ORACLES AND CHOICE SEQUENCES FOR TYPE-THEORETIC PRAGMATICS

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joint work with Darryl McAdams

INTRODUCTION

$$[\![\textit{A woman walked in.}]\!]$$

$$\triangledown$$

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$$\triangledown$$

$$(\Sigma p \in Woman)$$

$$[\![\textit{A woman walked in.}]\!]$$

$$\triangledown$$

$$(\Sigma p \in \mathit{Woman})\, \mathit{WalkedIn}(p)$$

$[\![\textit{She sat down}]\!]$

$\triangledown$

$$[\![\textit{She sat down}]\!]$$
$$\triangledown$$
$$\mathit{SatDown}(???)$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$
$$\triangledown$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$

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$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$

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$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, SatDown(???)$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$

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$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, SatDown(\pi_1(x))$$

$$[\![\textit{Every farmer who owns a donkey beats it.}]\!]$$

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$$(\Pi p \in (\Sigma x \in \mathit{Farmer})\,(\Sigma y \in \mathit{Donkey})\,\mathit{Owns}(x;y))$$

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$$(\Pi p \in (\Sigma x \in \mathit{Farmer})\,(\Sigma y \in \mathit{Donkey})\,\mathit{Owns}(x;y))\,\mathit{Beats}(???;???)$$

$$[\![\textit{Every farmer who owns a donkey beats it.}]\!]$$
$$\triangledown$$
$$(\Pi p \in (\Sigma x \in \mathit{Farmer})\,(\Sigma y \in \mathit{Donkey})\,\mathit{Owns}(x; y))\,\mathit{Beats}(\pi_1(p); \pi_1(\pi_2(p)))$$

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- terms for presuppositions

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- resolution of presuppositions

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THE **require** ORACLE: STATICS

require — FORMAL RULES

$$\textbf{require} : (0; 1) \qquad \text{(operator)}$$

$$\textbf{require}\ x : A\ \textbf{in}\ N \stackrel{\text{def}}{=} \textbf{require}(A; x.N) \qquad \text{(notation)}$$

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$$\frac{\Gamma \vdash A\ \textit{type} \quad \Gamma, x : A \vdash N \in B}{\Gamma \vdash \textbf{require}\ x : A\ \textbf{in}\ N \in B} \qquad (\textit{require})$$

$$[\![\textit{A woman walked in. She sat down}]\!]$$

$$\triangledown$$

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, SatDown(???)$$

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$$(\Sigma x \in (\Sigma p \in \mathit{Woman})\, \mathit{WalkedIn}(p))\ \mathbf{require}\ y : \mathit{Woman}\ \mathbf{in}\ \mathit{SatDown}(y)$$

The meaning of a sentence is a logical proposition.

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What we want:

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, \mathbf{require}\ y : Woman\ \mathbf{in}\ SatDown(y)$$

$$\sim$$

$$(\Sigma x \in (\Sigma p \in Woman)\, WalkedIn(p))\, \pi_1(x)$$

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$$(\Sigma x \in (\Sigma p \in \mathit{Woman})\, \mathit{WalkedIn}(p))\, \pi_1(x)$$

$$\boxed{\text{where } M \sim N \overset{\text{def}}{=} (M \preceq N) \wedge (N \preceq M)}$$

EVERY GRAMMATICAL SENTENCE HAS A MEANING

…but only some of them denote propositions (types)!

A NEGATIVE EXAMPLE

$$[\![\textit{The unicorn ran a marathon}]\!]$$

$$\triangledown$$

$$\llbracket \textit{The unicorn ran a marathon} \rrbracket$$

$$\triangledown$$

$$\textbf{require}\ x : \mathit{Unicorn}\ \textbf{in}\ (\Sigma y \in \mathit{Marathon})\, \mathit{Ran}(x; y)$$

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(not a proposition)

A POSITIVE EXAMPLE

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$[\![$ *The President ran a marathon* $]\!]$

$\triangledown$

$$[\![\ \textit{The President ran a marathon}\ ]\!]$$

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$$\textbf{require}\ x : \textit{President}\ \textbf{in}\ (\Sigma y \in \textit{Marathon})\,\textit{Ran}(x; y)$$

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$$\Downarrow$$

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Evaluation is now non-deterministic

Is **require** COMPUTATIONALLY EFFECTIVE?

