PS 5 Problem 2 Part 2 Parts a and b

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Part a

repeating steps from P1

$$\dot{p}_y(t) = -\frac{\partial H}{\partial y} = 0 \Rightarrow p_y(t) = \text{const} = c_1$$

From final BCs → $\because p_y(t_f) = -1 \Rightarrow p_y(t) = -1$

$$\dot{p}_v(t) = -\frac{\partial H}{\partial v} = -p_y(t) \Rightarrow p_v(t) = -c_1 t + c_2 = +t + c_2$$

$\because p_v(t_f) = 0 \Rightarrow c_2 = -t_f$

$$\Rightarrow p_v(t) = t - t_f \qquad p_y(t) = -1$$

$$\therefore \quad p_v(0) = -t_f \qquad p_y(0) = -1$$

Part b

$\because$ final time is arbitrary & H does not explicitly depend on t, $\mathcal{H} = 0 \ \forall \ t$

$$\mathcal{H}(t) = 0$$

$$\Rightarrow p_y(t) v(t) + p_v(t)\left(\frac{u(t)}{m(t)} - g\right) + p_m(t)\left(-b u(t)\right) = 0$$

$p_y(t) = -1$ $\quad p_v(t) = t - t_f$ $\quad u(0) = u_{max}$, $u(t_f) = 0$

@ $t = 0$

$$\Rightarrow -1 \, v_0 + (-t_f)\left(\frac{u_{max}}{m_0} - g\right) + p_m(0)\left(-b u_{max}\right) = 0$$

$$\Rightarrow \boxed{p_m(0) = \frac{-1}{b u_{max}}\left[v_0 + t_f\left(\frac{u_{max}}{m_0} - g\right)\right]}$$