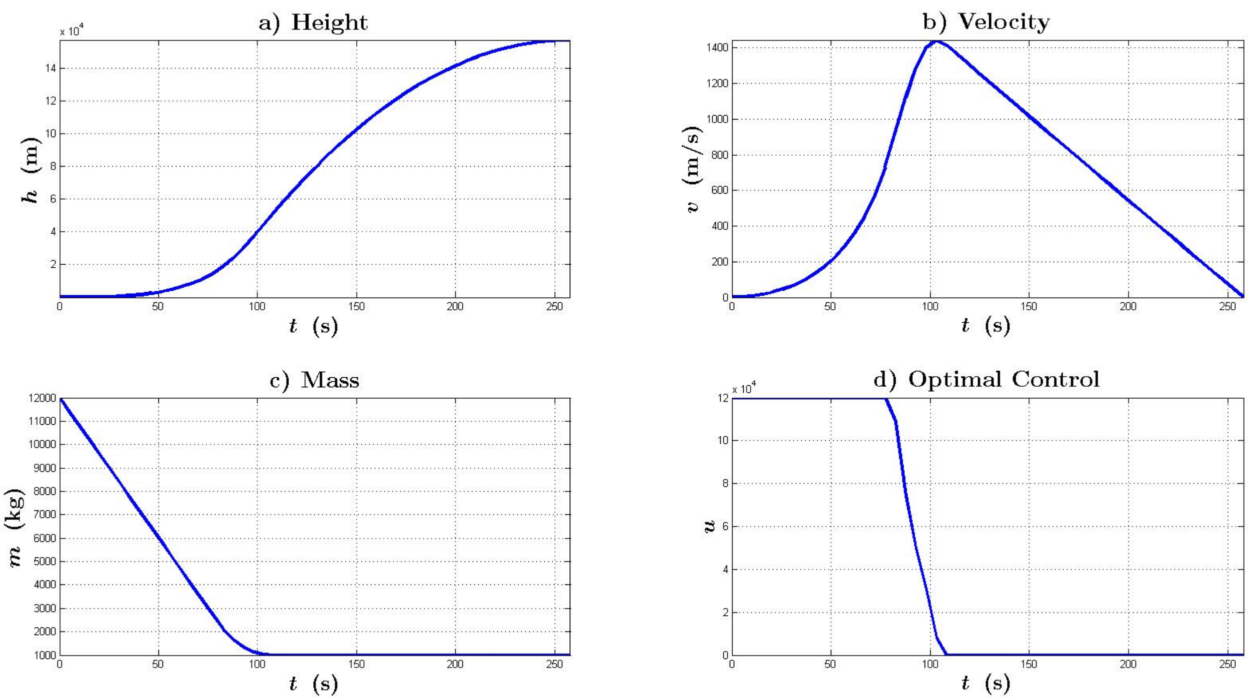
Problem 3

Part a

Following is the plot for the time evolution of the states and control subject to the trapezoidal rule direct method. Code attached on following pages.



```matlab
% Problem (OCP)_2 from Pset 6 - Trapezoidal Rule

clear all; clf; clc; format long;

% Parameters
global N; N = 50; % Choose here the number of discretization points
global mu; mu = 3.9915e14;
global rE; rE = 6378145;
global h0; h0 = 7500;
global D; D = 5e-3;
global b; b = 1e-3;
global uMax; uMax = 1.2e5;

% Scenario
global T; T = 258.;
global y0; y0 = 0.;
global v0; v0 = 0.;
global m0; m0 = 12000;
global mf; mf = 1000;

% Bound on the state: better conditioning the formulation (see below)
global yMax; yMax = 5e6;
global vMax; vMax = 2000;

% Since this optimal control problem is highly nonlinear, without
% an appropriate intialization direct methods unlikely converge.
% In the following lines, we provide such initialization by recalling the
% solution that we obtained for the simplified Goddard problem in the Pset 5.
% For the height, we just select a stright-line in time connecting y0 to
% 1.5e5 (which is more or less the final height that we found in Pset 5).
% For the velocity, we select the average v(t) = vMax/2 in [0,tf].
% For the mass, we select a straight-line in time between 0 and tSw, the
% switching time computed in Pset 5 (see below).
% Finally, for the control, we select the maximal value u(t) = uMax in [0,tf].

% Finding what index NSw the time tSw corresponds to
global tSw; tSw = (m0 - mf)/(b*uMax);
h = (1.0*T/(1.0*N));
NSw = 0; indexFound = 0; iterator = 0;
while indexFound == 0
    % If iterator*h <= tSw < iteartor*h + h, then we have found the index
    if iterator*h <= tSw && tSw < (iterator + 1)*h
        NSw = iterator + 1;
        indexFound = 1;
    end
    iterator = iterator + 1;
end
uInit = zeros(N+1,1);
yInit = zeros(N+1,1);
vInit = 0.5*vMax*ones(N+1,1);
mInit = mf*ones(N+1,1);
% Initialization exxplained above
for i=1:N+1
    yInit(i) = y0*(1. - (i-1)*1.0/N) + 1.5e5*(i-1)*1.0/N;
    if (i-1) <= NSw
        mInit(i) = m0*(1. - (i-1)*1.0/NSw) + mf*(i-1)*1.0/NSw;
    end
    if i<= N
        if (i-1)*1.0*T/N < tSw
            uInit(i) = uMax;
        end
    end
end
% Initialization for fmincon
varInit = [yInit; vInit; mInit; uInit];

% Lower and upper bounds.
```

```matlab
lb = zeros(4*N+4,1); ub = uMax*ones(4*N+4,1); % For the control: 0 \le u \le uMax
ub(1:N+1) = yMax; % For the state y : 0 \le y \le yMax
ub(N+2:2*N+2) = vMax; % For the state v : 0 \le v \le vMax
lb(2*N+3:3*N+3) = mf; ub(2*N+3:3*N+3) = m0; % For the state m : mf \le v \le m0

% Solving the problme via fmincon
options=optimoptions('fmincon','Display','iter','Algorithm','sqp','MaxFunEvals',100000,'MaxIter',10000);
% options=optimoptions('fmincon','Display','iter','Algorithm','sqp','MaxFunctionEvaluations',100000,'MaxIterations',10000);
[var,Fval,convergence] = fmincon(@cost,varInit,[],[],[],[],lb,ub,@constraint,options); % Solving the problem
convergence % = 1, good

% Collecting the solution. Note that var = [y;v;m;u]
y = var(1:N+1); v = var(N+2:2*N+2); m = var(2*N+3:3*N+3); u = var(3*N+4:4*N+4); % Collecting the solution
tState = zeros(N+1,1);
for i = 1:N
    tState(i+1) = tState(i) + (1.0*T/(1.0*N));
end
t = zeros(N+1,1);
for i = 1:N
    t(i+1) = t(i) + (1.0*T/(1.0*N));
end

% Plotting
