

ORACLES AND CHOICE SEQUENCES FOR TYPE-THEORETIC PRAGMATICS

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joint work with Darryl McAdams

INTRODUCTION

[[*A woman walked in.*]]



[[*A woman walked in.*]]

▽

($\Sigma p \in \textit{Woman}$)

[[*A woman walked in.*]]

∇

$(\Sigma p \in \textit{Woman}) \textit{WalkedIn}(p)$

[[*She sat down*]]



[[*She sat down*]]

▽

SatDown(???)

[[*A woman walked in. She sat down*]]



[[*A woman walked in. She sat down*]]

▽

$(\Sigma x \in (\Sigma p \in \textit{Woman}) \textit{WalkedIn}(p))$

[[A woman walked in. She sat down]]

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$(\Sigma x \in (\Sigma p \in \text{Woman}) \text{WalkedIn}(p)) \text{SatDown}(???)$

[[A woman walked in. She sat down]]

▽

$(\Sigma x \in (\Sigma p \in \text{Woman}) \text{WalkedIn}(p)) \text{SatDown}(\pi_1(x))$

THE “DONKEY SENTENCE”

[[*Every farmer who owns a donkey beats it.*]]

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$(\Pi p \in (\Sigma x \in \textit{Farmer}) (\Sigma y \in \textit{Donkey}) \textit{Owns}(x; y))$

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THE “DONKEY SENTENCE”

[[Every farmer who owns a donkey beats it.]]

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$(\Pi p \in (\Sigma x \in \text{Farmer}) (\Sigma y \in \text{Donkey}) \text{Owns}(x; y)) \text{Beats}(\pi_1(p); \pi_1(\pi_2(p)))$

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- terms for presuppositions (this talk)
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THE **require** ORACLE: STATICS

require : (0;1)

(operator)

require $x : A$ **in** $N \stackrel{\text{def}}{=} \textbf{require}(A; x.N)$

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(notation)

$$\frac{\Gamma \vdash A \text{ type} \quad \Gamma, x : A \vdash N \in B}{\Gamma \vdash \text{require } x : A \text{ in } N \in B}$$

(require)

[[A woman walked in. She sat down]]

∇

$(\Sigma x \in (\Sigma p \in Woman) WalkedIn(p)) SatDown(???)$

[[A woman walked in. She sat down]]

▽

$(\Sigma x \in (\Sigma p \in \text{Woman}) \text{WalkedIn}(p))$ **require** $y : \text{Woman}$ **in** $\text{SatDown}(y)$

The meaning of a sentence is a logical proposition.

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What we want:

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$(\Sigma x \in (\Sigma p \in \text{Woman}) \text{WalkedIn}(p)) \pi_1(x)$

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The meaning of a sentence is a type-theoretic expression which may evaluate to a canonical proposition.

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$(\Sigma x \in (\Sigma p \in \text{Woman}) \text{WalkedIn}(p)) \pi_1(x)$

where $M \sim N \stackrel{\text{def}}{=} (M \leq N) \wedge (N \leq M)$

EVERY GRAMMATICAL SENTENCE HAS A MEANING

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...but only some of them denote propositions (types)!

$$\frac{M \in A \quad [M/x] N \Downarrow N'}{\text{require } x : A \text{ in } N \Downarrow N'} \quad (??)$$

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1. impredicativity

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Can the above be made precise? There are two problems:

1. impredicativity
2. non-determinism

(HOLD THAT THOUGHT)

A POSITIVE EXAMPLE

[[*The President ran a marathon*]]



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▽

require $x : \textit{President}$ **in** $(\Sigma y \in \textit{Marathon}) \textit{Ran}(x; y)$

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require $x : \textit{President}$ **in** $(\Sigma y \in \textit{Marathon}) \textit{Ran}(x; y)$

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\Downarrow

$(\Sigma y \in \textit{Marathon}) \textit{Ran}(\textit{Obama}; y)$

A NEGATIVE EXAMPLE

[[*The unicorn ran a marathon*]]



$\llbracket \textit{The unicorn ran a marathon} \rrbracket$

∇

require $x : \textit{Unicorn}$ **in** $(\Sigma y \in \textit{Marathon}) \textit{Ran}(x; y)$

[[*The unicorn ran a marathon*]]

∇

require $x : \textit{Unicorn}$ **in** $(\Sigma y \in \textit{Marathon}) \textit{Ran}(x; y)$

(not a proposition)

IS require COMPUTATIONALLY EFFECTIVE?

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$$\boxed{\alpha \Vdash_w \mathcal{I}}$$

THE CREATING SUBJECT

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All mathematics is a mental construction performed by an idealized subject, **subject to the following observations about knowledge**:

1. experiences are never forgotten (**monotonicity, functoriality**)
2. at a point in time, the subject knows whether or not it has experienced a judgment (**decidability**)

Corollary

The meaning of a judgment \mathcal{J} must be explained in terms of its forcing condition, $w \Vdash \mathcal{J}$, for any stage/world w .

