DATE:

Practical -1 Calculate eigen vector and eigen values.

Q.1 Find all eigen values and corresponding eign vectors for the matrix $A = \begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix}$.

Solution –

$$A = \begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -6-\lambda & 3 \\ 4 & 5-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(-6-\lambda) - 12 = 0$$

$$-30 - 5\lambda + 6\lambda + \lambda^2 - 12 = 0$$

$$\lambda^2 + \lambda - 42 = 0$$

$$\lambda^2 + 7\lambda - 6\lambda - 42 = 0$$

$$\lambda(\lambda+7) - 6(\lambda+7) = 0$$

$$(\lambda+7)(\lambda-6) = 0$$

$\lambda + 7 = 0$ or $\lambda - 6 = 0$

$\lambda = -7$ or $\lambda = 6$

For, $\lambda = 6$

$A.V = \lambda.V$

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6x \\ 6y \end{bmatrix}$$

$$\begin{bmatrix} -6x + 3y \\ 4x + 5y \end{bmatrix} = \begin{bmatrix} 6x \\ 6y \end{bmatrix}$$

Now,

$-6x + 3y = 6x$ and $4x + 5y = 6y$

$12x - 3y = 0$ ....(i) and $4x - y = 0$ .....(ii)

Divide eq^n (i) by 3, we get

$4x - y = 0$ ......(iii)

From (ii) and (iii), we get

$4x - y = 0$

$\therefore 4x = y$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

for $\lambda = -7$

$A.V = \lambda.V$

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7x \\ -7y \end{bmatrix}$$

$$\begin{bmatrix} -6x + 3y \\ 4x + 5y \end{bmatrix} = \begin{bmatrix} -7x \\ -7y \end{bmatrix}$$

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Now,

$-6x + 3y = -7x$ and $4x + 5y = -7y$

$x + 3y = 0$ ..... (i) and $4x + 12y = 0$ ..... (ii)

Divide eq^n (ii) by 4, we get

$x + 3y = 0$ ..... (iii)

From (i) and (iii), we get

$x + 3y = 0$

$x = -3y$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

Q.2. Find eigen values and corresponding eigen vectors for the matrix $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$.

Solution — $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 4-\lambda & -1 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

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$(4-\lambda)(1-\lambda)+2=0$

$4-4\lambda-\lambda+\lambda^2+2=0$

$\lambda^2-5\lambda+6=0$

$\lambda^2-3\lambda-2\lambda+6=0$

$\lambda(\lambda-3)-2(\lambda-3)=0$

$(\lambda-2)(\lambda-3)=0$

$\lambda-2=0$ or $\lambda-3=0$

$\lambda=2$ or $\lambda=3$

For, $\lambda=2$

$A.V = \lambda.V$

$$\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

$$\begin{bmatrix} 4x-y \\ 2x+y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

Now,

$4x-y=2x$ and $2x+y=2y$

$2x-y=0$ .....(i) and $2x-y=0$ .....(ii)

From (i) and (ii), we get

$2x-y=0$

$2x=y$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

For, $\lambda=3$

$A.V. = \lambda.V$

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$$\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x \\ 3y \end{bmatrix}$$

$$\begin{bmatrix} 4x - y \\ 2x + y \end{bmatrix} = \begin{bmatrix} 3x \\ 3y \end{bmatrix}$$

Now,

$4x - y = 3x$ and $2x + y = 3y$

$x - y = 0$ ..... (i) and $2x - 2y = 0$ ..... (ii)

Divide eq^n (ii) by 2, we get

$x - y = 0$ ..... (iii)

From eq^n (i) and (iii), we get

$x - y = 0$

$x = y$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Q.3. Find all eigen values and corresponding eigen vectors for the matrix $A = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$

Solution — $A = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$

$|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 1-\lambda & 1 \\ -2 & -2-\lambda \end{vmatrix} = 0$$

$(1-\lambda)(-2-\lambda) + 2 = 0$

$-2-\lambda + 2\lambda + \lambda^2 + 2 = 0$

$\lambda^2 + \lambda = 0$

$\lambda(\lambda+1) = 0$

$\lambda = -1$ or $\lambda = 0$

For, $\lambda = -1$

$A.V = \lambda.V$

$$\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$$

$$\begin{bmatrix} x+y \\ -2x-2y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$$

Now,

$x + y = -x$ and $-2x - 2y = -y$

$2x + y = 0$ ....(i) and $2x + y = 0$ ....(ii)

From (i) and (ii), we get

$2x + y = 0$

$2x = -y$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

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For, $\lambda = 0$

$A.V = \lambda.V$

$$\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} x+y \\ -2x-2y \end{bmatrix} = 0$$

