# PS 2 Problem 5

Wednesday, April 17, 2019    1:04 PM

$$\delta s_{k+1} = A_k \delta s_k + B_k \delta u_k$$

$$\text{Cost} = \frac{1}{2}(s_N - s^*)^T Q_N (s_N - s^*) + \sum_{k=0}^{N-1}\left(\frac{1}{2}(s_k - s^*)^T Q (s_k - s^*) + \frac{1}{2}u_k^T R u_k\right)$$

$$s_N - \bar{s} = \delta s_N$$
$$s_N = \bar{s} + \delta s_N$$
$$s_N - s^* = (\bar{s} - s^*) + \delta s_N$$

$$\frac{1}{2}(s_N - s^*)^T Q_N (s_N - s^*) = \frac{1}{2}\Big[(\bar{s}-s^*)^T Q_N (\bar{s} - s^*)^T + 2(\bar{s}-s^*)^T Q_N \delta s_N + (\delta s_N)^T Q_N (\delta s_N)\Big]$$

$$= \frac{1}{2}(\delta s_N)^T Q_N (\delta s_N) + (\bar{s}-s^*)^T Q_N \delta s_N + \underbrace{\frac{1}{2}(\bar{s}-s^*)^T Q_N (\bar{s}-s^*)}_{\text{this term doesn't depend on } \delta s_N}$$

Matching to code

$$= \frac{1}{2}(dx)^T Q_N (dx) + (qf)^T dx + \text{const}$$

$$\Rightarrow \boxed{qf = Q_N^T(\bar{s}-s^*)}$$

Similarly,

$$\because s_k = \bar{s} + \delta s_k$$

$$\frac{1}{2}(s_k - s^*)^T Q (s_k - s^*) = \frac{1}{2}(\delta s_k)^T Q (\delta s_k) + (\bar{s}-s^*)^T Q \delta s_k + \underbrace{\frac{1}{2}(\bar{s}-s^*)^T Q (\bar{s}-s^*)}_{\text{this term doesn't depend on } \delta s_k}$$

$$\text{Cost} = \overset{\text{quad}}{\cdots} + q' \delta s_k + \overset{\text{const}}{\cdots}$$

$$\therefore \text{linear term} = \boxed{q = Q^T(\bar{s}-s^*)}$$

$$u_k - \bar{u} = \delta u_k$$

$$u_k = \bar{u} + \delta u_k$$

$$\frac{1}{2} u_k^T R u_k = \frac{1}{2}(\bar{u} + \delta u_k)^T R (\bar{u} + \delta u_k)$$

$$= \underbrace{\frac{1}{2} \bar{u}^T R \bar{u}}_{\text{const wrt } \delta u_k} + \underbrace{\bar{u}^T R \delta u_k}_{\text{linear}} \underbrace{+ \frac{1}{2} \delta u_k^T R \delta u_k}_{\text{quadratic}}$$

cost = quad + linear + const

$$(R^T \bar{u})^T \delta u_k$$

$$\therefore \boxed{r = R^T \bar{u}}$$