PS 4 Problem 3

Wednesday, May 1, 2019   11:11 AM

$$J = 2\pi \int_{-1}^{1} \underbrace{x(t)\sqrt{1+\dot{x}(t)^2}}_{g}\, dt$$

$(-1, 5)$ to $(1, 5)$

$\Rightarrow x(-1) = 5 \qquad x(1) = 5$

Could use Euler's eqn

$$\frac{\partial g}{\partial x} - \frac{d}{dt}\frac{dg}{d\dot{x}} = 0$$

but since $g(x, \dot{x})$ and not $g(x, \dot{x}, t)$, it might be easier to use Beltrami's eqn

$$g - \dot{x}\frac{dg}{\partial \dot{x}} = c$$

$$\Rightarrow x\sqrt{1+\dot{x}^2} - \dot{x}\frac{x\, 2\dot{x}}{2\sqrt{1+\dot{x}^2}} = c$$

$$\Rightarrow x(1+\dot{x}^2) - x\dot{x}^2 = c\sqrt{1+\dot{x}^2}$$

$$\Rightarrow x = c\sqrt{1+\dot{x}^2}$$

$$\Rightarrow \frac{x^2}{c} = 1 + \dot{x}^2$$

$$\Rightarrow \dot{x}^2 = kx^2 - 1$$

$$\Rightarrow \dot{x} = \sqrt{kx^2 - 1}$$

$$\Rightarrow \frac{dx}{dt} = \sqrt{kx^2 - 1}$$

$$\Rightarrow x(t) = \frac{e^{-c_1\sqrt{k} - \sqrt{k}\,t}\left(e^{2\sqrt{k}\,(c_1 + t)} + k\right)}{2k}$$

Now we need to plug in $t=-1, x=5$ and $t=1, x=5$

to solve for k and c_1