Now,

$x + y = 0$ .... (i) and $-2x - 2y = 0$ .... (ii)

Divide eq^n (ii) by -2, we get

$x + y = 0$ .... (iii)

From (i) and (iii), we get

$x + y = 0$

$x = -y$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Q.4 Find the eigen values and eigen vector for $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 9 \end{bmatrix}$

Solution —

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 9 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 9 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 4 & 9-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\begin{vmatrix} 3-\lambda & 4 \\ 4 & 9-\lambda \end{vmatrix} - 0\begin{vmatrix} 0 & 4 \\ 0 & 9-\lambda \end{vmatrix} + 0\begin{vmatrix} 0 & 3-\lambda \\ 0 & 4 \end{vmatrix} = 0$$

$(2-\lambda)\left[(3-\lambda)(9-\lambda)-16\right]=0$

$(2-\lambda)\left[27-3\lambda-9\lambda+\lambda^2-16\right]=0$

$(2-\lambda)\left[\lambda^2-12\lambda+11\right]=0$

$2\lambda^2-24\lambda+22-\lambda^3+12\lambda^2-11\lambda=0$

$-\lambda^3+14\lambda^2-35\lambda+22=0$

$-(\lambda-1)(\lambda-2)(\lambda-11)=0$

$\lambda=1$ or $\lambda=2$ or $\lambda=11$

For, $\lambda=1$

$A.V = \lambda.V$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 2x+0y+0z \\ 0x+3y+4z \\ 0x+4y+9z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Now,

$2x=x$ and $3y+4z=y$ and $4y+9z=z$

$x=0$ ....(i) and $2y+4z=0$ ....(ii) and $4y+8z=0$ ....(iii)

Divide eqn (ii) by 2 and (iii) by 4, we get

$y+2z=0$ ....(iv) and $y+2z=0$ ......(v)

From (iv) and (v), we get

$y + 2z = 0$

$y = -2z$

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \quad \text{Also, } x = 0 \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

For, $\lambda = 2$

$A.V = \lambda.V$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix}$$

$$\begin{bmatrix} 2x + 0y + 0z \\ 0x + 3y + 4z \\ 0x + 4y + 9z \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix}$$

Now,

$2x = 2x$ and $3y + 4z = 2y$ and $4y + 9z = 2z$

$2x - 2x = 0$ and $y + 4z = 0$ and $4y - 7z = 0$

$x = 1$ $\quad y = 0$ $\quad z = 0$

For, $\lambda = 11$

$A.V = \lambda.V$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11x \\ 11y \\ 11z \end{bmatrix}$$

$$\begin{bmatrix} 2x + 0y + 0z \\ 0x + 3y + 4z \\ 0x + 4y + 9z \end{bmatrix} = \begin{bmatrix} 11x \\ 11y \\ 11z \end{bmatrix}$$

Now,

$2x = 11x$ and $3y + 4z = 11y$ and $4y + 9z = 11z$

$9x = 0$ and $8y - 4z = 0$ and $4y - 2z = 0$

$x = 0 \quad \ldots (I)$ $\quad 8y - 4z = 0 \quad \ldots (II)$ $\quad 4y - 2z = 0 \quad \ldots (III)$

Divide eq^n (II) by 2, we get

$4y - 2z = 0 \quad \ldots (IV)$

From eq^n (III) and (IV), we get

$4y - 2z = 0$

$4y = 2z$

$2y = z$

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$ Also, $x = 0$ $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

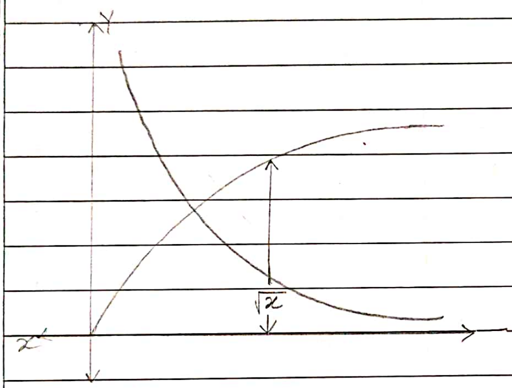
Practical-2 Graphical Representation of Functions

