
A causal discrete-time LTI system is described by the difference equation, $y[n] - \frac{12}{32}y[n-1] + \frac{1}{32}y[n-2] = 9x[n]$, where $x[n]$ and $y[n]$ are the input and output of the system respectively.

a) Find the system transfer function $H(z)$, and indicate the region of convergence in interval notation (e.g. (-INF,a), (a,b) or (b, INF)).

$H(z) =$ ________ $RoC:$ ___

b) Find the impulse response, $h[n]$, of the system.

$h[n] =$ ________

c) Find the step response, $s[n]$, of the system.

$s[n] =$ ________

In your answers, enter $z(n)$ for a discrete-time function $z[n]$ and enter $D(n)$ instead of $\delta[n]$. WebWork is unable to parse a function that uses square brackets.

Correct Answers:

- `9*z^2/[(z-0.125)*(z-0.25)]`
- `(0.25,infinity)`
- `9*u(n)*(-0.25)*(4*0.125^n-8*0.25^n)`
- `9*u(n)*[1.52381+4*0.125^n/28+8*0.25^n/(-12)]`

JY Note Apr 2, 2020: The stability responses for some students are incorrectly being marked wrong. I don't see an easy fix yet but if anybody believes this has negatively affected their grade, please contact me directly.

Consider an LTI system whose input $x[n]$ and output $y[n]$ are related by the difference equation $y[n-1] + \frac{10}{3}y[n] + \frac{8}{3}y[n+1] = x[n]$. Determine the three possible choices for the impulse response that makes this system 1) causal, 2) two-sided and 3) anti-causal. Then for each case, determine if the system is stable or not.

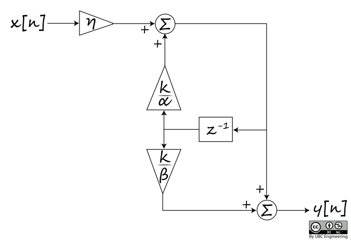
Causality	**Impulse Response**	**Stability**
Causal	____________	[?/Stable/Unstable]
two-sided	____________	[?/Stable/Unstable]
anti-Causal	____________	[?/Stable/Unstable]

In your answers, enter $z(n)$ for a discrete-time function $z[n]$. WebWork is unable to parse a function that uses square brackets.

Correct Answers:

- `(-1.5)*u(n)*(-0.75)^n+1.5*(-0.5)^n*u(n)`
- `Stable`
- `1.5*u(-n-1)*(-0.75)^n+1.5*(-0.5)^n*u(n)`
- `Unstable`
- `1.5*(-0.75)^n*u(-n-1)+(-1.5)*(-0.5)^n*u(-n-1)`
- `Unstable`

A causal discrete-time LTI system is described by the block-diagram below, where k is a real variable. Assume $\alpha = 9$, $\beta = 4$, and $\eta = 7$.



a) Find the transfer function, $H(z) = \frac{Y(z)}{X(z)}$, of the system in terms of parameter k (please express this as a rational polynomial function in POSITIVE powers of z).

$H(z) =$ ________

b) State the radius of convergance of this transfer function in interval notation (e.g. (-INF,a), (a,b) or (b, INF)).

$RoC:$ ___

c) Find the values of $|k|$ for which the system is BIBO stable. Enter your answer in interval notation.

___

Correct Answers:

- `7*(z+k/4)/(z-k/9)`
- `(k/9,infinity)`
- `(0,9)`

For the two discrete-time LTI systems described below, find the transfer function $H(z)$, if:

a) In system A, where an input-output signal pair is given by:

$$x[n] = \begin{cases} 6 & n = 0, 1 \\ 0 & otherwise \end{cases}$$

$$y[n] = \begin{cases} 2 & n = 0,3 \\ 7 & n = 1,5 \\ 0 & otherwise \end{cases}$$

$H(z) =$ ___

b) In system B, where an input-output signal pair is given by:

$x[n] = (-0.5)^n u[n]$

$$y[n] = \begin{cases} 0 & n < 0 \\ 4(n+1) & n = 0,1,2 \\ 2(-0.5)^n & n \geq 3 \end{cases}$$

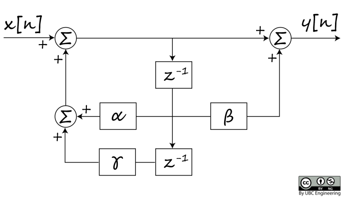
$H(z) =$ ___

Correct Answers:

- `[2+7*z^(-1)+2*z^(-3)+7*z^(-5)]/[6+6*z^(-1)]`
- `4*[1+2*z^(-1)+3*z^(-2)]*[1-(-0.5)*z^(-1)]+(-0.25)*z^(-3)`

JY Note Apr 5, 2020: One student has had parts (b) & (c) mistakenly graded as wrong. If you are confident that you were similarly mistakenly penalized, please contact me directly.