% subplot(221); plot(tState,y,'linewidth',3);
% title('\textbf{a) Height}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$h$} \ \textbf{(m)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(222); plot(tState,v,'linewidth',3) ;
% title('\textbf{b) Velocity}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$v$} \ \textbf{(m/s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(223); plot(tState,m,'linewidth',3) ;
% title('\textbf{c) Mass}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$m$} \ \textbf{(kg)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(224); plot(t,u,'linewidth',3);
% title('\textbf{d) Optimal Control}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$u$}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
```

```
                                                          Norm of First-order
 Iter F-count            f(x) Feasibility  Steplength        step  optimality
    0     205   -1.500000e+05   2.160e+03                           1.000e+00
    1     410   -1.383047e+05   3.602e+00   1.000e+00   1.075e+05   6.461e+04
    2     617   -1.418327e+05   4.102e+00   7.000e-01   5.057e+04   6.822e+04
    3     822   -1.483241e+05   3.523e+00   1.000e+00   4.833e+04   9.815e+04
    4    1027   -1.500931e+05   5.709e-01   1.000e+00   1.475e+04   1.013e+05
    5    1232   -1.502196e+05   3.835e-03   1.000e+00   1.127e+03   1.014e+05
    6    1437   -1.502204e+05   1.564e-07   1.000e+00   7.180e+00   5.868e+02
    7    1642   -1.502212e+05   1.856e-07   1.000e+00   7.238e+00   2.042e+02
    8    1847   -1.502250e+05   4.626e-06   1.000e+00   3.619e+01   2.042e+02
    9    2052   -1.502441e+05   1.158e-04   1.000e+00   1.810e+02   2.028e+02
   10    2257   -1.503397e+05   2.912e-03   1.000e+00   9.065e+02   1.650e+02
   11    2462   -1.507327e+05   4.969e-02   1.000e+00   3.724e+03   1.650e+02
   12    2667   -1.507329e+05   5.195e-07   1.000e+00   3.809e+00   1.650e+02
   13    2872   -1.507331e+05   7.126e-08   1.000e+00   1.418e+00   1.650e+02
```

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  14    3077   -1.507340e+05   2.389e-07   1.000e+00   5.171e+00   1.540e+02
  15    3282   -1.507346e+05   3.874e-07   1.000e+00   3.490e+00   1.169e+02
  16    3487   -1.507348e+05   6.257e-08   1.000e+00   1.416e+00   9.890e+01
  17    3692   -1.507349e+05   1.723e-09   1.000e+00   8.271e-01   8.842e+01
  18    3897   -1.507354e+05   4.257e-08   1.000e+00   4.100e+00   8.841e+01
  19    4102   -1.507357e+05   1.204e-08   1.000e+00   1.971e+00   7.893e+01
  20    4307   -1.507358e+05   9.076e-09   1.000e+00   1.576e+00   6.548e+01
  21    4512   -1.507364e+05   1.184e-07   1.000e+00   5.653e+00   6.349e+01
  22    4717   -1.507393e+05   2.959e-06   1.000e+00   2.827e+01   6.349e+01
  23    4922   -1.507538e+05   7.402e-05   1.000e+00   1.413e+02   6.348e+01
  24    5127   -1.508264e+05   1.857e-03   1.000e+00   7.072e+02   6.344e+01
  25    5332   -1.511901e+05   4.711e-02   1.000e+00   3.543e+03   5.901e+01
  26    5537   -1.522507e+05   4.113e-01   1.000e+00   1.029e+04   5.892e+01
  27    5742   -1.522599e+05   4.546e-06   1.000e+00   6.591e+01   5.893e+01
  28    5947   -1.522606e+05   1.423e-07   1.000e+00   5.303e+00   5.893e+01
  29    6152   -1.522620e+05   1.256e-07   1.000e+00   1.208e+01   5.893e+01
  30    6357   -1.522620e+05   4.735e-09   1.000e+00   3.541e-01   5.071e+01

