HW 8 Question 1

Monday, June 3, 2019 12:40 PM

$$x(t+1) = a\,x(t) + b\,x(t-1) + w(t)$$

Part a

$[x(t)]_{t=0}^{N}$ known

$$\min_{a,b} \sum_{t=1}^{N-1} \left(\bar{x}(t+1) - x(t+1)\right)^2$$

$$\bar{x}(t+1) = E\left(a\,x(t) + b\,x(t-1) + w(t)\right)$$

$$= a\,x(t) + b\,x(t-1)$$

$$\min_{a,b} \sum_{t=1}^{N-1} \left(x(t+1) - a\,x(t) - b\,x(t-1)\right)$$

$$\underbrace{\begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} x_1 & x_0 \\ x_2 & x_1 \\ \vdots & \\ x_{N-1} & x_{N-2} \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{\theta} + W, \quad W \sim N(0,\sigma)$$

$$\min_{a,b} \|Y - H\theta\|$$

$\Rightarrow$ if H is full rank

$$\theta^* = \left(H^T H^{-1}\right) H^T y$$

if H is not full rank, we can use MATLAB's left division

$$\theta^* = H \backslash y$$