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% Problem (OCP)_2 from Pset 6 - Trapezoidal Rule

clear all; clf; clc; format long;

% Parameters
global N; N = 50; % Choose here the number of discretization points
global mu; mu = 3.9915e14;
global rE; rE = 6378145;
global h0; h0 = 7500;
global D; D = 5e-3;
global b; b = 1e-3;
global uMax; uMax = 1.2e5;

% Scenario
global T; T = 258.;
global y0; y0 = 0.;
global v0; v0 = 0.;
global m0; m0 = 12000;
global mf; mf = 1000;

% Bound on the state: better conditioning the formulation (see below)
global yMax; yMax = 5e6;
global vMax; vMax = 2000;

% Since this optimal control problem is highly nonlinear, without
% an appropriate initialization direct methods unlikely converge.
% In the following lines, we provide such initialization by recalling the
% solution that we obtained for the simplified Goddard problem in the Pset 5.
% For the height, we just select a straight-line in time connecting y0 to
% 1.5e5 (which is more or less the final height that we found in Pset 5).
% For the velocity, we select the average  $v(t) = vMax/2$  in  $[0,tf]$ .
% For the mass, we select a straight-line in time between 0 and tSw, the
% switching time computed in Pset 5 (see below).
% Finally, for the control, we select the maximal value  $u(t) = uMax$  in  $[0,tf]$ .

% Finding what index NSw the time tSw corresponds to
global tSw; tSw = (m0 - mf)/(b*uMax);
h = (1.0*T/(1.0*N));
NSw = 0; indexFound = 0; iterator = 0;
while indexFound == 0
    % If iterator*h <= tSw < iteartor*h + h, then we have found the index
    if iterator*h <= tSw && tSw < (iterator + 1)*h
        NSw = iterator + 1;
        indexFound = 1;
    end
    iterator = iterator + 1;
end
uInit = zeros(N+1,1);
yInit = zeros(N+1,1);
vInit = 0.5*vMax*ones(N+1,1);
mInit = mf*ones(N+1,1);
% Initialization exxplained above
for i=1:N+1
    yInit(i) = y0*(1. - (i-1)*1.0/N) + 1.5e5*(i-1)*1.0/N;
    if (i-1) <= NSw
        mInit(i) = m0*(1. - (i-1)*1.0/NSw) + mf*(i-1)*1.0/NSw;
    end
    if i <= N
        if (i-1)*1.0*T/N < tSw
            uInit(i) = uMax;
        end
    end
end
% Initialization for fmincon
varInit = [yInit; vInit; mInit; uInit];

% Lower and upper bounds.

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lb = zeros(4*N+4,1); ub = uMax*ones(4*N+4,1); % For the control: 0 \le u \le uMax
ub(1:N+1) = yMax; % For the state y : 0 \le y \le yMax
ub(N+2:2*N+2) = vMax; % For the state v : 0 \le v \le vMax
lb(2*N+3:3*N+3) = mf; ub(2*N+3:3*N+3) = m0; % For the state m : mf \le v \le m0

% Solving the probleme via fmincon
options=optimoptions('fmincon','Display','iter','Algorithm','sqp','MaxFunEvals',10000,'MaxIter',10000);
% options=optimoptions('fmincon','Display','iter','Algorithm','sqp','MaxFunctionEvaluations',10000,'MaxIterations',10000);
[var,Fval,convergence] = fmincon(@cost,varInit,[],[],[],[],lb,ub,@constraint,options); % Solving the problem
convergence % = 1, good

% Collecting the solution. Note that var = [y;v;m;u]
y = var(1:N+1); v = var(N+2:2*N+2); m = var(2*N+3:3*N+3); u = var(3*N+4:4*N+4); % Collecting the solution
tState = zeros(N+1,1);
for i = 1:N
    tState(i+1) = tState(i) + (1.0*T/(1.0*N));
end
t = zeros(N+1,1);
for i = 1:N
    t(i+1) = t(i) + (1.0*T/(1.0*N));
end

