

PART 2

$$\text{Final Be: } \left[ \begin{pmatrix} p_y(t_f), p_v(t_f), p_m(t_f) \end{pmatrix} - \nabla h(y(t_f), v(t_f), m(t_f)) \right] \\ \cdot \ker \nabla F(y(t_f), v(t_f), m(t_f)) = 0$$

$$F(y, v, m) := m - m_f$$

$$\Rightarrow \nabla F = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{Let } w \in \ker \nabla F \Rightarrow (w_1 \ w_2 \ w_3) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow w_3 = 0$$

$$\Rightarrow \ker \nabla F = \{w \in \mathbb{R}^3 : w_3 = 0\}$$

$$h = -y(t_f)$$

$$\Rightarrow \nabla h = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \underbrace{\begin{pmatrix} p_y \\ p_v \\ p_m \end{pmatrix} - \nabla h}_{\text{ker } \nabla F} \cdot \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ 0 \end{bmatrix}}_{\text{ker } \nabla F} = 0$$

$$\Rightarrow w_1 (p_y(t_f) + 1) + w_2 (p_v(t_f)) = 0 \quad \forall w_1, w_2$$

$$\Rightarrow \boxed{p_y(t_f) = -1 \quad \text{and} \quad p_v(t_f) = 0}$$