Problem Set 3 Question 3

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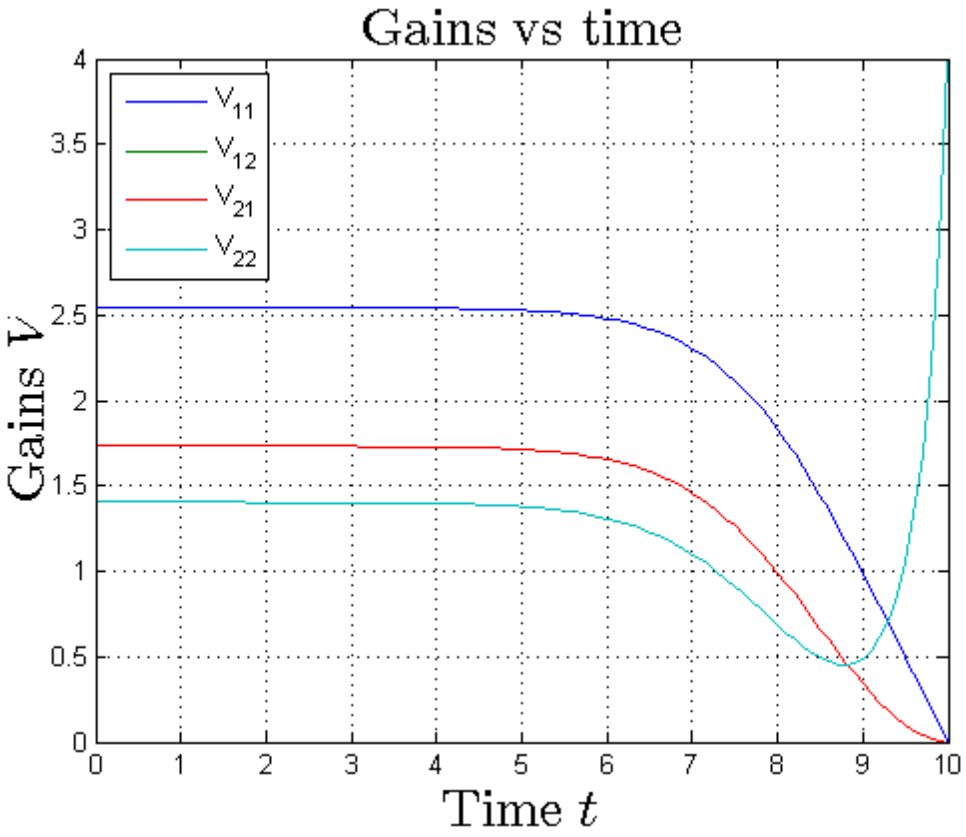
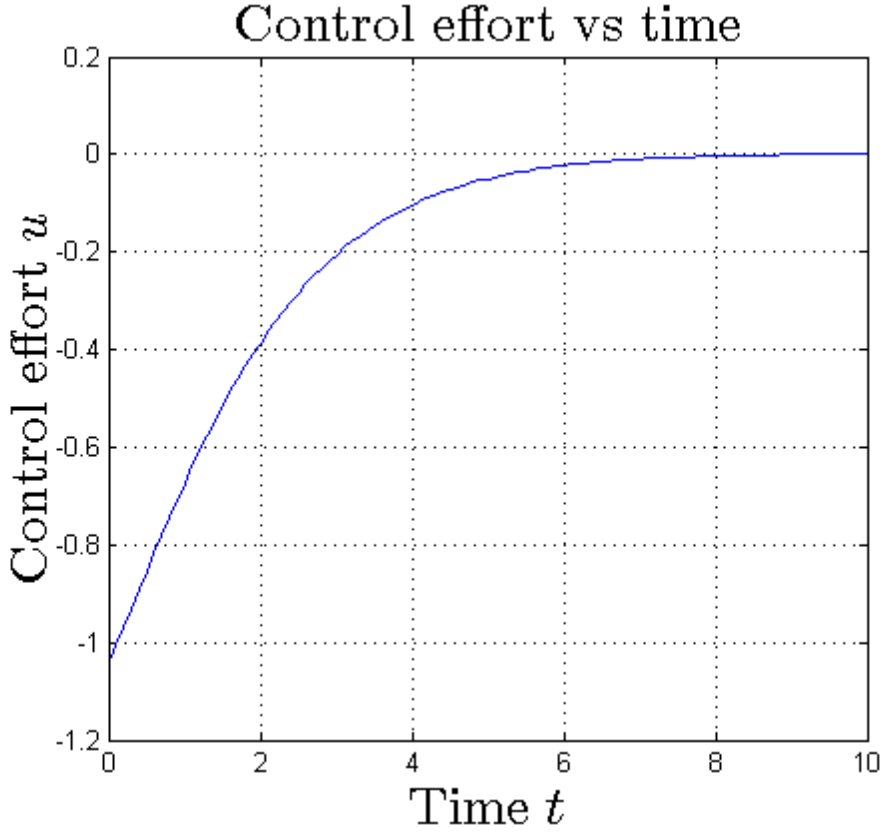
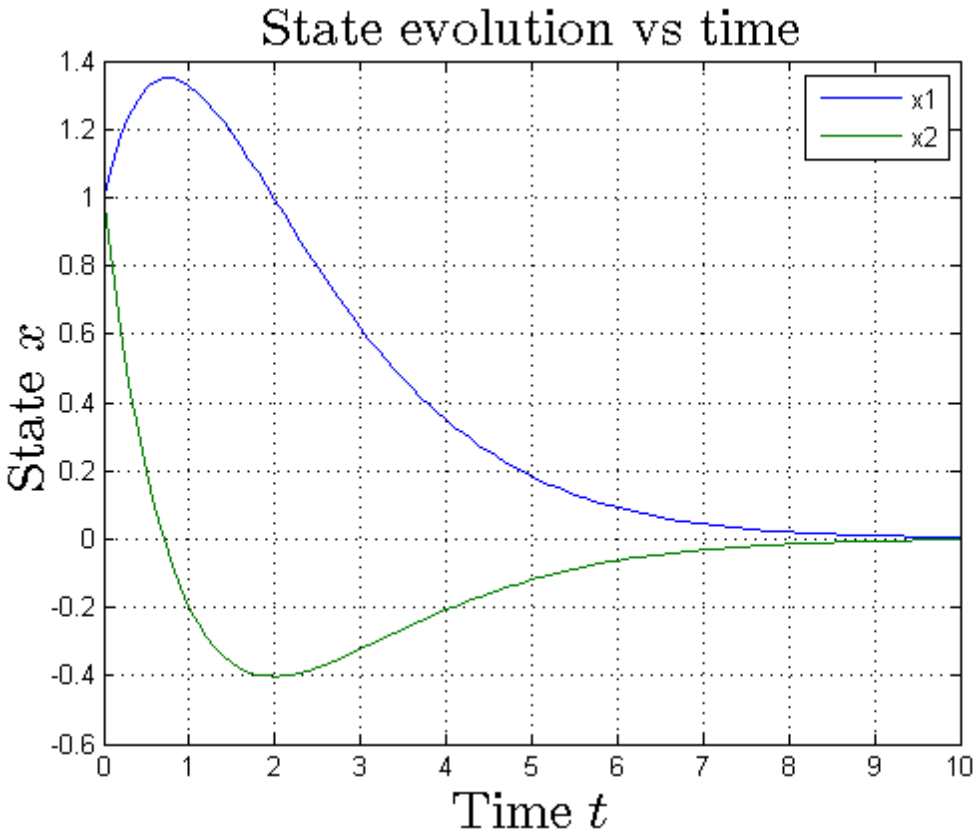
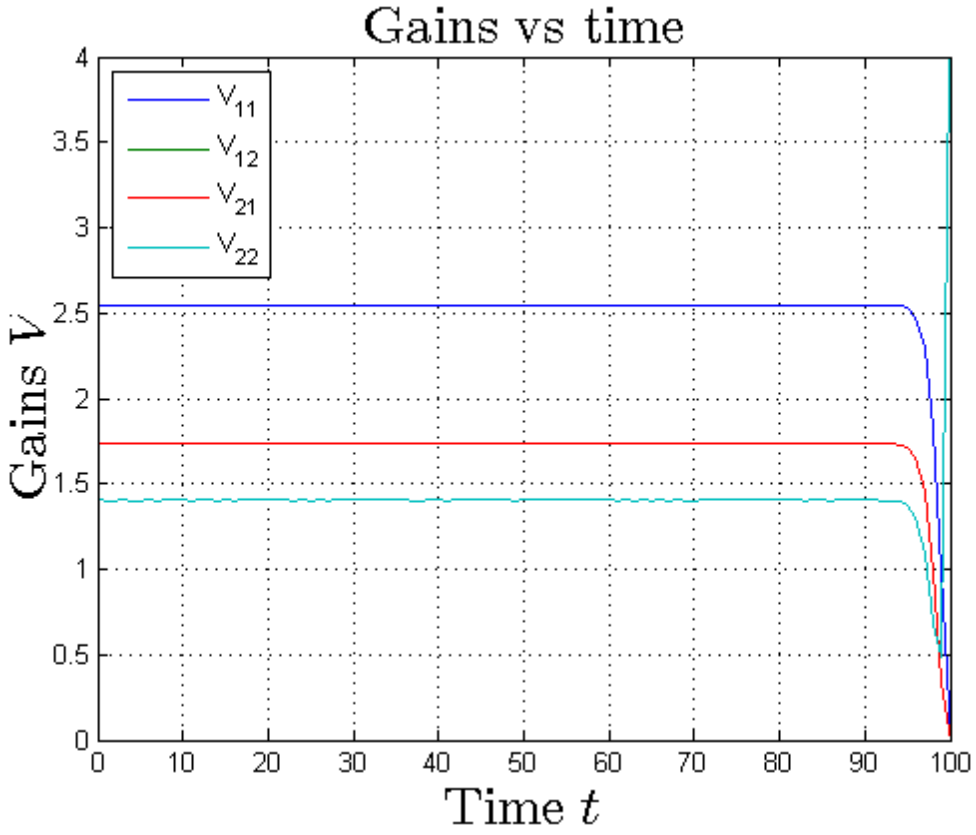
```
clc
clear all
close all

q = 1;
r = 3;
h = 4;
tf = 10;

A = [0 1; 0 -1];
B = [0;1];
Qf = [0 0; 0 h];
Q = [q 0; 0 0];
R = [r];

% initialize V[tf] as Qf
V_final = Qf(:);

for tf =[10,100]
    % Time in reverse
    dt = tf/100;
    rt = tf:-dt:0;

    [T, V] = ode45(@(t,V)mRiccati(t, V, A, B, Q, R), rt, V_final);

    [m, n] = size(V);
    VV = mat2cell(V, ones(m,1), n);
    fh_reshape = @(V)reshape(V,size(A));
    VV = cellfun(fh_reshape,VV,'UniformOutput',false);
    % Method inspired by https://www.mathworks.com/matlabcentral/answers/94722
    % -how-can-i-solve-the-matrix-riccati-differential-equation-within-matlab
    revV = flip(VV);

    uStar = zeros(size(revV,1),1);
    x = zeros(2,size(revV,1));
    x(:,1) = [1;1];
    for i = 1: size(revV,1)
        uStar(i) = -(R\(B.'))*revV{i}*x(:,i);
        if i ~= size(revV,1)
            dxdt = A*x(:,i) + B*uStar(i);
            x(:,i+1) = x(:,i) + dxdt * dt;
        end
    end

    forwardTime = fliplr(rt);

    matVV=cat(3, revV{:});

    V1 = matVV(1,1,:);
    V1 = V1(:);
    V2 = matVV(1,2,:);
    V2 = V2(:);
    V3 = matVV(2,1,:);
    V3 = V3(:);
```