                                                          Norm of First-order
 Iter F-count            f(x) Feasibility  Steplength        step  optimality
  31    6562   -1.522621e+05   3.479e-09   1.000e+00   1.029e+00   4.741e+01
  32    6767   -1.522623e+05   1.014e-08   1.000e+00   1.640e+00   3.837e+01
  33    6972   -1.522626e+05   4.522e-08   1.000e+00   3.428e+00   3.769e+01
  34    7177   -1.522642e+05   1.131e-06   1.000e+00   1.714e+01   3.769e+01
  35    7382   -1.522723e+05   2.830e-05   1.000e+00   8.570e+01   3.769e+01
  36    7587   -1.523127e+05   7.087e-04   1.000e+00   4.286e+02   3.768e+01
  37    7792   -1.525151e+05   1.787e-02   1.000e+00   2.145e+03   3.762e+01
  38    7997   -1.528313e+05   4.396e-02   1.000e+00   3.340e+03   3.752e+01
  39    8202   -1.528323e+05   2.736e-08   1.000e+00   7.553e+00   3.752e+01
  40    8407   -1.528324e+05   1.582e-09   1.000e+00   6.176e-01   3.140e+01
  41    8612   -1.528327e+05   3.925e-08   1.000e+00   3.088e+00   3.140e+01
  42    8817   -1.528343e+05   9.832e-07   1.000e+00   1.544e+01   3.140e+01
  43    9022   -1.528421e+05   2.460e-05   1.000e+00   7.721e+01   3.140e+01
  44    9227   -1.528809e+05   6.055e-04   1.000e+00   3.828e+02   3.139e+01
  45    9432   -1.528811e+05   1.684e-08   1.000e+00   1.984e+00   3.139e+01
  46    9637   -1.528820e+05   4.203e-07   1.000e+00   9.676e+00   3.139e+01
  47    9842   -1.528864e+05   1.051e-05   1.000e+00   4.838e+01   3.139e+01
  48   10047   -1.529083e+05   2.630e-04   1.000e+00   2.419e+02   3.138e+01
  49   10252   -1.530181e+05   6.610e-03   1.000e+00   1.210e+03   3.132e+01
  50   10457   -1.535694e+05   1.696e-01   1.000e+00   6.069e+03   3.104e+01
  51   10662   -1.539207e+05   6.877e-02   1.000e+00   3.811e+03   3.087e+01
  52   10867   -1.539221e+05   6.668e-08   1.000e+00   1.013e+01   3.087e+01
  53   11072   -1.539221e+05   2.402e-10   1.000e+00   2.677e-01   3.087e+01
  54   11277   -1.539222e+05   3.561e-09   1.000e+00   1.339e+00   3.087e+01
  55   11482   -1.539228e+05   8.981e-08   1.000e+00   6.693e+00   3.087e+01
  56   11687   -1.539239e+05   3.120e-07   1.000e+00   1.227e+01   3.087e+01
  57   11892   -1.539239e+05   1.071e-09   1.000e+00   5.357e-01   3.087e+01
  58   12097   -1.539241e+05   2.656e-08   1.000e+00   2.679e+00   3.087e+01
  59   12302   -1.539250e+05   6.632e-07   1.000e+00   1.339e+01   3.087e+01
  60   12507   -1.539295e+05   1.658e-05   1.000e+00   6.696e+01   3.086e+01

