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A multi-path delivery system with random refusal against online booking using Type-2 fuzzy logic-based fireworks algorithm

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ABSTRACT

With the development of the internet and the availability of smart phones, online shopping and the number of digital buyers have increased enormously under the present social context (COVID-19). The major pitfall of this marketing system is the random non-availability of/refusal by customers. Nowadays, with the development of infrastructure, there are several route connections among the different locations worldwide. Considering these, we formulate some multi-path routing for goods delivery with refusal problems (MPRfGDwRPs) against online booking having different route connections between nodes (customers) and solved by a novel Type-2 fuzzy logic based fireworks algorithm (T2FLFWA) developed for discrete problems with probability-based selection and generation based prime mutation. In T2FLFWA, inferences are drawn to obtain the amplitude coefficient against the assumed spark number and iteration ratio. Some instances from TSPLIB are solved and supremacy of T2FLFWA is established through ANOVA test. In MPRfGDwRP, a delivery man starts with goods from a warehouse against customers' online demands under cash-on-delivery or cash-on bookings systems and delivers the appropriate goods to the respective customers located at different locations. The random refusal or non-availability of customers is considered. The optimum travel plan and appropriate routes between the nodes are selected for minimum total cost. The models are illustrated numerically through a real-life problem. Some managerial conclusions are drawn. The novelty of the investigation is that for the first time, T2FLFWA for discrete problems is developed, delivery man's routing plan with multiple paths between nodes with refusals is formulated and its solution using T2FLFWA is presented.

1. Introduction and motivation

1.1. Motivation

In the modern era, due to online shopping, the number of digital buyers has been rapidly increasing worldwide in the last few years. According to Yang et al. [1], it is forecasted that online customers will increase from 1.52 to 2.14 billion in the period from 2019 to 2022. The study by Bubanja and Vidas-Bubanja [2] pointed out that e-commerce sales reached $4.2 trillion in the USA for the year 2021. In India, the e-commerce industry growth moves faster (cf. [3]) than in China, Indonesia, Malaysia, etc. Against the online bookings, e-commerce retailers such as Alibaba [4], Amazon [5], Wal-Mart [6], etc. follow TSP type delivery system starting from a depot/warehouse and coming back to the same place after delivering the appropriate goods to the respective customers at the customer's residences/nodes so that total delivery cost/time is minimum. In the e-channel product booking system, two types of cash transfer i.e. (i) cash-on booking (COB) and (ii) cash-on-delivery (COD) are considered. But on the flip-side, though there are a lot of advantages, the retailer e-commerce industry faces a major challenge during the delivery of goods due to the refusal of the products by the customers (cf. [7]). These refusals are uncertain, due to several reasons such as realization regarding quality after the order, delay in delivery, out of station of the customer, etc. So the present investigation focuses on the following queries.

Against online booking and delivery, how are e-commerce industries going to handle the refusal? Here, refusal may occur at different stages of the delivery system. If refusal occurs, how to optimize the

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transportation system and logistics costs be? What is the managerial strategic instruction to the salesman to tackle this situation?

In delivery problems, the researcher investigated with only a single route/path among the nodes (cf. [8–11]). In reality, nowadays, there is a lot of infrastructural development throughout the world, including the developing countries. Hence, several connecting routes between different places are available for transportation and business.

1.2. Introduction of the proposed problem

The above mentioned scenarios and facts motivated us to formulate and to investigate the proposed problem-MPRfGDwRP which consists of the delivery man's problems against different types of refusals in the context of presently available improved infra-structural facilities. Here we introduce more than one connecting path between the customers (nodes) for travel (multi-path/3D routing).

In the above logistic and distribution problems, the occurrence of refusal may be of five types such as (a) no refusal, (b) refusal at certain nodes, (c) probabilistic refusal at each node, (d) refusal is known at the customer's node after deliveryman's arrival, and (e) refusal known at the preceding node. In the last case, the delivery system is restructured. Taking the effects of these refusals on the cost, MPRfGDwRPs are formulated. In the face of the above real-life refusal problems faced by the e-commerce industry, now, the question arises what should be the appropriate routing schemes in different refusal circumstances so that total delivery cost is minimum?

Intelligence algorithms are used to get the useful findings efficiently for the routing (NP-hard) problems [12–16]. Tan and Zhu [17] developed a novel swarm intelligence (SI) named as FWA. It is one of the most promising (cf. [18]) swarm algorithms in continuous optimization for its fast convergence speed and optimization accuracy. Recently some discrete FWAs have been developed for discrete optimization problems ([19], etc.). The FWAs can be made more efficient, introducing some better stochastic operators and/or by using the imprecise relations (i.e., fuzzy logic with Type-1 and −2 fuzzy parameters) among its parameters such as sparks and iteration ratio. Barraza et al. [20,21] implemented one and two parameters fuzzy logics (T1FL and T2FL) in FWA for continuous problems. Till now, none used T1FL and T2FL in FWA for discrete problems. To fill up this vacuum, in the present investigation, we develop two parameters (spark and iteration ratio) Type-2 fuzzy logic for amplitude coefficient in FWA, termed as T2FLFWA for discrete problems like routing problems, which is used to solve MPRfGDwRPs.

In this paper, Multi-path routing for goods delivery with different types of refusal problems (MPRfGDwRPs) is formulated against online bookings having several route conditions between customers for travel. In MPRfGDwRP, a delivery man starts from a warehouse with goods to be delivered and delivers the goods to the respective customers, if available and at the end, comes back to the starting point so that the total system cost (travel cost + transport cost + cost due to refusal) is minimum. Five types of customer's refusal-(i) no refusal, (ii) refusal at certain nodes, (iii) random refusal/customer's non-availability at each node with a probability, (iv) refusal known at the time of delivery at a certain node and (v) refusal known at the preceding node. To solve MPRfGDwRP, T2FLFWA is developed along with probability-based selection (PBS) and generation based prime mutation (PM) to get better balance between exploitation and exploration of the search space. Twenty-five inferences between spark number and iteration ratio are drawn to derive amplitude coefficients. T2FLFWA is tested against some test functions from TSPLIB and a statistical test, ANOVA is performed. This algorithm is applied to solve MPRfGDwRP. The models are illustrated numerically and some managerial decisions are drawn. A real-life delivery problem is presented.

1.3. Contributions/novelties in this investigation are as follows:

- We study routing for goods delivery with different types of uncertain refusals
- A variant of uncertain refusal against cash-on booking and cash-on-delivery is studied
- Two parameters of Type-2 fuzzy logic-based fireworks algorithm (T2FLFWA) is considered for solution.
- probabilistic Selection, generation-dependent prime mutation in T2FLFWA
- The supremacy of T2FLFWA established through ANOVA test.

The paper is arranged as follows: In Section 1, a concise introduction and motivation are presented. A brief literature review is given in Section 2. Section 3 gives MPRfGDwRP model. Proposed T2FLFWA is explained in Section 4. Section 5 studies the solutions of MPRfGDwRPs using T2FLFWA. Numerical experiments of proposed models is presented in Section 6. In Section 7 a brief discussion is given. A real-life problem and managerial insights are presented in Sections 8 and 9 respectively. Finally, the conclusion and future scope are available in Section 10.

2. Literature review

2.1. Literature on shopping and delivery problems

Nowadays, online shopping and home delivery system are very demanding business areas (cf. [22–24]). Bräysy et al. [25] introduced a common home meal delivery problem formulating as a vehicle routing problem (VRP) with time windows. They formulated multiple TSP like problems and solved them by the tool based on VRP solver, SPIDER planner developed by SINTEF. Foroutan et al. [26] introduced a green VRP with reverse logistics (collecting returned goods) mixed integer non-linear programming model and solved using genetic algorithm (GA) and simulated annealing algorithms. They presented through sensitivity analysis, though collecting returned goods increases carbon emissions cost but it reduces overall system cost. Trachanatzi et al. [27] formulated a mathematical model on environmental Prize-Collecting VRP using the Firefly Algorithm. They considered the minimization of carbon emission and total system cost (fixed and variable cost). Ouyang et al. [28] formulated a multi-vehicle dynamic pick up and delivery problem with time constraints. They formulated a same-day delivery problem for online purchases and solved with a dynamic programming approach. Fathollahi-Fard et al. [29] solved home health care problem using set of efficient various metaheuristics and heuristics. Osaba et al. [30] solved real life drug distribution and pharmacological waste collection problem using the bat algorithm. Neves and Marques [31] introduced online shopping drivers and barriers for older adults (different ages and gender) to increase a better understanding of the drivers and barriers affecting older consumers intention to shop online. Ternova [32] represented the case of the Pizza delivery service to minimize the waiting time experienced by their service optimization in the Fast food industry. They focused on how to improve service level, delivery time and efficient utilization of the system. They used the Poisson assumption, and different hypotheses of the inter-arrival times having exponential probability distribution. Yu et al. [33] implemented an optimization model of the urgent order distribution and the delivery problem of online pharmacy with the objective function of minimizing the order fulfillment cost. They considered mainly order distribution, vehicle assignment and vehicle routing arrangement and focused on time and delivery. In spite of the above investigation, there is a scope of several investigations. In developing countries, refusals after placing the order are very common. Till now, not many investigations have been reported to tackle the above problem. An attempt has been made here to formulate and solve the delivery system with uncertain refusal against online purchases.

Table 1
Notation and description of parameters and decision variables.

Notation	Description
N	Number of node (customer locations)
i, j, k	Index sets
Q	Represents set of nodes $\{1, 2, 3, \dots, \mathrm{N}\}$
COD	Cash on delivery (value of the good with delivery cost is paid at the time of delivery)
COB	Cash on booking (value of the good with delivery cost is paid at the time of booking, if a COB customer refuses, he/she gets back the good's cost but looses the paid delivery charge)
x_{ij}	Decision variable, when traveling from ith city to jth city $x_{ij} = 1$, else, $x_{ij} = 0$
ξ_j	Decision variable, when probability of success in refusal jth city (coming from ith city) $\xi_j = 1$, else, $\xi_j = 0$
f_i	$= 0$, if ith node is COD $= 1$, if ith node is COB
$c(i, j)$	Traveling cost from ith city to jth city
$c(x_i, x_{i+1}, v_{qi})$	Traveling cost from ith city to $(i+1)$th city using (qi)th route per unit distance
$(v_1, v_2, \dots, v_P)$	Different routes, P is the number of routes
v_{qi}	Selected route at ith node to travel $(i+1)$th node, $q \in \{1, 2, 3, \dots, \mathrm{P}\}$, $i \in \{1, 2, 3, \dots, \mathrm{N}\}$
σ	Goods transportation charges per unit weight per unit distance
w_i	Carried weight for ith node, i = 1, 2, 3, …, N
$dis(x_i, x_{i+1}, v_{qi})$	Distance between ith city to $(i+1)$th city using (qi)th route
$dis(x_N, x_1, v_{qN})$	Distance between Nth city to depot using (qN)th route
d_i	Demand at ith node
D	Total demand
p_j	Refusal probability of the jth node
γ	Delivery charge for the particular ordered item
$(L+t)$th, u	Refusal node at $(L+t)$th node and uth node
$-i(a) - j(b)-$	Travel from node (i) to node (j) through path a, a = 0, 1, 2

2.2. Literature on FWA

FWA [17] is a comparatively new (in the year 2010), efficient heuristic algorithm. In FWA, sparks are generated to explore the solution space for finding optimal/near-optimal solutions. The main components of FWA are the spark numbers, the explosion of amplitude and perturbation. Tan and Zhu [17] first introduced FWA with RWS and Gaussian mutation. Initially, it was designed for the continuous problem [17]. Abdulmajeed and Ayob [34] presented a new direct vector representation scheme and a neighborhood operator consisting of 'random swap' and 'random move' to solve a capacitated VRP. The first attempt for a discrete version of FWA was developed by Abdulmajeed and Ayob [34], in which two kinds of sparks are generated. Also, a number of solutions are generated using 'random move' operators and a search within a local search space close to a current firework is conducted by a 'random swap' operator. Later, Tan [35] developed discrete FWA introducing changes in the explosion operator, selection strategy and mutation operator of FWA. The authors introduced two explosion operations-I and -II (2-opt and 3-opt respectively), identical mutation operator and adaptive selection strategy (similar to RWS) and solved the benchmark instances from TSPLIB [36] using this FWA. Taidi et al. [37] solved the TSPs following Abdul majeed and Ayob's FWA [34]. Luo et al. [38] solved a large-scale TSP following Tan's FWA using 2-opt and 3-opt operations. Best of our knowledge, till now none considered the generalization of discrete FWA.

