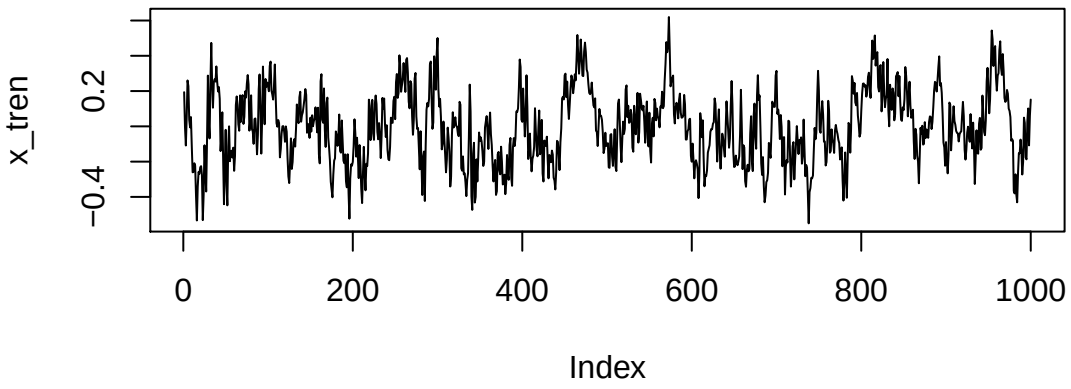
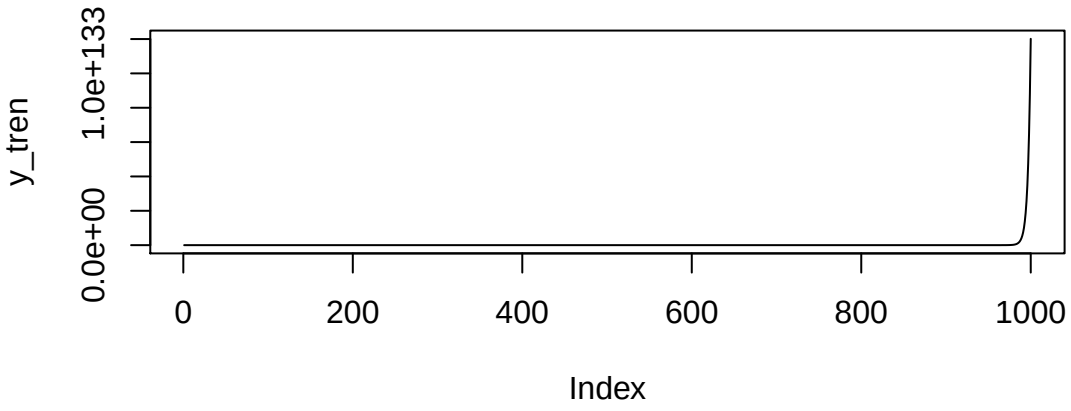
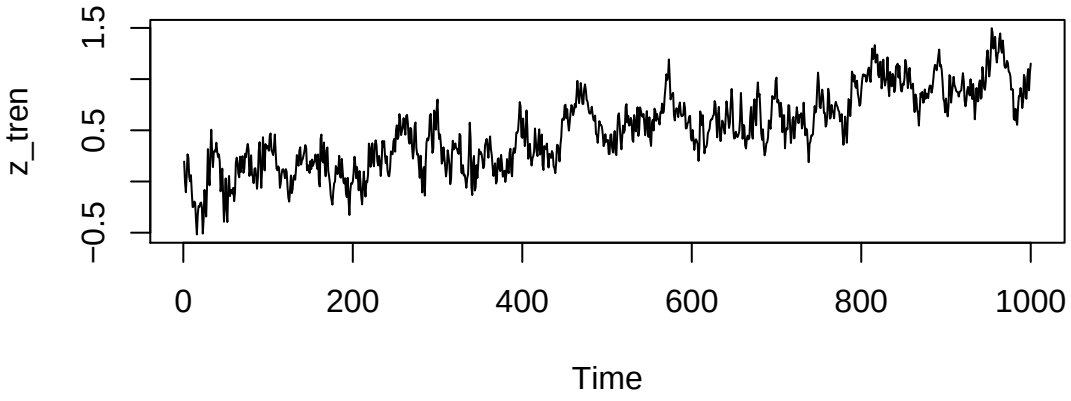
```r
# 3.1

# a)

eps <- rnorm(1013,0,0.1)
xt <- numeric(1013)
yt <- numeric(1013)

for(i in 4:1013){

  xt[i] <- 4/5*xt[i-1] - 2/5*xt[i-2] + 1/2*xt[i-3] + eps[i] + 1/4*eps[i-1]
  yt[i] <- 4/5*yt[i-1] + 2/5*yt[i-2] + 1/2*yt[i-3] + eps[i] + 1/4*eps[i-1]

}

xt <- xt[4:1013]
yt <- yt[4:1013]

x_tren <- xt[1:1000]
x_test <- xt[1001:1010]

y_tren <- yt[1:1000]
y_test <- yt[1001:1010]

plot(x_tren,type="l")
```



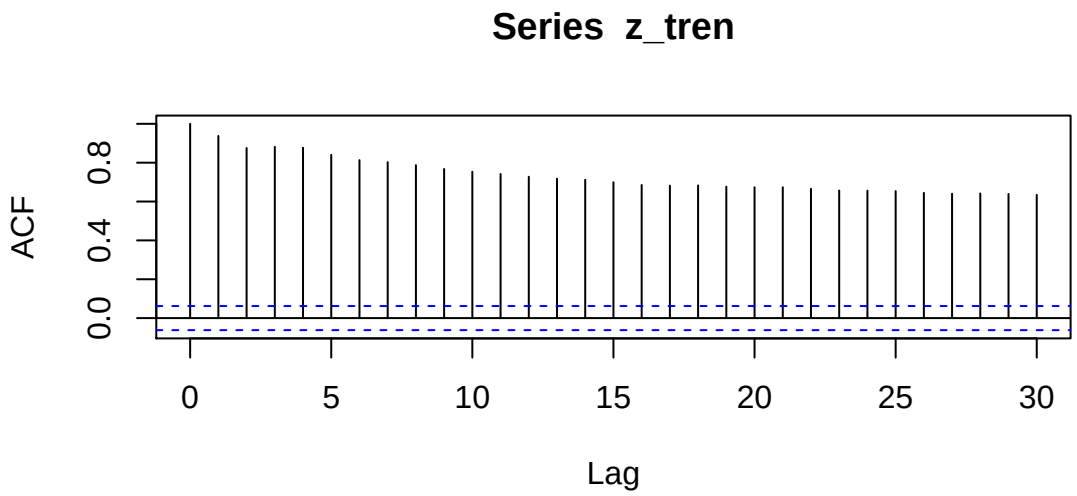

```r
plot(y_tren,type="l")
```



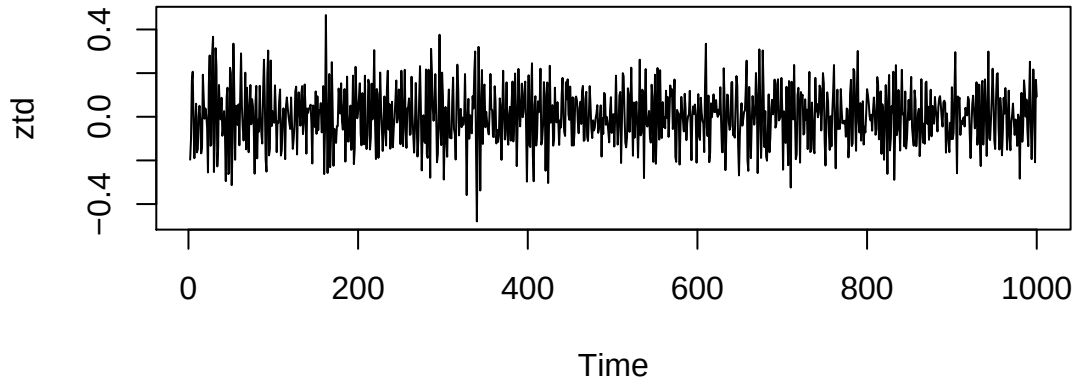

```r
# b)

zt <- numeric(1010)
for(i in 1:1010){
  zt[i] <- 1/1000*i + xt[i]
}

z_tren <- ts(zt[1:1000])
z_test <- ts(zt[1001:1010],start=1001,end=1010)

# c)

plot(z_tren,type="l")
```



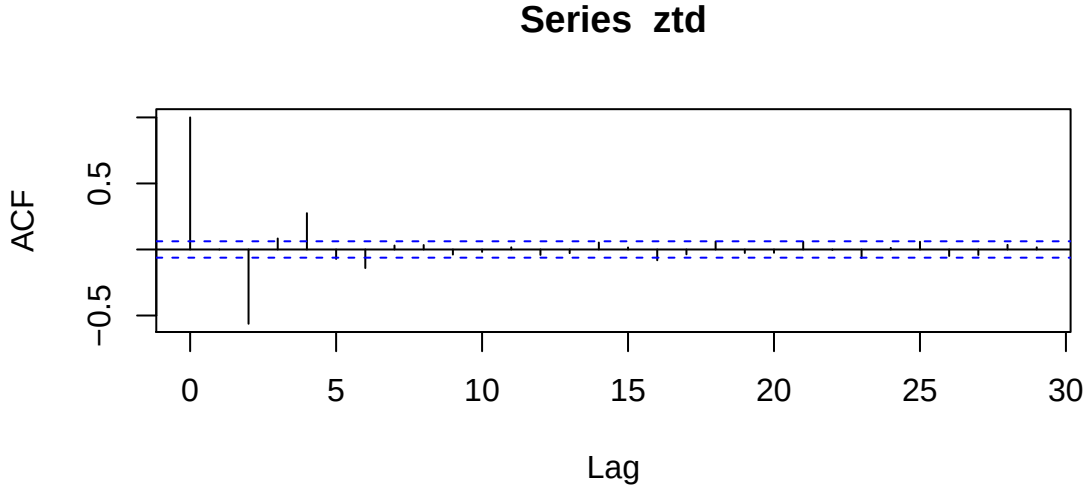

```r
acf(z_tren)
```



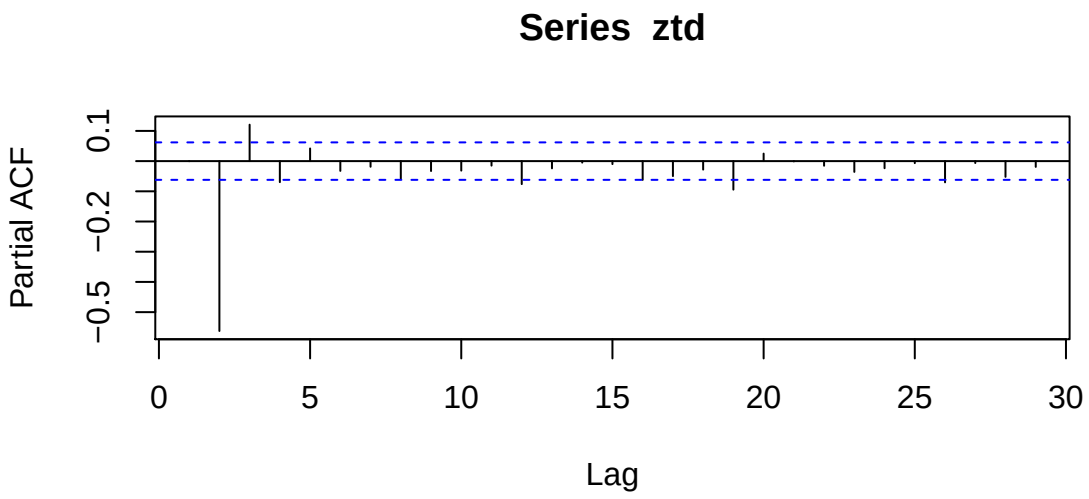

```
