ORACLES AND CHOICE SEQUENCES FOR TYPE-THEORETIC PRAGMATICS

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joint work with Darryl McAdams

INTRODUCTION

NATURAL LANGUAGE SEMANTICS AND ANAPHORA

$[\![$*A woman walked in.*$]\!]$

$\triangledown$

$$[\![\textit{A woman walked in.}]\!]$$

$$\triangledown$$

$$(\Sigma p \in Woman)$$

$$[\![\textit{A woman walked in.}]\!]$$

$$\triangledown$$

$$(\Sigma p \in Woman)\, WalkedIn(p)$$

$[\![\textit{She sat down}]\!]$

$\triangledown$

$$[\![\textit{She sat down}]\!]$$
$$\triangledown$$
$$\mathit{SatDown}(???)$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$

$$\triangledown$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$

$$\triangledown$$

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$

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$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, SatDown(???)$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$

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$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, SatDown(\pi_1(x))$$

$$[\![\textit{Every farmer who owns a donkey beats it.}]\!]$$
$$\triangledown$$
$$(\Pi p \in (\Sigma x \in \mathit{Farmer})\,(\Sigma y \in \mathit{Donkey})\,\mathit{Owns}(x;y))$$

$$[\![\textit{Every farmer who owns a donkey beats it.}]\!]$$

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$$(\Pi p \in (\Sigma x \in \mathit{Farmer})\,(\Sigma y \in \mathit{Donkey})\,\mathit{Owns}(x; y))\,\mathit{Beats}(???; ???)$$

$$[\![\textit{Every farmer who owns a donkey beats it.}]\!]$$

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$$(\Pi p \in (\Sigma x \in \mathit{Farmer})\,(\Sigma y \in \mathit{Donkey})\,\mathit{Owns}(x;y))\,\mathit{Beats}(\pi_1(p);\pi_1(\pi_2(p)))$$

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- terms for presuppositions (this talk)
- resolution of presuppositions

THE **require** ORACLE: STATICS

require — FORMAL RULES

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$$\mathbf{require} : (0; 1) \qquad \text{(operator)}$$

$$\mathbf{require}\ x : A\ \mathbf{in}\ N \stackrel{\text{def}}{=} \mathbf{require}(A; x.N) \qquad \text{(notation)}$$

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$$\frac{\Gamma \vdash A\ \mathit{type} \quad \Gamma, x : A \vdash N \in B}{\Gamma \vdash \textbf{require}\ x : A\ \textbf{in}\ N \in B} \qquad (\mathit{require})$$

require — EXAMPLES

$$[\![\textit{A woman walked in. She sat down}]\!]$$

$$\triangledown$$

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, SatDown(???)$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$

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$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\ \mathbf{require}\ y : Woman\ \mathbf{in}\ SatDown(y)$$

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What we want:

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, \textbf{require}\ y : Woman\ \textbf{in}\ SatDown(y)$$

$$\rightsquigarrow$$

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, \pi_1(x)$$

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$$\sim$$

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, \pi_1(x)$$

$$\boxed{\text{where } M \sim N \stackrel{\text{def}}{=} (M \preceq N) \wedge (N \preceq M)}$$

EVERY GRAMMATICAL SENTENCE HAS A MEANING

...but only some of them denote propositions (types)!

require—NAÏVE DYNAMICS

$$\frac{M \in A \quad [M/x]\,N \Downarrow N'}{\textbf{require}\ x : A\ \textbf{in}\ N \Downarrow N'} \tag{??}$$

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1. impredicativity

require—NAÏVE DYNAMICS

$$\frac{M \in A \quad [M/x]\,N \Downarrow N'}{\textbf{require}\ x : A\ \textbf{in}\ N \Downarrow N'} \qquad (??)$$

Can the above be made precise? There are two problems:

1. impredicativity
2. non-determinism

(HOLD THAT THOUGHT)

A POSITIVE EXAMPLE

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$$[\![\, \textit{The President ran a marathon} \,]\!]$$

$$\triangledown$$

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$$\textbf{require } x : \textit{President} \textbf{ in } (\Sigma y \in \textit{Marathon})\, \textit{Ran}(x; y)$$

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$$\Downarrow$$

$$(\Sigma y \in \mathit{Marathon})\, \mathit{Ran}(\mathit{Obama}; y)$$

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$$[\![\textit{The unicorn ran a marathon}]\!]$$

$$\triangledown$$

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$$[\![\ \textit{The unicorn ran a marathon}\ ]\!]$$

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$$\mathbf{require}\ x : \mathit{Unicorn}\ \mathbf{in}\ (\Sigma y \in \mathit{Marathon})\, \mathit{Ran}(x; y)$$

$$[\![\textit{The unicorn ran a marathon}]\!]$$

$$\triangledown$$

$$\mathbf{require}\ x : \mathit{Unicorn}\ \mathbf{in}\ (\Sigma y \in \mathit{Marathon})\, \mathit{Ran}(x; y)$$

(not a proposition)

Is **require** COMPUTATIONALLY EFFECTIVE?