                                                          Norm of First-order
 Iter F-count            f(x) Feasibility  Steplength        step  optimality
  61   12712   -1.539521e+05   4.151e-04   1.000e+00   3.348e+02   3.084e+01
  62   12917   -1.540654e+05   1.044e-02   1.000e+00   1.675e+03   3.071e+01
  63   13122   -1.546322e+05   2.676e-01   1.000e+00   8.370e+03   3.006e+01
  64   13327   -1.555254e+05   6.932e-01   1.000e+00   1.305e+04   2.899e+01
  65   13532   -1.555348e+05   1.843e-06   1.000e+00   8.475e+01   2.900e+01
  66   13737   -1.555349e+05   1.237e-07   1.000e+00   9.483e-01   2.900e+01
  67   13942   -1.555354e+05   4.536e-07   1.000e+00   3.057e+00   2.900e+01
  68   14147   -1.555355e+05   4.119e-08   1.000e+00   9.729e-01   2.900e+01
  69   14352   -1.555357e+05   1.850e-07   1.000e+00   1.643e+00   2.335e+01
  70   14557   -1.555358e+05   2.874e-09   1.000e+00   7.418e-01   1.829e+01
  71   14762   -1.555361e+05   6.435e-08   1.000e+00   3.709e+00   1.828e+01
  72   14967   -1.555377e+05   1.638e-06   1.000e+00   1.855e+01   1.828e+01
  73   15172   -1.555456e+05   4.105e-05   1.000e+00   9.273e+01   1.823e+01
  74   15377   -1.555510e+05   1.902e-05   1.000e+00   6.303e+01   1.820e+01
  75   15582   -1.555546e+05   2.011e-05   1.000e+00   2.248e+02   1.820e+01
  76   15787   -1.555727e+05   5.030e-04   1.000e+00   1.124e+03   1.820e+01
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  77   15992   -1.556376e+05   6.578e-03   1.000e+00   4.055e+03   1.820e+01
  78   16197   -1.556374e+05   1.322e-09   1.000e+00   6.623e-01   1.820e+01
  79   16402   -1.556374e+05   8.549e-11   1.000e+00   1.407e-01   1.820e+01
  80   16607   -1.556375e+05   1.488e-09   1.000e+00   7.035e-01   1.820e+01
  81   16812   -1.556377e+05   3.724e-08   1.000e+00   3.518e+00   1.820e+01
  82   17017   -1.556388e+05   9.350e-07   1.000e+00   1.759e+01   1.820e+01
  83   17222   -1.556442e+05   2.337e-05   1.000e+00   8.794e+01   1.820e+01
  84   17427   -1.556712e+05   5.850e-04   1.000e+00   4.397e+02   1.820e+01
  85   17632   -1.558062e+05   1.472e-02   1.000e+00   2.199e+03   1.820e+01
  86   17837   -1.560135e+05   3.516e-02   1.000e+00   3.374e+03   1.820e+01
  87   18042   -1.560136e+05   9.209e-09   1.000e+00   3.639e+00   1.820e+01
  88   18247   -1.560137e+05   1.456e-10   1.000e+00   2.128e-01   1.820e+01
  89   18452   -1.560137e+05   3.912e-09   1.000e+00   1.064e+00   1.820e+01
  90   18657   -1.560140e+05   9.877e-08   1.000e+00   5.319e+00   1.820e+01

                                                            Norm of First-order
 Iter F-count            f(x) Feasibility  Steplength          step  optimality
  91   18862   -1.560154e+05   2.471e-06   1.000e+00   2.660e+01   1.820e+01
  92   19067   -1.560223e+05   6.179e-05   1.000e+00   1.330e+02   1.820e+01
  93   19272   -1.560572e+05   1.548e-03   1.000e+00   6.649e+02   1.820e+01
  94   19477   -1.562312e+05   3.907e-02   1.000e+00   3.324e+03   1.820e+01
  95   19682   -1.570949e+05   1.011e+00   1.000e+00   1.650e+04   1.820e+01
  96   19887   -1.569294e+05   3.987e-02   1.000e+00   3.160e+03   1.820e+01
  97   20092   -1.569213e+05   9.737e-05   1.000e+00   1.574e+02   1.779e+01
  98   20297   -1.569213e+05   6.041e-10   1.000e+00   3.814e-01   1.780e+01
  99   20502   -1.569213e+05   1.145e-10   1.000e+00   2.031e-01   1.779e+01
 100   20707   -1.569213e+05   2.918e-09   1.000e+00   1.015e+00   1.779e+01
 101   20912   -1.569215e+05   7.216e-08   1.000e+00   5.076e+00   1.778e+01
 102   21117   -1.569222e+05   1.807e-06   1.000e+00   2.537e+01   1.770e+01
 103   21322   -1.569260e+05   4.493e-05   1.000e+00   1.266e+02   1.734e+01
 104   21527   -1.569444e+05   1.096e-03   1.000e+00   6.254e+02   1.553e+01
 105   21732   -1.570279e+05   2.425e-02   1.000e+00   2.955e+03   1.022e+01
 106   21937   -1.572442e+05   2.409e-01   1.000e+00   9.812e+03   2.595e+01
 107   22142   -1.571377e+05   1.251e-02   1.000e+00   1.920e+03   2.859e+01
 108   22347   -1.571363e+05   5.459e-04   1.000e+00   7.539e+02   1.567e+01
 109   22552   -1.571358e+05   6.095e-05   1.000e+00   2.531e+02   1.515e+00
 110   22757   -1.571357e+05   1.390e-08   1.000e+00   2.179e+00   8.903e-01
 111   22962   -1.571357e+05   2.620e-09   1.000e+00   1.592e+00   8.602e-01
 112   23167   -1.571357e+05   1.912e-08   1.000e+00   4.278e+00   8.508e-01
 113   23372   -1.571357e+05   5.591e-08   1.000e+00   7.316e+00   8.346e-01
 114   23577   -1.571357e+05   9.721e-08   1.000e+00   9.637e+00   8.131e-01
 115   23782   -1.571357e+05   3.434e-07   1.000e+00   1.809e+01   7.719e-01
 116   23987   -1.571357e+05   7.926e-07   1.000e+00   2.754e+01   7.173e-01
 117   24192   -1.571357e+05   2.168e-06   1.000e+00   4.574e+01   8.610e-01
 118   24397   -1.571358e+05   5.312e-06   1.000e+00   7.271e+01   1.084e+00
 119   24602   -1.571358e+05   1.320e-05   1.000e+00   1.199e+02   1.432e+00
 120   24807   -1.571360e+05   2.628e-05   1.000e+00   1.925e+02   1.908e+00