Q.1

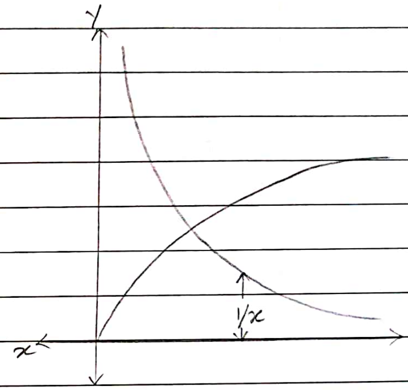
A] Arithmetic operation -

i) $y = \sqrt{x}$ and $y = 1/x$

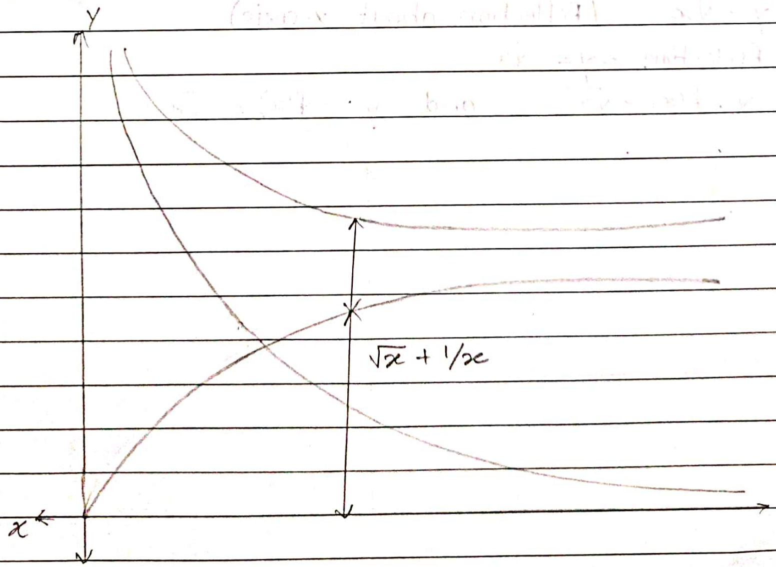
i) $y = \sqrt{x}$



ii) $y = 1/x$



Graph of $\sqrt{x} + 1/x$

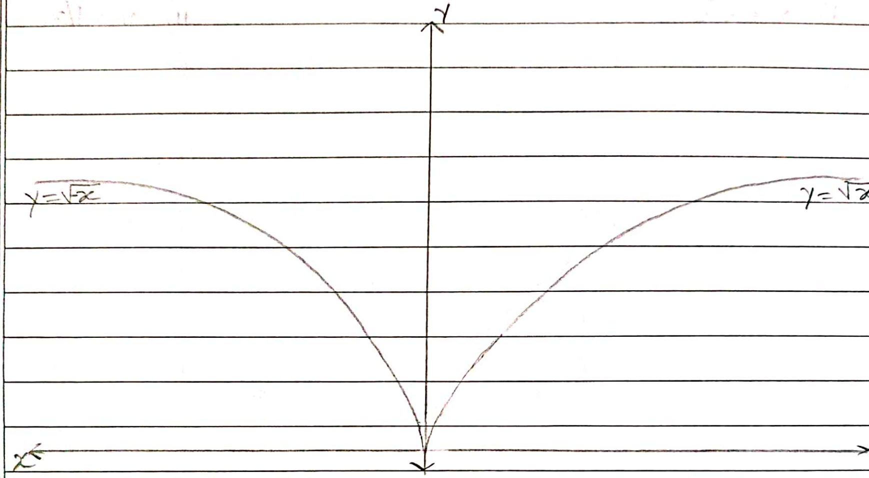


B] Reflection -

i $y = \sqrt{x}$ (Reflection about y-axis)

Reflection $= y = \sqrt{-x}$

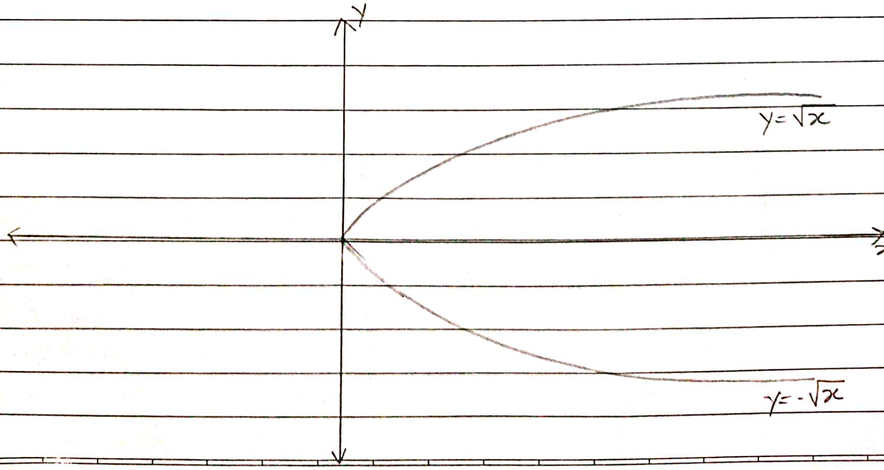
$y = f(x) = \sqrt{x}$ and $y = f(-x) = \sqrt{-x}$



ii $y = \sqrt{x}$ (Reflection about x-axis)

