EECE 261 - PROBLEM SET 7

SOLUTIONS

1

A toroidal core has a rectangular cross section defined by the surfaces $\rho = 2$ cm, $\rho = 3$ cm, $z = 4$ cm, and $z = 4.5$ cm. The core material has a relative permeability of 80. If the core is wound with a coil containing 8000 turns of wire, find its inductance: First we apply Ampere's circuital law to a circular loop of radius ρ in the interior of the toroid, and in the $\mathbf{a}_\phi$ direction.

$$\oint \mathbf{H} \cdot d\mathbf{L} = 2\pi\rho H_\phi = NI \quad \Rightarrow \quad H_\phi = \frac{NI}{2\pi\rho}$$

The flux in the toroid is then the integral over the cross section of $\mathbf{B}$:

$$\Phi = \int\int \mathbf{B} \cdot d\mathbf{L} = \int_{.04}^{.045} \int_{.02}^{.03} \frac{\mu_r \mu_0 NI}{2\pi\rho}\, d\rho\, dz = (.005)\frac{\mu_r \mu_0 NI}{2\pi} \ln\left(\frac{.03}{.02}\right)$$

The flux linkage is then given by $N\Phi$, and the inductance is

$$L = \frac{N\Phi}{I} = \frac{(.005)(80)(4\pi \times 10^{-7})(8000)^2}{2\pi} \ln(1.5) = \underline{2.08 \text{ H}}$$

2.

a) Find the current I that flows as a result of the motion of the sliding bar: The current is found through

$$I = \frac{1}{R}\oint \mathbf{E}\cdot d\mathbf{L} = -\frac{1}{R}\frac{d\Phi_m}{dt}$$

Taking the normal to the path integral as $\mathbf{a}_z$, the path direction will be counter-clockwise when viewed from above (in the $-\mathbf{a}_z$ direction). The minus sign in the equation indicates that the current will therefore flow *clockwise*, since the magnetic flux is increasing with time. The flux of $\mathbf{B}$ is $\Phi_m = Bdvt$, and so

$$|I| = \frac{1}{R}\frac{d\Phi_m}{dt} = \frac{Bdv}{R} \quad \text{(clockwise)}$$

b) The bar current results in a force exerted on the bar as it moves. Determine this force:

$$\mathbf{F} = \int Id\mathbf{L}\times\mathbf{B} = \int_0^d Idx\mathbf{a}_x \times B\mathbf{a}_z = \int_0^d \frac{Bdv}{R}\mathbf{a}_x \times B\mathbf{a}_z = -\frac{B^2d^2v}{R}\mathbf{a}_y \text{ N}$$

3.

A perfectly conducting filament containing a small 500-Ω resistor is formed into a square, as illustrated in Fig. 10.6. Find $I(t)$ if

a) $\mathbf{B} = 0.3\cos(120\pi t - 30°)\,\mathbf{a}_z$ T: First the flux through the loop is evaluated, where the unit normal to the loop is $\mathbf{a}_z$. We find

$$\Phi = \int_{\text{loop}} \mathbf{B}\cdot d\mathbf{S} = (0.3)(0.5)^2\cos(120\pi t - 30°) \text{ Wb}$$

Then the current will be

$$I(t) = \frac{\text{emf}}{R} = -\frac{1}{R}\frac{d\Phi}{dt} = \frac{(120\pi)(0.3)(0.25)}{500}\sin(120\pi t - 30°) = \underline{57\sin(120\pi t - 30°)\text{ mA}}$$

(3b) $B = 0.4 \cos[\pi(ct - y)]\hat{a}_z \ \mu T.$

$c = 3 \times 10^8 \ m/s.$

$$\phi = \iint_{x\,y} \bar{B} \cdot d\bar{s} = \int_0^{0.5}\int_0^{0.5} 0.4 \cos[\pi ct - \pi y] \cdot dx\,dy$$

$$= \underset{\substack{\uparrow \\ x}}{(0.4)(0.5)} \left[-\frac{1}{\pi} \sin[\pi ct - \pi y] \right]_0^{0.5}$$

$$= \frac{-0.2}{\pi} \left[\sin[\pi ct - \tfrac{\pi}{2}] - \sin(\pi ct) \right]$$

$$I(t) = \frac{emf}{R} = -\frac{1}{R}\frac{d\phi}{dt}$$

$$= \frac{0.2}{500\pi}\left(\pi c \cos(\pi ct - \pi/2) - \pi c \cos(\pi ct)\right)$$

$$= \frac{0.2c}{500}\left(\cos(\pi ct - \pi/2) - \cos(\pi ct)\right)$$

$$\frac{0.2c}{500}\left(\sin \pi ct - \cos \pi ct\right) \ \mu A$$

4. For a cylindrical structure, we know

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho} \hat{\rho} \quad V/m$$

We also have $V = -\int_b^a \bar{E} \cdot d\rho$

$$V = \frac{\rho_L}{2\pi\varepsilon_0} \ln(b/a)$$

isolate ρ_L:

$$\rho_L = \frac{V(2\pi\varepsilon_0)}{\ln(b/a)}$$

Sub into $\bar{E}$:

$$\bar{E} = \frac{V(2\pi\varepsilon_0)}{2\pi\varepsilon_0 \rho \ln(b/a)} \hat{\rho} \quad V/m$$

$$\bar{D} = \varepsilon_0 \bar{E} = \frac{\varepsilon_0 V}{\rho \ln(b/a)}$$

put $V = V_0 \cos \omega t$

$$\bar{D} = \frac{\varepsilon_0 V_0 \cos\omega t}{\rho \ln(b/a)}$$

$$\bar{J}_d = \frac{d\bar{D}}{dt} = \frac{-\omega \varepsilon_0 V_0 \sin\omega t}{\rho \ln(b/a)}$$

$$I_d = \int_s \bar{J}_d \cdot d\bar{S} = \frac{-\omega \varepsilon_0 V_0 \sin\omega t}{\ln(b/a)} \int_{z=0}^{l} \int_{\phi=0}^{2\pi} \frac{1}{\rho}\, dz\, \rho\, d\phi$$

$$= \frac{-2\pi \omega \varepsilon_0 l V_0 \sin\omega t}{\ln(b/a)}$$

whereas

$$I = C\frac{dV}{dt} = \frac{2\pi \varepsilon_0 l}{\ln(b/a)} \frac{d}{dt}\left(V_0 \cos\omega t\right)$$

$$= \frac{-2\pi \omega \varepsilon_0 l V_0 \sin\omega t}{\ln(b/a)}$$