2.3. Literature on Type-2 fuzzy logic

Though metaheuristic methods mainly depend on the stochastic operators, the performance of these methods is increased after the introduction of uncertainty/imprecise relations between the parameters of metaheuristic procedures (c.f [39,40]). One of the popular AI-based tools is Type-2 fuzzy logic [41]. The Type-2 fuzzy logic theory and applications were introduced by Castillo et al. [42] in the year of 2007. In the last few years, Type-2 fuzzy logic-based metaheuristic algorithms has been developed and shown their effectiveness. Interval type-2 fuzzy logic based ACO [43] and PSO [44] with the adaption of dynamic parameter are developed by Olivas et al. [45,46]. Also Castillo et al. [47] introduced dynamic fuzzy logic parameter in ACO and applied in fuzzy control of a mobile robot. Here, they used a fuzzy controller for maintaining diversity and avoid premature convergence.

Recently, Barraza et al. [20] introduced dispersion percent Iterative fuzzy FWA-I and -II for the better performance of FWA. In Fuzzy FWA-I and -II, one input and one output parameter Type-1 fuzzy inferences were implemented. In the same year, Barraza et al. [48,49] considered two sets of fuzzy interfaces-one between 'iteration', 'dispersion percent' against 'amplitude of explosion' (as output) and other between 'iteration' and 'dispersion' against 'amplitude of explosion' and 'amount of spark'. In another investigation (Barraza et al. [49]), they took inferences between 'evaluation', 'sparks' against 'amplitude coefficient'. All these improvements were incorporated in continuous FWA. To the best of our knowledge, none implemented fuzzy logic in discrete FWA.

3. Proposed multi-path routing for goods delivery with refusal problems (MPRfGDwRP) against online booking

3.1. Nomenclature

Table 1 presents the notations and description of a few important parameters which are used frequently in the following sections.

3.2. Assumptions

(i) Just before the commencement of the journey, the delivery man makes contact with all customers and ascertains the presence of the customers at their residence (node). (ii) In addition to (i), the delivery man also makes contact with the customer just after the delivery at the customer's preceding node. (iii) Instead of (i) and (ii) the delivery man commences his journey without any prior information assuming a certain probability of refusal at each node.

3.3. Classical TSP (2DTSP)

The classical TSP (CTSP) is mathematically represented as

$$\text{Minimize} \quad Z = \sum_{i \neq j} c(i,j) x_{ij} \tag{1}$$

$$\text{subject to} \quad \left.\begin{aligned} &\sum_{i=1}^{N} x_{ij} = 1 \;\text{ for } j = 1, 2, \dots, N \\ &\sum_{j=1}^{N} x_{ij} = 1 \;\text{ for } i = 1, 2, \dots, N \\ &\sum_{i \in S}^{N} \sum_{j \in S}^{N} x_{ij} \le |S| - 1, \forall S \subset Q \\ &where \quad x_{ij} \in \{0, 1\}, i, j = 1, 2, \dots, N. \end{aligned}\right\} \tag{2}$$

where Q = {1, 2, 3, ⋯, N} represents set of nodes, x_{ij} are the decision variable. If the salesman travels from ith city to jth city then $x_{ij} = 1$, else $x_{ij} = 0$. The first two constraints in Eq. (2) imply the visit of a node only once and the third constraint eliminates the sub route. Then the mentioned CTSP reduces to

$$\left.\begin{aligned} &\text{determine a complete tour} \quad (x_1, x_2, \cdots, x_N, x_1) \\ &\text{to minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}) + c(x_N, x_1) \\ &\qquad where \quad x_i \neq x_j, i, j = 1, 2, \dots, N. \end{aligned}\right\} \tag{3}$$

with the constraints Eq. (2).

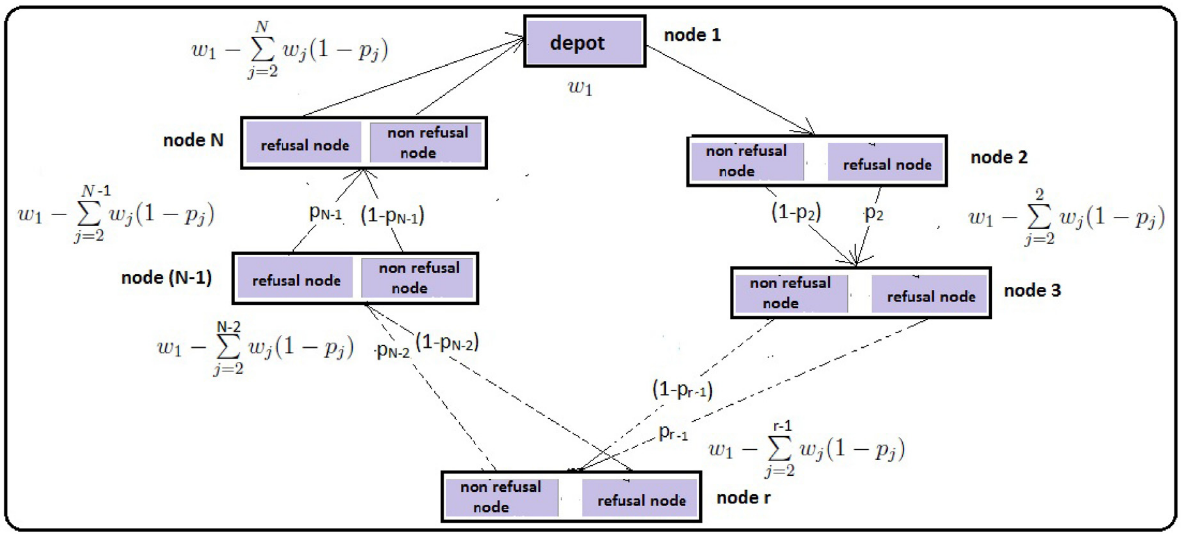


Fig. 1. Graphical representation of Probabilistic Refusal.

3.4. Multi-path TSP (3DTSP)

Let $c(x_i, x_{i+1}, v_{qi})$ is the traveling cost from ith city to $(i+1)$th city using (qi)th route per unit distance. The salesman determines entire tour $(x_1, x_2, \cdots, x_N, x_1)$ using the routes $(v_{q1}, v_{q2}, \cdots, v_{qN})$ for the tour from x_i to x_{i+1}, where $x_i \in \{1, 2, \ldots, N\}$ for $i = 1, 2, \ldots, N$, $v_{qi} \in \{1, 2, \ldots, P\}$. Then this problem is mathematically represented as:

$$\left.\begin{aligned} &\text{minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}, v_{qi}) * dis(x_i, x_{i+1}, v_{qi}) \\ &\qquad + c(x_N, x_1, v_{qN}) * dis(x_N, x_1, v_{qN}) \\ &where \quad x_i \neq x_j, i, j = 1, 2, \ldots, N, \quad v_k, v_l \in \{1, 2, \ldots, P\} \end{aligned}\right\} \tag{4}$$

3.5. Multi-path routing for goods delivery with refusal problems (MPRfGDwRP)

The refusal of booked goods at nodes may be due to several reasons. These may be due to the non-availability of the customer (at that time), long delay in delivery, casual booking, financial insufficiency (at that time), change of customer's decision, etc. Here, against these refusals, we propose some optimal routing plans and estimate/evaluate the minimum total cost. Two types of goods delivery concepts are considered-(i) without prior knowledge (Blind approach) (scenario 1), and (ii) with prior knowledge (scenarios 2, 3, 4, 5).

3.5.1. Scenario 1: Refusal at each node with certain probability

The chance of occurring the above refusals is not certain, rather can be expressed in terms of probability. As the refusal is obviously a potential economic loss to the e-commerce delivery company, this loss can be evaluated using the concept of probability (Fig. 1). Thus, instead of evaluating the crisp system cost, the expected total cost of the system is evaluated.

Sometimes, from previous records, the company may evaluate the probability of refusal depending on the areas of recipient customers etc. These probabilities of refusal, p_j, $j = 1, 2, \ldots, N$ can be assigned to the nodes.

Here, 1st node represents the depot/warehouse. The refusal may occur from 2nd node to Nth node. Let total goods weight is w_1. So weight of the goods w_1 is same between 1st and 2nd node. Let the total weight of the goods to be delivered be w_1. The delivery man starts his journey with weight w_1 from the depot (node 1). Therefore, w_i, i = 1, 2, 3, $\cdots$, N is the goods' weight which is carried along the path between the nodes (i) and (i + 1).

For the refusal at the 2nd node with probability p_2, the weight of the goods after the delivery at 2nd node

$$= w_1 p_2 + (w_1 - w_2)(1 - p_2)$$
$$= w_1 - w_2(1 - p_2)$$
$$= w_1 - \sum_{j=2}^{2} w_j(1 - p_j)$$

The weight of the goods, after the delivery at the 3rd node

$$= [w_1 - \sum_{j=2}^{3} w_j(1 - p_j)]p_3 + [w_1 - \sum_{j=2}^{3} w_j(1 - p_j)] - w_3(1 - p_3)$$
$$= w_1 - \sum_{j=2}^{3} w_j(1 - p_j)$$

Thus, after the delivery of article is at Nth node, the remaining goods

$$= w_1 - \sum_{j=2}^{N} w_j(1 - p_j) = w_N \text{ (say)}$$

For this scenario, we minimize the expected cost of the system i.e,

$$\left.\begin{aligned} &\text{minimize} \quad E(Z) = E[\sum_{i=1}^{N-1} c(x_i, x_{i+1}, v_{qi}) * dis(x_i, x_{i+1}, v_{qi}) * \xi_{i+1} \\ &\qquad + c(x_N, x_1, v_{qN}) * dis(x_N, x_1, v_{qN}) \\ &+ \sigma * w_1 * dis(x_1, x_2, v_{q1}) + \sum_{i=2}^{N-1} \sigma * [w_1 - \sum_{j=2}^{i} (w_j(1 - p_j))] \\ &* dis(x_i, x_{i+1}, v_{qi}) \\ &\qquad + (w_N * \sigma) * dis(x_N, x_1, v_{qN})] - p_i * \gamma * f_i \\ &\text{subject to} \quad w_1 = \sum_{j=1}^{N} d_j, w_N = w_1 - \sum_{j=2}^{N} w_j(1 - p_j) \\ &\qquad \xi_i = 1 - p_i, f_i, \xi_i \in \{0, 1\} \\ &\qquad where \quad x_i \neq x_j, i, j = 1, 2 \cdots N, v_{qi}, v_{qN} \in \{1, 2 \cdots, P\} \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} &\sum_{i=1}^{N} d_i = w_1 = D(say) \\ &d_1 = 0 \end{aligned}\right\} \tag{6}$$

The Eq. (5) consists of two parts, the first part is the traveling cost and the second part is the goods transportation cost. The Eq. (6) indicates the total demand is D and demand at 1st node/depot is 0 and d_i

represents demand at ith node. where, ξ_j represent probability of non-refusal, γ indicates the delivery charge for the particular ordered item (γ used when probabilistic refusal occurs). f_i is a binary variable to indicate whether the ordered materials is ordered under COD ($f_i = 1$)or COB ($f_i = 0$).

Here, $c(x_i, x_{i+1}, v_{qi})$ is the traveling cost from ith city to $(i+1)$th city using (qi)th route per unit distance. The salesman determines a complete tour $(x_1, x_2, \cdots, x_N, x_1)$ with routes $(v_{q1}, v_{q2}, \cdots, v_{qN})$, where $x_i \in \{1, 2, \dots, N\}$ for $i = 1, 2, \dots, N$ and $v_{qi} \in \{1, 2, \dots, P\}$ for $i = 1, 2, \dots, N$. Also $(w_i * \sigma)$ indicates the goods transport cost during the tour which decreases as the goods are handover to the customer in the respective node.

Also goods transportation cost depends on the weight of materials, so gradually weight and goods transportation cost decreases as salesman's visiting the node is continued. Here we consider discontinuous goods transportation cost in the form of All Unit Discount (AUD) which is defined as follows.

$$\sigma = \begin{cases} \sigma_1 & : w_2 < w_i \le w_1 \\ \sigma_2 & : w_3 < w_i \le w_2 \\ \sigma_3 & : w_4 < w_i \le w_3 \\ \sigma_4 & : 0 < w_i \le w_4 \end{cases} \tag{7}$$

where σ represents goods transportation charges per unit weight per unit distance. w_i indicates the remaining carried weight of those goods at ith node and w_1 represent total weight/demand. Here $\sigma_1 > \sigma_2 > \sigma_3 > \sigma_4$ and $w_1 > w_2 > w_3 > w_4$. But if a customer refuses to take materials, then the weight will remain same up to the next non-refused node. The goal of online shopping companies is to minimize the system cost.

