Interference

* The Superposition of the waves / Redistribution of energy

Interference :- The Phenomena of modification in intensity of resultant wave obtained due to superposition of two or more light waves traveling in the same direction and having constant phase difference between them is called Interference.

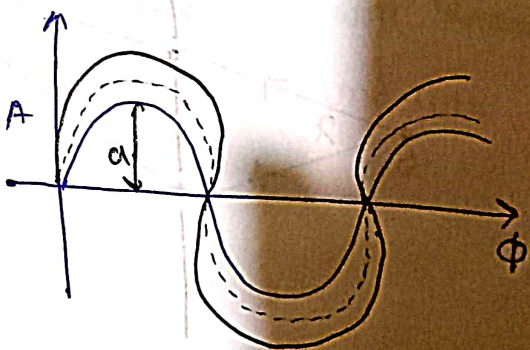
→ **Constructive Interference** :- CI occur at points where crest of one wave is falling on the crest of another wave so that the resultant wave has intensity more than the sum of intensities of superposing waves. In this case superposing waves are in same phase

$A > a_1$

$A > a_2$

$$\left[\begin{array}{l} I_1 \approx a_1^2 \\ I_2 \approx a_2^2 \end{array}\right.$$

$C = \nu\lambda$



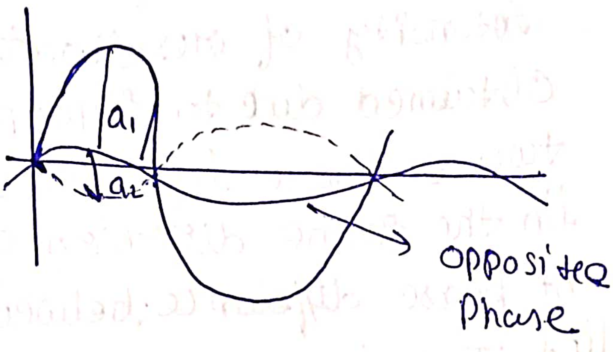
Resultant Intensity

$I >$ sum of Intensities of superposing wave

$I > (I_1 + I_2)$

↳ Destructive Interference :- DI occurs at a point where
(or Dark / minimum)

crest of one falling on the trough of another wave so that the resultant wave intensity is less



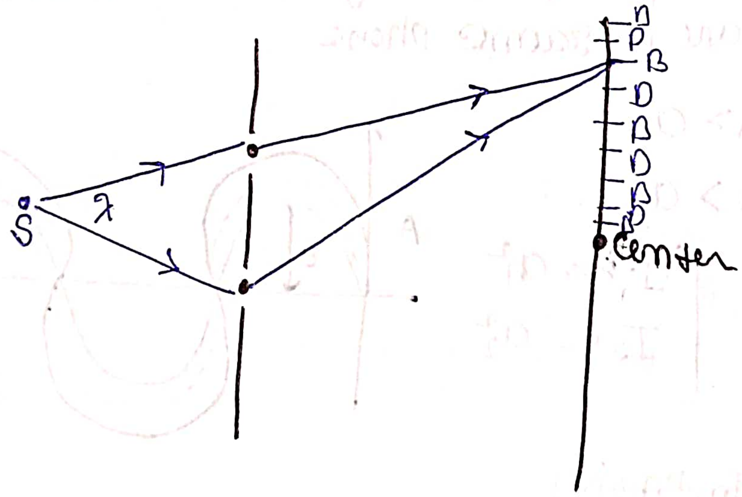
$A < a_1$

$A < a_2$

$I < (I_1 + I_2)$

Interference Pattern on Screen :-

Alternative points of constructive and destructive interference.



or

$$y = a_1 \sin\omega t + a_2 \sin(\omega t + \phi)$$

or

$$y = a_1 \sin\omega t + a_2 (\sin\omega t \cos\phi + \cos\omega t \sin\phi)$$

$$y = \sin\omega t \, (a_1 + a_2\cos\phi) + \cos\omega t \, (a_2 \sin\phi) \quad -(3)$$

