# Model Description (1 of 2)

- At any time t, Susceptible [S(t)] are the people who can get the infection. The infection spreads by contact. $\beta$ is the rate of contact with an infected person including probability of transmission.
- If the infected person is Asymptomatic [$I_A$(t)]or Quarantined [Q(t)], they may either have reduced infectivity ($\delta_1$) or a lower probability of coming in contact ($\delta_2$).
- After transmission, an individual moves from S(t) to Exposed [E(t)], or the incubation period of the disease. The probability of showing symptoms is $P_S$. The latency rate is defined as the rate of moving from E(t) to Infected [I(t)], $\kappa = (T_{incubation})^{-1}$. For e.g. if the average incubation period is 5 days, $\kappa = 0.2$
- Following incubation, an individual may be infected with symptoms [$I_S$(t)] or infected without symptoms [$I_A$(t)]
- An individual in $I_S$(t) may either be tested and quarantined [Q(t)], recover [R(t)], or die from infection [D(t)]. The rate of testing $\theta$, is inverse of the average time to testing after showing symptoms. $\theta = \left(T_{infect\ to\ test}\right)^{-1}$. For e.g. if it takes average 3 days to test and isolate a person after showing symptoms, $\theta = 0.33$
- The rate of recovery, $\gamma_1$, is inverse of the avg time to recover after showing symptoms. $\gamma_1 = \left(T_{infect\ to\ recover}\right)^{-1}$ For e.g. if it takes average 20 days to recover after showing symptoms, $\theta = 0.05$
- The mortality rate, $\mu_1$, is the fraction of infected who die per day, $\mu_1 = probability\ of\ death * \left(T_{infect\ to\ death}\right)^{-1}$ For e.g. if on average 12.5% of infected die and it takes 25 days to die after showing symptoms, $\mu_1 = \frac{0.125}{25} = 0.005$
- The respective recovery rates ($\gamma_i$) and mortality rates ($\mu_i$) from each compartment are similarly defined, and depend on the time to recovery or death from the stage of the disease.

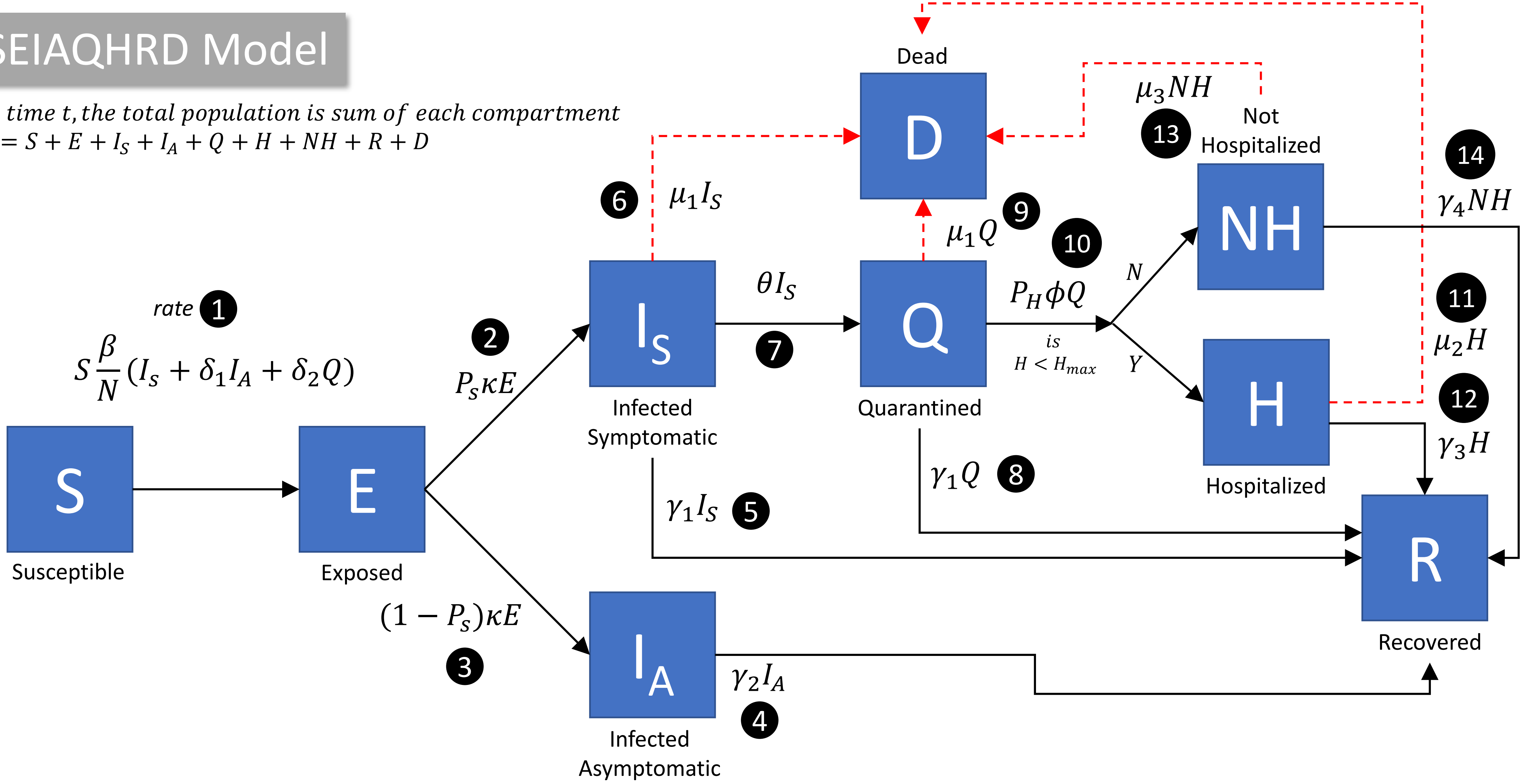
# Model Description (2 of 2)