% Plotting
% subplot(221); plot(tState,y,'linewidth',3);
% title('\textbf{a} Height','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{\$t\$} \ \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{\$h\$} \ \ \textbf{(m)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(222); plot(tState,v,'linewidth',3) ;
% title('\textbf{b} Velocity','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{\$t\$} \ \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{\$v\$} \ \ \textbf{(m/s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(223); plot(tState,m,'linewidth',3) ;
% title('\textbf{c} Mass','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{\$t\$} \ \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{\$m\$} \ \ \textbf{(kg)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(224); plot(t,u,'linewidth',3);
% title('\textbf{d} Optimal Control','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{\$t\$} \ \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{\$u\$}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;

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Iter	F-count				Norm of First-order	
		f(x)	Feasibility	Steplength	step	optimality
0	205	-1.500000e+05	2.160e+03			1.000e+00
1	410	-1.383047e+05	3.602e+00	1.000e+00	1.075e+05	6.461e+04
2	617	-1.418327e+05	4.102e+00	7.000e-01	5.057e+04	6.822e+04
3	822	-1.483241e+05	3.523e+00	1.000e+00	4.833e+04	9.815e+04
4	1027	-1.500931e+05	5.709e-01	1.000e+00	1.475e+04	1.013e+05
5	1232	-1.502196e+05	3.835e-03	1.000e+00	1.127e+03	1.014e+05
6	1437	-1.502204e+05	1.564e-07	1.000e+00	7.180e+00	5.868e+02
7	1642	-1.502212e+05	1.856e-07	1.000e+00	7.238e+00	2.042e+02
8	1847	-1.502250e+05	4.626e-06	1.000e+00	3.619e+01	2.042e+02
9	2052	-1.502441e+05	1.158e-04	1.000e+00	1.810e+02	2.028e+02
10	2257	-1.503397e+05	2.912e-03	1.000e+00	9.065e+02	1.650e+02
11	2462	-1.507327e+05	4.969e-02	1.000e+00	3.724e+03	1.650e+02
12	2667	-1.507329e+05	5.195e-07	1.000e+00	3.809e+00	1.650e+02
13	2872	-1.507331e+05	7.126e-08	1.000e+00	1.418e+00	1.650e+02

14	3077	-1.507340e+05	2.389e-07	1.000e+00	5.171e+00	1.540e+02
15	3282	-1.507346e+05	3.874e-07	1.000e+00	3.490e+00	1.169e+02
16	3487	-1.507348e+05	6.257e-08	1.000e+00	1.416e+00	9.890e+01
17	3692	-1.507349e+05	1.723e-09	1.000e+00	8.271e-01	8.842e+01
18	3897	-1.507354e+05	4.257e-08	1.000e+00	4.100e+00	8.841e+01
19	4102	-1.507357e+05	1.204e-08	1.000e+00	1.971e+00	7.893e+01
20	4307	-1.507358e+05	9.076e-09	1.000e+00	1.576e+00	6.548e+01
21	4512	-1.507364e+05	1.184e-07	1.000e+00	5.653e+00	6.349e+01
22	4717	-1.507393e+05	2.959e-06	1.000e+00	2.827e+01	6.349e+01
23	4922	-1.507538e+05	7.402e-05	1.000e+00	1.413e+02	6.348e+01
24	5127	-1.508264e+05	1.857e-03	1.000e+00	7.072e+02	6.344e+01
25	5332	-1.511901e+05	4.711e-02	1.000e+00	3.543e+03	5.901e+01
26	5537	-1.522507e+05	4.113e-01	1.000e+00	1.029e+04	5.892e+01
27	5742	-1.522599e+05	4.546e-06	1.000e+00	6.591e+01	5.893e+01
28	5947	-1.522606e+05	1.423e-07	1.000e+00	5.303e+00	5.893e+01
29	6152	-1.522620e+05	1.256e-07	1.000e+00	1.208e+01	5.893e+01
30	6357	-1.522620e+05	4.735e-09	1.000e+00	3.541e-01	5.071e+01

