ORACLES AND CHOICE SEQUENCES FOR TYPE-THEORETIC PRAGMATICS

Jon Sterling

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joint work with Darryl McAdams

INTRODUCTION

NATURAL LANGUAGE SEMANTICS AND ANAPHORA

$[\![$*A woman walked in.*$]\!]$

$\triangledown$

$$[\![\textit{A woman walked in.}]\!]$$

$$\bigtriangledown$$

$$(\Sigma p \in Woman)$$

$$[\![\textit{A woman walked in.}]\!]$$

$$\triangledown$$

$$(\Sigma p \in Woman)\, WalkedIn(p)$$

$$[\![\textit{She sat down}]\!]$$
$$\triangledown$$

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$$\triangledown$$

$$\mathit{SatDown}(???)$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$
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$$\triangledown$$

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))$$

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$$\triangledown$$
$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, SatDown(???)$$

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$$\triangledown$$

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, SatDown(\pi_1(x))$$

$$[\![\textit{Every farmer who owns a donkey beats it.}]\!]$$
$$\triangledown$$
$$(\Pi p \in (\Sigma x \in \mathit{Farmer})\,(\Sigma y \in \mathit{Donkey})\,\mathit{Owns}(x;y))$$

$$[\![\textit{Every farmer who owns a donkey beats it.}]\!]$$
$$\triangledown$$
$$(\Pi p \in (\Sigma x \in Farmer)\,(\Sigma y \in Donkey)\,Owns(x;y))\,Beats(???;???)$$

$$[\![\textit{Every farmer who owns a donkey beats it.}]\!]$$

$$\triangledown$$

$$(\Pi p \in (\Sigma x \in \mathit{Farmer})\,(\Sigma y \in \mathit{Donkey})\,\mathit{Owns}(x;y))\,\mathit{Beats}(\pi_1(p);\pi_1(\pi_2(p)))$$

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- terms for presuppositions (this talk)
- resolution of presuppositions

THE **require** ORACLE: STATICS

require — FORMAL RULES

$$\textbf{require} : (0; 1) \qquad \text{(operator)}$$

$$\textbf{require}\ x : A\ \textbf{in}\ N \stackrel{\text{def}}{=} \textbf{require}(A; x.N) \qquad \text{(notation)}$$

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$$\frac{\Gamma \vdash A\ \mathit{type} \quad \Gamma, x : A \vdash N \in B}{\Gamma \vdash \mathbf{require}\ x : A\ \mathbf{in}\ N \in B} \qquad (\mathit{require})$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$

$$\triangledown$$

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, SatDown(???)$$

require — EXAMPLES

$$[\![\textit{A woman walked in. She sat down}]\!]$$

$$\triangledown$$

$$(\Sigma x \in (\Sigma p \in \mathit{Woman})\, \mathit{WalkedIn}(p))\ \mathbf{require}\ y : \mathit{Woman}\ \mathbf{in}\ \mathit{SatDown}(y)$$

The meaning of a sentence is a logical proposition.

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What we want:

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, \mathbf{require}\; y : Woman\; \mathbf{in}\; SatDown(y)$$

$$\sim$$

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, \pi_1(x)$$

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$$\sim$$

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, \pi_1(x)$$

$$\boxed{\text{where } M \sim N \stackrel{\text{def}}{=} (M \leq N) \land (N \leq M)}$$

EVERY GRAMMATICAL SENTENCE HAS A MEANING

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...but only some of them denote propositions (types)!

require—NAÏVE DYNAMICS

$$\frac{M \in A \quad [M/x]\,N \Downarrow N'}{\textbf{require}\ x : A\ \textbf{in}\ N \Downarrow N'} \qquad (??)$$

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1. impredicativity

require—NAÏVE DYNAMICS

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Can the above be made precise? There are two problems:

1. impredicativity
2. non-determinism

(HOLD THAT THOUGHT)

A POSITIVE EXAMPLE

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$$[\![\textit{The President ran a marathon}]\!]$$

$$\triangledown$$

$$\llbracket \textit{ The President ran a marathon } \rrbracket$$
$$\triangledown$$
$$\textbf{require } x : \mathit{President} \textbf{ in } (\Sigma y \in \mathit{Marathon})\, \mathit{Ran}(x; y)$$

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$$\Downarrow$$

$$(\Sigma y \in \mathit{Marathon})\, \mathit{Ran}(\mathit{Obama}; y)$$

A NEGATIVE EXAMPLE

$$[\![\textit{The unicorn ran a marathon}]\!]$$

$$\triangledown$$

$$\llbracket\ \textit{The unicorn ran a marathon}\ \rrbracket$$

$$\triangledown$$

$$\textbf{require}\ x : \mathit{Unicorn}\ \textbf{in}\ (\Sigma y \in \mathit{Marathon})\, \mathit{Ran}(x; y)$$

$$[\![\ \textit{The unicorn ran a marathon}\ ]\!]$$

$$\triangledown$$

$$\textbf{require}\ x : \mathit{Unicorn}\ \textbf{in}\ (\Sigma y \in \mathit{Marathon})\,\mathit{Ran}(x;y)$$

(not a proposition)

Is **require** COMPUTATIONALLY EFFECTIVE?