Let $a_1 + a_2\cos\phi = A\cos\theta \quad -(4)$

$a_2 \sin\phi = A\sin\theta \quad -(5)$

where A and θ are new constants

Using eq's (4) and (5) in eq (3),

$$y = \sin\omega t (A\cos\theta) + \cos\omega t (A\sin\theta)$$

$$\boxed{y = A\sin(\omega t + \theta)} \quad -(6)$$

$$\left\{ \begin{array}{l} y_1 = a_1 \sin\omega t \\ y_2 = a_2 \sin(\omega t + \phi) \end{array} \right\}$$

This is the equation of Resultant wave

y = Resultant Displacement

A = Amplitude of Resultant wave

To find A, squaring & adding eq (4) & (5) :-

$$A^2\cos^2\theta + A^2\sin^2\theta = (a_1 + a_2\cos\phi)^2 + (a_2\sin\phi)^2$$

or

$$A^2 = a_1^2 + a_2^2\cos^2\phi + 2a_1a_2\cos\phi + a_2^2\sin^2\phi$$

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$$

$$\boxed{A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}} \quad -(7)$$

To find θ divide eq (5) by eq (4):-

$$\frac{\cancel{A}\sin\theta}{\cancel{A}\cos\theta} = \frac{a_2\sin\phi}{a_1+a_2\cos\phi}$$

or

$$\boxed{\tan\theta = \frac{a_2\sin\phi}{a_1+a_2\cos\phi}}$$

As Intensity $\propto$ (Amplitude)^2

or

Resultant Intensity $= KA^2$

K = Prop. Const

$$I = KA^2$$

$$I = K\left[a_1^2+a_2^2+2a_1a_2\cos\phi\right]$$

Taking $K=1$,

$$\boxed{I = a_1^2+a_2^2+2a_1a_2\cos\phi}$$

In Square Cosine form:-

$$I = a_1^2+a_2^2+2a_1a_2\left[2\cos^2\phi/2-1\right]$$

$$I = a_1^2+a_2^2-2a_1a_2+4a_1a_2\cos^2\phi/2$$

$$\boxed{I = (a_1-a_2)^2+4a_1a_2\cos^2\phi/2}$$

Conditions for Constructive Interference (Points of Max. Intensity)

Resultant $I = a_1^2+a_2^2+2a_1a_2\cos\phi$

for Maximum I, $\cos\phi = 1$

$\phi = 0, 2\pi, 4\pi \text{ --- } 2n\pi$

where $n = 0, 1, 2$ --

Phase difference $\phi = 2n\pi$, $n = 0, 1, 2 ---$

Phase change	Path change

$$\phi \qquad \Delta$$

$$2\pi \longrightarrow \lambda$$

$$1 \longrightarrow \lambda/2$$

$$\phi \longrightarrow \frac{\lambda}{2\pi}\phi$$

for $\phi = 2\pi n$, $\cos\phi = 1$

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2$$

$$= (a_1 + a_2)^2$$

If intensity of I^{st} wave $I_1 = a_1^2$

" " " II " $I_2 = a_2^2$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

Conditions for destructive Interference

for Minimum I, $\cos\phi = -1$

$$\phi = \pi, 3\pi, 5\pi ---$$

Phase diff $\phi = (2n-1)\pi$ $n = 1, 2 ---$

Path diff $\Delta = \frac{\lambda}{2\pi}\phi = \frac{\lambda}{2\cancel{\pi}} \times (2n-1)\cancel{\pi}$

$$= (2n-1)\frac{\lambda}{2}$$

for $\phi = (2n-1)\pi$, $\cos\phi = -1$

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2$$

$$= (a_1 - a_2)^2$$

If intensity of I ~~time~~ wave $\Rightarrow I_1 = a_1^2$

" " " II " $\Rightarrow I_2 = a_2^2$

$$(I_{min}) = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

Max	Min
Path $\Delta = 2n\lambda$	$\Delta = (2n-1)\lambda/2$
$= 0, \lambda, 2\pi$	$\lambda/2, 3\lambda/2 ---$

