Stanford University

CME 253A

Introduction to High Performance Computing and Parallel (GPU) Computing

Stanford Summer Session 4

3 July 2019 | Y2E2 111

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SIGMA

Session 4 - Converging Stokes flow

Today's agenda

- Lecture: Accelerating Stokes flow convergence

- Programming: 1/ Damping of the residuals
  2/ Tracking min and max

- Tasks: 1/ Viscous stokes: add convergence check to your code
  2/ Elastic (acoustic) wave: add P and S wave recording in the domain

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Accelerating Stokes flow convergence

Elastic waves	Viscous flow
$\frac{1}{k}\frac{\partial P}{\partial t} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right)$	$\frac{1}{k}\frac{\partial P}{\partial t} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right)$
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Pseudo-Transient iterations

A general approach

- e.g. solution to an elliptic problem:

$$C = \frac{\partial^2 A}{\partial x^2}$$

$$0 = \frac{\partial^2 A}{\partial x^2} - C = f_{\mathrm{A}}$$

Stokes residuals

$$f_{\mathrm{v}} = \nabla_j(\bar{\tau}_{ij} - \bar{p}\delta_{ij}) - \bar{\rho} g_i$$

$$f_{\bar{\mathrm{p}}} = \nabla_k v_k$$

- Naive iterations (1st order):

$$\cancelto{0}{\frac{\partial A}{\partial \tau_{\mathrm{A}}}} = f_{\mathrm{A}}$$

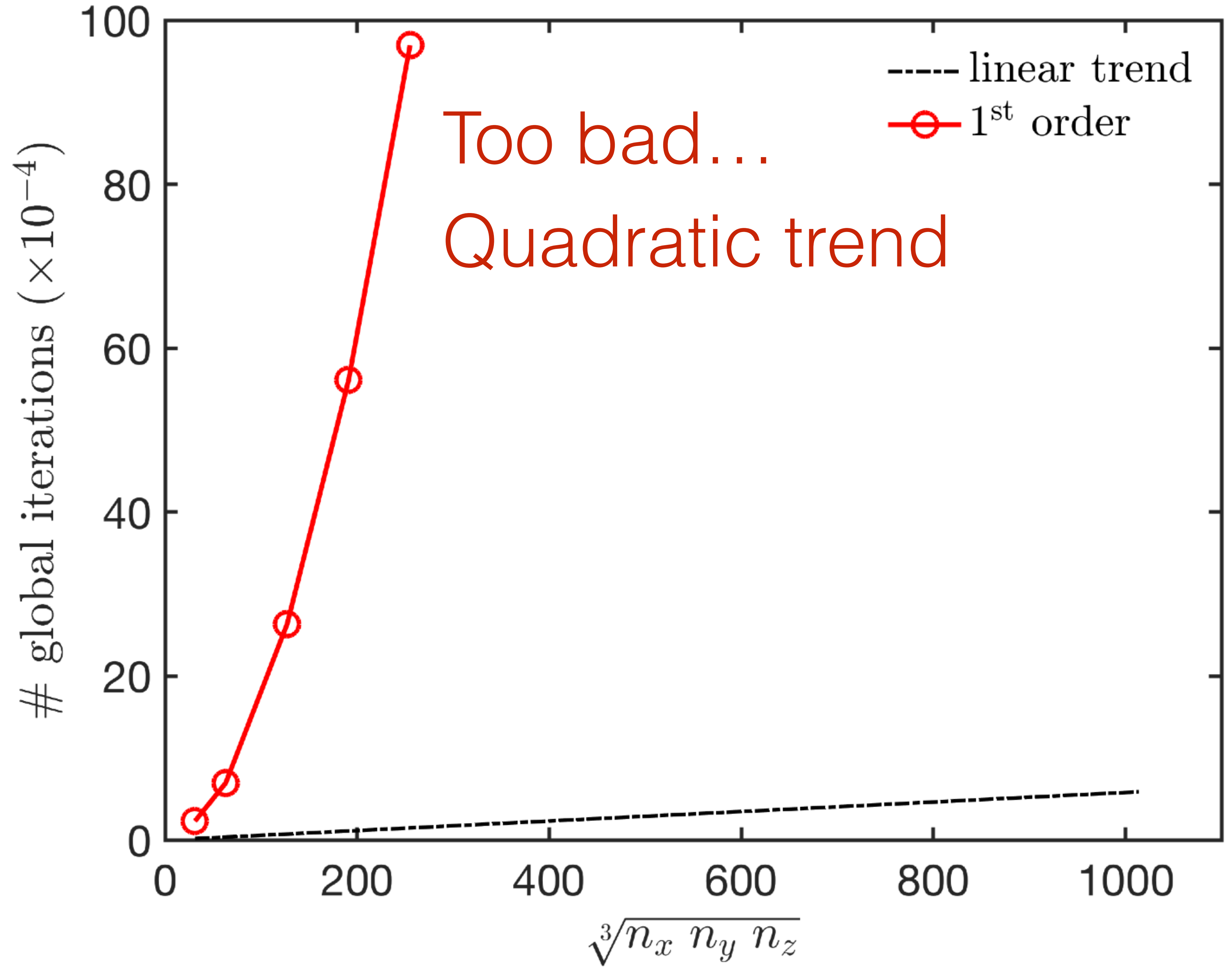
$$A^{[k+1]} = A^{[k]} + \Delta\tau_{\mathrm{A}} \cancelto{0}{f_{\mathrm{A}}^{[k]}}$$