Iter	F-count	f(x)	Feasibility	Steplength	Norm of First-order	
					step	optimality
31	6562	-1.522621e+05	3.479e-09	1.000e+00	1.029e+00	4.741e+01
32	6767	-1.522623e+05	1.014e-08	1.000e+00	1.640e+00	3.837e+01
33	6972	-1.522626e+05	4.522e-08	1.000e+00	3.428e+00	3.769e+01
34	7177	-1.522642e+05	1.131e-06	1.000e+00	1.714e+01	3.769e+01
35	7382	-1.522723e+05	2.830e-05	1.000e+00	8.570e+01	3.769e+01
36	7587	-1.523127e+05	7.087e-04	1.000e+00	4.286e+02	3.768e+01
37	7792	-1.525151e+05	1.787e-02	1.000e+00	2.145e+03	3.762e+01
38	7997	-1.528313e+05	4.396e-02	1.000e+00	3.340e+03	3.752e+01
39	8202	-1.528323e+05	2.736e-08	1.000e+00	7.553e+00	3.752e+01
40	8407	-1.528324e+05	1.582e-09	1.000e+00	6.176e-01	3.140e+01
41	8612	-1.528327e+05	3.925e-08	1.000e+00	3.088e+00	3.140e+01
42	8817	-1.528343e+05	9.832e-07	1.000e+00	1.544e+01	3.140e+01
43	9022	-1.528421e+05	2.460e-05	1.000e+00	7.721e+01	3.140e+01
44	9227	-1.528809e+05	6.055e-04	1.000e+00	3.828e+02	3.139e+01
45	9432	-1.528811e+05	1.684e-08	1.000e+00	1.984e+00	3.139e+01
46	9637	-1.528820e+05	4.203e-07	1.000e+00	9.676e+00	3.139e+01
47	9842	-1.528864e+05	1.051e-05	1.000e+00	4.838e+01	3.139e+01
48	10047	-1.529083e+05	2.630e-04	1.000e+00	2.419e+02	3.138e+01
49	10252	-1.530181e+05	6.610e-03	1.000e+00	1.210e+03	3.132e+01
50	10457	-1.535694e+05	1.696e-01	1.000e+00	6.069e+03	3.104e+01
51	10662	-1.539207e+05	6.877e-02	1.000e+00	3.811e+03	3.087e+01
52	10867	-1.539221e+05	6.668e-08	1.000e+00	1.013e+01	3.087e+01
53	11072	-1.539221e+05	2.402e-10	1.000e+00	2.677e-01	3.087e+01
54	11277	-1.539222e+05	3.561e-09	1.000e+00	1.339e+00	3.087e+01
55	11482	-1.539228e+05	8.981e-08	1.000e+00	6.693e+00	3.087e+01
56	11687	-1.539239e+05	3.120e-07	1.000e+00	1.227e+01	3.087e+01
57	11892	-1.539239e+05	1.071e-09	1.000e+00	5.357e-01	3.087e+01
58	12097	-1.539241e+05	2.656e-08	1.000e+00	2.679e+00	3.087e+01
59	12302	-1.539250e+05	6.632e-07	1.000e+00	1.339e+01	3.087e+01
60	12507	-1.539295e+05	1.658e-05	1.000e+00	6.696e+01	3.086e+01

Iter	F-count	f(x)	Feasibility	Steplength	Norm of First-order	
					step	optimality
61	12712	-1.539521e+05	4.151e-04	1.000e+00	3.348e+02	3.084e+01
62	12917	-1.540654e+05	1.044e-02	1.000e+00	1.675e+03	3.071e+01
63	13122	-1.546322e+05	2.676e-01	1.000e+00	8.370e+03	3.006e+01
64	13327	-1.555254e+05	6.932e-01	1.000e+00	1.305e+04	2.899e+01
65	13532	-1.555348e+05	1.843e-06	1.000e+00	8.475e+01	2.900e+01
66	13737	-1.555349e+05	1.237e-07	1.000e+00	9.483e-01	2.900e+01
67	13942	-1.555354e+05	4.536e-07	1.000e+00	3.057e+00	2.900e+01
68	14147	-1.555355e+05	4.119e-08	1.000e+00	9.729e-01	2.900e+01
69	14352	-1.555357e+05	1.850e-07	1.000e+00	1.643e+00	2.335e+01
70	14557	-1.555358e+05	2.874e-09	1.000e+00	7.418e-01	1.829e+01
71	14762	-1.555361e+05	6.435e-08	1.000e+00	3.709e+00	1.828e+01
72	14967	-1.555377e+05	1.638e-06	1.000e+00	1.855e+01	1.828e+01
73	15172	-1.555456e+05	4.105e-05	1.000e+00	9.273e+01	1.823e+01
74	15377	-1.555510e+05	1.902e-05	1.000e+00	6.303e+01	1.820e+01
75	15582	-1.555546e+05	2.011e-05	1.000e+00	2.248e+02	1.820e+01
76	15787	-1.555727e+05	5.030e-04	1.000e+00	1.124e+03	1.820e+01

