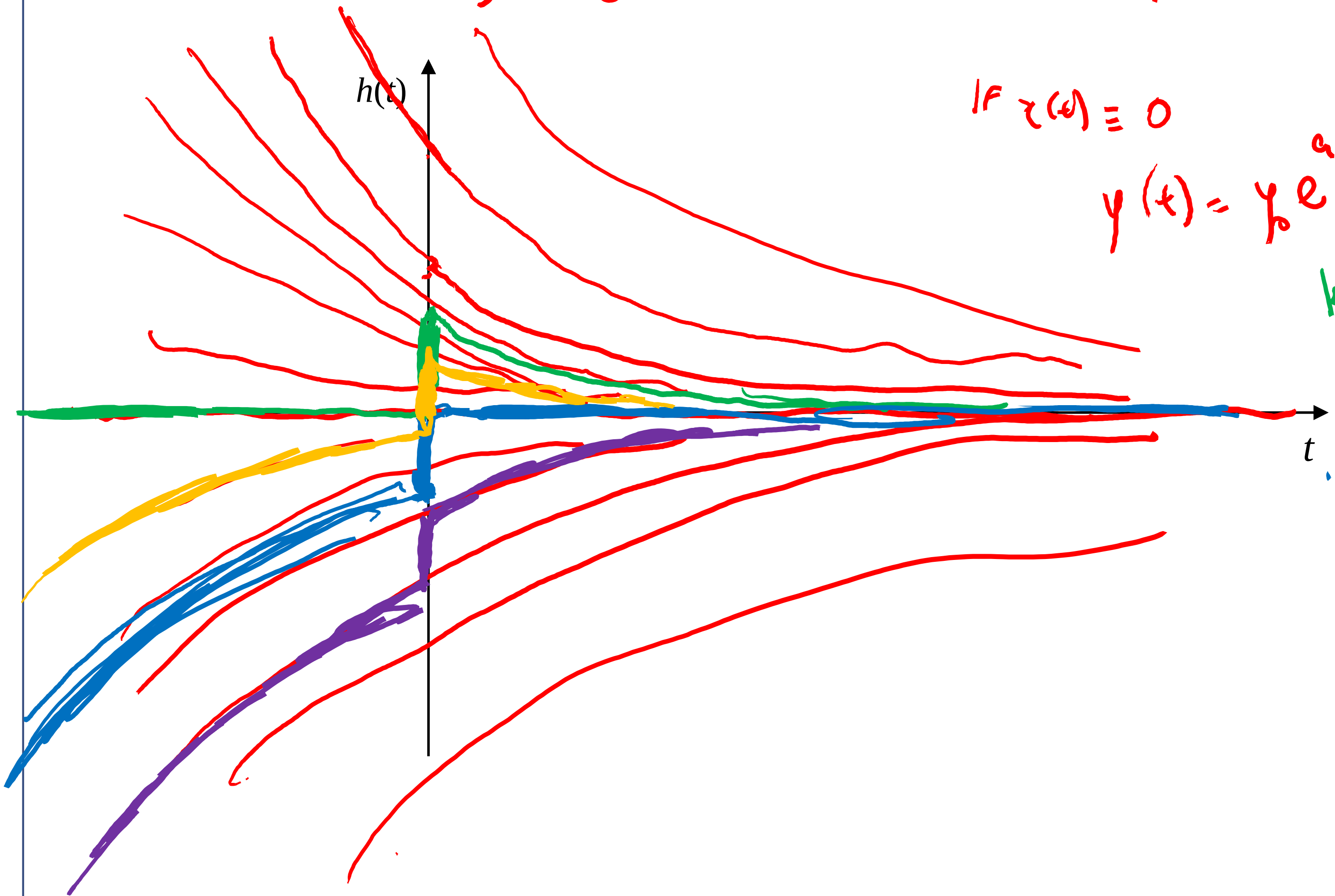
$a<0$

IF $x(t) \equiv 0$

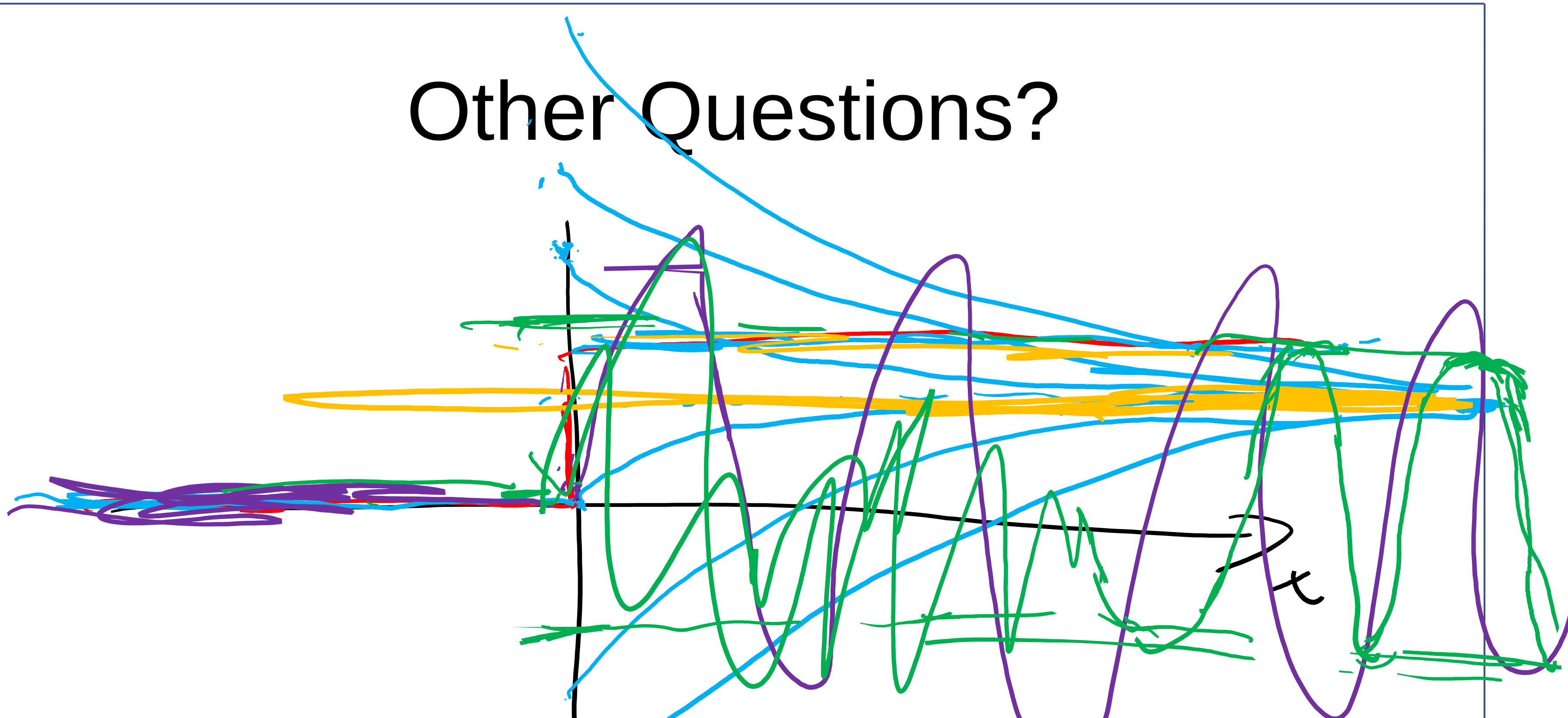
$y(t) = y_0 e^{at}$

IF CAUSAL

IF ANTICAUSAL



Other Questions?



$$x[n] \longrightarrow \boxed{\begin{matrix} h[n] \\ H(z) \end{matrix}} \longrightarrow y[n]$$

$$X(z) \qquad Y(z) = \underbrace{H(z)X(z)}_{ZSR} + \underbrace{\frac{I(z)}{A(z)}}_{ZIR}$$

Other Questions?