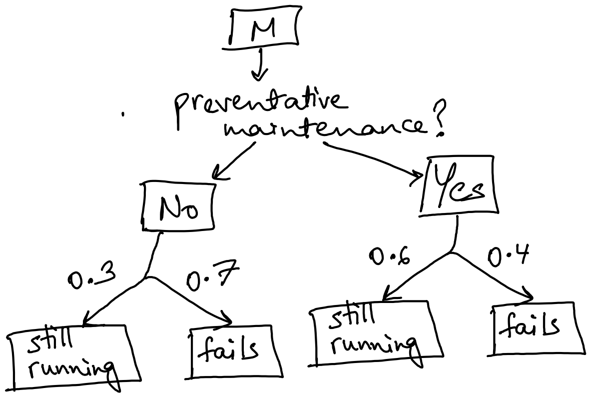
PS 3 Problem 1

Sunday, April 21, 2019 6:24 PM



Consider $J_4(x_4)$:= profit at end of week 4 when machine is in state x_4 at end of wk 4

$x_4 = 0 \longrightarrow$ broken

$x_4 = 1 \longrightarrow$ running

$J_4(0) = \$0$

$J_4(1) = \$100$

machine running at end of wk 3

$$J_3(1) = \$100 + \left(p(x_4=1)\,J_4(1) + p(x_4=0)\,J_4(0)\right)$$

Let $k=1$ if preventative maintenance done

$k=0$ if " " not done

$$= \$100 + \max_{k=0,1}\left(E(J_4)\right)$$

$$= \$100 + \max\Big(\underbrace{0}_{\text{no maintenance}} + (0.7)(0) + (0.3)(100),$$

$$\underbrace{-20}_{\text{maintenance cost}} + (0.4)(0) + (0.6)(100)\Big)$$

$$= 100 + \max(\underbrace{30}_{k=0}, \underbrace{40}_{k=1})$$

$\Rightarrow J_3(1) = \$140$ $\quad u^*(x_3=1) : k=1$ (do maintenance)

no profit for wk 3

$$J_3(0) = \$0 + \max_{f=0,1}\left(E(J_4)\right)$$

$f=0$: repair, $f=1$: replace

$$= \max\Big(-40 + 0.4\,(J_4(0)) + 0.6\,(J_4(1)),$$

$$-150 + 1.0\,(J_4(1))\Big)$$

$$= \max(-40 + 0 + 60,\ -150 + 100)$$

$\Rightarrow J_3(0) = \$20$ $\quad u^*(x_3=0) : f=0$ (repair)

$J_2(1) = \$100 + \max\big(0 + 0.7(J_3(0)) + 0.3(J_3(1)),$
$-20 + 0.4(J_3(0)) + 0.6(J_3(1))\big)$

$= 100 + \max(0 + 14 + 42,\ -20 + 8 + 84)$

$= 100 + \max(56, 72)$

$J_2(1) = \$172$ $\quad u^*(x_2=1) : k=1$ (do maintea)

$J_2(0) = \$0 + \max\big(-40 + 0.4(J_3(0)) + 0.6(J_3(1)),$
$-150 + 1.0(J_3(1))\big)$

$= \max(-40 + 8 + 84,\ -150 + 140)$

$= \max(52, -10)$

$J_2(0) = \$52$ $\quad u^*(x_2=0) : f=0$ (repair)

We start w/ new machine so it is guaranteed to not fail in 1st week.
$\Rightarrow x_1 = 0$ is impossible

$J_1(1) = \$100 + \max\big(0 + 0.7(J_2(0)) + 0.3(J_2(1)),$
$-20 + 0.4(J_2(0)) + 0.6(J_2(1))\big)$

$= 100 + \max(0 + 36.4 + 51.6,\ -20 + 20.8 + 103.2)$

$= 100 + \max(88, 104)$

$J_1(1) = \$204$ $\quad u^*(x_1=1) : k=1$ (do maintenance)

Optimal policy $\rightarrow u^*(x_1=1) = u^*(x_2=1) = u^*(x_3=1) : k=1$
i.e. Always do preventative maintenance if machine is running

($x_2=0$ impossible) $\quad u^*(x_2=0) = u^*(x_3=0) : f=0$
i.e. Always repair, don't replace a failed machine.

Maximized expected profit $= \$204$