Reflection $= y = -\sqrt{x}$

$y = f(x) = \sqrt{x}$ and $y = -f(x) = -\sqrt{x}$



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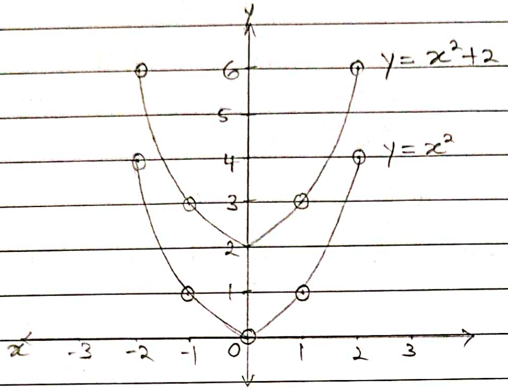
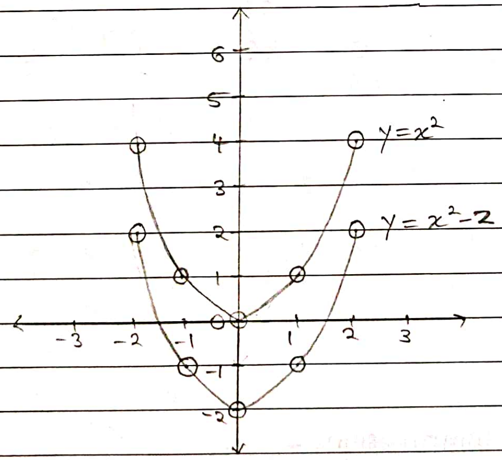
c] Translation - $f(x) = x^2$ and $c=2$.

1. $y = f(x) + c$



2. $y = f(x) - c$.



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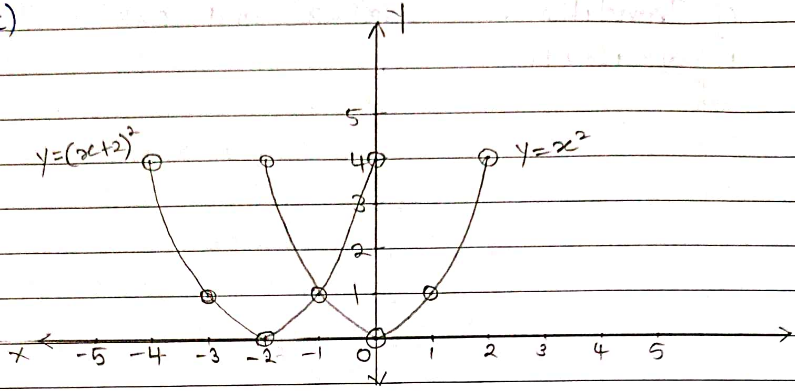
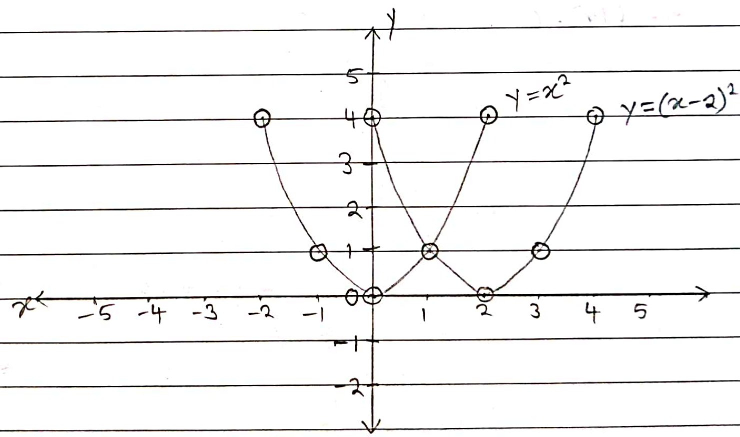
3. $y = f(x+c)$