Resultant Intensity due to superposition of two waves

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$$

If amplitudes of superposing waves are equal,

$$a_1 = a_2 = a$$

Resultant $I = a^2 + a^2 + 2a^2\cos\phi$

$$= 2a^2(1+\cos\phi)$$

$I_{max} = 4a^2$ for $\cos\phi = 1$

$\phi = 2n\pi$

$I_{min} = 0$ for $\cos\phi = -1$

$\phi = (2n-1)\pi$

Average Intensity of Resultant wave and law of conservation of Energy

$$\boxed{\text{Average Intensity } I_{av} = \frac{I_{max} + I_{min}}{2}}$$

Here $I_{max} = (a_1+a_2)^2$

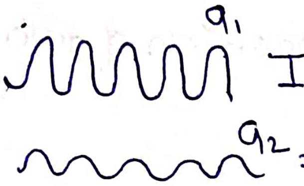
$I_{min} = (a_1-a_2)^2$

$$\therefore I_{av} = \frac{(a_1+a_2)^2 + (a_1-a_2)^2}{2}$$

$$= \frac{\cancel{2}(a_1^2+a_2^2)}{\cancel{2}} = a_1^2 + a_2^2$$

$I_{av} = I_1 + I_2$

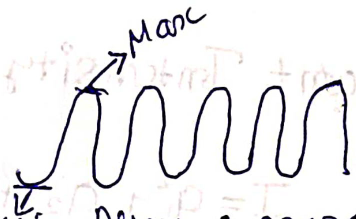
Average Intensity:- It is defined as the average of the maximum and minimum intensities of the resultant wave ~~opto~~ obtained after superposition, also Average Intensity is equal to the sum of intensities of the two superposing waves



$I_1 = a_1^2$

$\Rightarrow I_2 = a_2^2$

Before superposition



After superposition

$I = I_1 + I_2$

Total intensity

Resultant wave

$I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$

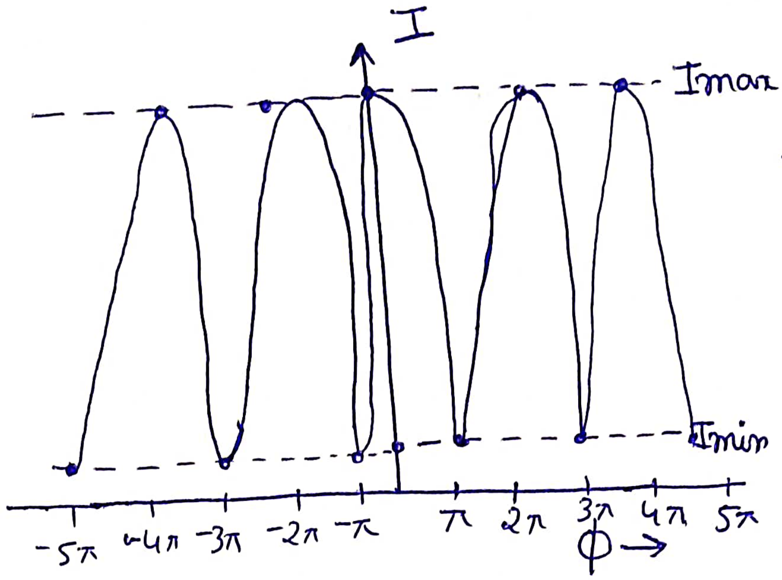
for $\cos\phi = 1$; $I_{max} = a_1^2 + a_2^2 + 2a_1a_2$

for $\cos\phi = -1$; $I_{min} = a_1^2 + a_2^2 - 2a_1a_2$

$I_{avg} = I_1 + I_2 = a_1^2 + a_2^2$

$\Rightarrow$ We can say that total energy is conserved in the phenomena of interference as average intensity of the resultant wave is equal to the sum of intensities of two superposing waves, Here a part of energy $(2a_1a_2)$ is transfering from minima to max so redistribution of energy takes place.

