THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering
ELEC 221 – Signals and Systems
Instructor: Dr. Joseph Yan
Final Exam for April 17, 2018

Three Double-sided 8.5"x11" sheets of notes are permitted. No calculators are permitted.

Time: 180 minutes.

This examination consists of 10 physical pages (last two pages intentionally blank for your overflow work). **Please check that you have a complete copy. You may use both sides of each sheet as needed.**

READ THIS

Surname First name

Student Number

#	MAX	GRADE
1	51	
2	21	
3	28	
4	22	
5	23	
6	35	
TOTAL	180	

IMPORTANT NOTE: The announcement "stop writing" will be made at the end of the examination. Anyone writing after this announcement will receive a score of 0. No exceptions, no excuses.

All writings must be on this booklet. The blank sides on the reverse of each page may also be used.

Each candidate should be prepared to produce, upon request, his/her Library/AMS card.

*Read and observe the following rules:**

No candidate shall be permitted to enter the examination room after the expiration of one-half hour, or to leave during the first half-hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination-questions.

***Caution** - Candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:*

Making use of any books, papers or memoranda, calculators, audio or visual cassette players or other memory aid devices, other than as authorized by the examiners.

Speaking or communicating with other candidates.

Purposely exposing written papers to the view of other candidates.

The plea of accident or forgetfulness shall not be received.

1. "Short" Answer Questions:

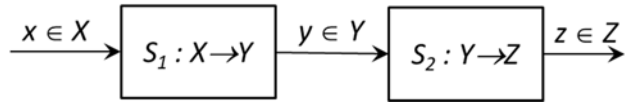
a) (3pts) Determine the z-transform (including RoC) of $x[n] = 2\delta[n] - 0.5\delta[n-2] + \delta[n-3]$.

b) (5pts) Consider two voltage signals $v_1(t) = A\cos(2000\pi\, t + \varphi_1)$ and $v_2(t) = A\cos(2000\pi\, t + \varphi_2)$ and their sum $v_3 = v_1 + v_2$. Describe quantitatively how the frequency and the amplitude of v_3 changes with the difference $\varphi_1 - \varphi_2$ (identifying specific notable values). Assume $A \in \mathbb{R}$ is constant.

c) (8pts) The Z-transform of an LTI system's impulse response is given by $\widehat{H}(z) = \frac{1}{z-0.5} + \frac{1}{z+2} + \frac{1}{z^2+2}$. List every possible RoC, indicating whether the resulting system is stable, causal, anti-causal, and/or two-sided.

d) (7pts) A continuous-time signal $x(t) = 4\cos(20\pi t) + 3\sin(30\pi t) + 5\cos(42\pi t + \pi/4)$ is sampled at 30 Hz. Determine the reconstructed signal $w = IdealInterpolator_T(Sampler_T(x))$ for T=(1/30)s (**NB:** The result is not as straightforward as in MT3 so consider carefully the effect of the phase shift. Note that the sampling occurs at integer multiples of T).

e) (6pts) Consider the cascade system in which S_1 is a time shifter such that $y(t)=x(t-2)$ and S_2 is an amplitude modulator such that $z(t)=y(t)\cos(\pi t)$. Find the mathematical relationship between the frequency-domain representations $X(\omega)$ and $Z(\omega)$.



f) (6pts) Find A^n using the CHT if A = $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ (i.e., set $A^n = \alpha_o I + \alpha_1 A$ and find α_o and α_1).

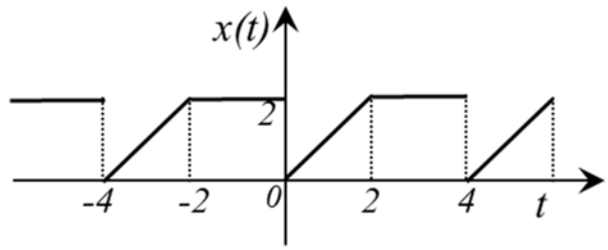
g) (9pts) Consider a discrete-time system, S, whose input x is related to output y through the given relationship $\forall n \in \mathbb{Z}$, $\sum_{m=0}^{M} a_m y(n-m) = \sum_{k=0}^{L} b_k x(n-k)$, where L and M are positive integers. Show whether S is linear, whether it is time-invariant, and determine an expression for the *order* of the system.

h) (7pts) Determine the z-transform (including the RoC) of the signal $x[n] = a^{-n}u[-n-1]$.