77	15992	-1.556376e+05	6.578e-03	1.000e+00	4.055e+03	1.820e+01
78	16197	-1.556374e+05	1.322e-09	1.000e+00	6.623e-01	1.820e+01
79	16402	-1.556374e+05	8.549e-11	1.000e+00	1.407e-01	1.820e+01
80	16607	-1.556375e+05	1.488e-09	1.000e+00	7.035e-01	1.820e+01
81	16812	-1.556377e+05	3.724e-08	1.000e+00	3.518e+00	1.820e+01
82	17017	-1.556388e+05	9.350e-07	1.000e+00	1.759e+01	1.820e+01
83	17222	-1.556442e+05	2.337e-05	1.000e+00	8.794e+01	1.820e+01
84	17427	-1.556712e+05	5.850e-04	1.000e+00	4.397e+02	1.820e+01
85	17632	-1.558062e+05	1.472e-02	1.000e+00	2.199e+03	1.820e+01
86	17837	-1.560135e+05	3.516e-02	1.000e+00	3.374e+03	1.820e+01
87	18042	-1.560136e+05	9.209e-09	1.000e+00	3.639e+00	1.820e+01
88	18247	-1.560137e+05	1.456e-10	1.000e+00	2.128e-01	1.820e+01
89	18452	-1.560137e+05	3.912e-09	1.000e+00	1.064e+00	1.820e+01
90	18657	-1.560140e+05	9.877e-08	1.000e+00	5.319e+00	1.820e+01

Iter	F-count	f(x)	Feasibility	Steplength	Norm of First-order	
					step	optimality
91	18862	-1.560154e+05	2.471e-06	1.000e+00	2.660e+01	1.820e+01
92	19067	-1.560223e+05	6.179e-05	1.000e+00	1.330e+02	1.820e+01
93	19272	-1.560572e+05	1.548e-03	1.000e+00	6.649e+02	1.820e+01
94	19477	-1.562312e+05	3.907e-02	1.000e+00	3.324e+03	1.820e+01
95	19682	-1.570949e+05	1.011e+00	1.000e+00	1.650e+04	1.820e+01
96	19887	-1.569294e+05	3.987e-02	1.000e+00	3.160e+03	1.820e+01
97	20092	-1.569213e+05	9.737e-05	1.000e+00	1.574e+02	1.779e+01
98	20297	-1.569213e+05	6.041e-10	1.000e+00	3.814e-01	1.780e+01
99	20502	-1.569213e+05	1.145e-10	1.000e+00	2.031e-01	1.779e+01
100	20707	-1.569213e+05	2.918e-09	1.000e+00	1.015e+00	1.779e+01
101	20912	-1.569215e+05	7.216e-08	1.000e+00	5.076e+00	1.778e+01
102	21117	-1.569222e+05	1.807e-06	1.000e+00	2.537e+01	1.770e+01
103	21322	-1.569260e+05	4.493e-05	1.000e+00	1.266e+02	1.734e+01
104	21527	-1.569444e+05	1.096e-03	1.000e+00	6.254e+02	1.553e+01
105	21732	-1.570279e+05	2.425e-02	1.000e+00	2.955e+03	1.022e+01
106	21937	-1.572442e+05	2.409e-01	1.000e+00	9.812e+03	2.595e+01
107	22142	-1.571377e+05	1.251e-02	1.000e+00	1.920e+03	2.859e+01
108	22347	-1.571363e+05	5.459e-04	1.000e+00	7.539e+02	1.567e+01
109	22552	-1.571358e+05	6.095e-05	1.000e+00	2.531e+02	1.515e+00
110	22757	-1.571357e+05	1.390e-08	1.000e+00	2.179e+00	8.903e-01
111	22962	-1.571357e+05	2.620e-09	1.000e+00	1.592e+00	8.602e-01
112	23167	-1.571357e+05	1.912e-08	1.000e+00	4.278e+00	8.508e-01
113	23372	-1.571357e+05	5.591e-08	1.000e+00	7.316e+00	8.346e-01
114	23577	-1.571357e+05	9.721e-08	1.000e+00	9.637e+00	8.131e-01
115	23782	-1.571357e+05	3.434e-07	1.000e+00	1.809e+01	7.719e-01
116	23987	-1.571357e+05	7.926e-07	1.000e+00	2.754e+01	7.173e-01
117	24192	-1.571357e+05	2.168e-06	1.000e+00	4.574e+01	8.610e-01
118	24397	-1.571358e+05	5.312e-06	1.000e+00	7.271e+01	1.084e+00
119	24602	-1.571358e+05	1.320e-05	1.000e+00	1.199e+02	1.432e+00
120	24807	-1.571360e+05	2.628e-05	1.000e+00	1.925e+02	1.908e+00

