Part 3

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$$\phi(t) = \frac{p_v(t)}{m(t)} - p_m(t)\, b$$

$$\Rightarrow \dot{\phi}(t) = \frac{\dot{p}_v(t)\, m(t) - \dot{m}(t) p_v(t)}{m(t)^2} - b\, \dot{p}_m(t)$$

From previous parts,

$p_y(t) = c_1 \quad p_y(t_f) = -1 \quad \Rightarrow c_1 = -1 \quad p_y(t) = -1$

$p_v(t) = -c_1 t + c_2 = t + c_2 \quad p_v(t_f) = 0$

$\Rightarrow c_2 = -t_f \quad \Rightarrow p_v(t) = t - t_f$

$$\dot{p}_m(t) = \frac{p_v(t)\, u(t)}{m(t)^2} = \frac{(t - t_f)\, u(t)}{m(t)^2}$$

$$\dot{\phi}(t) = \frac{\overbrace{\dot{p}_v(t)}^{-p_y(t)}\, m(t) - \overbrace{\dot{m}(t)}^{-bu(t)} p_v(t)}{m(t)^2} - b\, \overbrace{\dot{p}_m(t)}^{\frac{p_v(t)u(t)}{m(t)^2}}$$

$$\Rightarrow \dot{\phi}(t) = \frac{-p_y(t)\, m(t) + b u(t) p_v(t) - b\, p_v(t) u(t)}{m(t)^2}$$

$$\Rightarrow \boxed{\dot{\phi}(t) = \frac{-p_y(t)}{m(t)}}$$

We know $p_y(t) = -1 \quad \Rightarrow \dot{\phi}(t) = \frac{1}{m(t)}$

$\because m(t) > 0$ (mass can't be negative) $m(0) = m_0 \quad m(t_f) = m_f$

$\Rightarrow \dot{\phi}(t) > 0$

$\therefore$ $\boxed{\phi(t) \text{ can't be zero on any non-zero time interval } \because \dot{\phi}(t) > 0}$