# widoczny jest trend

# d)

ztd <- diff(z_tren)
ts.plot(ztd)
```




```
acf(ztd)
```



```
pacf(ztd)
```




```
# po roznicowaniu jest duzo lepiej

# e)

# proponuję ARMA(3,4)

model_moj <- arima(ztd, c(3,0,4))
Box.test(model_moj$resid,lag=20,type="Ljung")

##
##  Box-Ljung test
##
## data:  model_moj$resid
## X-squared = 12.62, df = 20, p-value = 0.8931

# funkcja ar() dopasowuje nam model AR i sama wybiera rzad modelu:

model_ar <- ar(ztd, aic=TRUE, method="mle", order.max=NULL)

model_ar$order        # rzad

## [1] 4

model_ar$ar           # wspolczynniki

## [1]  0.07479 -0.60409  0.12627 -0.07081

model_ar$x.mean       # wyraz wolny

## [1] 0.0009581

Box.test(model_ar$resid,lag=20,type="Ljung")    # moj model byl lepszy :D

##
##  Box-Ljung test
##
## data:  model_ar$resid
## X-squared = 27.87, df = 20, p-value = 0.1125
```

```r
# f)

aic <- matrix(0,7,7)
bic <- matrix(0,7,7)

for(p in 0:6){
   for(q in 0:6){