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$$\boxed{\alpha \Vdash_{w} \mathscr{J}}$$

ASSERTION ACTS IN TIME

assertion acts (judgments) are intensional (local)

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$$\mid_x \mathscr{J}(x) \qquad \text{(general judgment)}$$

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$$\mathscr{J}_2\ (\mathscr{J}_1) \qquad \text{(hypothetical judgment)}$$

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$\mathscr{J}_2\ (\mathscr{J}_1)$	(hypothetical judgment)
$M \Downarrow N$	(evaluation)

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$$M \Downarrow N \qquad \text{(evaluation)}$$
$$A = B\ \mathit{type} \qquad \text{(typehood)}$$

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$\vert_x \mathscr{J}(x)$	(general judgment)
$\mathscr{J}_2\ (\mathscr{J}_1)$	(hypothetical judgment)
$M \Downarrow N$	(evaluation)
$A = B\ \mathit{type}$	(typehood)
$A\ \mathit{true}$	(truth)

BETH–KRIPKE SEMANTICS FOR ASSERTIONS

assertion acts (judgments) are intensional (local)

$|_x \mathscr{J}(x)$	(general judgment)
$\mathscr{J}_2\ (\mathscr{J}_1)$	(hypothetical judgment)
$M \Downarrow N$	(evaluation)
$A = B\ \mathit{type}$	(typehood)
$A\ \mathit{true}$	(truth)
$M = N \in A$	(membership)

assertion acts (judgments) are intensional (local)

$$w \Vdash |_x\, \mathscr{J}(x) \qquad \text{(general judgment)}$$

$$w \Vdash \mathscr{J}_2\ (\mathscr{J}_1) \qquad \text{(hypothetical judgment)}$$

$$w \Vdash M \Downarrow N \qquad \text{(evaluation)}$$

$$w \Vdash A = B\ \mathit{type} \qquad \text{(typehood)}$$

$$w \Vdash A\ \mathit{true} \qquad \text{(truth)}$$

$$w \Vdash M = N \in A \qquad \text{(membership)}$$

$$w \Vdash |_x \mathscr{J}(x) \qquad \text{(general judgment)}$$

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$$
\begin{aligned}
w \Vdash |_x\, \mathscr{J}(x) \quad &\Longleftrightarrow \quad \cdots \\
w \Vdash \mathscr{J}_2\ (\mathscr{J}_1) \quad &\Longleftrightarrow \quad \cdots
\end{aligned}
$$

SEMANTICS OF HIGHER-ORDER ASSERTIONS

$$
\begin{aligned}
w \Vdash |_x\, \mathscr{I}(x) &\iff \forall u \geq w. \forall x \in \mathscr{M}_u.\ u \Vdash \mathscr{I}(x) \\
w \Vdash \mathscr{I}_2\ (\mathscr{I}_1) &\iff \forall u \geq w.\ u \Vdash \mathscr{I}_1 \Rightarrow u \Vdash \mathscr{I}_2
\end{aligned}
$$

$$
\begin{aligned}
w \Vdash |_x\, \mathscr{J}(x) \quad &\Longleftrightarrow \quad \forall u \geq w. \forall x \in \mathscr{M}_u.\ u \Vdash \mathscr{J}(x) \\
w \Vdash \mathscr{J}_2\ (\mathscr{J}_1) \quad &\Longleftrightarrow \quad \forall u \geq w.\ u \Vdash \mathscr{J}_1 \Rightarrow u \Vdash \mathscr{J}_2
\end{aligned}
$$

where $\mathscr{M}_w$ is the species of constructions that have been effected by stage w

intensional / ephemeral truth (kripke)

$$w \Vdash A \; \mathit{true} \iff$$

$$w \Vdash A\ \mathit{true} \iff \exists m \in \mathscr{M}_w.$$

intensional / ephemeral truth (kripke)

$$w \Vdash A\ \mathit{true} \iff \exists m \in \mathscr{M}_w.\ w \Vdash m = m \in A$$

$$w \Vdash A\ \mathit{true} \iff$$

$w \Vdash A\ \mathit{true} \iff \exists \mathfrak{B}\ \textbf{bars}\ w.$

$$w \Vdash A \; \mathit{true} \iff \exists \mathfrak{B} \; \textbf{bars} \; w. \forall u \in \mathfrak{B}.$$

INTENSIONAL / EPHEMERAL TRUTH (BETH)

$$w \Vdash A \; true \iff \exists \mathfrak{B} \; \textbf{bars} \; w. \forall u \in \mathfrak{B}. \exists m \in \mathscr{M}_u.$$

$$w \Vdash A\ \mathit{true} \iff \exists \mathfrak{B}\ \mathbf{bars}\ w. \forall u \in \mathfrak{B}. \exists m \in \mathscr{M}_u.\ u \Vdash m = m \in A$$

THE **require** ORACLE: DYNAMICS

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Remark

Contra Dummett, I by no means take the above as requiring that the following shall be true in a constructive metatheory, divorced from time:

$$\forall w. \forall \mathscr{J}.\ [\![w \Vdash \mathscr{J}]\!] \vee \neg [\![w \Vdash \mathscr{J}]\!] \qquad \text{(Dummett's infelicity)}$$

The above is impossible in a Beth model.

require — DYNAMICS

$$\frac{}{\mathfrak{S}(\langle\rangle)} \qquad \frac{\mathfrak{S}(\vec{u}) \quad |_n\ \rho(n) < n \ \ (n \in \mathbb{N}^+)}{\mathfrak{S}(\vec{u} \frown \rho)} \qquad \text{(spread law)}$$

$$\frac{\alpha \models_t A \Downarrow A' \quad |\varkappa_{A'}(t)| = \ell \quad \mathsf{hd}(\alpha)(\ell) = j \quad \mathsf{tl}(\alpha) \models_t \left[\varkappa_{A'}(j)/x\right] N \Downarrow N'}{\alpha \models_t \textbf{require}\ x : A\ \textbf{in}\ N \Downarrow N'}$$

(for $\alpha \in \mathfrak{S}$)

Questions?