3.5.2. Scenario 2: No refusal

In this scenario, it is assumed that there is no refusal by the customers. This case can be obtained as a particular case of scenario 1 putting p_i = probability of refusal at ith node = 0. Thus putting p = 0 in Eq. (5), we have

$$\left.\begin{aligned} &\text{minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}, v_{qi}) * dis(x_i, x_{i+1}, v_{qi}) + c(x_N, x_1, v_{qN}) \\ &\qquad * dis(x_N, x_1, v_{qN}) \\ &\qquad + \sum_{i=1}^{N-1} (w_i * \sigma) * dis(x_i, x_{i+1}, v_{qi}) + (w_N * \sigma) \\ &\qquad * dis(x_N, x_1, v_{qN}) \\ &\qquad\qquad \text{subject to} \quad w_i = D - \sum_{j=1}^{i} d_j \\ &\qquad where \quad x_i \ne x_j, i, j = 1, 2 \cdots N, and v_{qi}, v_{qN} \in \{1, 2 \cdots, P\} \end{aligned}\right\} \tag{8}$$

3.5.3. Scenario 3: Refusal known at beginning

In this scenario, it is assumed that, refusal is known at the beginning, i.e, when a delivery man starts routing for delivery. Say 10% refusal occurs and previously known uth $(1 < u \le N)$ node as the refusal node. Thus the customer point becomes N-1. The mathematical model formulated as

$$\left.\begin{aligned} &\text{minimize} \quad Z = \sum_{i=1}^{N-2} c(x_i, x_{i+1}, v_{qi}) * dis(x_i, x_{i+1}, v_{qi}) \\ &\qquad + c(x_{N-1}, x_1, v_{qN-1}) * dis(x_{N-1}, x_1, v_{qN-1}) \\ &+ \sum_{i=1}^{N-2} (w_i * \sigma) * dis(x_i, x_{i+1}, v_{qi}) + (w_{N-1} * \sigma) \\ &* dis(x_{N-1}, x_1, v_{qN-1}) \\ &\qquad\qquad \text{subject to} \quad w_i = D - \sum_{j=1}^{i} d_j \\ &where \quad x_i \ne x_j, i, j = 1, 2 \cdots N-1, and \ v_{qi}, v_{qN-1} \in \{1, 2 \cdots, P\} \end{aligned}\right\} \tag{9}$$

3.5.4. Scenario 4: Refusal known after reaching at the node

After reaching the customer point, if the refusal occurs, then the path of the salesman remains unaltered, and the weight of total goods remains the same. So the salesman has to carry the undelivered goods up to the depot and thus the carrying cost increases. Considering the uth $(1 < u \le N)$ node as the refusal node, the mathematical model becomes

$$\left.\begin{aligned} &\text{minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}, v_{qi}) * dis(x_i, x_{i+1}, v_{qi}) * \xi_{i+1} \\ &\qquad + c(x_N, x_1, v_{qN}) * dis(x_N, x_1, v_{qN}) \\ &+ \sum_{i=1}^{N} (\sigma * w_i) * dis(x_i, x_{i+1}, v_{qi}) + (w_N * \sigma) * dis(x_N, x_1, v_{qN}) - (\gamma * f_u) \\ &\qquad \text{subject to} \quad w_i = D - \sum_{j=1, j \ne u}^{i} d_j, \qquad f_u = 1 \\ &\quad where \quad x_i \ne x_j, i, j = 1, 2, \dots, N, v_{qi}, v_{qN} \in \{1, 2, \dots, P\} \end{aligned}\right\} \tag{10}$$

3.5.5. Scenario 5: Refusal known at the just preceding node

Normally a salesman, in the beginning, makes calls/messages to the customers to verify their presence at the nodes and after getting the assurance of presence, a travel plan is made following the TSP procedure. If at this stage, there is a refusal at mth node, the salesman makes the TSP travel plan for the rest of the nodes without mth node. In these cases, there is no problem in evaluating the objective function or expected objective function for the total cost. But, out of N nodes, after visiting few nodes as per the first TSP plan, if the salesman comes to know standing at a certain node that there will be refusal at a particular node, then he/she makes a new TSP plan (Fig. 2) taking the 'certain node' as the starting node and leaving the 'particular refusal node' out of the tour plan. In this case, it is assumed that the salesman verifies the presence of the next customers after each delivery. To illustrate this scenario with an example (Fig. 2), let, out of N (= 10) nodes 4th, 1st and 6th nodes as per the first TSP plan has been visited and at the 6th node, the salesman came to know that the customers at 5th node will not be available.

For a general mathematical model, let up to Lth positions, the goods have been delivered at the nodes and starting at Lth position, it is known that the customer at the $(L+1)$th position ((L + 1)<N) will not be available. In this case, a new TSP plan is made for the left out nodes after Lth position.

Hence, the objective function is,

$$\left.\begin{aligned} &\qquad \text{minimize} \quad Z = \sum_{i=1}^{L-1} c(x_i, x_{i+1}, v_{qi}) * dis(x_i, x_{i+1}, v_{qi}) \\ &+ c(x_L, x_{L+2}, v_{qL}) * dis(x_L, x_{L+2}, v_{qL}) + \sum_{i=L+2}^{N-1} c(x_i, x_{i+1}, v_{qi}) \\ &* dis(x_i, x_{i+1}, v_{qi}) \\ &\qquad + c(x_N, x_1, v_{qN}) * dis(x_N, x_1, v_{qN}) \\ &\quad + \sum_{i=1}^{L-1} (\sigma * w_i) * dis(x_i, x_{i+1}, v_{qi}) + (w_L * \sigma) * dis(x_L, x_{L+2}, v_{qL}) \\ &+ \sum_{i=L+2}^{N-1} (\sigma * w_i) * dis(x_i, x_{i+1}, v_{qi}) + (w_N * \sigma) * dis(x_N, x_1, v_{qN}) \\ &-(\gamma * f_{L+1}) \\ &\qquad\qquad \text{subject to} \quad w_i = D - \sum_{j=1, j \ne L+1}^{i} d_j \\ &\qquad where \quad x_i \ne x_j, i, j = 1, 2, \dots, N, v_{qi}, v_{qN} \in \{1, 2, \dots, P\} \end{aligned}\right\} \tag{11}$$

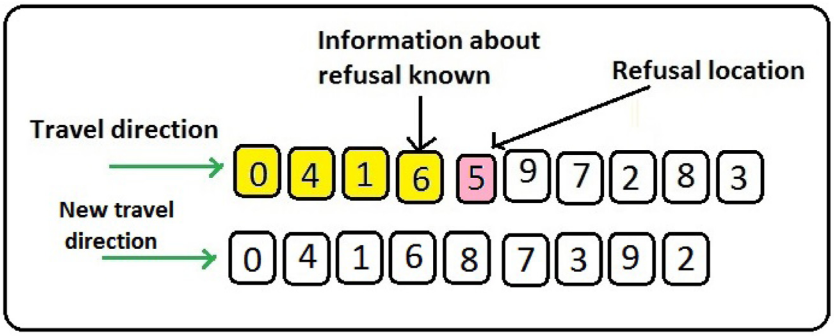


Fig. 2. Example of a refusal.

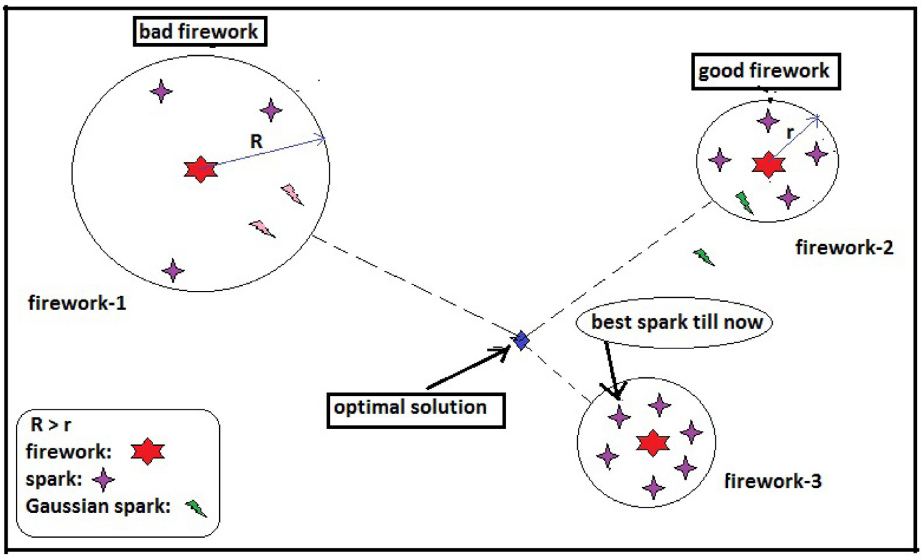


Fig. 3. Overall fireworks procedure.

4. Proposed Type-2 fuzzy logic based fireworks algorithm

4.1. Fireworks algorithm (FWA)

4.1.1. Initialization

The FWA has four steps. (i) $Initialization$, (ii) $Generation\ of\ sparks$, (iii) $Explosion\ amplitude$ and (iv) $Procedure\ of\ selection$.

In FWA, there are two explosions (search) processes being considered and the mechanisms are also designed to maintain the diversity of sparks. Fireworks, along with generated new sparks, indicate the solutions. An explosion process is done for every firework, and around it, the local space is filled by the shower of sparks. Thus, the firework explosion process indicates a search in the closet area near a certain point. Generally, two types of the explosion are seen in fireworks (Fig. 3), i.e., good fireworks generate more sparks, indicates the promising area, and creates the smaller explosion amplitude. This is close to the optimal solution and responsible for global search (exploration). On the other hand, bad fireworks (Fig. 3), generates few sparks, and bigger explosion amplitude, which is located far away from the optimal solutions and is responsible for local search (exploitation). The algorithm always focuses on maintaining the proper balance between exploration and exploitation. To find a solution, fireworks are set off in solution space until one spark targets or is close to the solution. In each generation of the explosion in the FWA, n locations (where location represents a solution in the feasible space) are selected n fireworks and their corresponding current sparks. The quality of sparks is then evaluated after each explosion. FWA will terminate when the stopping condition is met.

1	4	10	8	5	2	3	6	9	7

Fig. 4. Solution representation.

4.1.2. Representation

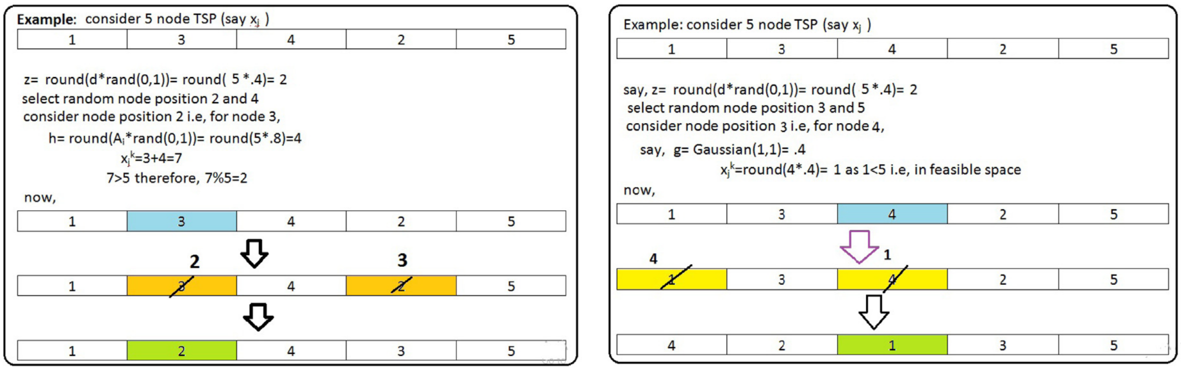
The present study follows the representation scheme of Abdulmajeed and Ayob [34]. The representation (Fig. 4 with 10 nodes) is in vector form, where each integer indicates a location/node where the booked item is to be delivered.

4.1.3. Generation of sparks

Spark numbers S_i [17] defined for each firework X_i are

$$S_i = M \times \frac{y_{max} - f(X_i) + \epsilon}{\sum_{i=1}^{n} (y_{max} - f(X_i)) + \epsilon}. \tag{12}$$

Here, $f(X_i)$ is the objective function of the ith firework to be optimized. M uses as a control parameter for the total number of sparks. $y_{max} = max(f(X_i)), i = 1, 2, 3, \ldots, n$; and ϵ is very small number for avoiding error occurred due to division by zero. From Eq. (12), a real value of



(a) Location of sparks (b) Mutated sparks

Fig. 5. Procedure for location and mutated sparks.

