Contents

AA 203 HW 8 Question 3

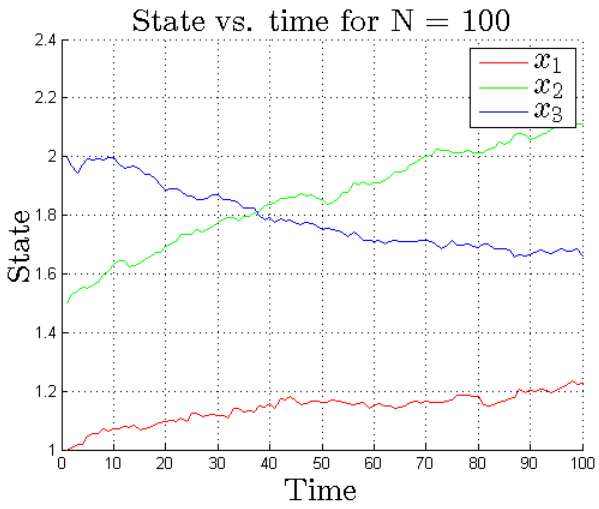
Somrita Banerjee

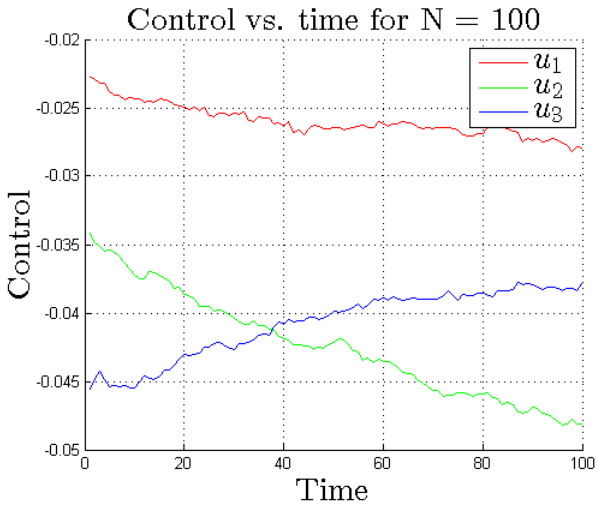
```
clc
clear all
close all
rng('default') % For reproducibility

A0 = [0.99 0 0;
    0 0.99 0;
    0 0 0.99];
B0 = [1 0 0;
    0 1 0;
    0 0 1];
A = [1.01 0.01 0;
    0.01 1.01 0.01;
    0 0.01 1.01];
B = B0;

noiseStdDev = 0.01;
Q = eye(3);
R = 1000*eye(3);

xInit = [1.0; 1.5; 2.0];
```

