CHENNAI MATHEMATICAL INSTITUTE

Discrete Mathematics

Assignment 1:
If you have discussed the solutions with anyone please write their names.
Do write the assignment yourself.
Jan 8, 2019.

(1) State and prove using induction the criterion of divisibility by 11.
(2) Let $a_1 = 1$ and define $a_{n+1} = 3a_n + 4$. Prove by induction that $a_n \leq 3^n$.
(3) Suppose $a_1, a_2, \ldots, a_n$ are positive integers such that $a_1 \leq a_2 \ldots \leq a_n$. Show that
$$\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n} = 1$$
implies $a_n < \frac{1}{2^{n!}}$.
(4) Prove that any Boolean expression in the variables $x_1, \ldots, x_n$ can be written in conjunctive normal form.
(5) Consider the algorithm $Euclid(a, b)$, which takes integers $a \geq b \geq 0$ and outputs integers x, y, d (in that order) such that $d = gcd(a, b)$ and $d = ax + by$.
If the input is $(a, 0)$ return $(1, 0, a)$, otherwise return the $Euclid(b, a$ mod $b)$. Prove that the algorithm is correct.
(6) Let V be a vector space over $\mathbb{C}$ and let $L : V \to V$ be a linear map. For $\lambda \in \mathbb{C}$ an eigen value of L define $E(\lambda)$ to be the subspace of vectors $v \in V$ such that $L(v) = \lambda v$. Let $\lambda_1 \ldots, \lambda_r$ be the distinct eigen values of the operator L. Prove by induction on r that V is a direct sum of $E(\lambda_1) \oplus E(\lambda_2) \cdots \oplus E(\lambda_r)$.