2. Consider the periodic signal shown with period p=4s.

a) (12pts) Expressing $x(t) = \sum_{m=-\infty}^{\infty} X_m \, e^{jm\omega_0 t}$, state the value of ω_0 and compute expressions for every one of the coefficients X_m.

b) (9pts) Let $y = x * h$ where $h(t) = \frac{\sin(4t)}{4t}$. Determine $y(t)$ and sketch $Y(\omega)$.



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3. You are to design the state machine for a drink vending machine to meet the following requirements:

- The machine only accepts quarters (25 cents) and loonies ($1).
- The vending machine dispenses 2 types of drinks: Drink A or Drink B.
- A sensor detects whether Drink A is in stock (returns '1') or out of stock (returns '0'). A separate but similar sensor detects whether Drink B is in stock.
- Drink A costs $1 and Drink B costs $1.25.
- The machine can only return 25 cents, return 50 cents, or return all the money.
- The maximum amount of money that the machine can have is $1.25. If after adding a coin the total amount exceeds $1.25, it should return the extra amount if it is less or equal to 50 cents, otherwise it should return all the money.
- Whenever a drink is selected by pressing one of the buttons, if the amount of money is insufficient it should give a message that tells the user that the amount is insufficient.
- When a drink is selected and the available amount of money is the correct one, it should check if the drink is available, and if it is available, it should dispense the drink and go back to the beginning.
- If the price of the selected drink is less than the available amount, it should return the difference, then check if the drink is available and dispense it if that is the case.
- If the available amount is the amount required for the selected drink, but this is not available, it should give a message telling that the drink is out of stock, and then go back to the state for the current amount of money.

a) (20pts) Draw the state machine diagram.

b) (8pts) Suppose that the machine has been reset and has one Drink A available, and two Drinks B available. Determine the trace and output response that would result from the following sequence of user interactions: insert quarter, insert quarter, press "Drink B", insert $1, insert quarter, insert $1, insert $1, press "Drink B", insert quarter, press "Drink A", insert $1, insert quarter, press "Drink A", insert quarter, press "Drink B".

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4. Consider the causal LTI system with $y[n] = 0.5y[n-1] + ay[n-2] + x[n], \forall n \in \mathbb{Z}$ relating input x to output y, where $a \in \mathbb{R}_+$ is a design parameter.

a) (9pts) Determine the system transfer function, $\hat{H}(z)$, its poles and sketch its region of convergence.

b) (9pts) Determine the output for $n \geq 0$ if $y[-1] = y[-2] = x[0] = 1$ and $x[n] = 0$ for $n>0$.

c) (4pts) Determine the values of a for which the system is stable.

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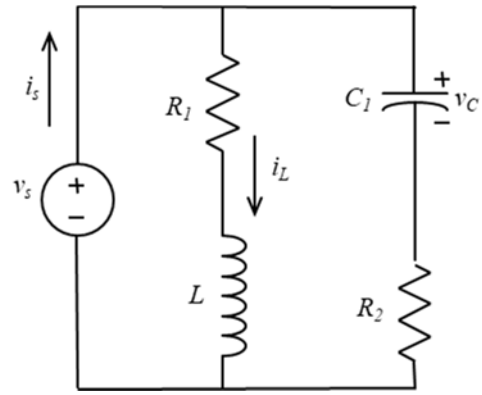
5. Consider the linear circuit shown in which all passive component values (i.e., R_1, R_2, C_1 and L) are positive and real.



a) (10pts) Find the [A,B,C,D] matrices for the circuit using $s(t) = \begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix}$ as the state, v_s as the input and i_s as the output.

b) (9pts) Explain whether the system is stable. Also determine the condition that would cause the system to lose controllability and provide an intuitive explanation.

c) (4pts) Write out the full expression for the output response at any time $t > 0$, in terms of the initial state $s(0)$, the input $v_s(t)$, and the [A,B,C,D] matrices.

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6. Consider a filter with difference equation $y[n] - \frac{1}{4}y[n-2] = x[n] + \frac{1}{2}x[n-1], \forall n \in \mathbb{Z}$.

a) (10pts) Determine both the frequency response and impulse response of this filter.

b) (5pts) For input $x[n] = \cos\left(\frac{\pi n}{2}\right), \forall n \epsilon \mathbb{Z}$, determine the output y.

c) (15pts) Find the Controllable Canonical Form realisation [A,B,C,D] for this system, sketch the associated block diagram and determine both its controllability and observability.

d) (5pts) The output y is passed through a second filter with frequency response $G(\omega) = 1 - \frac{1}{2}e^{-j\omega}$. Determine the relationship of the new output, w, to the original input x.

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