(pseudo) time step

Pseudo-Transient | naive

- 1^{st} order scheme | 3-D hydro-mechanics

Simple - but does not scale with resolution increase



Pseudo-Transient iterations

An improved approach

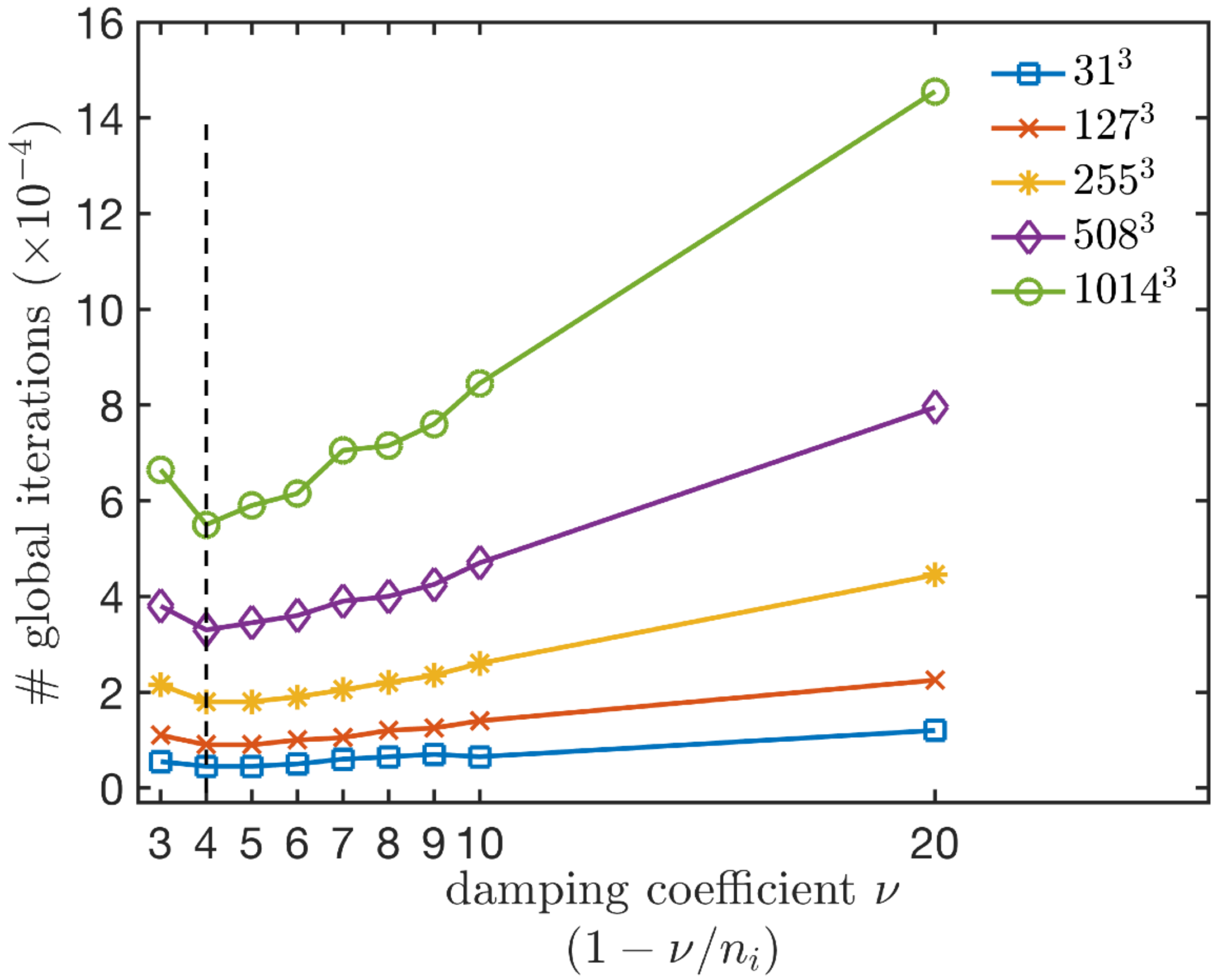
- Second order iterations: $$\alpha \cancelto{0}{\frac{\partial^2 A}{\partial \tau_{\mathrm{A}}^2}} + \cancelto{0}{\frac{\partial A}{\partial \tau_{\mathrm{A}}}} = f_{\mathrm{A}}$$