                                                            Norm of First-order
 Iter F-count            f(x) Feasibility  Steplength          step  optimality
 121   25012   -1.571365e+05   3.411e-05   1.000e+00   3.082e+02   2.442e+00
 122   25217   -1.571372e+05   1.147e-04   1.000e+00   4.507e+02   2.603e+00
 123   25422   -1.571379e+05   2.233e-04   1.000e+00   5.254e+02   2.255e+00
 124   25627   -1.571381e+05   1.041e-04   1.000e+00   3.927e+02   1.217e+00
 125   25832   -1.571380e+05   3.013e-05   1.000e+00   1.725e+02   2.977e-01
 126   26037   -1.571380e+05   5.790e-07   1.000e+00   4.400e+01   7.066e-02
 127   26242   -1.571380e+05   1.965e-08   1.000e+00   5.319e+00   4.792e-02
 128   26447   -1.571380e+05   9.919e-11   1.000e+00   4.016e-01   4.759e-02
 129   26652   -1.571380e+05   8.151e-11   1.000e+00   3.166e-01   4.723e-02
 130   26857   -1.571380e+05   3.601e-10   1.000e+00   7.526e-01   4.687e-02
 131   27062   -1.571380e+05   1.022e-09   1.000e+00   1.121e+00   4.555e-02
 132   27267   -1.571380e+05   2.996e-09   1.000e+00   1.989e+00   4.343e-02
 133   27472   -1.571380e+05   4.901e-09   1.000e+00   2.394e+00   4.024e-02
 134   27677   -1.571380e+05   2.455e-08   1.000e+00   5.542e+00   5.409e-02
 135   27882   -1.571380e+05   3.952e-08   1.000e+00   7.083e+00   7.049e-02
 136   28087   -1.571380e+05   1.242e-07   1.000e+00   1.227e+01   1.210e-01
 137   28292   -1.571380e+05   3.776e-07   1.000e+00   1.999e+01   2.099e-01
 138   28497   -1.571380e+05   8.627e-07   1.000e+00   3.034e+01   3.407e-01
 139   28702   -1.571380e+05   1.974e-06   1.000e+00   4.825e+01   5.104e-01
```

```
  140   28907   -1.571380e+05   5.414e-06   1.000e+00   7.498e+01   6.285e-01
  141   29112   -1.571381e+05   1.520e-05   1.000e+00   9.405e+01   5.331e-01
  142   29317   -1.571381e+05   8.756e-06   1.000e+00   7.580e+01   3.720e-01
  143   29522   -1.571381e+05   7.624e-07   1.000e+00   3.068e+01   1.273e-01
  144   29727   -1.571381e+05   4.451e-08   1.000e+00   7.298e+00   1.982e-02
  145   29932   -1.571381e+05   1.324e-09   1.000e+00   9.539e-01   2.543e-03
  146   30137   -1.571381e+05   2.319e-11   1.000e+00   7.672e-02   2.442e-03
  147   30139   -1.571381e+05   2.319e-11   4.900e-01   1.615e-02   1.018e-07

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.




convergence =

     1
```

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```
% Function providing equality and inequality constraints
% ceq(var) = 0 and c(var) \le 0

function [c,ceq] = constraint(var)

global N;
global T;

global y0;
global v0;
global m0;
global mf;

% Put here constraint inequalities
c = [];

% Note that var = [y;v;m;u]
y = var(1:N+1); v = var(N+2:2*N+2); m = var(2*N+3:3*N+3); u = var(3*N+4:4*N+4); % Note: var = [y;v;m;u]

% Computing dynamical constraints via the trapezoidal rule
h = 1.0*T/(1.0*N);
for i = 1:N
    % Provide here dynamical constraints via the trapeziodal formula
    [yDyn_i,vDyn_i,mDyn_i] = fDyn(y(i),v(i),m(i),u(i));
    [yDyn_ii,vDyn_ii,mDyn_ii] = fDyn(y(i+1),v(i+1),m(i+1),u(i+1));
    ceq(i) = y(i+1) - y(i) - h*(yDyn_i + yDyn_ii)/2;
    ceq(i+N) = v(i+1) - v(i) - h*(vDyn_i + vDyn_ii)/2;
    ceq(i+2*N) = m(i+1) - m(i) - h*(mDyn_i + mDyn_ii)/2;
end

% Put here initial and final conditions
ceq(1+3*N) = y(1) - y0;
ceq(2+3*N) = v(1) - v0;
ceq(3+3*N) = m(1) - m0;
ceq(4+3*N) = m(end) - mf;
```

```
% Cost of the problem

function c = cost(var)

global N;

% Note that var = [y;v;m;u]
y = var(1:N+1); v = var(N+2:2*N+2); m = var(2*N+3:3*N+3); u = var(3*N+4:4*N+3);

% Put here the cost
c = -y(end);
```

```
% Dynamics of the problem

function [yDyn,vDyn,mDyn] = fDyn(y,v,m,u)

global D;
global b;

g = gFunc(y);
rho = normRhoFunc(y);

% Put here the dynamics
yDyn = v;
vDyn = u/m - g - (D/m)*rho*v^2;
mDyn = -b*u;
```

