例6.4

而 $\hat{\theta}_1 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / n$

依 $E(x_i) = \mu$; $V(x_i) = \sigma^2 = E(x_i^2) - \mu^2$

則 $E(\bar{x}) = \mu$, $V(\bar{x}) = \frac{\sigma^2}{n} = E(\bar{x}^2) - \mu^2$

$$E(\hat{\theta}_1) = E\left(\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}\right) = \frac{1}{n}E\left(\sum_{i=1}^{n} x_i^2 - n x^2\right)$$

$$= \frac{1}{n}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \frac{n-1}{n}\sigma^2$$

$$E(\hat{\theta}_2) = E\left(\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}\right) = \frac{1}{n-1}E\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n-1}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$$

因此，$\hat{\theta}_2 = \sum_{i=1}^{n}(x_i - \bar{x})^2 / (n-1)$ 為母變異數 σ^2 之不偏估計量，