Intensity Distribution curve I versus ϕ (phase diff) $a_1 \neq a_2$



$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$$

$I_{max} = (a_1+a_2)^2$ for $\phi = 0, 2\pi, 4\pi, 6\pi$ -

$I_{min} = (a_1-a_2)^2$ for $\phi = \pi, 3\pi, 5\pi$

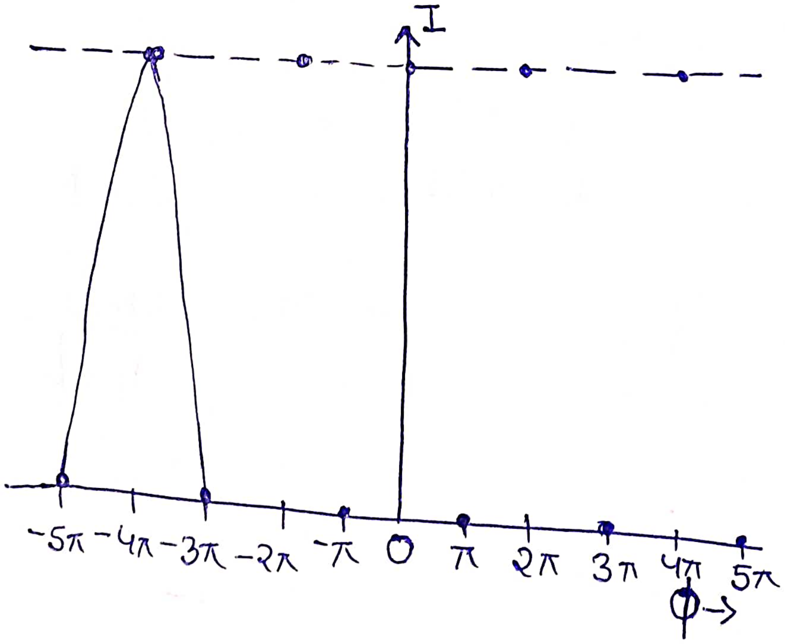
$$I_{avg} = a_1^2 + a_2^2$$

$a_1 = a_2 = a$

$$I = 2a^2(1+\cos\phi)$$

$I_{max} = 4a^2$

$I_{min} = 0$, $I_{av} = 2a^2$



Que I In an interference two light waves having a amplitude ration of 5 : 3 find the Ration of max - min Intensity of resultant wave gets superposed.

Sol $\frac{I_{max}}{I_{min}} = ?$ Let $a_1 = 5x$, $a_2 = 3x$

$$\frac{a_1}{a_2} = \frac{5}{3}$$

$$= \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{(5x+3x)^2}{(5x-3x)^2} = \frac{(8x)^2}{(2x)^2} = \frac{64x}{4x}$$

$$= 16 : 1$$

Que If in an interference the ratio of max and minimum Intensity of resultant wave 36 : 1 find the ratio of amplitude and Intensities of the superposing waves

Sol $\frac{I_{max}}{I_{min}} = \frac{36}{1}$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} \Rightarrow \frac{36}{1} = \frac{(a_1+a_2)^2}{(a_1+a_2)^2}$$

$$\Rightarrow \frac{6}{1} = \frac{a_1+a_2}{a_1-a_2} \Rightarrow 6a_1 - 6a_2 = a_1 + a_2$$

$$6a_1 - a_1 = a_2 + 6a_2$$

$$5a_1 = 7a_2$$

$$\frac{a_1}{a_2} = \frac{7}{5}$$

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{49}{25}$$

Que Find the resultant Intensity of the wave obtained due to the superposition of two waves having amplitude 1 unit and 2 unit and Phase difference is 180°?

Sol $I = 1 + 4 \times 2 \times 2 \times \cos 180°$

$= 5 + 4(-1)$

$5 - 4 = 1$ units.

Que If $I_1 : I_2 = \alpha : 1$ Prove that

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{\alpha}}{\alpha + 1}$$

Sol
$$\frac{(a_1+a_2)^2 - (a_1-a_2)^2}{(a_1+a_2)^2 + (a_1 - a_2)^2} = \frac{2\sqrt{\alpha}}{\alpha+1}$$

$$\Rightarrow \frac{\cancel{a_1^2} + \cancel{a_2^2} + 2a_1a_2 - \cancel{a_1^2} - \cancel{a_2^2} + 2a_1a_2}{a_1^2 + a_2^2 + \cancel{2a_1a_2} + a_1^2 + a_2^2 - \cancel{2a_1a_2}} = \frac{2\sqrt{\alpha}}{\alpha+1}$$

