

PART 4

$\because \phi(t) \neq 0$ on any non-zero time interval,

$$\begin{aligned}
 u^*(t) &= \arg \min_u \mathcal{H}(p_y, p_v, p_m, y, v, m) \\
 &= \arg \min_u \left(p_y v + p_v \left(\frac{u}{m} - g \right) + p_m (-bu) \right) \\
 &= \arg \min_u \left(\underbrace{-1 \cdot v(t)}_{\leftarrow p_y(t)} + u(t) \left(\underbrace{\frac{p_v(t)}{m(t)} - b p_m(t)}_{\leftarrow \text{indep of } u(t)} \right) - \underbrace{g(t-t_f)}_{\leftarrow p_v(t)} \right)
 \end{aligned}$$

$$= \arg \min_u \left(u(t) \phi(t) \right)$$

$$0 \leq u(t) \leq u_{\max}$$

$$\therefore u^*(t) = \begin{cases} 0 & \text{if } \phi(t) > 0 \\ u_{\max} & \text{if } \phi(t) < 0 \end{cases}$$