...

2. at a point in time, the subject knows whether or not it has experienced a judgment (decidability)

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Remark

Contra Dummett, I by no means take the above as requiring that the following shall be true in a constructive metatheory, divorced from time:

$$\forall w. \forall \mathcal{J}. \llbracket w \Vdash \mathcal{J} \rrbracket \vee \neg \llbracket w \Vdash \mathcal{J} \rrbracket \quad (\text{Dummett's infelicity})$$

The above is impossible in a Beth model.

logical consequence \Rightarrow semantic consequence

Brouwer?, Martin-Löf \Rightarrow *Brouwer?, Heyting, Allen*

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proof conditions \Rightarrow assertion conditions

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Brouwer?, Martin-Löf \Rightarrow *Brouwer?, Heyting, Allen*

proof conditions \Rightarrow assertion conditions

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global meaning explanation \Rightarrow local meaning explanation

Husserl, Dummett, Martin-Löf \Rightarrow *Brouwer?, Beth, Kripke*

assertion acts (judgments) are intensional (local)

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$|_x \mathcal{J}(x)$

(general judgment)

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$\mathcal{I}_2 (\mathcal{I}_1)$

(hypothetical judgment)

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 $M \Downarrow N$

(evaluation)

assertion acts (judgments) are intensional (local)

$ _x \mathcal{I}(x)$	(general judgment)
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$M \Downarrow N$	(evaluation)
$A = B \text{ type}$	(typehood)

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$A = B \text{ type}$	(typehood)
$A \text{ true}$	(truth)
$M = N \in A$	(membership)

assertion acts (judgments) are intensional (local)

$w \Vdash _x \mathcal{J}(x)$	(general judgment)
$w \Vdash \mathcal{J}_2 (\mathcal{J}_1)$	(hypothetical judgment)
$w \Vdash M \Downarrow N$	(evaluation)
$w \Vdash A = B \text{ type}$	(typehood)
$w \Vdash A \text{ true}$	(truth)
$w \Vdash M = N \in A$	(membership)

$w \Vdash |_x \mathcal{I}(x)$

(general judgment)

$w \Vdash \mathcal{I}_2 (\mathcal{I}_1)$

(hypothetical judgment)

$$w \Vdash |_x \mathcal{I}(x) \quad \Longleftrightarrow \quad \dots$$

$$w \Vdash \mathcal{I}_2 (\mathcal{I}_1) \quad \Longleftrightarrow \quad \dots$$

$$w \Vdash |_x \mathcal{I}(x) \iff \forall u \succeq w. \forall x \in \mathcal{M}_u. u \Vdash \mathcal{I}(x)$$

$$w \Vdash \mathcal{I}_2 (\mathcal{I}_1) \iff \forall u \succeq w. u \Vdash \mathcal{I}_1 \Rightarrow u \Vdash \mathcal{I}_2$$

$$\begin{aligned}
w \Vdash |_x \mathcal{I}(x) &\iff \forall u \succeq w. \forall x \in \mathcal{M}_u. u \Vdash \mathcal{I}(x) \\
w \Vdash \mathcal{I}_2 (\mathcal{I}_1) &\iff \forall u \succeq w. u \Vdash \mathcal{I}_1 \Rightarrow u \Vdash \mathcal{I}_2
\end{aligned}$$

where \mathcal{M}_w is the species of constructions that have been effected by stage w

$$w \Vdash A \text{ true} \iff$$

$$w \Vdash A \text{ true} \iff \exists m \in \mathcal{M}_w.$$

$$w \Vdash A \text{ true} \iff \exists m \in \mathcal{M}_w. w \Vdash m = m \in A$$

THE require ORACLE: DYNAMICS

$$\frac{\overline{\mathfrak{S}(\langle \rangle)}}{\mathfrak{S}(\vec{u})} \quad \frac{\mathfrak{S}(\vec{u}) \quad |_n \rho(n) < n \quad (n \in \mathbb{N}^+)}{\mathfrak{S}(\vec{u} \sim \rho)} \quad (\text{spread law})$$

$$\frac{\alpha \Vdash_t A \Downarrow A' \quad |\mathcal{A}_{A'}(t)| = \ell \quad \text{hd}(\alpha)(\ell) = j \quad \text{tl}(\alpha) \Vdash_t [\mathcal{A}_{A'}(j)/x] N \Downarrow N'}{\alpha \Vdash_t \text{require } x : A \text{ in } N \Downarrow N'} \quad (\text{for } \alpha \in \mathfrak{S})$$

QUESTIONS?