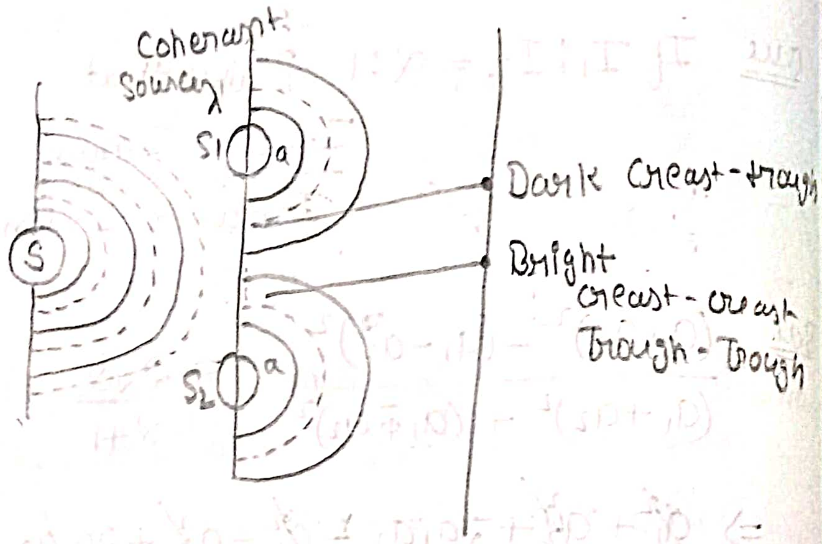
$$\Rightarrow \frac{\cancel{4}^{2} a_1 a_2}{\cancel{2}(a_1^2 + a_2^2)} = \frac{2\sqrt{\alpha}}{\alpha+1}$$

Methods of obtaining Interference Pattern

1) Division of wavefront :- Fresnel's Biprism Expt ; YDSE

2) Division of Amplitude :- Newton's Ring Expt ; thin films

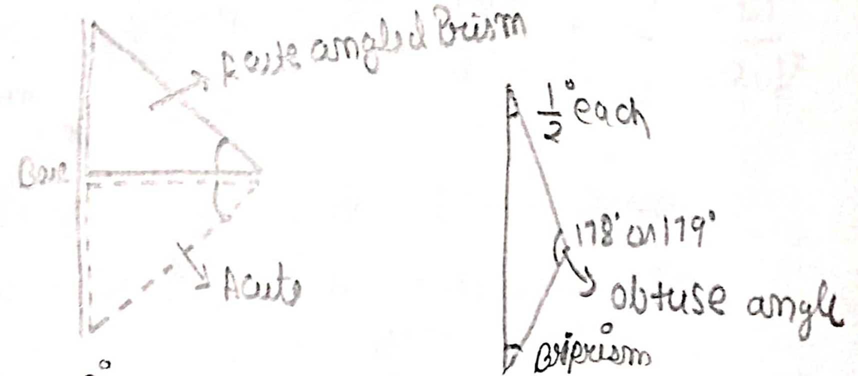
Fresnel's Biprism Expt :- (Division of wavefront)

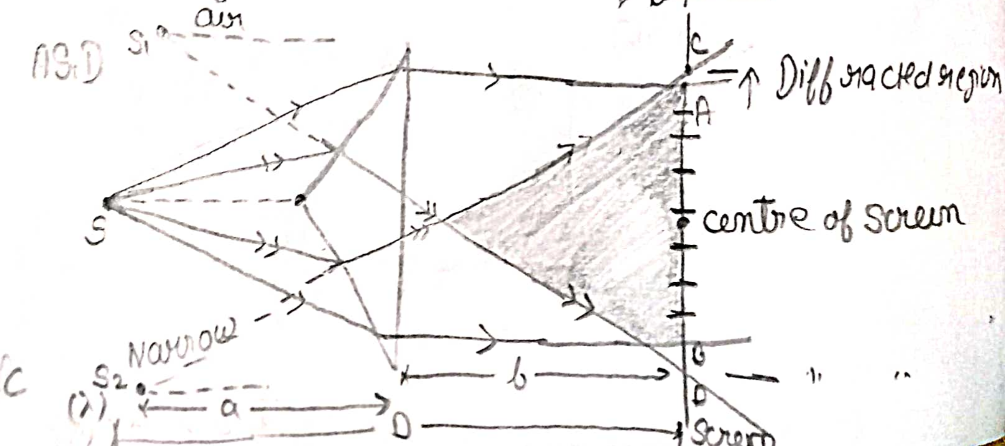


Biprism :- Combination of two acute angled Prism

⇒ Optical device (Glass)