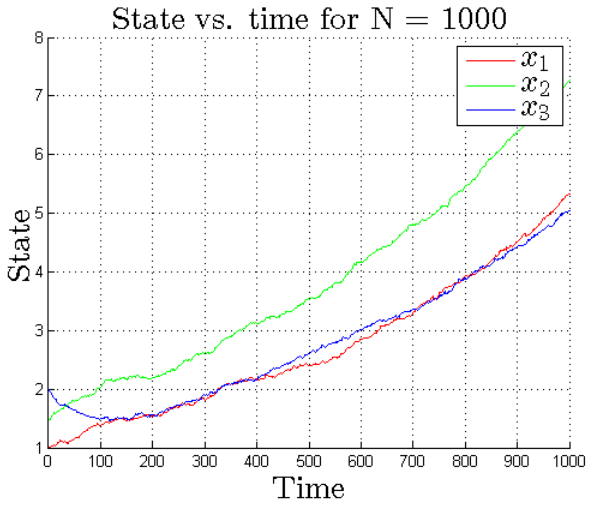
Part a - Plain LQR

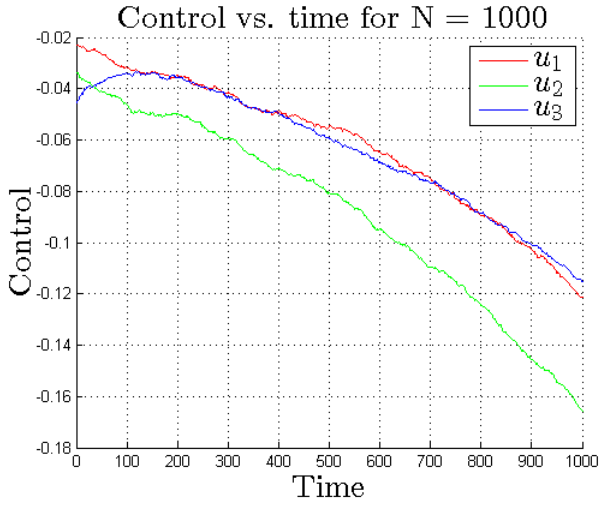
```
[K,S,e] = dlqr(A0,B0,Q,R);
N_vals = [100 1000 10000];
for N = N_vals
    x = zeros(3, N);
    u = zeros(3, N);
    cost = 0;
    for i = 1:N
        if i == 1
            x(:,i) = xInit;
        else
            noise = normrnd(0, noiseStdDev, [3,1]);
            x(:,i) = A*x(:,i-1) + B*u(:,i-1) + noise;
        end
        u(:,i) = -K*x(:,i);
        cost = cost + x(:,i)'*Q*x(:,i) + u(:,i)'*R*u(:,i);
    end

    fprintf('Cost for N = %d is %.2f \n',N, cost)

    figure
    hold on
    plot([1:N], x(1,:),'r')
    plot([1:N], x(2,:),'g')
    plot([1:N], x(3,:),'b')
    titl = sprintf('State vs. time for N = %d',N);
    legend({'$$x_1$$','$$x_2$$','$$x_3$$'},'Interpreter','latex','FontSize',20);
    xlabel('Time','Interpreter','latex','FontSize',20);
    ylabel('State','Interpreter','latex','FontSize',20);
    title(titl,'Interpreter','latex','FontSize',20);
    grid on

    figure
    hold on
    plot([1:N], u(1,:),'r')
    plot([1:N], u(2,:),'g')
    plot([1:N], u(3,:),'b')
    titl = sprintf('Control vs. time for N = %d',N);
    legend({'$$u_1$$','$$u_2$$','$$u_3$$'},'Interpreter','latex','FontSize',20);
    xlabel('Time','Interpreter','latex','FontSize',20);
    ylabel('Control','Interpreter','latex','FontSize',20);
    title(titl,'Interpreter','latex','FontSize',20);
    grid on
end
```

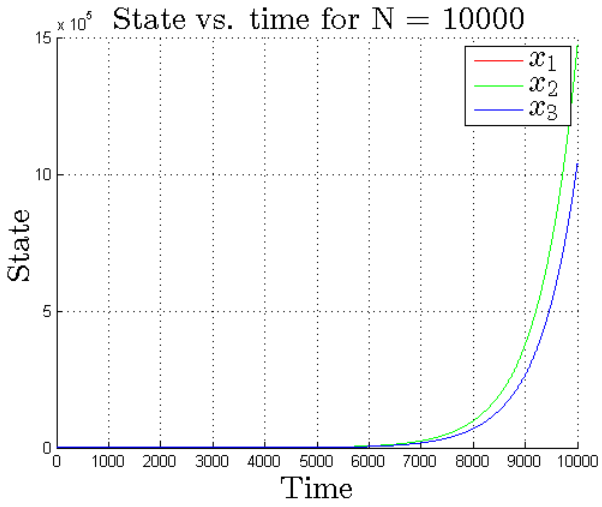
```
Cost for N = 100 is 1216.89
Cost for N = 1000 is 53255.71
Cost for N = 10000 is 2418241054959084.00
```