4. $y = f(x-c)$



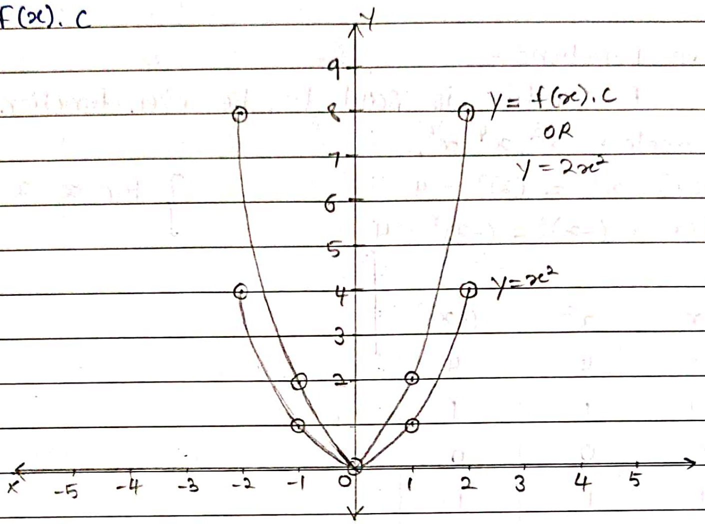
D] Scretches and compressions -

1. $y = f(x).c$
(Multiplying on output variable)

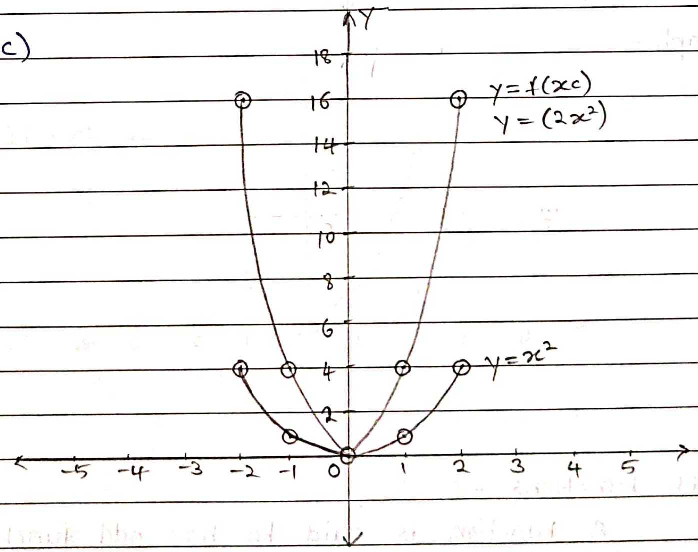
2. $y = f(x.c)$
(Multiplying on input variable)

let, $y = x^2$ and $c = 2$

1. $y = f(x).c$



2. $y = f(x.c)$



Practical - 3 Even and Odd functions.

1. Even functions -

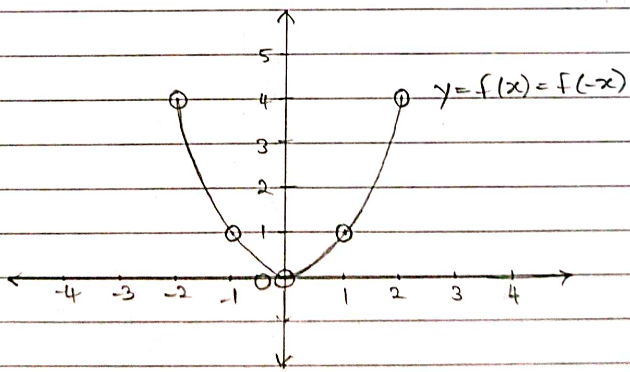
A function is said to be even function if $f(-x) = f(x)$