Consider a causal discrete-time LTI system whose input, $y[n]$, and output, $x[n]$, are related by the block diagram given in the figure below. Assume $\alpha = \frac{-1}{10}$, $\beta = 7$, and $\gamma = \frac{2}{10}$.



a) Find the difference equation that describes this system

___

In your answers, enter $z(n)$ for a discrete-time function $z[n]$. WebWork is unable to parse a function that uses square brackets.

b) Find the transfer function, $H(z)$, of this system

$H[z] =$ ___

c) M/C Question: Is the system stable?

[?/Yes/No]

*Part **c** will only be marked correct if the answer to part **b** is correct.*

Correct Answers:

- `y(n) = x(n)+7*x(n-1)+(-0.1)*y(n-1)+0.2*y(n-2)`
- `[1+7*z^(-1)]/[1-(-0.1)*z^(-1)-0.2*z^(-2)]`
- `Yes`

JY Note Apr 5, 2020: If you receive full marks for parts (a) & (c) but your (b) is marked incorrect, please contact me directly to have this rectified.

Consider a system whose input, $x[n]$ and output $y[n]$ are related by $6y[n-1] + 3y[n] = 8x[n]$.

a) Determine the zero-input response, $y_{zi}[n]$, of the system if $y[-1] = 7$

$y_{zi}[n] =$ ___

b) Determine the zero-state response, $y_{zs}[n]$, of the system if $x[n] = \left(\frac{1}{4}\right)^n u[n]$

$y_{zs}[n] =$ ___

c) Determine the output, $y[n]$, of the system for $n \geq 0$, if $y[-1] = 7$ and $x[n] = \left(\frac{1}{4}\right)^n u[n]$.

$y[n] =$ ___

In your answers, enter $z(n)$ for a discrete-time function $z[n]$. WebWork is unable to parse a function that uses square brackets.

Correct Answers:

- `-14*(-2)^n*u(n)`
- `2.37037*(-2)^n*u(n)+0.296296*0.25^n*u(n)`
- `0.296296*0.25^n*u(n)+(-2)^n*u(n)*(-11.6296)`

The Z-transform of the unit-step response of a causal LTI system is given by $S(z) = \frac{7}{1-z^{-1}} - \frac{7.7}{1-1.1z^{-1}}$. Determine the impulse response, $h[n]$ of the system. Simplify your answer as much as possible and write it in terms of $\delta[n]$ and $u[n-1]$.

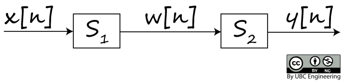
$h[n] =$ ___

In your answer, enter $z(n)$ for a discrete-time function $z[n]$ and enter $D(n)$ instead of $\delta[n]$. WebWork is unable to parse a function that uses square brackets.

Correct Answers:

- `-0.7*D(n)+(-0.7)*1.1^n*u(n-1)`

Suppose we cascade two systems as shown in the figure below, where the systems are characterized by the following input-output equations:



$S_1: \quad w[n] = 3(x[n] - x[n-1])$

$S_2: \quad y[n] = \frac{4y[n-1]}{9} + \frac{w[n]}{9}$

a) If $x[n] = u[n]$ find $y[n]$, assuming zero initial conditions.

$y[n] =$ ______

b) Determine the steady-state response $y_{ss}[n]$ to input $x[n] = 14u[n] + sin(n\frac{\pi}{3})u[n]$ (include at least 4 significant figures for the phase).

$y_{ss}[n] =$ ______

In your answers, enter $z(n)$ for a discrete-time function $z[n]$. WebWork is unable to parse a function that uses square brackets.

Correct Answers:

- `0.333333*0.444444^n*u(n)`
- `0.384111*sin(n*pi/3+0.587661)`