ENERALIZED

IFT EXISTS FOR $x(t)$ SIGNALS

R POWER USE GENERALIZED

$\Rightarrow X(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - k2\pi)$

$\Rightarrow S_x(t)$

$T = 1, \Omega = WT \Rightarrow \Omega = W$

$u[n] \Leftrightarrow \frac{1}{1-e^{-j\Omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\Omega - k2\pi)$

COMPLEX EXPONENTIAL

$1 \Leftrightarrow 2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - k2\pi)$

$e^{j\Omega_0 n} \Leftrightarrow 2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - \Omega_0 - k2\pi)$

COSINE

$\cos(\Omega_0 n) \Leftrightarrow \pi \sum_{k=-\infty}^{\infty} \delta(\Omega + \Omega_0 - k2\pi) +$

$\cdots \delta(\Omega - \Omega_0 - k2\pi)]$

RIODIC SIGNALS

$n]$, PERIOD N

$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\Omega_0 n}$

$X(e^{j\Omega}) = \sum_{k=\langle N \rangle} a_k \left(2\pi \sum_{l=-\infty}^{\infty} \delta(\Omega - \Omega_0 k - l2\pi) \right)$

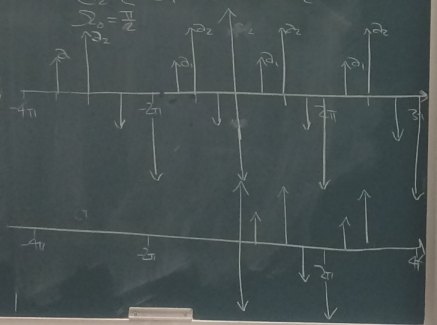
$a_k = a_{k+N}$

$\Rightarrow X(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\Omega - k\Omega_0)$

$a_1 = 1 \quad a_3 = -1 \quad a_0 = a_4$

$a_2 = 2 \quad a_4 = -2 \quad a_6 = a_2$

$\Omega_0 = \frac{\pi}{2}$



CONVOLUTION

$x[n] * y[n] = Y(e^{j\Omega}) \cdot Y(e^{j\Omega})$

PROOF

$z[n] = x[n] * y[n] = \sum_{m=-\infty}^{\infty} x[m] y[n-m]$

$y[n-m] = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y(e^{j\Omega}) e^{j\Omega(n-m)} d\Omega$

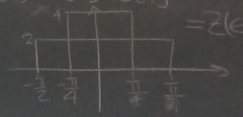
$z[n] = \sum_{m=-\infty}^{\infty} x[m] \frac{1}{2\pi} \int_{-\pi}^{\pi} Y(e^{j\Omega}) e^{-j\Omega(n-m)} d\Omega$

$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\sum_{m=-\infty}^{\infty} x[m] e^{-j\Omega m} \right) Y(e^{j\Omega}) e^{j\Omega n} d\Omega$

$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\Omega}) Y(e^{j\Omega}) e^{j\Omega n} d\Omega$

$x[n] = \text{sinc}\left(\frac{n}{2}\right), \quad y[n] = \text{sinc}\left(\frac{n}{4}\right)$

$z[n] = x[n] * y[n]$



$\Rightarrow z[n] = 2y[n] = 2\text{sinc}\left(\frac{n}{4}\right)$

MULTIPLICATION IN TIME

$x[n] \cdot y[n] \Leftrightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta}) Y(e^{j(\Omega-\theta)}) d\theta$

PROOF:

$z(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] y[n] e^{-j\Omega n}$

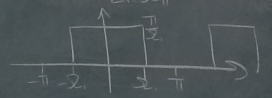
$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta}) e^{-j\theta n} d\theta \right) y[n] e^{j\Omega n}$

$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta}) \left(\sum_{n=-\infty}^{\infty} y[n] e^{jn(\Omega-\theta)} \right) d\theta$

$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta}) Y(e^{j(\Omega-\theta)}) d\theta$

$z[n] = \text{sinc}^2\left(\frac{\Omega_1}{\pi} n\right)$

$z(e^{j\Omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta}) Y(e^{j(\Omega-\theta)}) d\theta$



$\text{sinc}^2\left(\frac{\Omega_1}{\pi} n\right) \Leftrightarrow \frac{\pi}{\Omega_1} \sum_{k=-\infty}^{\infty} \Delta\left(\frac{\Omega - k}{\Omega}\right)$