

Linear Algebra and Vector Calculus

Notes and Exercises

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January 14, 2020

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Chapter 1

Linear Algebra Basics

1.1 Linear Functions

A function $L: \mathbf{R}^m \rightarrow \mathbf{R}^n$ is a linear function if for all $\mathbf{x}, \mathbf{y} \in \mathbf{R}^m$ and for all $a, b \in \mathbf{R}$

$$L(a\mathbf{x} + b\mathbf{y}) = aL(\mathbf{x}) + bL(\mathbf{y}).$$

It follows (by induction) that for all $\mathbf{x}_1, \dots, \mathbf{x}_r \in \mathbf{R}^m$ and all $a_1, \dots, a_r \in \mathbf{R}$

$$L(a_1\mathbf{x}_1 + \dots + a_r\mathbf{x}_r) = a_1L(\mathbf{x}_1) + \dots + a_rL(\mathbf{x}_r).$$

Theorem 1.1. *A linear function $L: \mathbf{R}^m \rightarrow \mathbf{R}^n$ is completely determined by its effect on the standard basis vectors $\mathbf{e}_1, \dots, \mathbf{e}_m$ of \mathbf{R}^m . An arbitrary choice of vectors $L(\mathbf{e}_1), \dots, L(\mathbf{e}_m)$ of \mathbf{R}^n determines a linear function from \mathbf{R}^m to \mathbf{R}^n .*