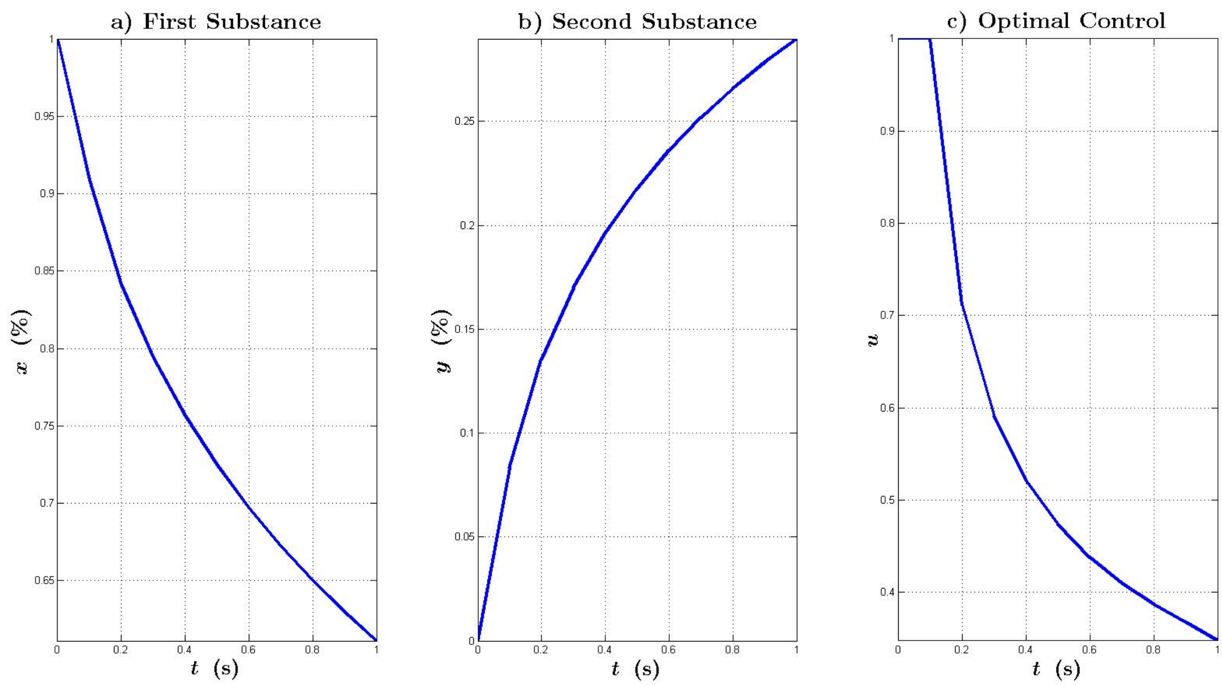
Problem 2

Part a

Following is the plot for the time evolution of the states and control subject to the trapezoidal rule direct method. Code attached on following pages.



```matlab
% Problem (OCP)_1 from Pset 6

clear all; clf; clc; format long;

% Parameters and scenario
global N; N = 10;
global T; T = 1.;
global uMax; uMax = 1.0;
global x0; x0 = 1.;
global y0; y0 = 0.;

% Random initialization
uInit = 0.5*uMax*ones(N+1,1);
xInit = ones(N+1,1); yInit = zeros(N+1,1);
varInit = [xInit; yInit; uInit];

% Lower and upper bounds.
lb = zeros(3*N+3,1); ub = uMax*ones(3*N+3,1); % For the control: 0 \le u \le uMax
ub(1:N+1) = 1.; % For the state x : 0 \le x \le 1
ub(N+2:2*N+2) = 1.; % For the state y : 0 \le y \le 1

% Solving the problme via fmincon
options=optimoptions('fmincon','Display','iter','Algorithm','sqp','MaxFunEvals',100000,'MaxIter',10000);
% options=optimoptions('fmincon','Display','iter','Algorithm','sqp','MaxFunctionEvaluations',100000,'MaxIterations',10000);
[var,Fval,convergence] = fmincon(@cost,varInit,[],[],[],[],lb,ub,@constraint,options); % Solving the problem
convergence % = 1, good

% Collecting the solution. Note that var = [x;y;u]
x = var(1:N+1); y = var(N+2:2*N+2); u = var(2*N+3:3*N+3);
tState = zeros(N+1,1);
t = zeros(N+1,1);
for i = 1:N
    t(i+1) = t(i) + (1.0*T/(1.0*N));
end

% Plotting
fprintf('Optimal Final Quantity for the Second Substance = %f\n\n',Fval);
% subplot(131); plot(t,x,'linewidth',3);
% title('\textbf{a) First Substance}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$x$} \ \textbf{(\%)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(132); plot(t,y,'linewidth',3) ;
% title('\textbf{b) Second Substance}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$y$} \ \textbf{(\%)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(133); plot(t,u,'linewidth',3);
% title('\textbf{c) Optimal Control}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$u$}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
```

```
                                                          Norm of First-order
 Iter F-count            f(x) Feasibility  Steplength        step  optimality
    0      34   -0.000000e+00   5.000e-02                           1.000e+00
    1      68   -2.355430e-01   6.015e-03   1.000e+00   8.404e-01   2.996e-01
    2     102   -2.595025e-01   1.487e-04   1.000e+00   9.635e-02   2.447e-02
    3     136   -2.691728e-01   4.202e-04   1.000e+00   1.658e-01   2.300e-02
    4     171   -2.802969e-01   2.296e-04   1.000e+00   4.903e-01   1.343e-02
```

```
    5     205   -2.814745e-01   9.095e-05   1.000e+00   7.340e-02   1.337e-02
    6     239   -2.848175e-01   6.548e-04   1.000e+00   1.986e-01   1.322e-02
    7     273   -2.865277e-01   2.701e-04   1.000e+00   1.551e-01   1.946e-02
    8     307   -2.875543e-01   2.145e-04   1.000e+00   1.164e-01   9.424e-03
    9     341   -2.873440e-01   5.010e-05   1.000e+00   5.320e-02   7.636e-03
   10     375   -2.877465e-01   4.403e-05   1.000e+00   5.583e-02   7.914e-03
   11     409   -2.887661e-01   9.233e-05   1.000e+00   1.204e-01   1.138e-02
   12     443   -2.893493e-01   1.236e-04   1.000e+00   1.105e-01   1.082e-02
   13     477   -2.894912e-01   2.966e-05   1.000e+00   3.790e-02   7.160e-03
   14     511   -2.896184e-01   2.723e-05   1.000e+00   3.681e-02   3.220e-03