- Assumed that an individual without symptoms in $I_A(t)$ may only recover [R(t)], i.e. will not be detected without indiscriminate or random testing nor die without first exhibiting symptoms. It is also possible that $I_A$ may have higher rate of recovery, $\gamma_2 \leq \gamma_1$
- Individuals who are quarantined [Q(t)], may either recover [R(t)] on their own, need hospitalization [H(t)], or die from infection [D(t)]. The rate of recovery and mortality are assumed same as $I_S$, $\gamma_1$ and $\mu_1$
- The probability of high risk or severe cases requiring Hospitalization is assumed to be $P_H$ , and the rate of hospitalization individual $\phi$ , is inverse of the average time to hospitalization after testing. $\phi = \left(T_{test\ to\ hosp}\right)^{-1}$ . For e.g. if it takes average 4 days to disease to turn serious, $\phi = 0.25$
- Hospital beds may be limited ($H_{max}$). If beds are available at time t, then individual will be hospitalized. The rate of recovery and mortality are assumed, $\gamma_3$ and $\mu_2$. For simplicity, they may also be assumed the same as $I_S$ and Q.
- If beds are NOT available at time t, the individual will be Not Hospitalized [NH(t)]. The rate of recovery and mortality are assumed, $\gamma_4$ and $\mu_3$. It is likely that the recovery rate will be lower and mortality rate higher for this compartment than others.
- All recovered individuals from $I_S$, $I_A$, Q, H, NH go to R(t) compartment.
- All dead individuals from $I_S$, $I_A$, Q, H, NH go to D(t) compartment.

# SEIAQHRD Model

*At time t, the total population is sum of each compartment*

$N = S + E + I_S + I_A + Q + H + NH + R + D$




$\beta$ = *contact rate*
$\delta_i$ = *reduced infectivity*
$P_S$ = *probablity of symptoms*

$\kappa$ = *latency rate* = $(T_{incubation})^{-1}$
$\mu_i$ = *mortality rate*
$\gamma_i$ = *recovery rate*

$\theta$ = *testing rate* = $(T_{infect\ to\ test})^{-1}$
$P_H$ = *probability of hospitalization*
$\phi$ = *hospitalization rate* = $(T_{test\ to\ hosp})^{-1}$

# System of Equations

1. $\frac{dS}{dt} = -S\frac{\beta}{N}(I_s + \delta_1 I_A + \delta_2 Q)$
2. $\frac{dE}{dt} = S\frac{\beta}{N}(I_s + \delta_1 I_A + \delta_2 Q) - \kappa E$
3. $\frac{dI_S}{dt} = P_S\kappa E - \theta I_S - \gamma_1 I_S - \mu_1 I_S$
4. $\frac{dI_A}{dt} = (1 - P_S)\kappa E - \gamma_2 I_A$
5. $\frac{dQ}{dt} = \theta I_S - P_H\phi Q - \gamma_1 Q - \mu_1 Q$
6. $if\ H \leq H_{max}$: $\frac{dH}{dt} = P_H\phi Q - \gamma_3 H - \mu_2 H$
7. $if\ H > H_{max}$: $\frac{dNH}{dt} = P_H\phi Q - \gamma_4 NH - \mu_3 NH$
8. $\frac{dR}{dt} = \gamma_1 I_S + \gamma_2 I_A + \gamma_1 Q + \gamma_3 H + \gamma_4 NH$
9. $\frac{dD}{dt} = \mu_1 I_S + \mu_1 Q + \mu_2 H + \mu_3 NH$