

$t_0 =$	1	1	1	1	1	1	2	2	2	3	3	3	3	3
	2	2	2	4	4	4	4	4	4	5	5	5	5	6
	3	5	5	6	6	6	6	6	7	7	7	7	7	7

$t_1 =$	1	1	1	1	1	1	2	2	2	3	3	3	3	3
	2	2	2	4	4	4	4	4	4	5	5	5	5	5
	3	5	6	6	6	6	6	6	7	7	7	7	7	7

### 0.1 Factoring $t_0, t_3, t_4$

We will use following plucker relations:

$$R_1: \quad 0 = \begin{array}{|c|c|} \hline 2 & 3 \\ \hline 5 & 4 \\ \hline 7 & 7 \\ \hline \end{array} - \begin{array}{|c|c|} \hline 2 & 3 \\ \hline 4 & 5 \\ \hline 7 & 7 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 2 & 4 \\ \hline 3 & 5 \\ \hline 7 & 7 \\ \hline \end{array}$$

On  $[3, 6, 7]$ , last monomial in  $R_1$  is 0.

Applying  $R_1$  on  $t_0, t_3, t_4$  we get

$t_0$	=	<table> <tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>2</td><td>2</td><td>2</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr> <tr><td>2</td><td>2</td><td>2</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>5</td><td>4</td><td>5</td><td>5</td><td>5</td><td>6</td></tr> <tr><td>3</td><td>5</td><td>5</td><td>6</td><td>6</td><td>6</td><td>6</td><td>6</td><td>7</td><td>7</td><td>7</td><td>7</td><td>7</td><td>7</td></tr> </table>	1	1	1	1	1	1	2	2	2	3	3	3	3	3	2	2	2	4	4	4	4	4	5	4	5	5	5	6	3	5	5	6	6	6	6	6	7	7	7	7	7	7
1	1	1	1	1	1	2	2	2	3	3	3	3	3																															
2	2	2	4	4	4	4	4	5	4	5	5	5	6																															
3	5	5	6	6	6	6	6	7	7	7	7	7	7																															
$t_3$	=	<table> <tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>2</td><td>2</td><td>2</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr> <tr><td>2</td><td>2</td><td>2</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>5</td><td>4</td><td>5</td><td>6</td><td>6</td><td>6</td></tr> <tr><td>3</td><td>5</td><td>5</td><td>5</td><td>5</td><td>6</td><td>6</td><td>6</td><td>7</td><td>7</td><td>7</td><td>7</td><td>7</td><td>7</td></tr> </table>	1	1	1	1	1	1	2	2	2	3	3	3	3	3	2	2	2	4	4	4	4	4	5	4	5	6	6	6	3	5	5	5	5	6	6	6	7	7	7	7	7	7
1	1	1	1	1	1	2	2	2	3	3	3	3	3																															
2	2	2	4	4	4	4	4	5	4	5	6	6	6																															
3	5	5	5	5	6	6	6	7	7	7	7	7	7																															
$t_4$	=	<table> <tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>2</td><td>2</td><td>2</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr> <tr><td>2</td><td>2</td><td>2</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>5</td><td>4</td><td>5</td><td>5</td><td>6</td><td>6</td></tr> <tr><td>3</td><td>5</td><td>5</td><td>5</td><td>6</td><td>6</td><td>6</td><td>6</td><td>7</td><td>7</td><td>7</td><td>7</td><td>7</td><td>7</td></tr> </table>	1	1	1	1	1	1	2	2	2	3	3	3	3	3	2	2	2	4	4	4	4	4	5	4	5	5	6	6	3	5	5	5	6	6	6	6	7	7	7	7	7	7
1	1	1	1	1	1	2	2	2	3	3	3	3	3																															
2	2	2	4	4	4	4	4	5	4	5	5	6	6																															
3	5	5	5	6	6	6	6	7	7	7	7	7	7																															

Note following tableau is factor of all of above tableaux

$$f_{034} = \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 2 & 3 & 3 & 3 \\ \hline 2 & 2 & 4 & 4 & 4 & 5 & 6 \\ \hline 5 & 5 & 6 & 6 & 7 & 7 & 7 \\ \hline \end{array}$$