Example - $x^2, x^4, x^6, \ldots$

$f(x) = x^2 = (2)^2 = 4$
$f(-x) = (-x)^2 = (-2)^2 = 4$ } for $x = 2$

x	x^2	$(-x)^2$
-2	4	4
-1	1	1
0	0	0
1	1	1
2	4	4

Graph -



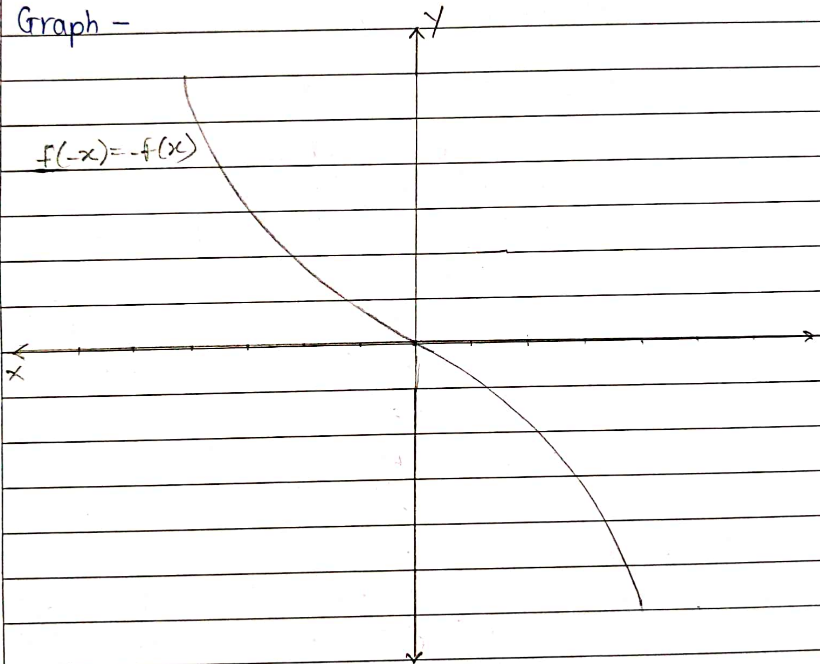
2. Odd Functions -

A function is said to be odd functions if $f(-x) = -f(x)$

$f(-x) = (-x^3) = (-2)^3 = -8$
$-f(x) = -(x)^3 = -(2)^3 = -8$ } for $x = 2$

x	$(-x)^3$	$-(x^3)$
-2	8	8
-1	1	1
0	0	0
1	-1	-1
2	-8	-8

Graph -



$$\left.\begin{aligned} -f(x) &= -(2^3) = -8 \\ f(-x) &= (-2)^3 = -8 \end{aligned}\right\} \text{ for } x = 2$$

Practical-4 Find Limits.

Q.1 $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2}$

Solution –

$$\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2} \cdot \frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3}$$

$$= \lim_{x\to 0} \frac{(\sqrt{x^2+9})^2-(3)^2}{x^2(\sqrt{x^2+9}+3)}$$

$$= \lim_{x\to 0} \frac{x^2+9-9}{x^2(\sqrt{x^2+9}+3)}$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}}{\cancel{x^2}(\sqrt{x^2+9}+3)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x^2+9}+3}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3}$$

$$= \frac{1}{6}$$

Q.2 $\lim_{x\to 2} \frac{5x^3+4}{x-3}$

Solution –

$$\lim_{x\to 2} \frac{5x^3+4}{x-3} = \frac{5\lim_{x\to 2} x^3 + \lim_{x\to 2} 4}{\lim_{x\to 2} x - \lim_{x\to 2} 3}$$

$$= \frac{5(2)^3+4}{2-3}$$

$$= \frac{(5\times 8)+4}{-1}$$

$$= \frac{40+4}{-1} = -44$$

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Q.3 $\lim_{x \to 1^-} \frac{x^2+2x-3}{|x-1|}$

Solution –

$x < 1$

$x - 1 < 0$

If $x-1 < 0$, then, $|x-1| = -(x-1)$

$$\therefore \lim_{x \to 1^-} \frac{x^2+2x-3}{|x-1|} = \lim_{x \to 1^-} \frac{x^2+2x-3}{-(x-1)}$$

$$= \lim_{x \to 1^-} \frac{x^2+3x-x-3}{-(x-1)}$$

$$= \lim_{x \to 1^-} \frac{x(x+3)-1(x+3)}{-(x-1)}$$

$$= \lim_{x \to 1^-} \frac{\cancel{(x-1)}(x+3)}{-\cancel{(x-1)}}$$

$$= \lim_{x \to 1^-} -(x+3)$$

$$= \lim_{x \to 1^-} -(x+3)$$

$$= -(1+3)$$

$$= -4$$

Q.4 $\lim_{x \to 2} \frac{x^2+4x-12}{|x-2|}$

Solution – $x < 2$

$x - 2 < 0$

If $x-2 < 0$, then, $|x-2| = -(x-2)$

$$\lim_{x \to 2^-} \frac{x^2+4x-12}{|x-2|} = \lim_{x \to 2^-} \frac{x^2+6x-2x-12}{-(x-2)}$$

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$$= \lim_{x \to 2^-} \frac{x(x+6) - 2(x+6)}{-(x-2)}$$

$$= \lim_{x \to 2^-} \frac{\cancel{(x-2)}(x+6)}{-\cancel{(x-2)}}$$

$$= \lim_{x \to 2^-} -(x+6)$$

$$= -(2+6)$$

$$= -8$$

$x > 2$

$x - 2 > 0$

If $x-2 > 0$, then, $|x-2| = (x-2)$

$$\lim_{x \to 2^+} \frac{x^2+4x-12}{|x-2|} = \lim_{x \to 2^+} \frac{x^2+4x-12}{(x-2)}$$

$$= \lim_{x \to 2^+} \frac{x^2+6x-2x-12}{(x-2)}$$

$$= \lim_{x \to 2^+} \frac{x(x+6) - 2(x+6)}{(x-2)}$$

$$= \lim_{x \to 2^+} \frac{\cancel{(x-2)}(x+6)}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2^+} (x+6)$$

$$= 2+6$$

$$= 8$$

The limit from the right of 2 and the limit from the left of 2 are not equal.