S_i is evaluated and it is converted into integer number as:

$$S_i = \begin{cases} round(c * M) & : S_i < cM \\ round(d * M) & : S_i > dM, \quad c < d < 1 \\ round(S_i) & : otherwise \end{cases}$$

Here, c and d are constant parameter.

4.1.4. Amplitude of explosion

For ith firework, amplitudes of the exploded sparks are evaluated [17] as:

$$A_i = A \times \frac{f(X_i) - y_{min} + \epsilon}{\sum_{i=1}^{n} (f(X_i) - y_{min}) + \epsilon} \tag{13}$$

where A i.e, amplitude coefficient is a constant parameter that controls the maximum amplitude of each firework, and $y_{min} = min(f(X_i)), i = 1, 2, 3, \dots, n$. To make this constant parameter in dynamic, we use Type-2 fuzzy logic (Section 4.2) for better performance. In each iteration best fireworks are used to generate huge sparks with low amplitude for local search. On the flip side, worse fireworks are used to produce small number of sparks with high amplitude for global search.

4.1.5. Sparks generation

In the explosion, sparks are generated randomly in different directions. These random directions (dimensions), say z is defined as

$$z = round(N * rand(0, 1)) \tag{14}$$

Here, N indicates the total number of cities, and z produces any discrete value between 1 to N.

4.1.6. Displacement

For specific node positions generated from the Eq. (14) different displacements generate based on their corresponding amplitude. These random displacements, say h is defined as

$$h = round(A_i * rand(0, 1)) \tag{15}$$

where, A_i is the amplitude of the explosion of the ith firework, round used for non-fractional value of displacement (h).

4.1.7. Location of sparks

Spark locations are obtained by the following Algorithm 1. By this algorithm, if a location is out of the feasible space, it is brought back to the feasible space Fig. 5(a).

Algorithm 1: Location of sparks

Input: Location of firework
Output: Location of spark
Initialize $x_j = x_i$ //location of spark
z=round(d * rand(0, 1))
Select randomly z directions (dimensions) of x_j
h = round(A_i * rand(0, 1)) //displacement
for *each dimension* $x_j^k \leftarrow 1$ ***to*** *pre-assigned z dimensions of* x_j **do**
  $x_k^j = x_k^j$+h // new position
  if $(x_k^j < x_k^{min} || x_k^j > x_k^{max})$ **then**
    $x_k^j = x_k^{min} + |x_k^j| \% (x_k^{max} - x_k^{min})$ // back to feasible space
    Fit non repeated x_k^j as shown in Fig. 5(a)

4.1.8. Mutation

(a) Prime Mutation (generation dependent):

Instead of deterministic (predefined) p_m, in the present study a dynamic adaptation of p_m is introduced. In the present study p_m is evaluated depending on generation by given function. Here, we use generation dependent prime mutation as

$$p_m = \frac{k}{\sqrt{prime\ number\ corresponding\ current\ generation\ number}}, \quad k \in [0, 1].$$

For prime numbers 2, 3, 5, 7, ⋯, p_m will be $\frac{k}{\sqrt{2}}, \frac{k}{\sqrt{3}}, \frac{k}{\sqrt{5}}, \cdots$ as generation progress.

(b) Mutation process:

If $r < p_m$, $r \in [0, 1]$ is satisfied, then the corresponding solution is selected for mutation. Finally conventional random mutation is performed depending on the value of p_m.

4.1.9. Probabilistic selection (p_s) and Boltzmann-probability (p_B)

In the present study, a predefined value named as the probability of selection parameter (p_s) is used. The Boltzmann-Probability function (p_B) is

$$p_B = e^{((y_{min} - f(X_i))/T)},$$

$T = T_0(1\text{-}a)^k$, k = (1 + 100*(g/G)), g indicates the present generation number, G for the highest generation, a = rand(0,1), T_0 = rand(15,150), f(X_i) for the fitness corresponding X_i. Procedures described by Maity et al. [50] are followed in the present study.

4.2. Type-2 fuzzy logic based fireworks algorithm (T2FLFWA)

The fuzzy inference system is introduced using linguistic variables depending on membership functions of input variables (Iteration-ratio

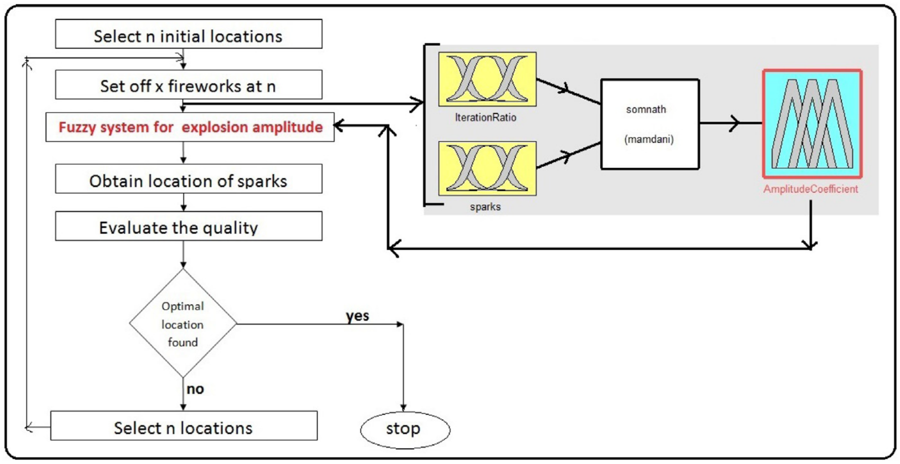
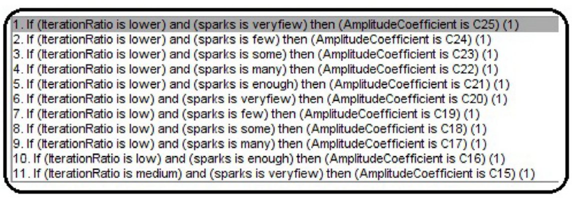
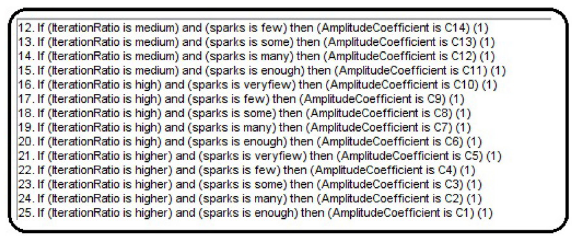


Fig. 6. Flowchart for Type-2 fuzzy logic based FWA.



(a) 25 rules (1-11)



(b) 25 rules (12-25)

Fig. 7. Rules for Type-2 fuzzy logic.

and sparks) to generate one output variable (amplitude coefficient).

$$\text{Iteration-ratio} = \frac{Current\ \ Iteration\ \ number}{Total\ \ Iteration\ \ number}$$

Iteration-ratio is categorized into five parts as lower, low, medium, high and higher (membership function) in the range [0,1]. This is because of normalization used in the range between 0 and 1. In Fig. 6, the generalized flowchart of fuzzy FWA is presented and explicitly shows how the output variable i.e., amplitude coefficient is extracted.

In this fuzzy system (Fig. B.1), (i) Mamdani Type, (ii) Iteration-ratio [0,1] and sparks [1,48] as two input variables, (iii) amplitude coefficient [0,50] as the output variable.

4.2.1. Flowchart for Type-2 fuzzy logic based FWA

A flowchart of the proposed algorithm illustrating the flow of operations in Fig. 6.

4.2.2. Rules for Type-2 fuzzy logic

All the rules and figs. are design using type-2 fuzzy logic toolbox [41]. For controlling the amplitude coefficient, we used 25 rules (Fig. 7) and the corresponding fuzzy linguistic values (Figs. B.2 and B.3) are given. The system used is shown in Fig. B.1. Here, amplitude coefficients are evaluated using the rules and linguistic values for Type-2 fuzzy logic presented in Fig. 6. Uses of Type-2 values for linguistic variables provide Type-2 fuzzy logic respectively.

4.2.3. Algorithm of T2FLFWA

A complete step wise description is provided in Algorithm 2.

4.3. Parameter setting for T1FLFWA and T2FLFWA (c.f [17])

(a) Number of fireworks $(N) = 10$

(b) For every firework, maximum number of explosion sparks $(M) = 48$

(c) Control Parameter $(CP) = 50$

(d) Constant $a = 0.04$ and $b = 0.80$

(e) Maximum explosion amplitude $(A) = 40$

(f) Number of mutation $(N') = 10$

(g) Termination criteria: Maximum number of function evaluations/generation = 2500

(h) Input variables sparks $[1, 48]$

(i) Output variable amplitude coefficient $[0, 50]$

4.4. Fuzzy linguistics input/output values for T2FLFWA

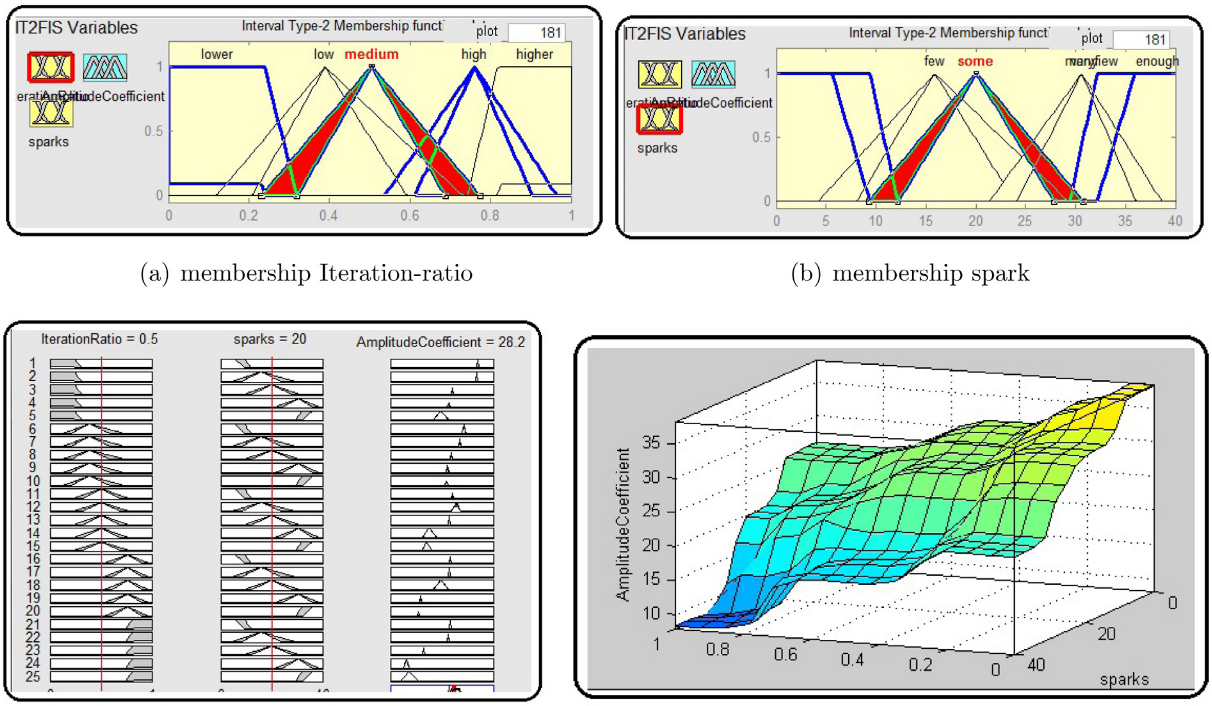
Here, fuzzy linguistic values input data for T2FLFWA is given in (Fig. B.2). The corresponding outputs of T2FLFWA is given in Fig. B.3.

4.5. Pictorial representations of amplitude coefficients for Type-2 FWA

All the results are taken using type-2 fuzzy logic toolbox [41]. The amplitude coefficients are evaluated using the rules and linguistic values for Type-2 fuzzy logic and presented in Fig. 8.

It is seen that the amplitude coefficient becomes more dynamic (in Fig. 8(d)) using the set of rules in the Type-2 fuzzy logic. Figs. 8(a) and 8(b) represent the membership of iteration-ratio and spark respectively. Using the interval value viewer, for different values of iteration-ratio and number of sparks, we get the corresponding amplitude coefficients from Fig. 8(c).

The developed T2FLFWA is tested with standard functions from TSP Library and its supremacy over other algorithms (T1FLFWA, FWA, GA and ACO) is established using ANOVA. All these are presented in Appendix A.



(a) membership Iteration-ratio (b) membership spark

(c) interval rule viewer (d) surface view

Fig. 8. Graphical view of Type-2 fuzzy logic.