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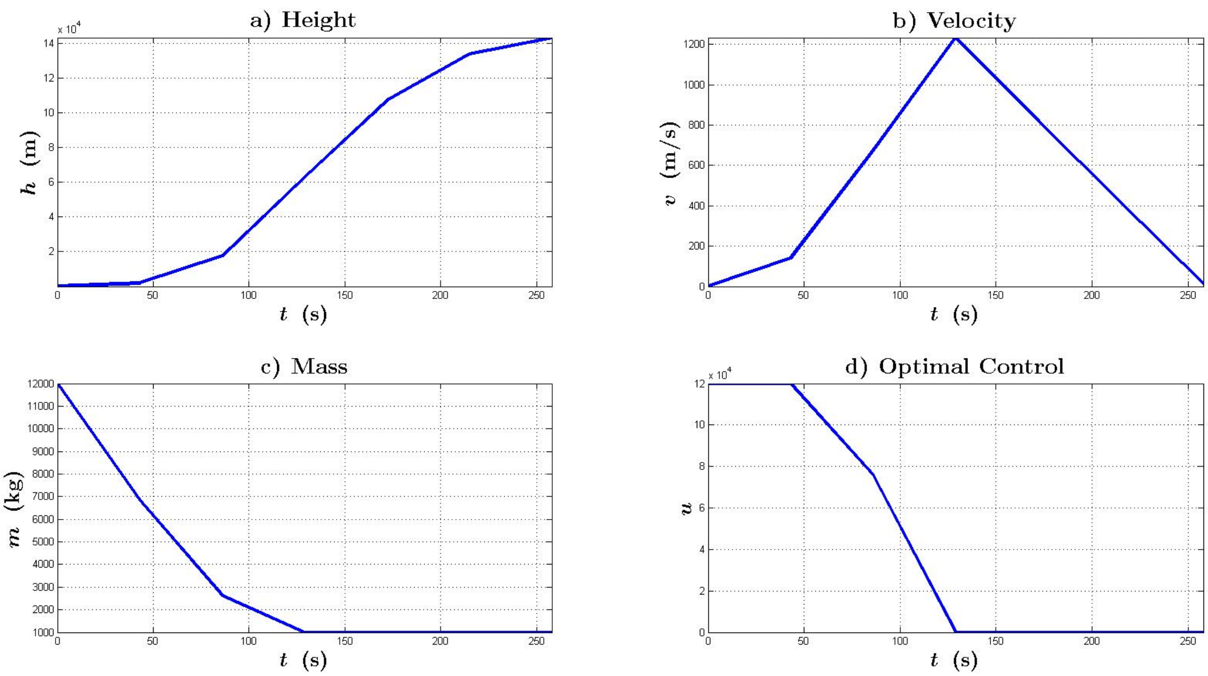
Part b

The minimum number of discretization points N required to get the MATLAB function using trapezoidal rule direct method to converge (with exitflag = 1) was **50**.

Part c

Following is the plot for the time evolution of the states and control subject to the Hermite-Simpson rule direct method.