Iter	F-count	f(x)	Feasibility	Steplength	Norm of First-order	
					step	optimality
121	25012	-1.571365e+05	3.411e-05	1.000e+00	3.082e+02	2.442e+00
122	25217	-1.571372e+05	1.147e-04	1.000e+00	4.507e+02	2.603e+00
123	25422	-1.571379e+05	2.233e-04	1.000e+00	5.254e+02	2.255e+00
124	25627	-1.571381e+05	1.041e-04	1.000e+00	3.927e+02	1.217e+00
125	25832	-1.571380e+05	3.013e-05	1.000e+00	1.725e+02	2.977e-01
126	26037	-1.571380e+05	5.790e-07	1.000e+00	4.400e+01	7.066e-02
127	26242	-1.571380e+05	1.965e-08	1.000e+00	5.319e+00	4.792e-02
128	26447	-1.571380e+05	9.919e-11	1.000e+00	4.016e-01	4.759e-02
129	26652	-1.571380e+05	8.151e-11	1.000e+00	3.166e-01	4.723e-02
130	26857	-1.571380e+05	3.601e-10	1.000e+00	7.526e-01	4.687e-02
131	27062	-1.571380e+05	1.022e-09	1.000e+00	1.121e+00	4.555e-02
132	27267	-1.571380e+05	2.996e-09	1.000e+00	1.989e+00	4.343e-02
133	27472	-1.571380e+05	4.901e-09	1.000e+00	2.394e+00	4.024e-02
134	27677	-1.571380e+05	2.455e-08	1.000e+00	5.542e+00	5.409e-02
135	27882	-1.571380e+05	3.952e-08	1.000e+00	7.083e+00	7.049e-02
136	28087	-1.571380e+05	1.242e-07	1.000e+00	1.227e+01	1.210e-01
137	28292	-1.571380e+05	3.776e-07	1.000e+00	1.999e+01	2.099e-01
138	28497	-1.571380e+05	8.627e-07	1.000e+00	3.034e+01	3.407e-01
139	28702	-1.571380e+05	1.974e-06	1.000e+00	4.825e+01	5.104e-01

140	28907	-1.571380e+05	5.414e-06	1.000e+00	7.498e+01	6.285e-01
141	29112	-1.571381e+05	1.520e-05	1.000e+00	9.405e+01	5.331e-01
142	29317	-1.571381e+05	8.756e-06	1.000e+00	7.580e+01	3.720e-01
143	29522	-1.571381e+05	7.624e-07	1.000e+00	3.068e+01	1.273e-01
144	29727	-1.571381e+05	4.451e-08	1.000e+00	7.298e+00	1.982e-02
145	29932	-1.571381e+05	1.324e-09	1.000e+00	9.539e-01	2.543e-03
146	30137	-1.571381e+05	2.319e-11	1.000e+00	7.672e-02	2.442e-03
147	30139	-1.571381e+05	2.319e-11	4.900e-01	1.615e-02	1.018e-07

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible directions, to within the default value of the function tolerance, and constraints are satisfied to within the default value of the constraint tolerance.

convergence =

1