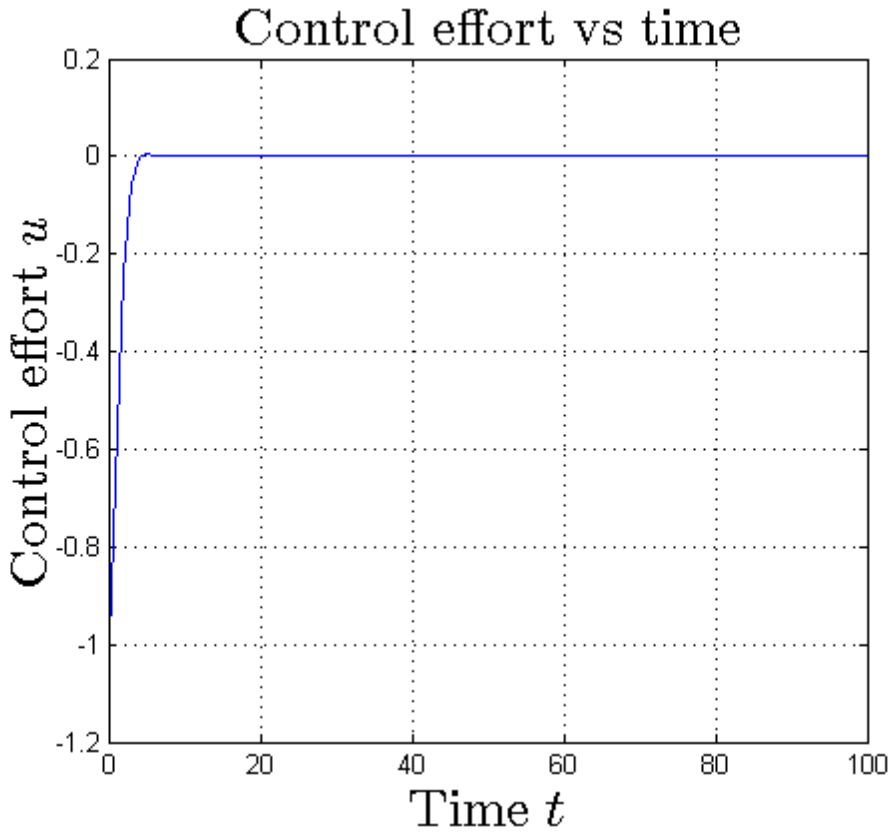
```
    V4 = matVV(2,2,:);
    V4 = V4(:);
    figure
    plot(forwardTime,V1,forwardTime,V2,forwardTime,V3,forwardTime,V4)
    title('Gains vs time','Interpreter','latex','FontSize',20)
    xlabel('Time $$t$$','Interpreter','latex','FontSize',20)
    ylabel('Gains $$V$$','Interpreter','latex','FontSize',20)
    legend({'V_{11}','V_{12}','V_{21}','V_{22}'},'location','northwest');
    grid on

    figure
    plot(forwardTime,uStar)
    title('Control effort vs time','Interpreter','latex','FontSize',20)
    xlabel('Time $$t$$','Interpreter','latex','FontSize',20)
    ylabel('Control effort $$u$$','Interpreter','latex','FontSize',20)
    grid on

    figure
    plot(forwardTime,x)
    title('State evolution vs time','Interpreter','latex','FontSize',20)
    xlabel('Time $$t$$','Interpreter','latex','FontSize',20)
    ylabel('State $$x$$','Interpreter','latex','FontSize',20)
    legend('x1','x2');
    grid on
end
```