⇒ To create virtual coherent source





D → Distance b/w virtual sources & screen

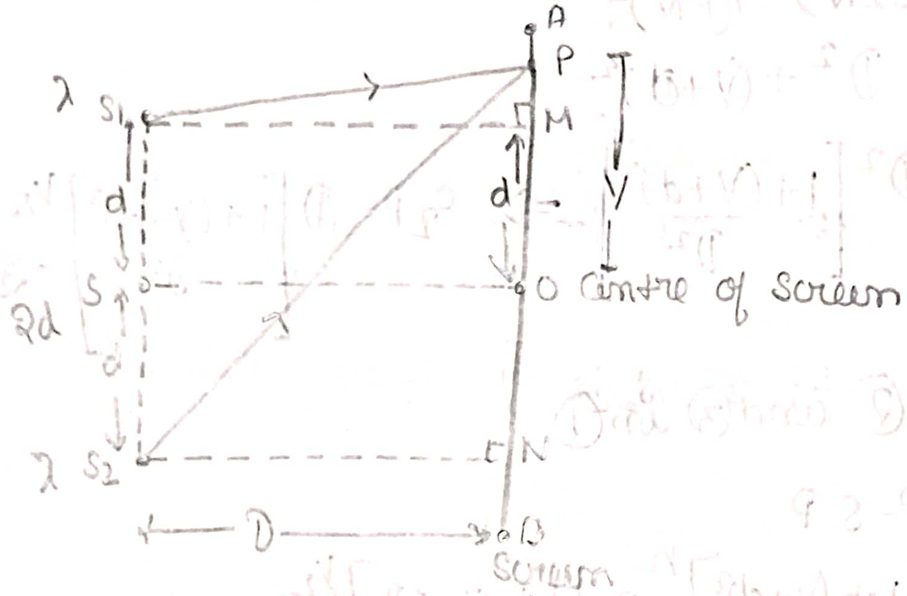
$D = a + b$

where

a = distance b/w Source & Biprism

b = distance b/w Biprism & screen

⇒ To determine the position of dark & Bright fringes on screen using Biprism Expt or YDSE expt and also find Expression of Fringe width :-



Now

<u>Draw</u>

$S_1M \perp AB$ (Screen)

$S_2N \perp AB$ (")

$SO \perp AB$ (")

$S_1S_2 = 2d$

D = Distance b/w Set & screen

Path difference b/w light rays meeting at point P on screen

$\Delta = S_2P - S_1P \quad -(1)$

from right angled ΔS_1PM,

$(S_1P)^2 = (S_1M)^2 + (PM)^2$

$(S_1P)^2 = D^2 + (y-d)^2$

$$(S_1P)^2 = \left[D^2 + (y-d)^2\right]$$

$$(S_1P)^2 = D^2\left[1 + \frac{(y-d)^2}{D^2}\right]$$

$$S_1P = D\left[1 + \frac{(y-d)^2}{D^2}\right]^{1/2} \quad -(2)$$

From ΔS_2PN_1

$$(S_2P)^2 = (S_2N)^2 + (PN)^2$$

$$D^2 + (y+d)^2$$

$$(S_2P)^2 = D^2\left[1 + \frac{(y+d)^2}{D^2}\right] \Rightarrow S_2P = D\left[1 + \frac{(y+d)^2}{D^2}\right]^{1/2} \quad -(3)$$

Put eq^n (2) and (3) in (1)

$$\Delta = S_2P - S_1P$$

$$\Delta = D\left[1 + \frac{(y+d)^2}{D^2}\right]^{1/2} - D\left[1 + \frac{(y-d)^2}{D^2}\right]^{1/2}$$

$$\Delta = D\left[\left[1 + \frac{(y+d)^2}{D^2}\right]^{1/2} - \left[1 + \frac{(y-d)^2}{D^2}\right]^{1/2}\right]$$