Frankel, 1950

$$A^{[k+1]} = A^{[k]} + \Delta\tau_{\mathrm{A}} \cancelto{0}{g_{\mathrm{A}}^{[k]}}$$

$$g_{\mathrm{A}}^{[k]} = \cancelto{0}{f_{\mathrm{A}}^{[k]}} + (1 - \nu/n_i) g_{\mathrm{A}}^{[k-1]}$$

damping



free parameter ν is resolution independent

Räss et al., 2019. Geophys. Journal.Int.

Pseudo-Transient iterations

An improved approach

- Second order iterations:

  Frankel, 1950

$$\alpha \cancelto{0}{\frac{\partial^2 A}{\partial \tau_A^2}} + \cancelto{0}{\frac{\partial A}{\partial \tau_A}} = f_A$$

$$A^{[k+1]} = A^{[k]} + \Delta\tau_A\, \cancelto{0}{g_A^{[k]}}$$

$$g_A^{[k]} = \cancelto{0}{f_A^{[k]}} + (1 - \nu / n_i)\, g_A^{[k-1]}$$

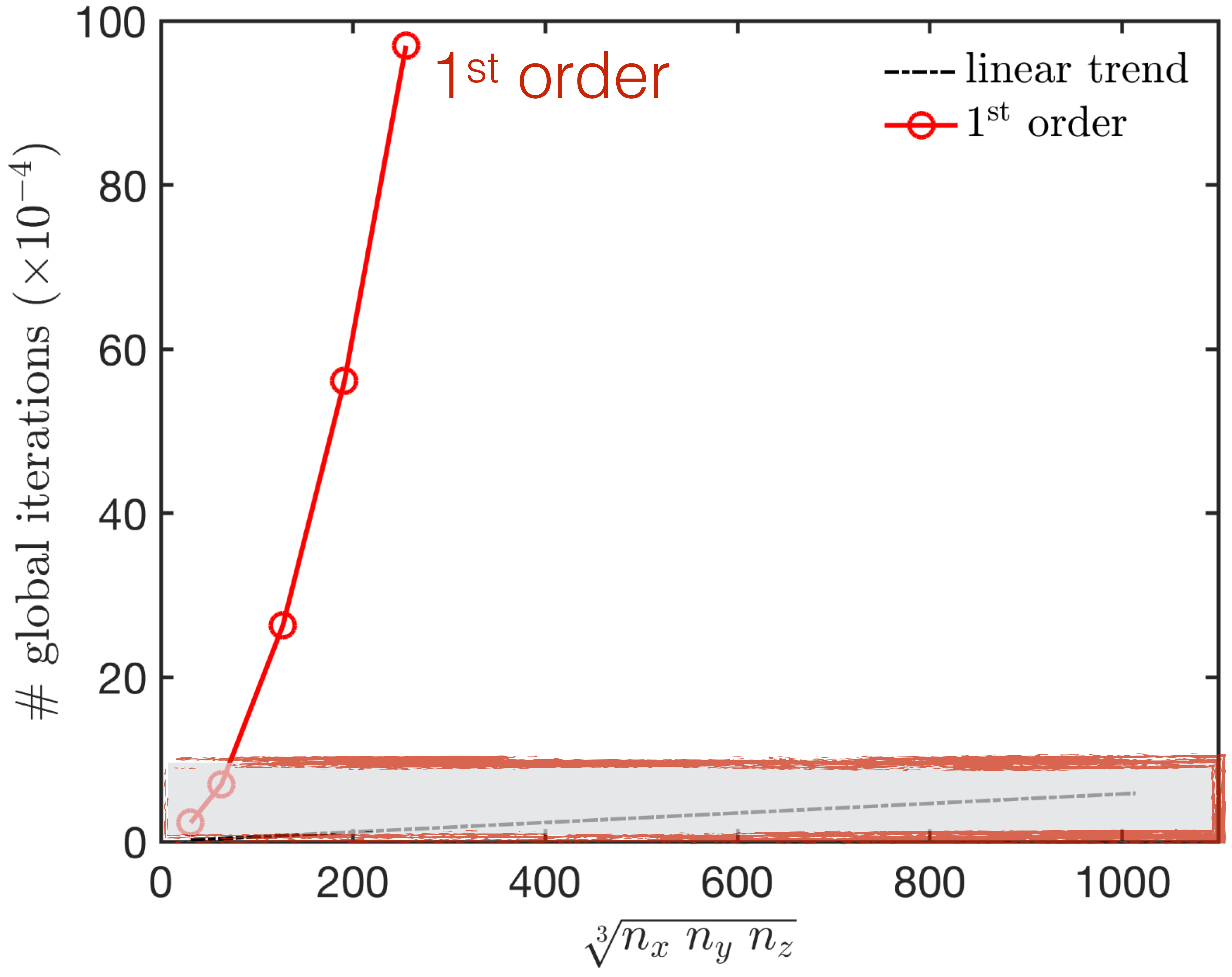
damping

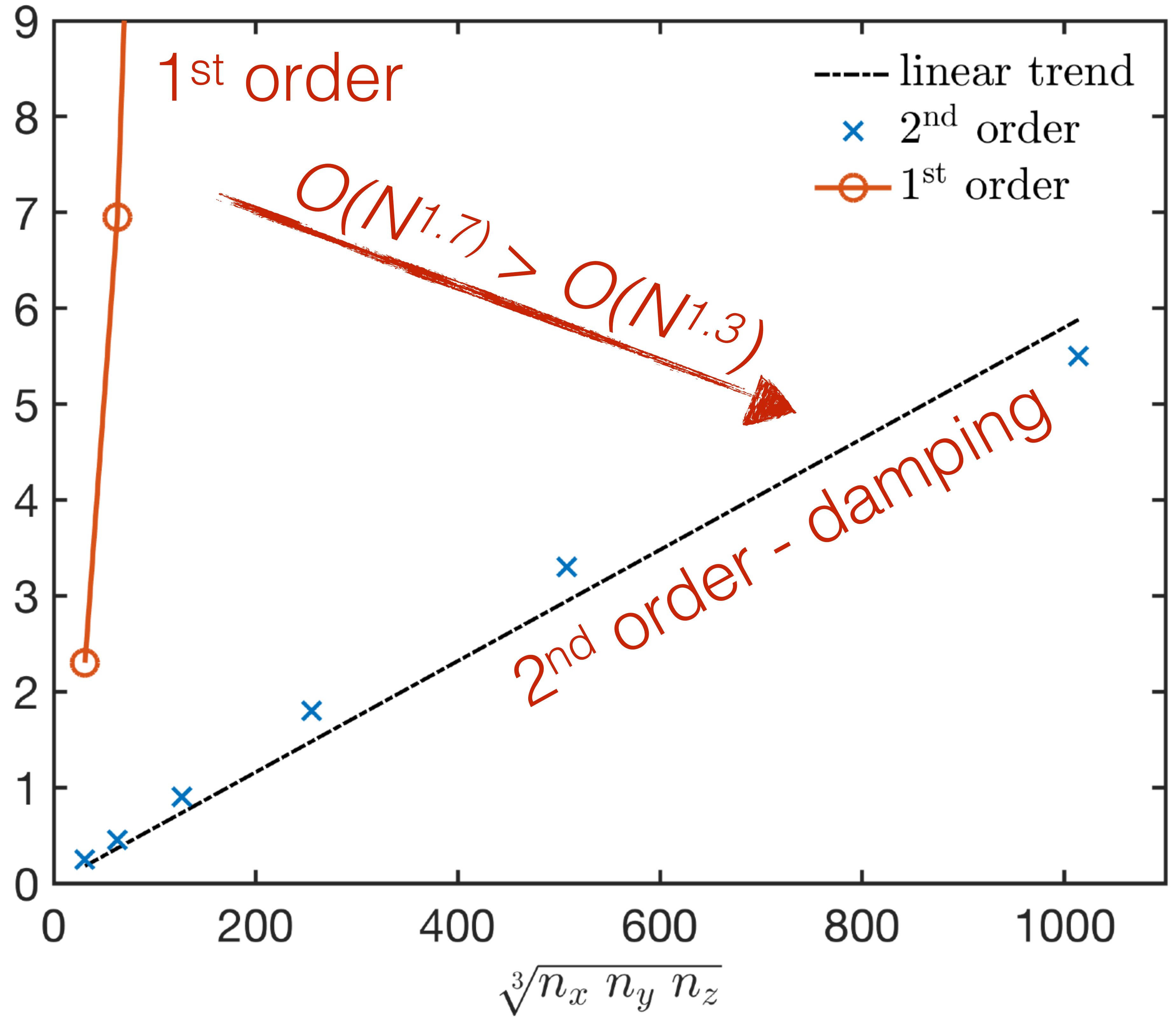
- No global reduction: $\Delta\tau_A$ is local (diagonal preconditionner)
- Only local communication: like propagation of physical information
- Same workflow on each grid point: perfect for GPUs !

