PS 3 Problem 4

Sunday, April 21, 2019 9:21 PM

Part 1

$$\mathcal{T} = \left\{ (h,v,m) : \left| h - \frac{150{,}000}{c} \right| \leq \frac{500}{c}, \right.$$
$$\left| v - \frac{28}{c} \right| \leq \frac{2.8}{c},$$
$$\left. 250 \leq m \leq 500 \right\}$$

$$l(h,v,m) = \max\left(h - \frac{149500}{c},\ v - \frac{25.2}{c},\ m - 250,\ \frac{150{,}500}{c} - h,\ \frac{30.8}{c} - v,\ 500 - m \right)$$

$$l(h,v,m) \leq 0 \iff (h,v,m) \in \mathcal{T}$$

Part 2

$V(h,v,m,t)$ satisfies HJI PDE

$$\frac{\partial V}{\partial t}(h,v,m,t) + \min_{u \in U} \max_{d \in D} \nabla V(h,v,m,t)' f(h,v,m,u,d) = 0$$

$$\nabla V(h,v,m,t)' f(h,v,m,u,d)$$
$$= \begin{bmatrix} \frac{\partial V}{\partial h} & \frac{\partial V}{\partial v} & \frac{\partial V}{\partial m} \end{bmatrix} \begin{bmatrix} v \\ \frac{u}{m} - g + d \\ -bu \end{bmatrix}$$

$$= \frac{\partial V}{\partial h} v + \frac{\partial V}{\partial v}\left(\frac{u}{m} - g + d\right) + \frac{\partial V}{\partial m}(-bu)$$
$$= u\left(\frac{1}{m}\frac{\partial V}{\partial v} - b\frac{\partial V}{\partial m}\right) + \frac{\partial V}{\partial v} d + \frac{\partial V}{\partial h} v - \frac{\partial V}{\partial v} g$$

$\because$ u & d are linear terms here

$$\Rightarrow u^* = \arg\min_{u \in U} \left(\frac{1}{m}\frac{\partial V}{\partial v} - b\frac{\partial V}{\partial m}\right) u$$

$$\boxed{u^* = \begin{cases} 0 & \text{if } \frac{1}{m}\frac{\partial V}{\partial v} - b\frac{\partial V}{\partial m} \geq 0 \\ \frac{10000}{c} & \text{if } \frac{1}{m}\frac{\partial V}{\partial v} - b\frac{\partial V}{\partial m} < 0 \end{cases}}$$

$$d^* = \arg\max_{d \in D} \frac{\partial V}{\partial v} d$$

$$\boxed{d^* = \begin{cases} 1/c & \text{if } \frac{\partial V}{\partial v} \geq 0 \\ -1/c & \text{if } \frac{\partial V}{\partial v} < 0 \end{cases}}$$