The “cost” and “fdyn” functions remain unchanged. The code for the “collocation” and “constraint” functions are attached on following pages.

```matlab
% Problem (OCP)_2 from Pset 6 - Hermite-Simpson Rule

clear all; clf; clc; format long;

% Parameters
global N; N = 6; % Choose here the number of discretization points
global mu; mu = 3.9915e14;
global rE; rE = 6378145;
global h0; h0 = 7500;
global D; D = 5e-3;
global b; b = 1e-3;
global uMax; uMax = 1.2e5;

% Scenario
global T; T = 258.;
global y0; y0 = 0.;
global v0; v0 = 0.;
global m0; m0 = 12000;
global mf; mf = 1000;

% Bound on the state: better conditioning the formulation (see below)
global yMax; yMax = 5e6;
global vMax; vMax = 2000;

% Since this optimal control problem is highly nonlinear, without
% an appropriate intialization direct methods unlikely converge.
% In the following lines, we provide such initialization by recalling the
% solution that we obtained for the simplified Goddard problem in the Pset 5.
% For the height, we just select a stright-line in time connecting y0 to
% 1.5e5 (which is more or less the final height that we found in Pset 5).
% For the velocity, we select the average v(t) = vMax/2 in [0,tf].
% For the mass, we select a straight-line in time between 0 and tSw, the
% switching time computed in Pset 5 (see below).
% Finally, for the control, we select the maximal value u(t) = uMax in [0,tf].

% Finding what index NSw the time tSw corresponds to
global tSw; tSw = (m0 - mf)/(b*uMax);
h = (1.0*T/(1.0*N));
NSw = 0; indexFound = 0; iterator = 0;
while indexFound == 0
    % If iterator*h <= tSw < iteartor*h + h, then we have found the index
    if iterator*h <= tSw && tSw < (iterator + 1)*h
        NSw = iterator + 1;
        indexFound = 1;
    end
    iterator = iterator + 1;
end
uInit = zeros(N+1,1);
yInit = zeros(N+1,1);
vInit = 0.5*vMax*ones(N+1,1);
mInit = mf*ones(N+1,1);
% Initialization exxplained above
for i=1:N+1
    yInit(i) = y0*(1. - (i-1)*1.0/N) + 1.5e5*(i-1)*1.0/N;
    if (i-1) <= NSw
        mInit(i) = m0*(1. - (i-1)*1.0/NSw) + mf*(i-1)*1.0/NSw;
    end
    if i<= N
        if (i-1)*1.0*T/N < tSw
            uInit(i) = uMax;
        end
    end
end
% Initialization for fmincon
varInit = [yInit; vInit; mInit; uInit];

% Lower and upper bounds.
```

```
lb = zeros(4*N+4,1); ub = uMax*ones(4*N+4,1); % For the control: 0 \le u \le uMax
ub(1:N+1) = yMax; % For the state y : 0 \le y \le yMax
ub(N+2:2*N+2) = vMax; % For the state v : 0 \le v \le vMax
lb(2*N+3:3*N+3) = mf; ub(2*N+3:3*N+3) = m0; % For the state m : mf \le v \le m0

% Solving the problme via fmincon
options=optimoptions('fmincon','Display','iter','Algorithm','sqp','MaxFunEvals',100000,'MaxIter',10000);
% options=optimoptions('fmincon','Display','iter','Algorithm','sqp','MaxFunctionEvaluations',100000,'MaxIterations',10000);
[var,Fval,convergence] = fmincon(@cost,varInit,[],[],[],[],lb,ub,@constraint,options); % Solving the problem
convergence % = 1, good

% Collecting the solution. Note that var = [y;v;m;u]
y = var(1:N+1); v = var(N+2:2*N+2); m = var(2*N+3:3*N+3); u = var(3*N+4:4*N+4); % Collecting the solution
tState = zeros(N+1,1);
for i = 1:N
    tState(i+1) = tState(i) + (1.0*T/(1.0*N));
end
t = zeros(N+1,1);
for i = 1:N
    t(i+1) = t(i) + (1.0*T/(1.0*N));
end

% Plotting
% subplot(221); plot(tState,y,'linewidth',3);
% title('\textbf{a) Height}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$h$} \ \textbf{(m)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(222); plot(tState,v,'linewidth',3) ;
% title('\textbf{b) Velocity}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$v$} \ \textbf{(m/s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(223); plot(tState,m,'linewidth',3) ;
% title('\textbf{c) Mass}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$m$} \ \textbf{(kg)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(224); plot(t,u,'linewidth',3);
% title('\textbf{d) Optimal Control}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$u$}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
```

```
                                                          Norm of First-order
 Iter F-count            f(x) Feasibility  Steplength        step  optimality
    0      29   -1.500000e+05   2.195e+04                            1.000e+00
    1      58   -1.300265e+05   1.408e+02   1.000e+00   6.641e+04   1.483e+06
    2      87   -1.302402e+05   5.410e-01   1.000e+00   1.256e+03   1.517e+04
    3     116   -1.302413e+05   1.637e-05   1.000e+00   3.926e+00   2.542e+01
    4     145   -1.302419e+05   3.305e-07   1.000e+00   8.138e-01   4.686e+00
    5     174   -1.302449e+05   6.053e-06   1.000e+00   4.069e+00   4.686e+00
    6     203   -1.302600e+05   1.458e-04   1.000e+00   2.034e+01   4.686e+00
    7     232   -1.303357e+05   3.643e-03   1.000e+00   1.017e+02   4.686e+00
    8     261   -1.305159e+05   2.088e-02   1.000e+00   2.422e+02   4.685e+00
    9     290   -1.305188e+05   1.050e-05   1.000e+00   4.154e+00   4.685e+00
   10     319   -1.305325e+05   2.386e-04   1.000e+00   2.021e+01   4.685e+00
   11     348   -1.306013e+05   6.002e-03   1.000e+00   1.011e+02   4.685e+00
   12     377   -1.309454e+05   1.512e-01   1.000e+00   5.056e+02   4.685e+00
   13     406   -1.310556e+05   1.555e-02   1.000e+00   1.618e+02   4.685e+00
```

```
    14     435   -1.310562e+05   3.920e-07   1.000e+00   1.055e+00   4.685e+00
    15     464   -1.310589e+05   1.172e-05   1.000e+00   4.982e+00   4.686e+00
    16     493   -1.310720e+05   2.956e-04   1.000e+00   2.491e+01   4.686e+00
    17     522   -1.311378e+05   7.465e-03   1.000e+00   1.246e+02   4.691e+00
    18     551   -1.314671e+05   1.878e-01   1.000e+00   6.232e+02   4.714e+00
    19     580   -1.324321e+05   1.631e+00   1.000e+00   1.825e+03   4.781e+00
    20     609   -1.324546e+05   6.150e-04   1.000e+00   4.081e+01   4.782e+00
    21     638   -1.325160e+05   7.243e-03   1.000e+00   1.159e+02   4.787e+00
    22     667   -1.328238e+05   1.825e-01   1.000e+00   5.813e+02   4.808e+00
    23     696   -1.338126e+05   1.903e+00   1.000e+00   1.866e+03   4.879e+00
    24     725   -1.338182e+05   7.383e-08   1.000e+00   9.419e+00   4.865e+00
    25     754   -1.338186e+05   1.038e-07   1.000e+00   8.835e-01   4.865e+00
    26     783   -1.338207e+05   2.724e-06   1.000e+00   4.418e+00   4.867e+00
    27     812   -1.338308e+05   6.745e-05   1.000e+00   2.209e+01   4.876e+00
    28     841   -1.338816e+05   1.686e-03   1.000e+00   1.105e+02   4.921e+00
    29     870   -1.341357e+05   4.229e-02   1.000e+00   5.527e+02   5.148e+00
    30     899   -1.354107e+05   1.075e+00   1.000e+00   2.771e+03   6.304e+00

                                                          Norm of First-order
  Iter F-count            f(x) Feasibility  Steplength        step  optimality
    31     928   -1.413389e+05   2.438e+01   1.000e+00   1.283e+04   1.211e+01
    32     957   -1.420058e+05   1.624e-01   1.000e+00   1.316e+03   1.260e+01
    33     986   -1.420188e+05   4.979e-05   1.000e+00   3.210e+01   1.261e+01
    34    1015   -1.420192e+05   2.789e-07   1.000e+00   8.408e-01   1.261e+01
    35    1044   -1.420213e+05   7.602e-06   1.000e+00   4.199e+00   1.261e+01
    36    1073   -1.420319e+05   1.884e-04   1.000e+00   2.100e+01   1.263e+01
    37    1102   -1.420849e+05   4.593e-03   1.000e+00   1.050e+02   1.271e+01
    38    1131   -1.423506e+05   1.157e-01   1.000e+00   5.262e+02   1.313e+01
    39    1160   -1.432112e+05   1.219e+00   1.000e+00   1.701e+03   5.343e+00
    40    1189   -1.432192e+05   1.178e-07   1.000e+00   1.221e+01   5.042e+00
    41    1218   -1.432192e+05   1.819e-11   1.000e+00   8.017e-07   5.684e-14

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.




convergence =

     1
```