      model <- arima(ztd, c(p,0,q),method="ML",
                     optim.control=list(maxit=10^5))
      aic[p+1,q+1] <- AIC(model)
      bic[p+1,q+1] <- AIC(model,k=log(1000))
   }
}

wym_aic <- which(aic==min(aic),arr.ind=TRUE)-1
wym_bic <- which(bic==min(bic),arr.ind=TRUE)-1

wym_aic
```

```
##      row col
## [1,]   6   1
```

```r
wym_bic
```

```
##      row col
## [1,]   4   2
```

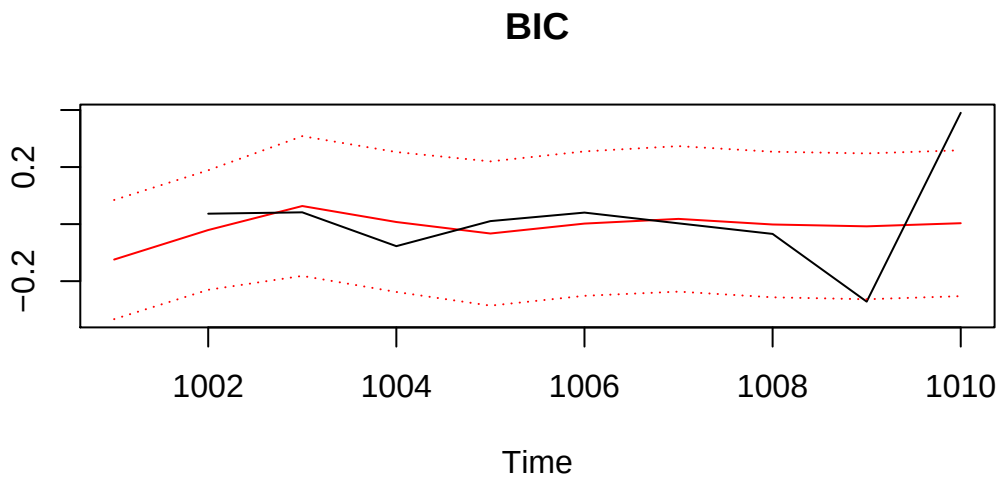
```r
model_aic <- arima(ztd,c(wym_aic[1],0,wym_aic[2]))
model_bic <- arima(ztd,c(wym_bic[1],0,wym_bic[2]))

Box.test(model_aic$resid,lag=20,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  model_aic$resid
## X-squared = 11.86, df = 20, p-value = 0.9209
```

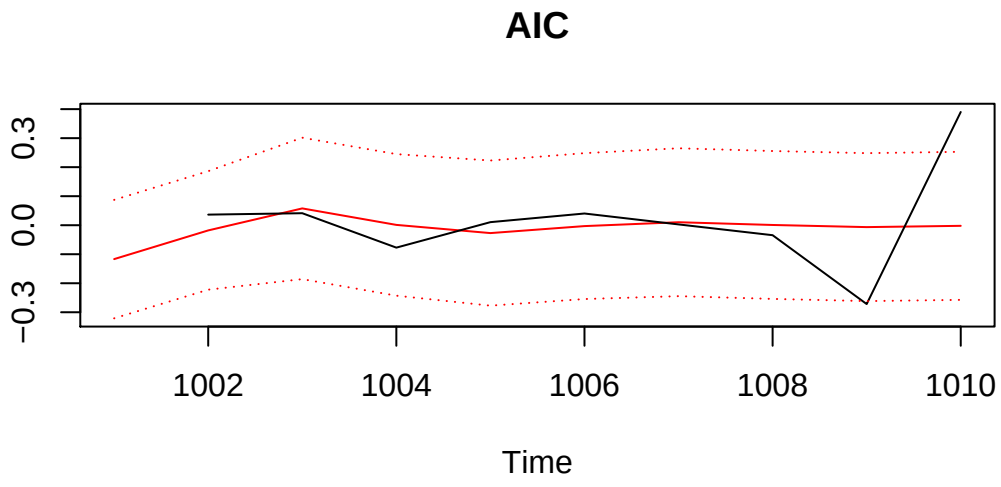
```r
Box.test(model_bic$resid,lag=20,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  model_bic$resid
## X-squared = 23.58, df = 20, p-value = 0.261
```

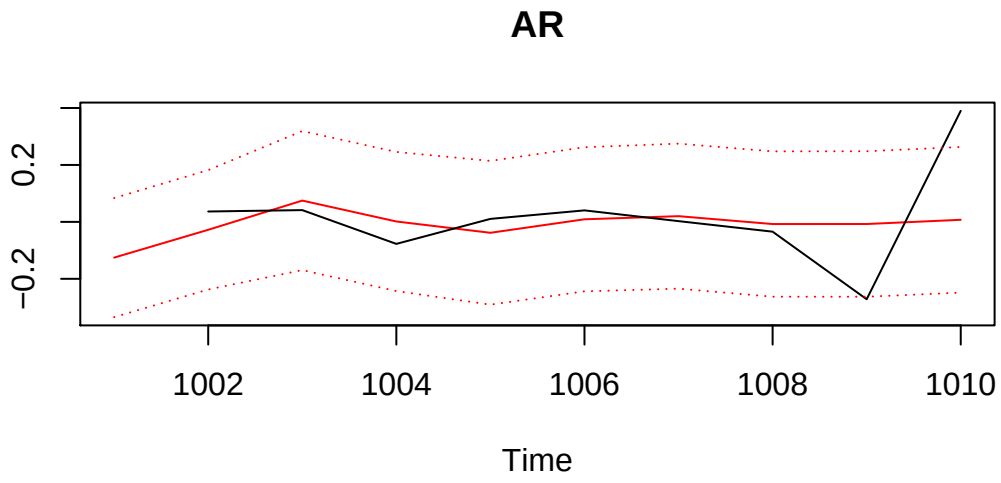
```r
