PS 1 Problem 2

Thursday, April 4, 2019 5:05 PM

$$\min \quad x_1 + x_2$$
$$\text{st} \quad x_1^2 + x_2^2 = 2$$

$$\mathcal{L} = x_1 + x_2 + \lambda(x_1^2 + x_2^2 - 2)$$

Let x^*, λ^* be local min^m & Lagrange multiplier

$$\nabla_x \mathcal{L} = \begin{bmatrix} 1 + 2\lambda x_1 \\ 1 + 2\lambda x_2 \end{bmatrix} = 0$$

$$\Rightarrow x_1 = -\frac{1}{2\lambda} = x_2$$

$$\nabla_\lambda \mathcal{L} = x_1^2 + x_2^2 - 2 = 0$$

$$\Rightarrow \frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} = 2$$

$$\Rightarrow 4\lambda^2 = 1$$

$$\Rightarrow \lambda = +1/2 \quad \text{or} \quad -1/2$$

$$x_1 = x_2 = -1 \qquad x_1 = x_2 = 1$$

Candidates for optimality $\rightarrow$ $x_1 = 1, x_2 = 1$
$\rightarrow$ $x_1 = -1, x_2 = -1$

At $(1,1)$ $\quad x_1 + x_2 = 2$

At $(-1,-1)$ $\quad x_1 + x_2 = -2$

$\boxed{\therefore x_1 = 1 \;\; x_2 = 1 \text{ is the unique global max}^m \;\; \& \;\; x_1 = -1 \;\; x_2 = -1 \text{ " " " " min}^m}$

They are unique $\because$ only 2 candidate pts.