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```
% Function providing equality and inequality constraints
% ceq(var) = 0 and c(var) \le 0

function [c,ceq] = constraint(var)

global N;
global T;

global y0;
global v0;
global m0;
global mf;

% Put here constraint inequalities
c = [];

% Note that var = [y;v;m;u]
y = var(1:N+1); v = var(N+2:2*N+2); m = var(2*N+3:3*N+3); u = var(3*N+4:4*N+4); % Note: var = [y;v;m;u]

% Computing dynamical constraints via the Hermite-Simpson rule
h = 1.0*T/(1.0*N);
for i = 1:N
    % Provide here dynamical constraints via the Hermite-Simpson formula
    [yDyn_i,vDyn_i,mDyn_i] = fDyn(y(i),v(i),m(i),u(i));
    [yDyn_ii,vDyn_ii,mDyn_ii] = fDyn(y(i+1),v(i+1),m(i+1),u(i+1));

    y_ic = (1./2.)*(y(i) + y(i+1)) + (1.0*T/(1.0*N))/8.*(yDyn_i - yDyn_ii); % Evaluating state and control at collocation points via the Hermite-Simpson formula
    v_ic = (1./2.)*(v(i) + v(i+1)) + (1.0*T/(1.0*N))/8.*(vDyn_i - vDyn_ii);
    m_ic = (1./2.)*(m(i) + m(i+1)) + (1.0*T/(1.0*N))/8.*(mDyn_i - mDyn_ii);
    u_ic = (u(i) + u(i+1))/2.;

    [yDyn_ic,vDyn_ic,mDyn_ic] = fDyn(y_ic,v_ic,m_ic,u_ic); % Evaluating dynamics at collocation points

    ceq(i) = y(i+1) - y(i) - ((1.0*T/(1.0*N))/6.0)*(yDyn_i + 4*yDyn_ic + yDyn_ii);
    ceq(i+N) = v(i+1) - v(i) - ((1.0*T/(1.0*N))/6.0)*(vDyn_i + 4*vDyn_ic + vDyn_ii);;
    ceq(i+2*N) = m(i+1) - m(i) - ((1.0*T/(1.0*N))/6.0)*(mDyn_i + 4*mDyn_ic + mDyn_ii);;
end

% Put here initial and final conditions
ceq(1+3*N) = y(1) - y0;
ceq(2+3*N) = v(1) - v0;
ceq(3+3*N) = m(1) - m0;
ceq(4+3*N) = m(end) - mf;
```

Part d

The minimum number of discretization points N required to get the MATLAB function using Hermite-Simpson rule direct method to converge (with exitflag = 1) was **6**.