Table 2
Input data: Distance (km) and traveling cost ($) per unit distance Matrix.

i/j	0	1	2	3	4	5	6	7	8	9
0	∞	(28,26,32)	(31,29,26)	(36,33,31)	(17,25,20)	(17,15,13)	(36,39,34)	(38,34,31)	(41,46,43)	(31,28,25)
	∞	(9,10,7)	(8,7,10)	(7,8,9)	(8,6,10)	(7,8,9)	(10,8,7)	(8,7,10)	(6,8,10)	(8,9,6)
1	(28,26,32)	∞	(21,25,27)	(29,25,30)	(36,35,37)	(47,45,42)	(31,35,32)	(43,45,46)	(29,25,24)	(15,14,16)
	(9,10,7)	∞	(7,8,7)	(8,9,10)	(7,9,8)	(6,8,9)	(10,9,8)	(7,8,7)	(6,9,7)	(10,9,8)
2	(31,29,26)	(21,25,27)	∞	(31,35,38)	(26,25,22)	(36,38,40)	(10,15,14)	(33,37,34)	(41,43,46)	(31,36,33)
	(8,7,10)	(7,8,7)	∞	(7,9,9)	(8,6,10)	(9,8,7)	(10,8,8)	(9,8,9)	(7,7,7)	(8,7,9)
3	(36,33,31)	(29,25,30)	(31,35,38)	∞	(21,25,27)	(26,25,27)	(31,35,34)	(36,35,39)	(23,25,21)	(38,35,33)
	(7,8,9)	(8,9,10)	(7,9,9)	∞	(8,7,9)	(6,8,8)	(7,9,8)	(8,8,9)	(7,8,9)	(8,7,8)
4	(17,25,20)	(36,35,37)	(26,25,22)	(21,25,27)	∞	(21,25,23)	(22,25,27)	(31,35,37)	(15,15,17)	(29,35,30)
	(8,6,10)	(7,8,9)	(8,6,10)	(8,7,9)	∞	(9,8,9)	(8,7,6)	(8,9,7)	(6,8,8)	(7,9,8)
5	(17,15,13)	(47,45,42)	(36,38,40)	(26,25,27)	(21,25,23)	∞	(33,35,37)	(41,45,46)	(33,35,37)	(31,35,29)
	(7,8,9)	(6,8,9)	(9,8,7)	(6,8,8)	(9,8,9)	∞	(6,7,7)	(8,8,10)	(9,8,7)	(7,8,8)
6	(36,39,34)	(31,35,32)	(10,15,14)	(31,35,34)	(22,25,27)	(33,35,37)	∞	(11,15,12)	(33,35,31)	(21,25,19)
	(10,8,7)	(10,9,8)	(10,8,8)	(7,9,8)	(8,7,6)	(6,7,7)	∞	(7,8,7)	(9,9,8)	(10,8,7)
7	(38,34,31)	(43,45,46)	(33,37,34)	(36,35,39)	(31,35,37)	(41,45,46)	(11,15,12)	∞	(23,25,24)	(38,35,41)
	(8,7,10)	(7,8,7)	(9,8,9)	(8,8,9)	(8,9,7)	(8,8,10)	(7,8,7)	∞	(8,7,9)	(9,8,10)
8	(41,46,43)	(29,25,24)	(41,43,46)	(23,25,21)	(15,15,17)	(33,35,37)	(33,35,31)	(23,25,24)	∞	(29,25,24)
	(6,8,10)	(6,9,7)	(7,7,7)	(7,8,9)	(6,8,8)	(9,8,7)	(9,9,8)	(8,7,9)	∞	(8,10,9)
9	(31,28,25)	(15,14,16)	(31,36,33)	(38,35,33)	(29,35,30)	(31,35,29)	(21,25,19)	(38,35,41)	(29,25,24)	∞
	(8,9,6)	(10,9,8)	(8,7,9)	(8,7,8)	(7,9,8)	(7,8,8)	(10,8,7)	(9,8,10)	(8,10,9)	∞

Table 3
Input data: Demand matrix, Payment system and Probability of refusal.

Matrix(1 × 10)										
i/j	0	1	2	3	4	5	6	7	8	9
Demand (kg)	0	16	10	8	12	15	7	15	10	12
Payment system	–	COD	COB	COB	COD	COB	COD	COB	COD	COB
Probability of refusal	0	0.2	0.4	1	0.3	0.6	1	0.5	0.2	1

5. Solutions of MPRfGDwRPs using T2FLFWA

As developed T2FLFWA is better than T1FLFWA (c.f Appendix A.1), we solve the proposed the Multi-path delivery problems (MPRfGDwRPs) presented through Eqs. (5)–(9) using T2FLFWA in a PC having IntelCorei5-4210U CPU with 1.70 GHz processor and 8.0 GB of RAM.

6. Numerical experiments of proposed models

6.1. Input data

The distance and traveling cost per unit distance matrices (Row-wise 1st and 2nd sets correspond to distances and costs respectfully) for MPRfGDwRPs are presented in Table 2.

Here, we consider three types of routes between two nodes with the corresponding distance and traveling cost matrices along different routes for the models. For the distance and traveling cost (a,b,c) (say), the values a, b and c are for the 1st, 2nd and 3rd routes respectively. The demand matrix (in weight) of materials is given in Table 3. Total

Algorithm 2: Algorithm of T2FLFWA

Input: Number of fireworks (n), Max Gen (G), Problem data
Output: Optimal/near optimal solutions

1 Set initialization g ← 1 // g: generation number, Initialize $x_j = x_i$ //location of spark
2 n locations randomly select
3 **while** $(g \leq Max\ Gen\ )$ **do**
4   set off n fireworks at n locations
5   **for** $x_i \leftarrow 1$ ***to*** n **do**
6   determine the spark numbers $S_i = M \times \frac{y_{max} - f(X_i) + \epsilon}{\sum_{i=1}^{n}(y_{max} - f(X_i)) + \epsilon}$
7   Iteration-ratio=$\frac{Current \quad Iteration \quad number(g)}{Total \quad Iteration \quad number(G)}$
8   evaluate amplitude explosion (A_i) using Type-2 fuzzy logic (subsection 4.2.2)
9   **//sparks generation**
10   Initialize $x_j = x_i$ (//location of spark), z=round(d * rand(0, 1))
11   Select randomly z directions (dimensions) of x_j
12   h = round(A_i * rand(0, 1)) //displacement
13   **for** *each dimension* $x_j^k \leftarrow 1$ ***to*** *pre-assigned z dimensions of* x_j **do**
14     $x_k^j = x_k^j$+h // new position
15   **if** $(x_k^j < x_k^{min} || x_k^j > x_k^{max})$ **then**
16       $x_k^j = x_k^{min} + |x_k^j| \ \%\ (x_k^{max} - x_k^{min}\ )$ // back to feasible space
17   Adjust non repeated x_k^j as shown in Fig. 5(a)
18   Total solutions (T_s)=number of firework (n)+number of sparks (s)
19   **//Generation dependent prime mutation**
20   $p_m = \frac{k}{\sqrt{p_g}}$, k∈(0, 1) //$p_g$ is prime number corresponding current generation number
21   **for** $i \leftarrow 0$ ***to*** T_s **do**
22     $r = rand(0, 1)$
23   **if** $r < p_m$ **then**
24   Choose current solution, $a_1 = rand[1, N]$, $b_1 = rand[1, N]$
25   **if** $(a_1 {==} b_1)$ **then**
26   goto step 28
27   **for** $j \leftarrow 1$ ***to*** N **do**
28   **if** *(x[j]==a_1)* **then**
29   p=j
30   **if** *(x[j]==b_1)* **then**
31   q=j
32   x[p]=b_1; x[q]=a_1; //replace a_1 by b_1 and b_1 by a_1.
33   **//Select the best fitted spark and other (n-1) sparks using probabilistic selection**
34   **for** $i \leftarrow 1$ ***to*** T_s **do**
35     $r = rand(0, 1)$ and T_0= rand(15, 150), $a = rand(0, 1)$ and $C = rand(0, 100)$
36     $k = (1 + C * rand(0, 1))$ and $T = T_0$(1-a)^k, $p_B = e^{((g/G)*(y_{min} - f(X_i))/K*T)}$
37   **if** $(r < Max(p_s, p_B)\ )$ **then**
38   Choose the corresponding firework
39   **else**
40   Select the firework corresponding to y_{min}
41 Store the global optimal results

goods weight gradually decreases after the delivery of materials at each node and hence, transporting cost also gradually decreases. The customers at different nodes choose the payment system, either COD or COB. Table 3 also furnishes the payment option matrix against each customer. The probability of refusal of booked goods for each customer is calculated based on past records on locality basis and these probabilities are also presented in Table 3. The discontinuous goods transportation cost in the form of AUD following the expression (7) are given in Table 4.

Table 4
Values of weight parameters in expression (7).

Values of parameters							
σ_4	σ_3	σ_2	σ_1	w_4	w_3	w_2	w_1
0.2	0.3	0.4	0.6	25 kg	50 kg	75 kg	105 kg

6.2. Experimental outcomes: Results of MPRfGDwRP under different scenarios

With the above data, results for the models (MPRfGDwRPs) solved by T2FLFWA under the scenarios-1, 2, 3, 4 and 5 are presented at Table 5.

Particular case: Instead of multiple routes, taking a single route between different nodes for the two dimensional routing for goods delivery with refusal problems, say 2D RfGDwRPs, the results are obtained different scenarios and presented in Table 5. This shows the importance of having multiple route connections between nodes.

7. Discussions

In Table 5, against every scenario, optimal routing plan with appropriate routes between nodes, total distance(km), traveling cost($) of salesman, transportation cost($) and overall system cost($) are presented.

7.1. Comparison of results for MPRfGDwRP (3D routing) and 2D RfGDwRP

In scenario 1, against the probabilistic refusals given in Table 3 (from past records), for MPRfGDwRP, the optimal routing plan with appropriate paths is 0(2)-3(1)-7(1)-8(0)-1(1)-9(2)-6(1)-2(1)-4(1)-5(1), total distance and overall cost are 233 km and $10 283.27 (traveling cost($1757) + transportation cost($8606.27) - delivery charge($80)) respectively. With the same data for 2D RfGDwRP, the optimal routing plan is 0(0)-4(0)-8(0)-7(0)-6(0)-2(0)-1(0)-3(0)-5(0)-9(0), distance and overall cost are 244 km and $10 723.83 (traveling cost($1817) + transportation cost($8986.83) - delivery charge($80)) respectively. Here, for 3D MPRfGDwRP, the refusals occur at 3rd, 6th and 9th nodes where 3rd and 9th nodes are under COB, and hence the delivery charge $40*2 = $80 is accumulated in company account. The refusal at 6th node is under the COD system, and hence no cost is incurred in the company's account.

In scenario 2, when no customer refuses i.e., the salesman delivers the items as per demand at each node. Here optimal path for MPRfGDwRP is 0(1)-3(0)-5(2)-8(0)-4(2)-7(2)-6(0)-2(2)-1(1)-9(2), distance and overall cost are 246 km and $6330.19(traveling cost($1737) + transportation cost($4593.19) respectively, and those for 2D RfGDwRP are 0(0)-5(0)-9(0)-3(0)-8(0)-2(0)-1(0)-7(0)-6(0)-4(0), 294 km and $7256.89 (traveling cost($2186) + transportation cost ($5070.89))respectively.

In scenario 3, when assuming that, refusal is known at the beginning. Say refusal occur and previously known 4th node as the refusal node. Thus the customer point becomes $10 - 1 = 9$. Now, the salesman delivers the items as per demand at each node except refusal 4th node. Here optimal path for MPRfGDwRP is 0(2)-3(1)-8(2)-5(0)-7(1)-1(2)-6(0)-2(1)-9(0), distance and overall cost are 219 km and $5842.25 (traveling cost($1671) + transportation cost($4171.25) respectively,

Table 5
Results of all Scenarios of MPRfGDwRP by T2FLFWA.

