Neural Networks and Deep Learning: Exercises

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Chapter 1

Using Neural Networks to Recognize Handwritten Digits

Exercise 1. Consider a network of perceptrons. Suppose that we multiply all weights and biases by a positive constant $c > 0$. Show that the behaviour of the network does not change.

Solution. First consider a single perceptron. Assume that weights and bias are $w_1, \ldots, w_n$ and b, respectively. Then $\sum_i w_i \cdot x_i + b$ and $c \cdot (\sum_i w_i \cdot x_i + b)$ have exactly the same sign and hence multiplying the weights and the bias by c will not change the behaviour of this single perceptron. Now if all perceptrons in a network have their weights and biases multiplied by $c > 0$, then each individual perceptron behaves as before and hence the network behaves as before. ■

Exercise 2. Suppose that we have network of perceptrons with a chosen input value $\mathbf{x}$. We won't need the actual input value, we just need the input to have been fixed. Suppose the weights and biases are such that all $\mathbf{w} \cdot \mathbf{x} + b \neq 0$ for the input $\mathbf{x}$ to any particular perceptron in the network. Now replace all the perceptrons in the network by sigmoid neurons, and multiply the weights and biases of the network by a positive constant $c > 0$. Show that in the limit as $c \to \infty$, the of behaviour of this network of sigmoid neurons in exactly the same as the network of perceptrons. How can this fail when $\mathbf{w} \cdot \mathbf{x} + b = 0$ for one of the perceptrons?

Solution. ■