Pseudo-Transient | improved

- 1st versus 2nd order scheme | Stokes flow

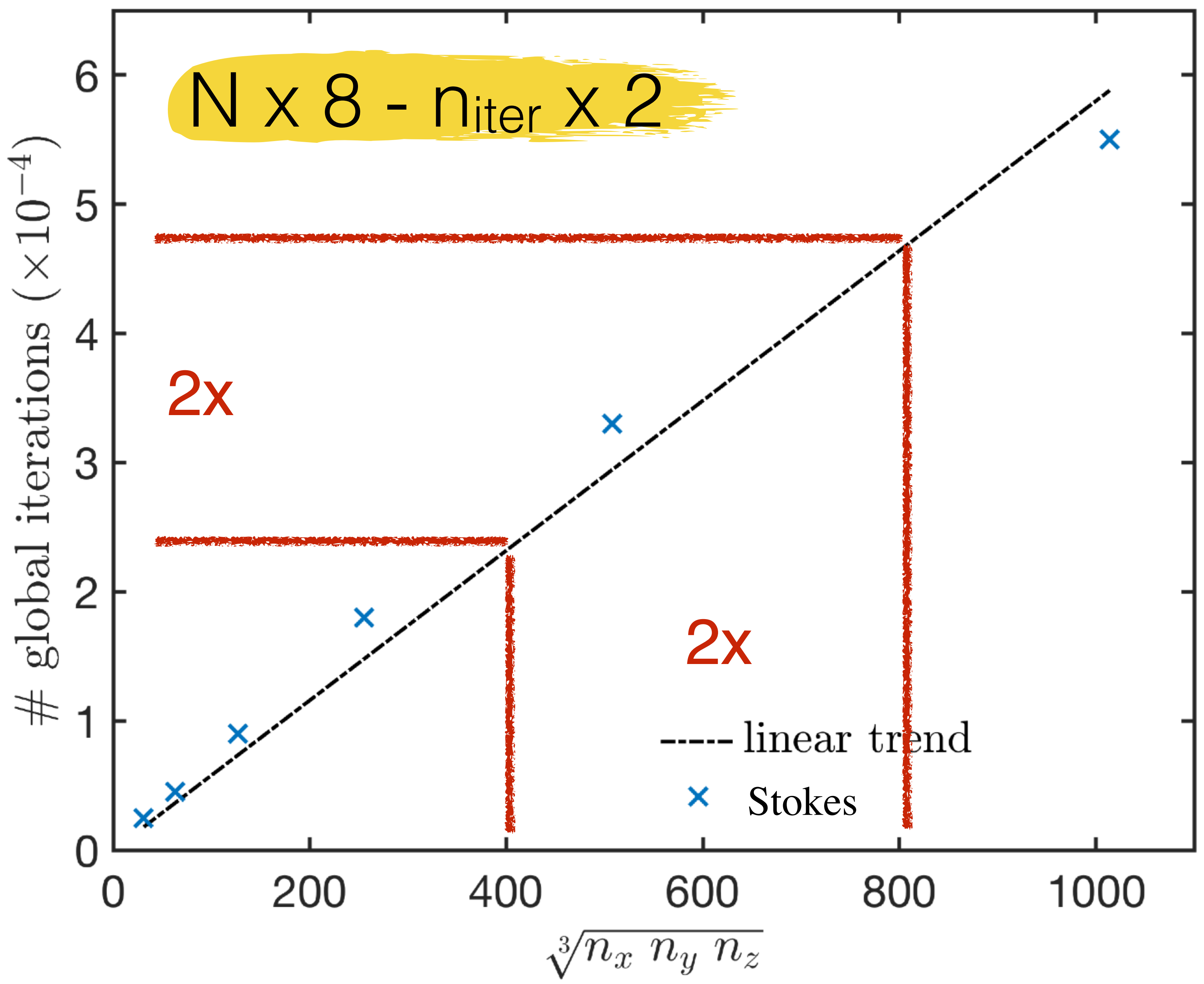
Scales $O(N^{1.3})$

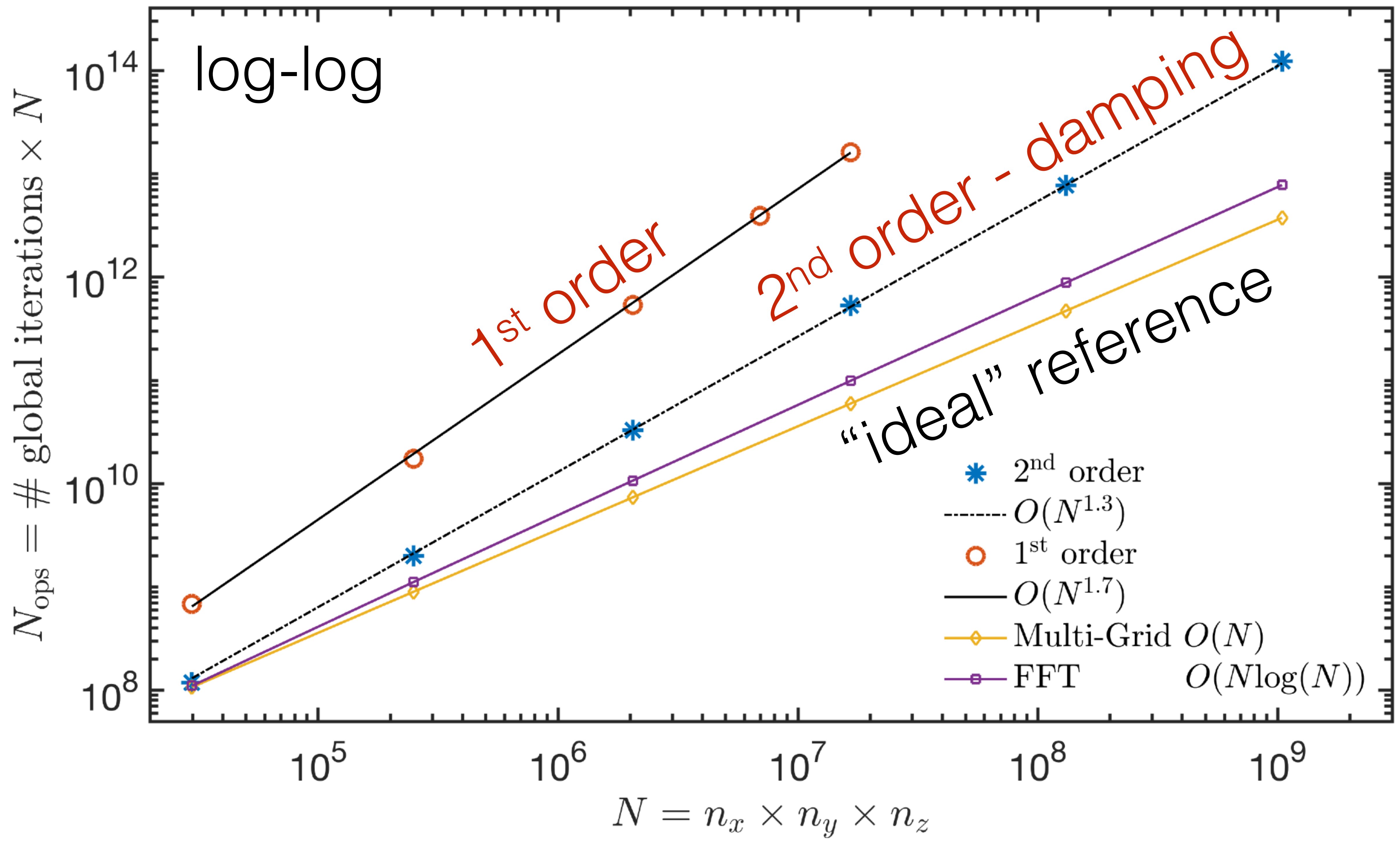




Pseudo-Transient | improved

- 2nd order scheme | implications





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3D equations

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Projects

Projects due date: August 1, 2019 - (midnight)

- Objective: a/ 3D viscous Stokes flow: Buoyancy driven rising sphere setup
  - or -
  b/ 3D elastic wave propagation: initial gaussian pressure anomaly

- Results: 1/ Report performance (MTP), convergence with resolution increase
  2a/ Viscous stokes: add convergence check to your code
  2b/ Elastic (acoustic) wave: add P and S wave recording in a place of the domain

- Nice plots and fancy 3D graphics is a plus

- Hand in a report (max 5 pages) including: 1/ intro, 2/ motivation, 3/ mathematical model, 4/ numerical approach, 5/ results, 6/ discussion (personal thoughts on the topic) and conclusion.

- Hand in a zip file including the codes used to generate the results.

Outlook - Session 5

- Topic: CUDA MPI - 1D wave on distributed memory (multi-GPUs)
- Programming: 1D multi-GPU acoustic wave
- Tasks: Projects
- Discussion on projects and overall Q&A.

Suggested references

- Performance of stencil codes + MPI

  https://on-demand.gputechconf.com/gtc/2019/video/_/S9368/

- Iterative method for solving large 3D problems on GPUs

  http://www.nature.com/articles/s41598-018-29485-5

  https://doi.org/10.1093/gji/ggz239

  https://doi.org/10.1093/gji/ggy434

That's it for today

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