PS 4 Problem 1

Sunday, April 28, 2019 11:40 PM

Find x^* to minimize

$$J = \int_0^1 \Big[\underbrace{\tfrac{1}{2}\dot{x}^2 + 5x\dot{x} + x^2 + 5x}_{g}\Big]dt$$

Using Euler eq^n;

$$\frac{\partial g}{\partial x} - \frac{d}{dt}\frac{\partial g}{\partial \dot{x}} = 0$$

$$\Rightarrow 5\dot{x} + 2x + 5 - \frac{d}{dt}(\dot{x} + 5x) = 0$$

$$\Rightarrow \cancel{5\dot{x}} + 2x + 5 - \ddot{x} - \cancel{5\dot{x}} = 0$$

$$\Rightarrow \ddot{x} = 2x + 5$$

Let $x = e^{rt} \Rightarrow r^2 = 2 \Rightarrow r = \pm\sqrt{2}$

For $\ddot{x} = 2x + 5$ try $x = c_1 e^{\sqrt{2}t} + c_2 e^{-\sqrt{2}t} + c_3$

$$2c_1 e^{\sqrt{2}t} + 2c_2 e^{-\sqrt{2}t} = 2c_1 e^{\sqrt{2}t} + 2c_2 e^{-\sqrt{2}t} + 2c_3 + 5$$

$$\Rightarrow c_3 = -5/2$$

$$x(t) = c_1 e^{\sqrt{2}t} + c_2 e^{-\sqrt{2}t} - 5/2$$

$x(0) = 1 \Rightarrow c_1 + c_2 - 5/2 = 1 \Rightarrow c_1 + c_2 = 7/2$

$x(1) = 3 \Rightarrow c_1 e^{\sqrt{2}} + c_2 e^{-\sqrt{2}} - 5/2 = 3$

$$c_1 e^{2\sqrt{2}} + c_2 = \frac{11}{2} e^{\sqrt{2}}$$

$$c_1 (e^{2\sqrt{2}} - 1) = \frac{11}{2} e^{\sqrt{2}} - \frac{7}{2}$$

$$\Rightarrow c_1 \approx 1.2013$$

$$\Rightarrow c_2 = 7/2 - c_1$$

$$\Rightarrow c_2 \approx 2.2987$$

$$\boxed{x^*(t) = 1.2013 e^{\sqrt{2}t} + 2.2987 e^{-\sqrt{2}t} - 2.5}$$