PS 5 Problem 1 Part 1

Wednesday, May 8, 2019 10:54 AM

Objective: $\underset{u(\cdot)}{\text{maximize}}\ y(t_f)$

$\Rightarrow \min_{u(\cdot)} J = -y(t_f)$

$\therefore h = -y(t_f) \quad g = 0 \quad [J = h + \int g\,dt]$

dynamics: $\dot{y}(t) = v(t) \quad \dot{v}(t) = \frac{u(t)}{m(t)} - g$

$\dot{m}(t) = -bu(t)$

PART 1

$H = g + p_y \dot{y} + p_v \dot{v} + p_m \dot{m}$

$= 0 + p_y v + p_v\left(\frac{u}{m} - g\right) + p_m(-bu)$

(Omitting dep on t for clarity)

By NOC,

$\dot{p}_y = -\frac{\partial H}{\partial y} = 0 \quad \Rightarrow p_y = \text{const} = c_1$

$\dot{p}_v = -\frac{\partial H}{\partial v} = -p_y \quad \Rightarrow p_v = -c_1 t + c_2$

$\dot{p}_m = -\frac{\partial H}{\partial m} = -\left(-\frac{p_v u}{m^2}\right) \quad \Rightarrow \dot{p}_m(t) = \frac{(-c_1 t + c_2)u(t)}{m(t)^2}$