$\cancel{\Delta = \mathscr{D}}$

Expending Binomially upto 1st term as $y \ll D$

$$= D\left[1 + \frac{1}{2}\frac{(y+d)^2}{D^2} + \cdots\right] - D\left[1 + \frac{1}{2}\frac{(y-d)^2}{D^2} + \cdots\right]$$

$$\frac{(y+d)^2}{2D} - \frac{(y-d)^2}{2D} \Rightarrow \frac{\cancel{y^2} + \cancel{d^2} + 2yd - \cancel{y^2} - \cancel{d^2} + 2yd}{2D}$$

$$\boxed{\Delta = \frac{2yd}{D}} \quad -(4)$$ [using $(1+x)^n = 1+nx+ \ldots$]

Position of bright fringes (Points of constructive Interference)

For constructive interference ; $\Delta = n\lambda$

$$\frac{2yd}{D} = n\lambda$$

where $n = 0, 1, 2 \ldots$ (order)

Position of nth bright fringe on screen from the centre

$$\boxed{y = \frac{nD\lambda}{2d}} \quad -(5)$$

$n = 0, 1, 2 \ldots$

central fringe or zero order fring ;

$$y_1 = \frac{D\lambda}{2d}, \quad y_2 = \frac{2D\lambda}{2d}$$

Position of dark fringes (Points of destructive Interference)

For destructive interference ; $\Delta = (2n-1)\frac{\lambda}{2}$; $n = 1, 2, 3 \ldots$

or

$$\frac{2yd}{D} = (2n-1)\frac{\lambda}{2}$$

Position of dark fringes from centre

$$\boxed{y = (2n-1)\frac{D\lambda}{4d}} \quad -(6)$$

$n = 1, 2, 3, \ldots$

$$y_1 = (2\times1-1)\frac{D\lambda}{4d} \Rightarrow \frac{D\lambda}{4d}, \quad y_2 = \frac{3D\lambda}{4d}$$

Fringe width β :- The distance b/w two consecutive bright fringes and two consecutive dark fringes.

Bright

$y_0 = 0$

$y_1 = \frac{D\lambda}{2d}$

$y_2 = \frac{2D\lambda}{2d}$

If y_1 and y_2 are position of I & II bright fringes on screen,

from eq (5) $y_1 = \frac{D\lambda}{2d}$; $y_2 = \frac{2D\lambda}{2d}$

So fring width $\beta = y_2 - y_1 = \frac{2D\lambda}{2d} - \frac{D\lambda}{2d}$

$$\boxed{\beta = \frac{D\lambda}{2d}} \quad -(7)$$

If y_1 and y_2 are position of I & II dark fringes on screen, from eq (6)

$y_1 = \frac{D\lambda}{4d}$, $y_2 = \frac{3D\lambda}{4d}$

$\beta = (y_2 - y_1) \Rightarrow \frac{\cancel{2}D\lambda}{\cancel{4}_2 d} = \frac{D\lambda}{2d}$

$\beta = \frac{D\lambda}{2d}$

$\beta \propto D$

$\beta \propto \frac{1}{2d}$

$\beta \propto \lambda$

no effect of n in β

Methods of determination of '2d'

a) Deviation method
↓
(angle of deviation prism)

a = Distance b/w source & biprism

δ = Angle of deviation

d = distance b/w S and S_1

$$\tan\delta = \frac{d}{a}$$

Here

$$d << a$$

$$\tan\delta \approx \delta = \frac{d}{a}$$

Angle of deviation

$$\delta = (\mu - 1)\alpha$$

where μ = refractive index of biprism

$\therefore \frac{d}{a} = (\mu-1)\alpha$

$d = a(\mu-1)\alpha$

$$\boxed{2d = 2a(\mu-1)\alpha}$$

Que In biprism Exp light of $\lambda = 5000Å$ is incident on the biprism placed at a distance of 10cm from the source, if the distance b/w source and the screen is (D) = 100cm find the fringe width for fringes formed on the screen. If $\mu = 1.5$ and $\alpha = 1°$