$\therefore$ The given limit does not exists.

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Practical-5 Prove Contuinity.

Q.1 Determine whether the function $f(x) = 2x+3$ is continous at $x=-4$.

Solution —

Condition 1 :

$f(c) = f(-4) = 2(-4)+3 = -8+3 = -5$

Condition 1 Satisfied.

Condition 2 :

$$\lim_{x\to -4} 2x+3 = 2\lim_{x\to -4} x + \lim_{x\to -4} 3$$

$$= 2(-4)+3$$

$$= -5$$

Condition 2 Satisfied.

Condition 3 :

$$\lim_{x\to -4} 2x+3 = f(-4)$$

Condition 3 satisfied.

Hence, the given function is continous at $x=-4$.

Q.2 Determine whether the given function $f(x) = \dfrac{x^2-4}{x-2}$ is continous at $x=2$.

Solution —

$$f(x) = \frac{x^2-4}{x-2}$$

Condition 1 :

$$f(c) = f(2) = \frac{(2)^2-4}{2-2} = \frac{0}{0} = \text{undefined}$$

hence, the given function is not continous.

Q.3 Determine whether the given function

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & ; x \neq 2 \\ 4 & ; x = 2 \end{cases}$$

is continous at $x = 2$

Solution —

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & ; x \neq 2 \\ 4 & , x = 2 \end{cases}$$

Condition 1 :

$f(2) = 4$

Condition 1 Satisfied.

Condition 2:

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2} (x+2)$$

$$= 4$$

Condition 2 Satisfied.

Condition 3 :

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = f(2)$$

Condition 3 Satisfied.

hence, the given function is continous at $x=2$.

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Q.4. Discuss the continuity of

$$f(x) = \begin{cases} 5-2x & , x<-3 \\ x^2+2 & , x \geq -3 \end{cases} \quad \text{at } x=-3$$

Solution—

Condition 1 :

$f(-3) = (-3)^2+2 = 9+2 = 11$

Condition 1 satisfied.

Condition 2:

$$\lim_{x\to -3} (x^2+2) = \lim_{x\to -3} x^2 + \lim_{x\to -3} 2.$$

$$= (-3)^2 + 2$$

$$= 11$$

Condition 2 satisfied.

Condition 3:

$$\lim_{x\to -3} x^2+2 = f(-3)$$

Condition 3 satisfied.

hence, the given function is satisfied at $x=-3$.

DATE:

Practical - 6 Derive Product rule and Quotient rule.

Q.1 Derivation of product rule

Solution – $\frac{d}{dx}\left[f(x) \cdot g(x)\right] = f(x)\, g'(x) + g(x)\, f'(x)$

w.k.t

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)\, g(x+h) - f(x)\, g(x)}{h}$$

Adding and subtracting $f(x+h)\, g(x)$ on numerator, we get

$$= \lim_{h \to 0} \frac{f(x+h)\, g(x+h) - f(x+h)\, g(x) + f(x+h)\, g(x) - f(x)\, g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)\left[g(x+h) - g(x)\right] + g(x)\left[f(x+h) - f(x)\right]}{h}$$

$$= f(x) \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} + g(x) \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$$

$$= f(x)\, g'(x) + g(x)\, f'(x)$$

hence, proved.

Q. 2. Derivation of Quotient rule.

Solution – $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\, f'(x) + f(x)\, g'(x)}{[g(x)]^2}$

w. k. t

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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$$= \frac{\lim_{h\to 0} \frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$= \frac{\lim_{h\to 0} \frac{f(x+h)\,g(x) - g(x+h)\,f(x)}{g(x+h)\,g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x) - g(x+h)f(x)}{hg(x)\,g(x+h)}$$

Adding and subtracting $f(x)\,g(x)$ on numerator, we get

$$= \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - g(x+h)f(x)}{hg(x)\,g(x+h)}$$

$$= \lim_{h\to 0} \frac{g(x)\,[f(x+h) - f(x)] - f(x)[g(x+h) - g(x)]}{h\,g(x)\,g(x+h)}$$

$$= \frac{g(x) \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} - f(x) \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}}{g(x) \lim_{h\to 0} g(x+h)}$$

$$= \frac{g(x)\,.\frac{d}{dx}\,f(x) - f(x)\,\frac{d}{dx}\,g(x)}{g(x)\,.\,g(x)}$$

$$= \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

hence, proved.