Scenario	Refusal Position	Dimension	Path (route)	Distance (km)	Travel. cost ($)	Transp. cost ($)	Refunded ($)	Total cost ($)
	Probabilistic	2D	0(0)-4(0)-8(0)-7(0)-6(0)-2(0)-1(0)-3(0)-5(0)-9(0)	244	1817	8986.83	80	10 723.83
1	Refusal	3D	0(2)-3(1)-7(1)-8(0)-1(1)-9(2)-6(1)-2(1)-4(1)-5(1)	233	1757	8606.27	80	10 283.27
	No	2D	0(0)-5(0)-9(0)-3(0)-8(0)-2(0)-1(0)-7(0)-6(0)-4(0)	294	2186	5070.89	–	7 256.89
2	Refusal	3D	0(1)-3(0)-5(2)-8(0)-4(2)-7(2)-6(0)-2(2)-1(1)-9(2)	246	1737	4593.19	–	6 330.19
	Refusal known (4th node)	2D	0(0)-7(0)-3(0)-1(0)-5(0)-8(0)-2(0)-6(0)-9(0)	277	2098	4614.12	–	6 712.12
3	At beginning	3D	0(2)-3(1)-8(2)-5(0)-7(1)-1(2)-6(0)-2(1)-9(0)	219	1671	4171.25	–	5 842.25
	Refusal after reaching	2D	0(0)-5(0)-9(0)-3(0)-8(0)-2(0)-1(0)-7(0)-6(0)-4(0)	294	2186	8957.90	40	11 103.90
4	6th node position	3D	0(1)-3(0)-5(2)-8(0)-4(2)-7(2)-6(0)-2(2)-1(1)-9(2)	246	1737	8080.39	40	9 777.39
	Refusal at 6th position	2D	0(0)-5(0)-9(0)-3(0)-8(0)-1(0)-4(0)-6(0)-7(0)	280	2181	6907.50	40	9 048.50
5	Known at 5th node position	3D	0(2)-3(0)-5(2)-8(0)-4(2)-6(1)-1(0)-2(2)-9(0)	226	1689	6714.10	40	8 363.10

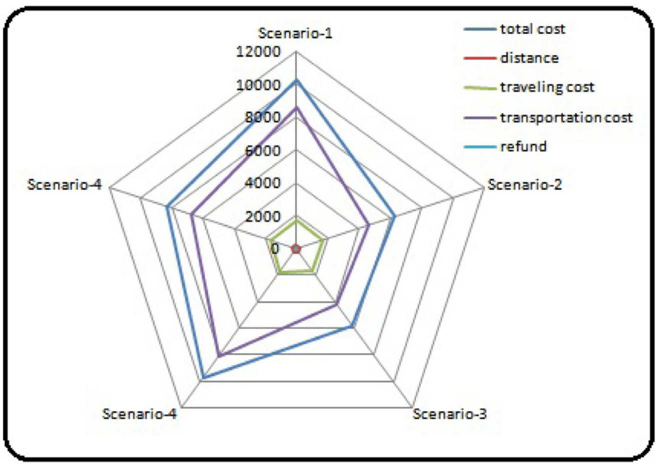


Fig. 9. Graphical result for all scenarios.

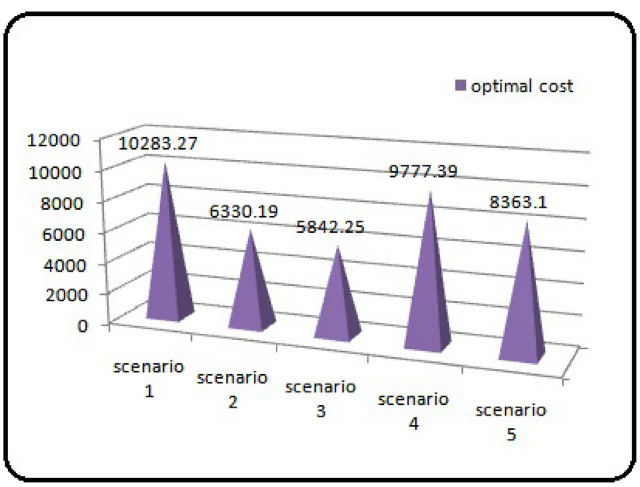


Fig. 10. Overall System cost for all cases.

and those for 2D RfGDwRP are 0(0)-7(0)-3(0)-1(0)-5(0)-8(0)-2(0)-6(0)-9(0), 277 km and $6712.12 (traveling cost($2098) + transportation cost($4614.12))respectively.

In scenario 4, let refusal node position be the 6th. The salesman reached this node as he did not have the refusal information (refusal known after reaching the node). So, refusal item (demand at 6th node) is extra weight for the remaining paths. Thus only transportation cost will increase but the optimal path, total distance, and traveling cost remain unaltered as in scenario 4. Here delivery charge $40 remains with the supplier as the system is under COB mode. Therefore, the overall cost for MPRfGDwRP and 2D RfGDwRP are $9777.39 and $11 103.90 respectively which are more compared to scenario 4 (with out refusal).

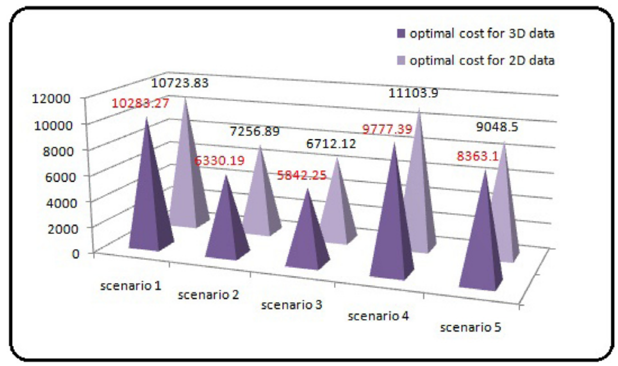


Fig. 11. Comparison of 2D and 3D result.

In scenario 5, let the refusal be at 6th position of the optimum routing plan (0-3-5-8-4-7-6-2-1-9-0 with no refusal) i.e, 7th node and it is known at node '4' (5th position). At this position, the salesman makes a new routing plan for the remaining nodes (nodes 6, 2, 1, 9) starting from the node '4', ending at the node '0' and excluding the node '7'. Thus in this scenario, optimal path for MPRfGDwRP is 0(2)-3(0)-5(2)-8(0)-4(2)-6(1)-1(0)-2(2)-9(0), distance and overall cost are respectively 226 km and $8363.10 (traveling cost($1689) + transportation cost($6714.10) - delivery charge($40)). In the same way, the optimal path for 2D RfGDwRP is 0(0)-5(0)-9(0)-3(0)-8(0)-1(0)-4(0)-6(0)-7(0), distance and overall cost are 280 km and $9048.50 (traveling cost($2181) + transportation cost($6907.50) - delivery charge($40)) respectively. Therefore the overall cost is less compared to scenario 4 (refusal known to reach the node).

In Fig. 9, graphically the optimal cost, distance, traveling cost, transportation cost and refund cost (when refusal occurs under COB) are presented for 3D routing plan. Fig. 10, represents the optimal cost of all scenarios along with special case (when refusal at all nodes is known after reaching). From Fig. 11 and Table 5, it is clear that for all the scenarios, results of MPRfGDwRP are better than 2D RfGDwRP results.

7.2. Justification of multi-paths

The use of multi-path in an routing model, is more realistic due to infrastructural development throughout the world. Considering the availability of different connecting paths among each and every node, the experimental study shows that this gives better results than the single path among the nodes (2D) (cf. Table 5).

Very few researchers have considered multi-paths in the routing problems. The justification of taking alternate paths in this investigation is as follows. In Table 5, in scenario 4 (no refusal), the optimal

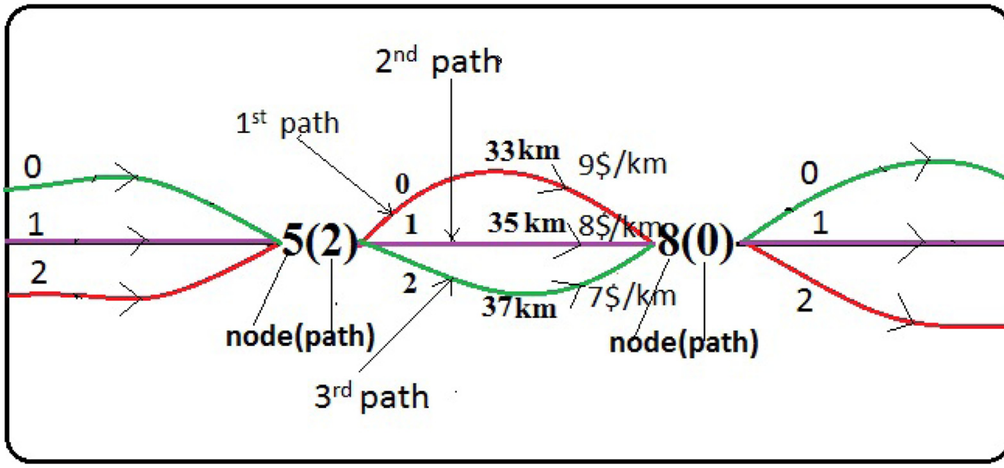


Fig. 12. Justification of multi-path.

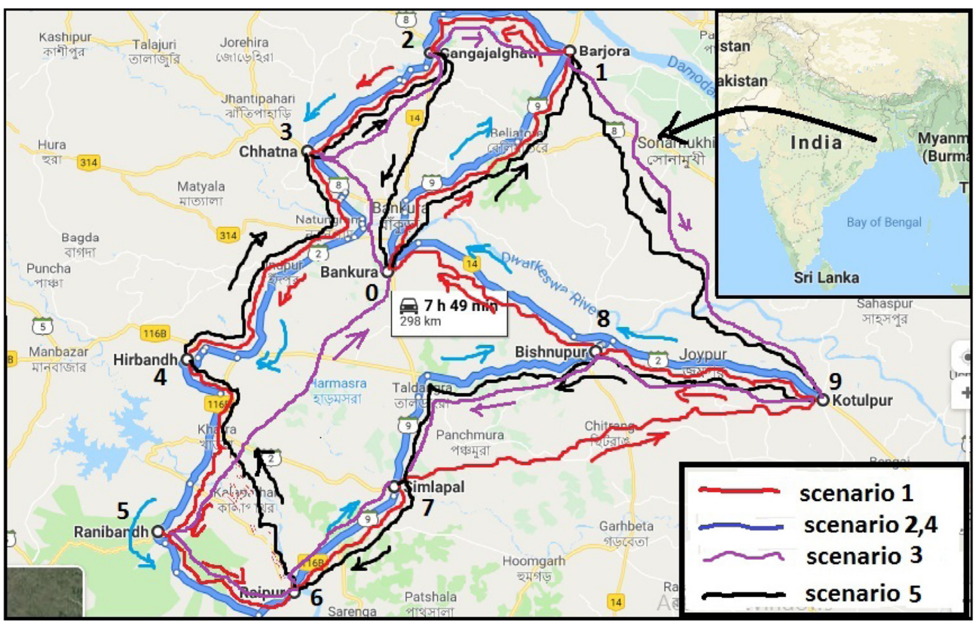


Fig. 13. Optimal paths (refusal and non-refusal) of Flipkart company in Bankura. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

path for MPRfGDwRP is 0(1)-3(0)-5(2)-8(0)-4(2)-7(2)-6(0)-2(2)-1(1)-9(2), distance and overall cost are 246 km and $6330.19(traveling cost($1737) + transportation cost($4593.19) respectively.

Here (in Fig. 12), the path is between 5(2)-8(0) i.e, 3rd path is chosen among three alternate paths between nodes 5 and 8. If we consider only a single path between 5 and 8 nodes, say the first alternate path (0), distance is reduced by 4 km but cost increases by $38, so overall cost becomes $6368.19. For the second alternate path (1) distance is reduced by 2 km but cost increases by $21, so overall cost becomes $6351.19. Traveling cost depends on 'distance' and 'traveling cost per unit distance', so multi-path/alternate path impacts on routing. For accurate and realistic findings, multi-path has an important role in routing models.

8. Practical implementation

This section illustrates one small real-life multi-path goods delivery problem with random refusal and the application of the developed algorithms for the solution. We select an e-commerce company say Flipkart, performing in the Bankura District, West Bengal, India. The company collects orders throughout the e-channel and supplies the materials to distant customers in time. The customers' details from different locations in Bankura district (cf. Fig. 13) are collected and probabilities of refusals are determined from the past data. Transportation costs are evaluated from the distances measured through google map and other information are captured through private contacts. Let the company follows the procedure, as mentioned in the proposed MPRfGDwRP system. Let there were 10 distant customers, as pointed out in Fig. 13 and which is a replica of the MPRfGDwRP model and solved by the proposed T2FLFWA. To solve this real-life problem, the input data are presented in Table 6 for 3D distance and cost matrices. Table 7 furnishes demand, payment system and the probability of refusal. AUD values are the same as Table 4. We consider scenarios-1, 2, 3, and 4 for this real life problem and evaluate the corresponding minimum costs.

Here, For scenario 1, considering the probabilistic refusal and using T2FLFWA, we evaluate the overall optimal cost and path (red line) as $9753.10 and Bankura(0)-Barjora(1)-Gangajalghati(2)-Chhatna(3)-Hirbandh(4)-Ranibandh(5)-Raipur(6)-Simlapal(7)-Kotulpur(9)-Bishnupur(8)-Bankura(0) respectively.