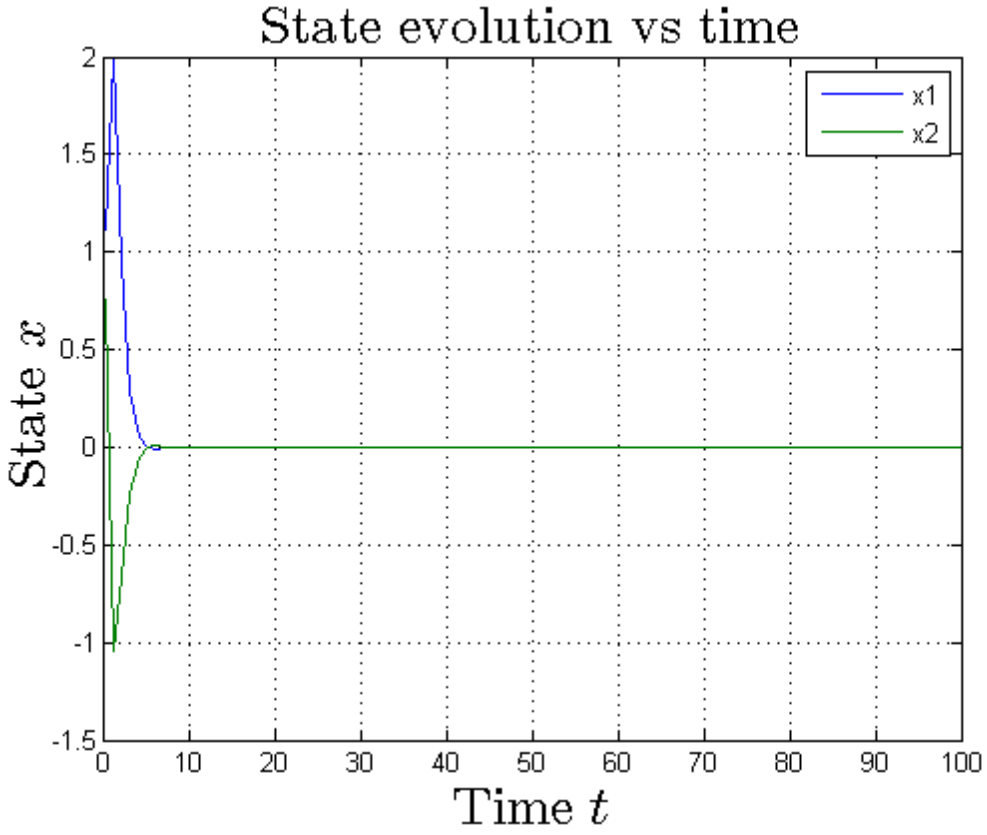

Gains vs time
V_{11}
V_{12}
V_{21}
V_{22}
Gains V
Time t
0
0.5
1
1.5
2
2.5
3
3.5
4
0
1
2
3
4
5
6
7
8
9
10


Control effort vs time
Control effort u
Time t
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
2
4
6
8
10


State evolution vs time
x1
x2
State x
Time t
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
1
2
3
4
5
6
7
8
9
10


Gains vs time
V_{11}
V_{12}
V_{21}
V_{22}
Gains V
Time t
4
3.5
3
2.5
2
1.5
1
0.5
0
0
10
20
30
40
50
60
70
80
90
100





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