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Practical-7 Implicit differentiation

Q.1 Use implicit differentiation to find dy/dx for the following equation.

$$(x-y)^2 = (x+y-1)$$

Solution —

Apply differentiation w.r.t x on both sides, we get

$$\frac{d}{dx}(x-y)^2 = \frac{d}{dx}(x+y-1)$$

$$2(x-y)\left(1-\frac{dy}{dx}\right) = 1+\frac{dy}{dx}$$

$$(2x-2y)\left(1-\frac{dy}{dx}\right) = 1+\frac{dy}{dx}$$

$$(2x-2y)+(2y-2x)\frac{dy}{dx} = 1+\frac{dy}{dx}$$

$$(2y-2x-1)\frac{dy}{dx} = (2y-2x+1)$$

$$\frac{dy}{dx} = \frac{2y-2x+1}{2y-2x-1}$$

Q.2 Use implicit differentiation to find dy/dx of $4x^2-2y^2=9$.

Solution —

$$4x^2-2y^2=9$$

$$4\frac{d}{dx}x^2 - 2\frac{d}{dx}y^2 = \frac{d}{dx}9.$$

$$8x-4y\frac{dy}{dx}=0$$

$$4y\frac{dy}{dx}=8x$$

$$\frac{dy}{dx}=\frac{8x}{4y}$$

$$\frac{dy}{dx} = \frac{2x}{y}$$

Q.3 Use implicit differentiation to find dy/dx of $x^2+xy+y^2=1$

Solution –

$x^2 + xy + y^2 = 1$

$$\frac{d}{dx}(x^2+xy+y^2) = \frac{d}{dx}1.$$

$$\frac{d}{dx}x^2 + \frac{d}{dx}xy + \frac{d}{dx}y^2 = 0$$

$$2x + \left[x\frac{dy}{dx} + y\frac{d}{dx}x\right] + 2y\frac{dy}{dx} = 0$$

$$2x + \left[x\frac{dy}{dx} + y\right] + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}[x+2y] = -2x-y$$

$$\therefore \frac{dy}{dx} = \frac{-(2x+y)}{(x+2y)}$$

Q.4 Use implicit differentiation to find dy/dx of

$$\frac{x-y^3}{y+x^2} = x+2$$

Solution –

$$\frac{x-y^3}{y+x^2} = x+2$$

$$(x-y^3) = (x+2)(y+x^2)$$

$$(x-y^3) = xy + x^3 + 2y + 2x^2$$

$$\frac{d}{dx}(x-y^3) = \frac{d}{dx}(xy+x^3+2y+2x^2)$$

$$\frac{d}{dx}x - \frac{d}{dx}y^3 = \frac{d}{dx}xy + \frac{d}{dx}x^3 + 2\frac{dy}{dx} + 2\frac{dx^2}{dx}$$

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$$1 - 3y^2 \frac{dy}{dx} = \left[x \frac{d}{dx} y + y(1) \right] + 3x^2 + 2\frac{dy}{dx} + 4x$$

$$1 - 3y^2 \frac{dy}{dx} = x \frac{dy}{dx} + y + 3x^2 + 2\frac{dy}{dx} + 4x$$

$$1 - y - 3x^2 - 4x = \frac{dy}{dx}(x + 3y^2 + 2)$$

$$\therefore \frac{dy}{dx} = \frac{(1 - y - 3x^2 - 4x)}{(x + 3y^2 + 2)}$$

Practical-8 Integration by Substitution

Q.1 Solve $\int (x+1)^3 \, dx$ using substitution.

Solution –

$\int (x+1)^3 \, dx$

let, $(x+1) = u$

$\int (x+1)^3 . 1 \, dx = u^3 \, du$

$= \frac{u^{3+1}}{3+1} + C$

$= \frac{u^4}{4} + C$

Now, put $u = x+1$

$\int (x+1)^3 \, dx = \frac{(x+1)^4}{4} + C$

Q.2 Solve $\int (5x+2)^7 \, dx$ using substitution

Solution –

$\int (5x+2)^7 \, dx$

let $u = 5x+2$

Multiply and divide by 5, we get

$= \frac{1}{5} \int (5x+2)^7 . 5 \, dx$

$= \frac{1}{5} \int u^7 . 5 \, dx = \frac{1}{5} \int u^7 . du$ ...let, $5dx = du$

$= \frac{1}{5} . \frac{u^8}{8} + C = \frac{u^8}{40} + C$ ...Now put $u = 5x+2$

$\int (5x+2)^7 \, dx = \frac{(5x+2)^8}{40} + C$