## 0.2 Factoring $t_2$

We will use following plucker relations:

$$R_2 : \quad 0 = \begin{array}{|c|c|} \hline 2 & 3 \\ \hline 6 & 4 \\ \hline 7 & 7 \\ \hline \end{array} - \begin{array}{|c|c|} \hline 2 & 3 \\ \hline 4 & 6 \\ \hline 7 & 7 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 2 & 4 \\ \hline 3 & 6 \\ \hline 7 & 7 \\ \hline \end{array}$$

On  $[3, 6, 7]$ , last monomial in  $R_2$  is 0.

Applying  $R_2$  on  $t_2$  we get

$$t_2 = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \\ \hline 2 & 2 & 2 & 4 & 4 & 4 & 4 & 4 & 6 & 5 & 4 & 6 & 6 & 6 \\ \hline 3 & 5 & 5 & 5 & 5 & 5 & 6 & 6 & 7 & 7 & 7 & 7 & 7 & 7 \\ \hline \end{array}$$

Note following tableau is factor of above tableau

$$f_{034} = \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 2 & 3 & 3 & 3 \\ \hline 2 & 2 & 4 & 4 & 4 & 6 & 6 \\ \hline 5 & 5 & 5 & 6 & 7 & 7 & 7 \\ \hline \end{array}$$

## 0.3 Factoring $t_5$

We define one more pluckre relation

$$R_3 : \quad 0 = \begin{array}{|c|c|} \hline 2 & 3 \\ \hline 5 & 4 \\ \hline 6 & 7 \\ \hline \end{array} - \begin{array}{|c|c|} \hline 2 & 3 \\ \hline 4 & 5 \\ \hline 6 & 7 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 2 & 4 \\ \hline 3 & 5 \\ \hline 6 & 7 \\ \hline \end{array} - \begin{array}{|c|c|} \hline 2 & 4 \\ \hline 3 & 6 \\ \hline 5 & 7 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 2 & 3 \\ \hline 4 & 6 \\ \hline 5 & 7 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 2 & 5 \\ \hline 3 & 6 \\ \hline 4 & 7 \\ \hline \end{array}$$

Observe that  $3^{rd}, 4^{th}$  and  $6^{th}$  monomials are 0 on  $[3, 6, 7]$  Hence after multiplying following tableau

$$\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \\ \hline 2 & 2 & 2 & 4 & 4 & 4 & 4 & 4 & 6 & 6 & 6 & 6 & 6 & 6 \\ \hline 3 & 5 & 5 & 5 & 5 & 5 & 6 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\ \hline \end{array}$$

to above relation in empty columns, we get

$$\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \\ \hline 2 & 2 & 2 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 6 & 6 & 6 & 6 \\ \hline 3 & 5 & 5 & 5 & 5 & 5 & 6 & 6 & 7 & 7 & 7 & 7 & 7 & 7 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \\ \hline 2 & 2 & 2 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 6 & 6 & 6 & 6 \\ \hline 3 & 5 & 5 & 5 & 5 & 5 & 6 & 6 & 7 & 7 & 7 & 7 & 7 & 7 \\ \hline \end{array} - \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \\ \hline 2 & 2 & 2 & 4 & 4 & 4 & 4 & 4 & 4 & 6 & 6 & 6 & 6 & 6 \\ \hline 3 & 5 & 5 & 5 & 5 & 5 & 5 & 6 & 7 & 7 & 7 & 7 & 7 & 7 \\ \hline \end{array} = t_2 - t_5$$

Observe that following is factor of left hand side tableau

$$f_5 = \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 2 & 3 & 3 & 3 \\ \hline 2 & 2 & 4 & 4 & 4 & 6 & 6 \\ \hline 5 & 5 & 5 & 6 & 7 & 7 & 7 \\ \hline \end{array}$$

Hence  $t_5$  factored.