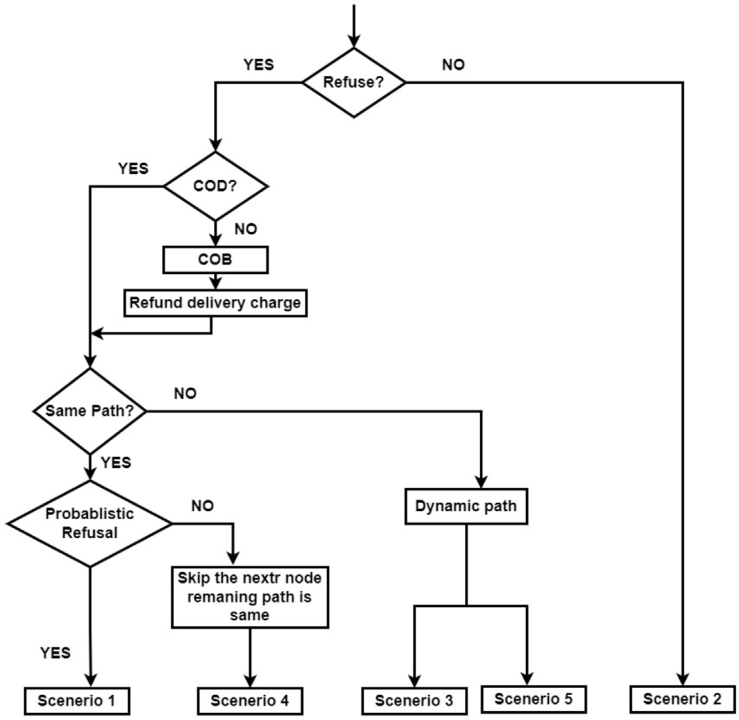


Fig. 14. Flowchart of proposed model.

Table 6
Input data: Distance (km) and traveling cost ($) per unit distance matrix for Flipkart in Bankura.

i/j	Bankura(0)	Barjora(1)	Gangajalghati(2)	Chhatna(3)	Hirbandh(4)	Ranibandh(5)	Raipur(6)	Simlapal(7)	Bishnupur(8)	Kotulpur(9)
Bankura(0)	∞ ∞	(28,30.64,35) (9,10,7)	(22.59,26,27) (8,7,10)	(12,14.12,16) (7,8,9)	(31,33.02,35) (8,6,10)	(47,49,50.25) (7,8,9)	(47,50.96,52) (10,8,7)	(36,39.93,41) (8,7,10)	(33,32,30.42) (6,8,10)	(52,57.02,61) (8,9,6)
Barjora(1)	(28,30.64,35) (9,10,7)	∞ ∞	(29,32,19.13) (7,8,7)	(32,39,36.30) (8,9,10)	(62.71,65,69) (7,9,8)	(80.18,85,77) (6,8,9)	(86,93,77.54) (10,9,8)	(55,61.58,65) (7,8,7)	(53,41.21,39) (6,9,7)	(69,62,55.86) (10,9,8)
Gangajalghati(2)	(22.59,26,27) (6,7,9)	(29,32,19.13) (8,8,9)	∞ ∞	(23,19.68,17) (7,8,9)	(52,58,49.97) (7,6,10)	(65,68,67.47) (8,8,9)	(82,72.36,69) (9,8,8)	(47,69,57.07) (9,10,9)	(53,56,43.23) (7,8,7)	(69,78,67.56) (8,8,9)
Chhatna(3)	(12,14.12,16) (6,8,9)	(32,39,36.30) (8,10,10)	(23,19.68,17) (7,8,9)	∞ ∞	(36,31,31.43) (8,8,9)	(52,57,51.71) (6,7,8)	(65,69,57.04) (7,10,8)	(57,51,44.15) (8,8,9)	(45.15,53,56) (7,8,9)	(72.70,78,74) (8,9,8)
Hirbandh(4)	(31,33.02,35) (6,7,10)	(62.71,65,69) (7,8,9)	(52,58,49.97) (8,6,10)	(36,31,31.43) (9,7,9)	∞ ∞	(25,20.91,22) (9,10,9)	(41,31.18,36) (8,7,6)	(36,37,31.33) (8,9,7)	(52,50.87,58) (6,8,8)	(84,81,78.30) (7,8,8)
Ranibandh(5)	(47,49,50.25) (8,10,9)	(80.18,85,77) (6,8,9)	(65,68,67.47) (9,8,7)	(52,57,51.71) (6,8,8)	(25,20.91,22) (9,8,9)	∞ ∞	(26,23,16.39) (6,7,7)	(35,32,29.23) (8,8,10)	(68,67,58.23) (9,6,7)	(93,83.07,87) (7,8,8)
Raipur(6)	(47,50.96,52) (9,8,7)	(86,93,77.54) (8,9,8)	(82,72.36,69) (10,8,8)	(65,69,57.04) (7,9,8)	(41,31.18,36) (8,7,6)	(26,23,16.39) (6,7,7)	∞ ∞	(22,18.89,26) (7,8,7)	(55,50.16,53) (9,9,8)	(76,75,70.54) (10,8,7)
Simlapal(7)	(36,39.93,41) (7,9,10)	(55,61.58,65) (7,8,7)	(47,69,57.07) (9,8,9)	(57,51,44.15) (8,8,9)	(36,37,31.33) (8,9,7)	(35,32,29.23) (8,8,10)	(22,18.89,26) (7,8,7)	∞ ∞	(26,28.68,32) (8,7,9)	(55,52.36,62) (9,8,10)
Bishnupur(8)	(33,32,30.42) (8,6,10)	(53,41.21,39) (6,9,7)	(53,56,43.23) (7,7,7)	(45.15,53,56) (7,9,9)	(52,50.87,58) (6,8,8)	(68,67,58.23) (9,8,7)	(55,50.16,53) (9,9,8)	(26,28.68,32) (8,7,9)	∞ ∞	(35,45,30.26) (8,10,9)
Kotulpur(9)	(52,57.02,61) (9,10,6)	(69,62,55.86) (8,9,8)	(69,78,67.56) (8,7,9)	(72.70,78,74) (8,7,8)	(84,81,78.30) (7,9,8)	(93,83.07,87) (8,9,8)	(76,75,70.54) (10,8,7)	(55,52.36,62) (9,8,10)	(35,45,30.26) (8,10,9)	∞ ∞

For scenario 2, using T2FLFWA, we evaluate the overall optimal cost and optimal path (blue line) as $5582.60 and Bankura(0)-Barjora(1)-Gangajalghati(2)-Chhatna(3)-Hirbandh(4)-Ranibandh(5)-Raipur(6)-Simlapal(7)-Bishnupur(8)-Kotulpur(9)-Bankura(0) respectively.

For scenario 3, using T2FLFWA, we evaluate the overall optimal cost and optimal path (purple line) as $5147.40 and Bankura(0)-Chhatna(3)-Gangajalghati(2)-Barjora(1)-Kotulpur(9)-Bishnupur(8)-Simlapal(7)-Raipur(6)-Ranibandh(5)-Bankura(0) respectively.

In scenario 4, considering the refusal node at the 6th position and then using T2FLFWA, the evaluated overall optimal cost is $8753.10 and the optimum path is same (blue line) as scenario 4.

For scenario 5, let the refusal node be the 6th position node which is known at the 2nd position node. Then using T2FLFWA, the evaluated overall optimal cost and path (black line) are $7641.45 and Bankura(0)-Barjora(1)-Kotulpur(9)-Bishnupur(8)-Simlapal(7)-Raipur(6)-Hirbandh (4)-Chhatna(3)-Gangajalghati(2)-Bankura(0) respectively.

We observe that, in scenario 3, the overall optimal cost is minimum compared to scenarios- 1, 2, 4 and 5.

9. Managerial insights

This section indicates how management can make optimal decisions to minimize overall costs when refusal occurs. From our investigation, scenario 3 is more effective, i.e., when the rejection is known to the salesman at a previous node. So, management can develop an automated system to help the salesman if customers' refusals occur. So salesmen can contact the head office to generate the dynamic path to reduce the overall cost. Also it is easily understood from scenario 3 that the loss of the company is less if the refusal node is known to the salesman as early as possible. Also, the overall cost depends on traveling cost as well as transportation cost. So refusal node's position and how much amount of refused demand at that node are very much

Table 7
Input Data: Demand matrix (kg), Payment system (COD/COB) and Probability of refusal (random).

Matrix(1 × 10)										
i/j	0	1	2	3	4	5	6	7	8	9
Demand (kg)	0	24	10	12	16	14	18	12	15	9
Payment system	–	COD	COB	COB	COD	COB	COD	COB	COD	COB
Probability of refusal	0	0.2	0.4	0.1	0.3	1	.3	0.5	0.2	.6

effective for all the scenarios. All the possible scenarios are visible in Fig. 14 and the management can make an appropriate decision based on the actual scenario that happened and informed by the salesman.

10. Conclusions, limitations and future scope

In the present investigation, a Type-2 fuzzy fireworks algorithm (T2FLFWA) is developed and using it, a practical TSP type multi-path delivery problems with refusal is solved. The impreciseness of the parameters of FWA is outlined through Type-2 fuzzy logic. The improvement of FWA is made by incorporating probabilistic selection and prime-generation dependent mutation. Also, a practical, real-life problem faced by today's e-commerce industries/MNCs is mathematically formulated and solved T2FLFWA. The customer refusal for a delivery system changes the routing plan, which affects the system costs. The present article shows the effectiveness of the model under different refusal scenarios. The developed algorithm also finds its supremacy against the FWA, GA, ACO which is established by the statistical test, ANOVA. We introduce the Type-2 fuzzy logic in FWA, an extension of the fuzzy FWA. Some managerial insights are drawn in for the organizations. Thus, in the present investigation, we have introduced a new heuristic method, T2FLFWA for discrete problems in the literature and answered the questions/problems faced by the delivery man in the case of refusals.

Thus, the novelties of the investigations are two folds—Development of (i) Multi-path routing for goods delivery with uncertain/random refusal problems (MPRfGDwRPs) and (ii) Discrete fireworks algorithm with Type-2 fuzzy logic (T2FLFWA). T2FLFWA has solved these MPRfGDwRPs.

Still, there are some limitations to the proposed model and algorithm. The amplitude coefficients depend on many parameters, but only two are considered here. The model has been illustrated for only 10 nodes, it can be formulated and solved for large set of data. A heuristic optimization method involves both exploration and exploitation. In Fireworks, these are generation of sparks (S_i/M) and brightness of sparks (h/A) respectively. It is difficult to set these parameters appropriately in FW algorithm.

In the future, both the method and model can be extended. The T2FLFWA has been developed in general form for discrete NP-hard problems. It can be applied to solve other TSP type routing problems such as Facility location problem, Traveling purchaser problem, etc. Moreover hybrid metaheuristic methods incorporating T2FLFWA with GA, ACO, PSO, etc. (i.e, GA-T2FLFWA, ACO-T2FLFWA, etc.) can be developed and used. For the first time, 3D delivery system (with multi-routes between nodes) has been introduced and this realistic extension can be made for similar TSP type problems such Bombay Dabbawala problem, etc. The model can be formulated with a time window concerning the customers or with time constraints. It can also be formulated a multi-objective problem taking time minimization and/or emission minimization.

CRediT authorship contribution statement

Somnath Maji: Conceptualization, Methodology, Software, Writing – original draft. **Samir Maity:** Conceptualization, Methodology, Writing – original draft. **Debasis Giri:** Supervision, Validation, Writing – review & editing. **Oscar Castillo:** Supervision, Software, Methodology. **Manoranjan Maiti:** Supervision, Validation, Writing – review & editing.

Table A.1
Results for Standard TSP Problem (TSPLIB).

Instances	OS	Error (%)			
		T2FLFWA	T1FLFWA	FWA	GA
us16	6859	0.12	0.17	0.25	0.32
gr17	2085	0.18	0.21	0.38	0.27
gr21	2707	1.23	1.47	1.89	1.97
fri26	937	1.36	1.67	2.17	2.38
bays29	2020	0.92	1.28	2.65	2.41
dantzig42	699	2.18	2.47	3.28	3.19
eil51	426	2.37	2.94	4.12	4.08
berlin52	7542	1.96	2.13	3.74	4.19
st70	675	2.63	3.27	4.15	3.92
eil76	538	3.28	3.82	4.87	4.66
pr76	108 159	3.19	4.27	6.28	5.81
rat99	1211	1.79	3.28	6.95	7.28
eil101	629	2.85	4.67	8.19	7.69
kroA100	21 282	4.26	6.45	9.13	11.12
kroC100	20 749	4.79	5.42	12.14	16.23
kroA150	26 524	5.23	6.85	11.17	10.29
kroB200	29 437	6.72	7.29	14.67	13.28
a280	2579	5.93	6.28	16.18	17.93
pr299	48 191	6.86	8.67	18.11	14.38
lin318	42 029	7.23	9.12	20.12	17.66

Table A.2
Parameters for T2FLFWA and T1FLFWA.