# model_aic wyglada na najlepszy ze wszystkich

# g)

pr_bic <- predict(model_bic,n.ahead=10)$pred
se_bic <- predict(model_bic,n.ahead=10)$se

ts.plot(pr_bic, diff(z_test), pr_bic + 2*se_bic, pr_bic - 2*se_bic,
        col=c("red","black","red","red"), lty=c(1,1,3,3), main="BIC")
```




```
pr_aic <- predict(model_aic,n.ahead=10)$pred
se_aic <- predict(model_aic,n.ahead=10)$se

ts.plot(pr_aic, diff(z_test), pr_aic + 2*se_aic, pr_aic - 2*se_aic,
        col=c("red","black","red","red"), lty=c(1,1,3,3), main="AIC")
```




```
pr_ar <- predict(model_ar,n.ahead=10)$pred
se_ar <- predict(model_ar,n.ahead=10)$se

ts.plot(pr_ar, diff(z_test), pr_ar + 2*se_ar, pr_ar - 2*se_ar,
        col=c("red","black","red","red"), lty=c(1,1,3,3), main="AR")
```




```
# h)

blad_ar <- sum((pr_ar[-1] - diff(z_test))^2)
blad_bic <- sum((pr_bic[-1] - diff(z_test))^2)
blad_aic <- sum((pr_aic[-1] - diff(z_test))^2)

blad_ar
```

```
## [1] 0.2323
```

```
blad_bic
```

```
## [1] 0.2351
```

```
blad_aic
```

```
## [1] 0.2382
```

```
# i)

model_true <- arima(diff(z_tren),c(3,0,2),include.mean=TRUE)
model_true$coef
```

```
##        ar1        ar2        ar3        ma1        ma2  intercept
## -0.6407516 -0.5900645 -0.2775594  0.7204050  0.0366392  0.0009614
```

```
# zgadzaja sie z teoretycznymi :D

Box.test(model_ar$resid,lag=20,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  model_ar$resid
## X-squared = 27.87, df = 20, p-value = 0.1125
```

```
Box.test(model_bic$resid,lag=20,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  model_bic$resid
## X-squared = 23.58, df = 20, p-value = 0.261
```

```r
Box.test(model_aic$resid,lag=20,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  model_aic$resid
## X-squared = 11.86, df = 20, p-value = 0.9209
```

```r
Box.test(model_true$resid,lag=20,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  model_true$resid
## X-squared = 23.15, df = 20, p-value = 0.2817
```