Sol $\frac{3.14}{180} = 0.0174$

$2d = 2 \times 10(1.5-1)\, 0.0174$

$2d = 2\cancel{0}\frac{(.5)}{1\cancel{0}} \times 0.0174$

$= 1\cancel{0} \times \frac{0.0174}{1000\cancel{0}}$

$2d = 0.174$

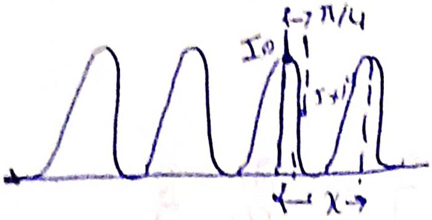
$\beta = \frac{D\lambda}{0.174} = \frac{100 \times 0.0174}{0.174}$

$\beta = \frac{100 \times 5000 \times 10^{-8}}{0.0174}$

$= 0.02873$ cm

Tut

Qu 2



The Phase difference two consecutive bright fringes is 2π

$2n\pi$

$n = 0, 1, 2, 3 \ldots$

$x \rightarrow 2\pi$

$1 \rightarrow \frac{2\pi}{x}$

$\frac{x}{4} - \frac{2\pi}{\cancel{x}} \times \frac{\cancel{x}}{\cancel{4}2} = \frac{\pi}{2}$

$\phi_1 = 0$

$\phi_2 = 2\pi$

$\phi_2 - \phi_1 = 2\pi$

$I_0 = a^2 + a^2 + 2 \cdot a \cdot a \cos 0^\circ = 4a^2$

$I_{x/4} = a^2 + a^2 + 2a \cdot a \cos\frac{\pi}{2} = 2a^2$

$I_0 : I_{x/4} = 4a^2 : 2a^2$
$= 2:1$ Ans

Qu 3 $I_{max} - I_{avg} = \frac{5}{100} I_{avg}$

$I_{avg} = \frac{I_{max} + I_{min}}{2}$ $\Rightarrow I_{max} = I_{avg} + \frac{5}{100} I_{avg}$

$\Rightarrow I_{max} = \frac{105}{100} I_{avg}$

$\Rightarrow 100 I_{max} = 105 \left(\frac{I_{max} + I_{min}}{2}\right)$

$\Rightarrow 200 I_{max} = 105 I_{max} + 105 I_{min}$

$\Rightarrow 95 I_{max} = 105 I_{min}$

$\Rightarrow \frac{I_{max}}{I_{min}} = \frac{105}{95} \Rightarrow \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{105}{95}$

$\Rightarrow \frac{a_1+a_2}{a_1-a_2} = \frac{10.25}{9.75} \Rightarrow \frac{a_1+a_2+a_1-a_2}{(a_1+a_2-a_1+a_2} = \frac{20}{0.5} = \frac{40}{1}$

$\frac{a_1}{a_2} = \frac{40}{1} \Rightarrow \frac{I_1}{I_2} = \frac{1600}{1}$

Que 4 $\dfrac{I_{max} - I_{min}}{I_{max} + I_{min}} =$?

$$\frac{I_1}{I_2} = \frac{9}{1} \Rightarrow \frac{a_1^2}{a_2^2} = \frac{9}{1} \Rightarrow \frac{a_1}{a_2} = \frac{3}{1}$$

$$\frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{(3+1)^2}{(3-1)^2} = \frac{(4)^2}{(2)^2} = \frac{16}{4} = \frac{4}{1}$$

$$\frac{I_{max}}{I_{min}} = \frac{4}{1}$$

$$\therefore \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{4-1}{4+1} = \frac{3}{5}$$

$\lambda = 7000 \times 10^{-10}$

Que 5 $\beta = \dfrac{\lambda D}{2d}$

$$\left[\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}\right] \text{--(i)}$$

$\Rightarrow$ again $n_1\beta_1 = n_2\beta_2$ $\therefore \left[\dfrac{\beta_1}{\beta_2} = \dfrac{n_2}{n_1}\right]$ --(ii)

so using (i) & (ii)

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} \Rightarrow \therefore n_2 = \left(\frac{\lambda_1}{\lambda_2}\right) n_1$$

$$= \frac{7000}{5000} \times 10 = 14$$