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$$\boxed{\alpha \Vdash_{w} \mathscr{J}}$$

ASSERTION ACTS IN TIME

assertion acts (judgments) are intensional (local)

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BETH-KRIPKE SEMANTICS FOR ASSERTIONS

assertion acts (judgments) are intensional (local)

$\mid_x \mathscr{J}(x)$	(general judgment)
$\mathscr{J}_2\ (\mathscr{J}_1)$	(hypothetical judgment)
$M \Downarrow N$	(evaluation)

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$M = N \in A$	(membership)

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$w \Vdash M \Downarrow N$	(evaluation)
$w \Vdash A = B \ \mathit{type}$	(typehood)
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SEMANTICS OF HIGHER-ORDER ASSERTIONS

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SEMANTICS OF HIGHER-ORDER ASSERTIONS

$$
\begin{aligned}
w \Vdash |_x\, \mathscr{J}(x) \quad &\iff \quad \forall u \geq w. \forall x \in \mathscr{M}_u.\ u \Vdash \mathscr{J}(x) \\
w \Vdash \mathscr{J}_2\ (\mathscr{J}_1) \quad &\iff \quad \forall u \geq w.\ u \Vdash \mathscr{J}_1 \Rightarrow u \Vdash \mathscr{J}_2
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\end{aligned}
$$

where $\mathscr{M}_w$ is the species of constructions that have been effected by stage w

$$w \Vdash A \ \mathit{true} \iff$$

intensional / ephemeral truth (kripke)

$$w \Vdash A\ \mathit{true} \iff \exists m \in \mathscr{M}_w.$$

INTENSIONAL / EPHEMERAL TRUTH (KRIPKE)

$$w \Vdash A\ \mathit{true} \iff \exists m \in \mathscr{M}_w.\ w \Vdash m = m \in A$$

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$w \Vdash A\ \mathit{true} \iff \exists \mathfrak{B}\ \textbf{bars}\ w.$

$$w \Vdash A \; \mathit{true} \iff \exists \mathfrak{B} \; \mathbf{bars} \; w. \forall u \in \mathfrak{B}.$$

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$$w \Vdash A \ \mathit{true} \iff \exists \mathfrak{B} \ \mathbf{bars} \ w . \forall u \in \mathfrak{B} . \exists m \in \mathscr{M}_u . \ u \Vdash m = m \in A$$

THE **require** ORACLE: DYNAMICS

We need to explain what it means for the following judgment to be forced:

$$\mathbf{require}\ x : A\ \mathbf{in}\ N \Downarrow N'$$

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Non-determinism may be eliminated via by stating the judgments of type theory relative to a choice sequence, subject to a spread law.

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All mathematics is a mental construction in the mind of an idealized subject, subject to the following laws:

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Remark

Contra Dummett, I by no means take the above as requiring that the following shall be true in a constructive metatheory, divorced from time:

$$\forall w.\forall \mathscr{J}.\ [\![w \Vdash \mathscr{J}]\!] \vee \neg [\![w \Vdash \mathscr{J}]\!] \tag{infelicity}$$

The above is impossible in a Beth model.

$$\frac{}{\mathfrak{S}(\langle\rangle)} \qquad \frac{\mathfrak{S}(\vec{u}) \quad |_n\, \rho(n) < n \;\; (n \in \mathbb{N}^+)}{\mathfrak{S}(\vec{u} \frown \rho)}$$

(spread law)

$$\frac{\alpha \models_t A \Downarrow A' \quad |\varkappa_{A'}(t)| = \ell \quad \mathsf{hd}(\alpha)(\ell) = j \quad \mathsf{tl}(\alpha) \models_t \left[\varkappa_{A'}(j)/x\right] N \Downarrow N'}{\alpha \models_t \textbf{require } x : A \textbf{ in } N \Downarrow N'}$$

(for $\alpha \in \mathfrak{S}$)

Questions?