Node (N)	Maxgen	$\text{Iter}_{T2FLFWA}$	$\text{Iter}_{T1FLFWA}$	p_s	p_m
N ≤ 50	500	250	300		
$50 < N \leq 110$	1500	900	1400		
$110 < N \leq 160$	2000	1200	1800		
$160 < N \leq 210$	2000	1400	1900	.60	prime-gd
$210 < N \leq 260$	2000	1600	1900		
$260 < N \leq 310$	2500	1800	2000		
$310 < N \leq 360$	2500	2000	2000		

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Funding

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Ethics approval:

This article does not contain any studies with human participants or animals performed by any of the authors.

Availability of data and material:

All of the information is real-life data, which we gathered from both local and online sources. We also addressed an existing problem for comparison, which we cited and discussed in the article.

Code availability:

System: Windows 2010, CPU: CORE i5, RAM: 4 GB, Software: C++ (Code Block), MATLAB,.

Table A.3
For different algorithms number of wins.

Problem	us16	gr17	gr21	fri26	bays29	dantzig42	eil51	berlin52	st70	eil76	pr76	rat99
T2FLFWA	80	88	75	83	78	87	91	86	79	82	78	76
T1FLFWA	68	77	67	79	66	69	74	70	72	66	69	71
FWA	62	68	65	67	64	60	70	68	69	64	66	63
GA	58	66	59	62	61	52	68	57	67	56	61	59
ACO	56	63	58	60	58	50	66	52	63	54	60	58

Table A.4
Subtracted table from Table A.3.

Problem	us16	gr17	gr21	fri26	bays29	dantzig42	eil51	berlin52	st70	eil76	pr76	rat99	Mean
X_1	15	23	10	18	13	22	26	21	14	17	13	11	$\overline{X}_1 = 16.91$
X_2	3	12	2	14	1	4	9	5	7	1	4	6	$\overline{X}_2 = 5.66$
X_3	−3	3	0	2	−1	−5	5	3	4	−1	1	−2	$\overline{X}_3 = 0.50$
X_4	−7	1	−6	−3	−4	−13	3	−8	2	−9	−4	−6	$\overline{X}_4 = -4.75$
X_5	−9	−2	−7	−5	−7	−15	1	−13	−2	−11	−5	−7	$\overline{X}_5 = -6.83$

Table A.5
ANOVA summary table.

Source of variation	Sum of square	df	Mean of square	F
Between groups	$SS_B = 4294.40$	$J - 1 = 4$	$MS_B = \frac{SS_B}{J-1} = 1073.60$	
Within groups	$SS_W = 1071.25$	$J(I - 1) = 55$	$MS_W = \frac{SS_W}{J(I-1)} = 19.47$	$\frac{MS_B}{MS_W} = 55.12$
Total	$SS_T = 5365.66$	$IJ - 1 = 59$		

Informed consent:

Informed consent was obtained from all individual participants included in the study.

Consent for publication:

The manuscript has not been sent to any other journal for publication.

Appendix A. Test results for T2FLFWA

The performance of the proposed T2FLFWA is established by solving 20 standard benchmark problems from TSPLIB [36]. The results of T2FLFWA along with T1FLFWA (Type-1 fuzzy logic based fireworks algorithm), FWA and GA have been presented in Table A.1. Problems are compared in terms of percentage of error. All the results for all algorithms are the presented using 100 independent runs. In Table A.1, OS indicates the optimal solution. Among all the algorithms T2FLFWA shows efficiency with less percentage of error.

For different nodes of different TSP, the parameters for T2FLFWA are set in Table A.2. As the nodes number of the TSP increase, then there is an increase in the maximum generation number.

A.1. Supremacy of T2FLFWA through ANOVA

In this investigation, assumptions of ANOVA (normal population distribution, distributions having homogeneous variance and independent data) are satisfied. So, to determine the effectiveness of the proposed T2FLFWA, we use the statistical test ANOVA. The ANOVA test is applied to determine whether there are any statistically significant differences between the means of two or more independent groups. Here, for five algorithms T2FLFWA, T1FLFWA, FWA, GA, ACO with Maxgen = 2500 are used. For the statistical test, 12 standard test functions (benchmark problems) are considered and above five algorithms are tested against these functions. The number of wins (obtaining the correct results) out of 100 independent runs of the above algorithms against the said test functions are recorded (cf. Table A.3). Thus, for the comparison of a set of algorithms, to test the differences between more than two related sample means, the most common statistical test method, ANOVA is performed.

Different steps of ANOVA are calculated and without loss of generality, 65 is subtracted from each number in Table A.3 and thus, we obtain the Table A.4.

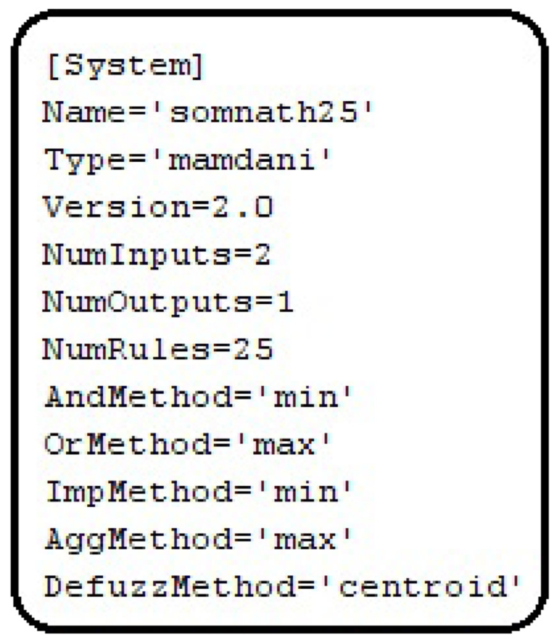
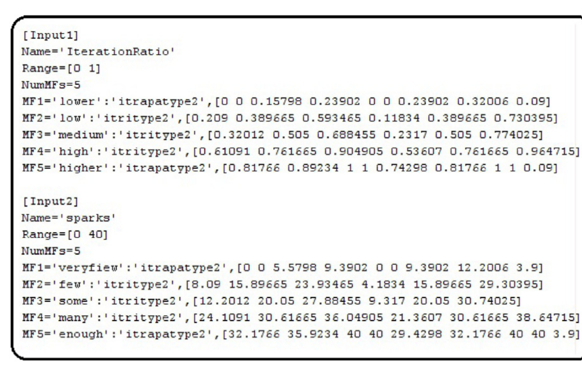

```
[System]
Name='somnath25'
Type='mamdani'
Version=2.0
NumInputs=2
NumOutputs=1
NumRules=25
AndMethod='min'
OrMethod='max'
ImpMethod='min'
AggMethod='max'
DefuzzMethod='centroid'
```

Fig. B.1. System used for Type-2 fuzzy logic.


```
[Input1]
Name='IterationRatio'
Range=[0 1]
NumMFs=5
MF1='lower':'itrapatype2',[0 0 0.15798 0.23902 0 0 0.23902 0.32006 0.09]
MF2='low':'itritype2',[0.209 0.389665 0.593465 0.11834 0.389665 0.730395]
MF3='medium':'itritype2',[0.32012 0.505 0.688455 0.2317 0.505 0.774025]
MF4='high':'itritype2',[0.61091 0.761665 0.904905 0.53607 0.761665 0.964715]
MF5='higher':'itrapatype2',[0.81766 0.89234 1 1 0.74298 0.81766 1 1 0.09]

[Input2]
Name='sparks'
Range=[0 40]
NumMFs=5
MF1='veryfiew':'itrapatype2',[0 0 5.5798 9.3902 0 0 9.3902 12.2006 3.9]
MF2='few':'itritype2',[8.09 15.89665 23.93465 4.1834 15.89665 29.30395]
MF3='some':'itritype2',[12.2012 20.05 27.88455 9.317 20.05 30.74025]
MF4='many':'itritype2',[24.1091 30.61665 36.04905 21.3607 30.61665 38.64715]
MF5='enough':'itrapatype2',[32.1766 35.9234 40 40 29.4298 32.1766 40 40 3.9]
```

Fig. B.2. Fuzzy linguistics values for rule 25 (input data).

```
[Output1]
Name='AmplitudeCoefficient'
Range=[0 45]
NumMFs=25
MF1='C1':'itristype2',[4.542 12.0043 0.0577875 0.0577875]
MF2='C2':'itristype2',[5.8512 8.3479 0.0620875 0.0620875]
MF3='C3':'itristype2',[14.0076 14.6347 0.0783874999999999 0.0783874999999999]
MF4='C4':'itristype2',[25.0045 25.6046 0.0750124999999999 0.0750124999999999]
MF5='C5':'itristype2',[25.8572 26.406 0.0686 0.0686]
MF6='C6':'itristype2',[12.0667 12.5683 0.0627 0.0627]
MF7='C7':'itristype2',[12.859 13.4628 0.075475 0.075475]
MF8='C8':'itristype2',[18.8439 25.4706 0.0783375000000001 0.0783375000000001]
MF9='C9':'itristype2',[25.3805 25.9429 0.0703 0.0703]
MF10='C10':'itristype2',[25.9247 26.4648 0.0675125000000001 0.0675125000000001]
MF11='C11':'itristype2',[13.7161 18.3554 0.0799125000000001 0.0799125000000001]
MF12='C12':'itristype2',[13.8443 20.4902 0.0807375 0.0807375]
MF13='C13':'itristype2',[25.3449 25.8986 0.0692125 0.0692125]
MF14='C14':'itristype2',[26.5792 31.0699 0.0613374999999999 0.0613374999999999]
MF15='C15':'itristype2',[26.6022 27.0998 0.0622 0.0622]
MF16='C16':'itristype2',[24.2213 24.8085 0.0734 0.0734]
MF17='C17':'itristype2',[24.4085 25.3922 0.1229625 0.1229625]
MF18='C18':'itristype2',[26.3829 26.889 0.0632625 0.0632625]
MF19='C19':'itristype2',[30.4373 30.8841 0.0558500000000001 0.0558500000000001]
MF20='C20':'itristype2',[31.6237 33.0556 0.0539875 0.0539875]
MF21='C21':'itristype2',[18.8291 25.4329 0.075475 0.075475]
MF22='C22':'itristype2',[25.1887 25.7557 0.070875 0.070875]
MF23='C23':'itristype2',[26.6488 27.1413 0.0615625000000001 0.0615625000000001]
MF24='C24':'itristype2',[37.6145 38.0523 0.0547250000000001 0.0547250000000001]
MF25='C25':'itristype2',[38.025 38.4443 0.0524125 0.0524125]
```

Fig. B.3. Fuzzy linguistics values for rule 25 (output data).

Here, total number of problems I = 12 and total number of algorithm is J = 5. Mean of the sample means, $\overline{\overline{X}} = 2.29$.

From the standard table, the Critical F-values, $F_{0.05(4,55)} \approx 2.54$. This value is much less than the calculated F (= 55.12 from Table A.5). Thus it is calculated that significant differences exist between the groups. Now, as F-ratio is observed to be significant in an ANOVA test with more than two groups, it is essential to find which group-means differ significantly from each other and for that, a multiple comparison test is performed. Here, for this purpose, we perform Scheffe's multiple comparison F- test to determine whether the groups (T2FLFWA and T1FLFWA) and/or (T2FLFWA and FWA) and/or (T2FLFWA and GA) and/or (T2FLFWA and ACO) are significant. Taking the first pair, we calculate F following $F = \frac{(\overline{X}_1-\overline{X}_2)^2}{MS_W(\frac{1}{I}+\frac{1}{J})} = 34.68$. Similarly, F = 91.23 for the second group (T2FLFWA and T1FLFWA), F = 112.97 for the third group (T2FLFWA and GA) and F = 140.32 for the fourth group (T2FLFWA and ACO). Here all the calculated values (34.68, 91.23, 112.97 and 140.32) of F are higher than the tabulated value of F (2.54) and hence significant differences for both the groups exist. But, the mean ($\overline{X}_1$) of X_1 is higher than the other means, $\overline{X}_2$, $\overline{X}_3$, $\overline{X}_4$ and $\overline{X}_5$ (cf. Table A.4). Therefore, it is concluded that T2FLFWA is better compared to the other two algorithms-T1FLFWA, FWA, GA and ACO.

Appendix B. Input data

See Figs. B.1–B.3.

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