#### 0.4 Factoring $t_7$

Next we factor  $t_7$ , we will use following plucker relation:

$$R_4: \quad 0 = \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 3 & 5 \\ \hline 7 & 6 \\ \hline \end{array} - \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 3 & 5 \\ \hline 6 & 7 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 3 & 6 \\ \hline 5 & 7 \\ \hline \end{array}$$

We multiply following tableau to above relation (in empty columns)

1	1	1	1	1	2	2	2	3	3	3	3
2	2	2	4	4	4	5	5	6	6	6	6
4	4	4	5	5	5	6	7	7	7	7	7

then we get following

$$\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \\ \hline 2 & 2 & 2 & 3 & 4 & 4 & 4 & 5 & 5 & 6 & 6 & 6 & 6 & 6 \\ \hline 4 & 4 & 4 & 5 & 5 & 5 & 5 & 6 & 7 & 7 & 7 & 7 & 7 & 7 \\ \hline \end{array} = - \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \\ \hline 2 & 2 & 2 & 3 & 4 & 4 & 4 & 5 & 5 & 5 & 6 & 6 & 6 & 6 \\ \hline 4 & 4 & 4 & 7 & 5 & 5 & 5 & 6 & 7 & 6 & 7 & 7 & 7 & 7 \\ \hline \end{array} + \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \\ \hline 2 & 2 & 2 & 3 & 4 & 4 & 4 & 5 & 5 & 5 & 6 & 6 & 6 & 6 \\ \hline 4 & 4 & 4 & 6 & 5 & 5 & 5 & 6 & 7 & 7 & 7 & 7 & 7 & 7 \\ \hline \end{array}$$

Observe that tableau on left hand side is  $t_7$ , also observe that following two tableaux are factors of two tableaux on right side in above relation

$$f_{71} = \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 2 & 2 & 3 & 3 \\ \hline 2 & 3 & 4 & 4 & 5 & 6 & 6 \\ \hline 4 & 7 & 5 & 5 & 6 & 7 & 7 \\ \hline \end{array} \quad f_{72} = \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 2 & 2 & 3 & 3 \\ \hline 2 & 3 & 4 & 4 & 5 & 6 & 6 \\ \hline 4 & 6 & 5 & 5 & 7 & 7 & 7 \\ \hline \end{array}$$

#### 0.5 Factoring $t_6$

Next we will use following plucker relation to factor  $t_6$

$$R_5: \quad 0 = \begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 3 \\ \hline 5 & 4 \\ \hline \end{array} - \begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 3 \\ \hline 4 & 5 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 4 \\ \hline 3 & 5 \\ \hline \end{array}$$

We multiply following tableau to above relation (in empty columns)

1	1	1	1	1	2	2	2	3	3	3	3	3
2	2	4	4	4	5	5	6	6	6	6	6	6
4	4	5	5	5	6	7	7	7	7	7	7	7

We get

1	1	1	1	1	1	2	2	2	3	3	3	3	3
2	2	2	3	4	4	4	5	5	6	6	6	6	6
5	4	4	4	5	5	5	6	7	7	7	7	7	7

---


$$\begin{aligned}
&= \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \\ \hline 2 & 2 & 2 & 3 & 4 & 4 & 4 & 5 & 5 & 6 & 6 & 6 & 6 & 6 \\ \hline 4 & 4 & 4 & 5 & 5 & 5 & 5 & 6 & 7 & 7 & 7 & 7 & 7 & 7 \\ \hline \end{array} - \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \\ \hline 2 & 2 & 2 & 4 & 4 & 4 & 4 & 5 & 5 & 6 & 6 & 6 & 6 & 6 \\ \hline 3 & 4 & 4 & 5 & 5 & 5 & 5 & 6 & 7 & 7 & 7 & 7 & 7 & 7 \\ \hline \end{array} \\
&= t_7 - t_6
\end{aligned}$$

Observe that following tableau is factor of left hand side tableau in above relation

$$f_6 = \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 2 & 3 & 3 & 3 \\ \hline 2 & 2 & 4 & 4 & 6 & 6 & 6 \\ \hline 4 & 5 & 5 & 5 & 7 & 7 & 7 \\ \hline \end{array}$$