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$$\boxed{\alpha \models_{w} \mathscr{J}}$$

THE CREATING SUBJECT

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All mathematics is a mental construction performed by an idealized subject, subject to the following observations about knowledge:

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Corollary
The meaning of a judgment $\mathscr{J}$ must be explained in terms of its forcing condition, $\boxed{w \Vdash \mathscr{J}}$*, for any stage/world* w*.*

REMARK ON DECIDABILITY

...

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Remark

Contra Dummett, I <u>by no means</u> take the above as requiring that the following shall be true in a constructive metatheory, divorced from time:

$$\forall w. \forall \mathscr{J}\,.\ [\![w \Vdash \mathscr{J}]\!] \vee \neg [\![w \Vdash \mathscr{J}]\!] \qquad \text{(Dummett's infelicity)}$$

The above is impossible in a Beth model.

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$$|_x \mathscr{J}(x) \qquad \text{(general judgment)}$$

$$\mathscr{J}_2 \ (\mathscr{J}_1) \qquad \text{(hypothetical judgment)}$$

BETH-KRIPKE SEMANTICS FOR ASSERTIONS

assertion acts (judgments) are intensional (local)

$\vert_x \mathscr{J}(x)$	(general judgment)
$\mathscr{J}_2\ (\mathscr{J}_1)$	(hypothetical judgment)
$M \Downarrow N$	(evaluation)

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$M \Downarrow N$	(evaluation)
$A = B\ \mathit{type}$	(typehood)

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$A\ \mathit{true}$	(truth)
$M = N \in A$	(membership)

assertion acts (judgments) are intensional (local)

$$w \Vdash \,|_x\, \mathscr{J}(x) \qquad \text{(general judgment)}$$

$$w \Vdash \mathscr{J}_2 \ (\mathscr{J}_1) \qquad \text{(hypothetical judgment)}$$

$$w \Vdash M \Downarrow N \qquad \text{(evaluation)}$$

$$w \Vdash A = B \ \mathit{type} \qquad \text{(typehood)}$$

$$w \Vdash A \ \mathit{true} \qquad \text{(truth)}$$

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SEMANTICS OF HIGHER-ORDER ASSERTIONS

$$
\begin{aligned}
w \Vdash \vert_x\, \mathscr{I}(x) \quad &\iff \quad \cdots \\
w \Vdash \mathscr{I}_2\ (\mathscr{I}_1) \quad &\iff \quad \cdots
\end{aligned}
$$

SEMANTICS OF HIGHER-ORDER ASSERTIONS

$$
\begin{aligned}
w \Vdash |_x\, \mathscr{J}(x) \quad &\iff \quad \forall u \geq w. \forall x \in \mathscr{M}_u.\ u \Vdash \mathscr{J}(x) \\
w \Vdash \mathscr{J}_2\ (\mathscr{J}_1) \quad &\iff \quad \forall u \geq w.\ u \Vdash \mathscr{J}_1 \Rightarrow u \Vdash \mathscr{J}_2
\end{aligned}
$$

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w \Vdash |_x \mathscr{J}(x) &\iff \forall u \geq w. \forall x \in \mathscr{M}_u.\ u \Vdash \mathscr{J}(x) \\
w \Vdash \mathscr{J}_2\ (\mathscr{J}_1) &\iff \forall u \geq w.\ u \Vdash \mathscr{J}_1 \Rightarrow u \Vdash \mathscr{J}_2
\end{aligned}
$$

where $\mathscr{M}_w$ is the species of constructions that have been effected by stage w

$$w \Vdash A\ \mathit{true} \iff$$

intensional / ephemeral truth (kripke)

$$w \Vdash A\ \mathit{true} \iff \exists m \in \mathscr{M}_w.$$

$$w \Vdash A\ \mathit{true} \iff \exists m \in \mathscr{M}_w.\ w \Vdash m = m \in A$$

$$w \Vdash A \ \mathit{true} \iff$$

intensional / ephemeral truth (beth)

$$w \Vdash A\ \mathit{true} \iff \exists \mathfrak{B}\ \textbf{bars}\ w.$$

$$w \Vdash A\ \mathit{true} \iff \exists \mathfrak{B}\ \textbf{bars}\ w. \forall u \in \mathfrak{B}.$$

$$w \Vdash A\ \mathit{true} \iff \exists \mathfrak{B}\ \textbf{bars}\ w.\forall u \in \mathfrak{B}.\exists m \in \mathscr{M}_u.$$

$$w \Vdash A\ \mathit{true} \Longleftrightarrow \exists \mathfrak{B}\ \mathbf{bars}\ w. \forall u \in \mathfrak{B}. \exists m \in \mathscr{M}_u.\ u \Vdash m = m \in A$$

THE **require** ORACLE: DYNAMICS

require — DYNAMICS

$$\frac{}{\mathfrak{S}(\langle\rangle)} \qquad \frac{\mathfrak{S}(\vec{u}) \quad |_n\, \rho(n) < n \ \ (n \in \mathbb{N}^+)}{\mathfrak{S}(\vec{u} \frown \rho)} \qquad \text{(spread law)}$$

$$\frac{\alpha \models_t A \Downarrow A' \quad |\varkappa_{A'}(t)| = \ell \quad \mathsf{hd}(\alpha)(\ell) = j \quad \mathsf{tl}(\alpha) \models_t \left[\varkappa_{A'}(j)/x\right] N \Downarrow N'}{\alpha \models_t \mathbf{require}\ x : A\ \mathbf{in}\ N \Downarrow N'}$$

(for $\alpha \in \mathfrak{S}$)

QUESTIONS?