   15     545   -2.896203e-01   5.757e-06   1.000e+00   1.866e-02   2.536e-03
   16     579   -2.896686e-01   7.119e-06   1.000e+00   1.912e-02   2.495e-03
   17     613   -2.896967e-01   9.225e-06   1.000e+00   2.050e-02   1.997e-03
   18     647   -2.896969e-01   4.026e-06   1.000e+00   1.278e-02   1.219e-03
   19     681   -2.896988e-01   9.327e-07   1.000e+00   7.252e-03   1.153e-03
   20     715   -2.897080e-01   1.363e-06   1.000e+00   1.031e-02   1.593e-03
   21     749   -2.897188e-01   2.664e-06   1.000e+00   1.341e-02   1.973e-03
   22     783   -2.897193e-01   4.064e-07   1.000e+00   4.369e-03   1.452e-03
   23     817   -2.897202e-01   3.408e-07   1.000e+00   4.207e-03   2.091e-04
   24     851   -2.897196e-01   1.149e-08   1.000e+00   7.273e-04   1.729e-04
   25     885   -2.897203e-01   1.137e-07   1.000e+00   2.768e-03   3.421e-04
   26     919   -2.897203e-01   4.886e-08   1.000e+00   1.673e-03   2.749e-04
   27     953   -2.897203e-01   2.363e-08   1.000e+00   1.112e-03   1.060e-04
   28     987   -2.897203e-01   1.187e-09   1.000e+00   2.958e-04   3.901e-05
   29    1021   -2.897203e-01   9.043e-10   1.000e+00   2.835e-04   4.250e-05
   30    1055   -2.897203e-01   1.477e-09   1.000e+00   3.157e-04   4.239e-05

                                                          Norm of First-order
 Iter F-count            f(x) Feasibility  Steplength        step  optimality
   31    1089   -2.897203e-01   1.560e-09   1.000e+00   2.744e-04   3.140e-05
   32    1123   -2.897203e-01   4.151e-10   1.000e+00   1.342e-04   1.208e-05
   33    1157   -2.897203e-01   3.489e-11   1.000e+00   4.701e-05   6.886e-06
   34    1191   -2.897203e-01   3.684e-11   1.000e+00   4.258e-05   5.896e-06
   35    1225   -2.897203e-01   3.106e-11   1.000e+00   4.442e-05   5.937e-06
   36    1259   -2.897203e-01   1.542e-11   1.000e+00   3.203e-05   2.789e-06
   37    1293   -2.897203e-01   2.077e-12   1.000e+00   1.215e-05   9.103e-07

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.




convergence =

     1

Optimal Final Quantity for the Second Substance = -0.289720
```

```
% Function providing equality and inequality constraints
% ceq(var) = 0 and c(var) \le 0

function [c,ceq] = constraint(var)

global N;
global T;

global x0;
global y0;

% Put here constraint inequalities
c = [];

% Note that var = [x;y;u]
x = var(1:N+1); y = var(N+2:2*N+2); u = var(2*N+3:3*N+3);

% Computing dynamical constraints via the trapezoidal rule
h = 1.0*T/(1.0*N);
for i = 1:N
    % Provide here dynamical constraints via the trapeziodal formula
    [xDyn_i,yDyn_i] = fDyn(x(i),y(i),u(i));
    [xDyn_ii,yDyn_ii] = fDyn(x(i+1),y(i+1),u(i+1));
    ceq(i) = x(i+1) - x(i) - h*(xDyn_i + xDyn_ii)/2;
    ceq(i+N) = y(i+1) - y(i) - h*(yDyn_i + yDyn_ii)/2;
end

% Put here initial conditions
ceq(1+2*N) = x(1) - 1;
ceq(2+2*N) = y(1) - 0;
```

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```
% Cost of the problem

function c = cost(var)

global N;

% Note that var = [x;y;u]
x = var(1:N+1); y = var(N+2:2*N+2); u = var(2*N+3:3*N+3);

% Put here the cost
c = -y(end);
```

```
% Dynamics of the problem

function [xDyn,yDyn] = fDyn(x,y,u)

% Put here the dynamics
xDyn = -x*u + y*u^2;
yDyn = x*u - 3*y*u^2;
```

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Part b

The final cost at $t_f = 1$ is given by -0.289720 (see output of collocation function). Cost $= -y_f^*$. Therefore, $\boldsymbol{y_f^* = 28.97}\%$.