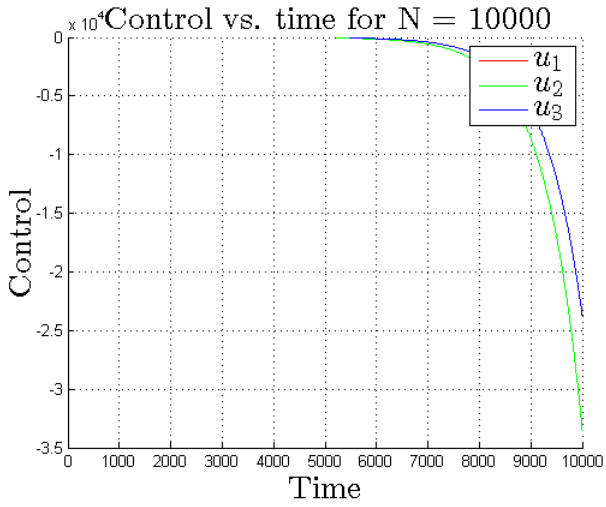










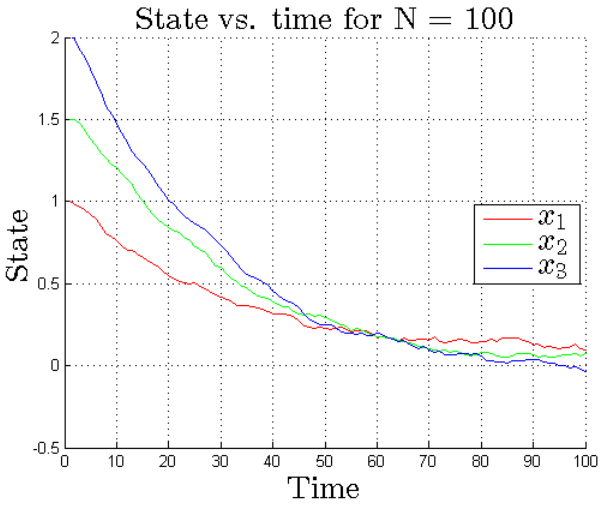


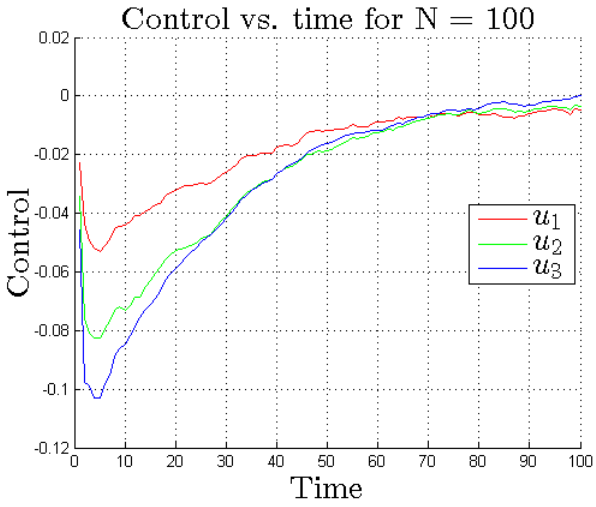
Part b- Certainty equivalent adaptive LQR controller

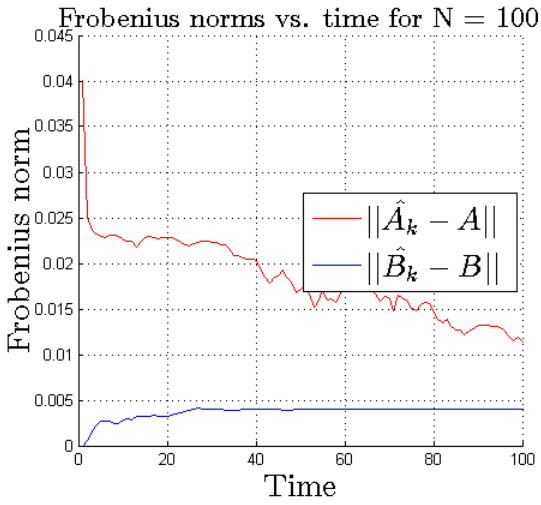
```
N_vals = [100 1000 10000];
for N = N_vals
    x = zeros(3, N);
    u = zeros(3, N);
    A_vals = zeros(3,3,N);
    B_vals = zeros(3,3,N);
    L_vals = zeros(6,6,N);
    Q_vals = zeros(6,3,N);
    A_fro_vals = zeros(1,N);
    B_fro_vals = zeros(1,N);
    cost = 0;
    for i = 1:N
        if i == 1
            x(:,i) = xInit;
            A_vals(:,:,i) = A0;
            B_vals(:,:,i) = B0;
```

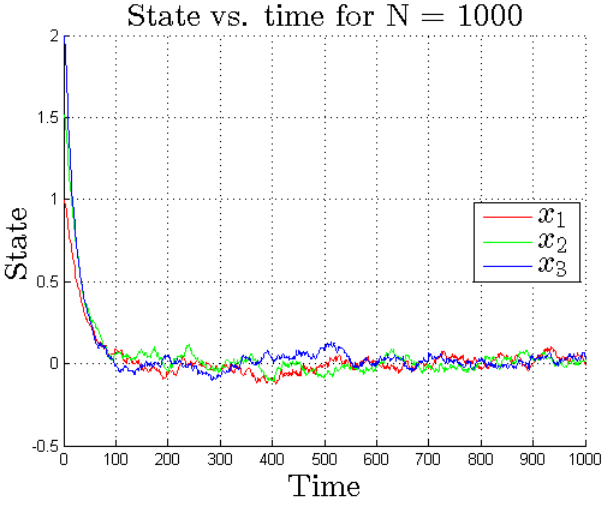
```
            L_vals(:,:,i) = eye(6);
            Q_vals(:,:,i) = [A0' B0']';
        else
            noise = normrnd(0, noiseStdDev, [3,1]);
            xk = x(:,i-1);
            uk = u(:,i-1);
            Ak = A_vals(:,:,i-1);
            Bk = B_vals(:,:,i-1);
            x(:,i) = A*xk + B*uk + noise;

            % Update L, Q, A, B vals for ith
            xbar = [xk' uk']';
            Lk = L_vals(:,:,i-1);
            Qk = Q_vals(:,:,i-1);
            Lknext = Lk - (1/(1+xbar'*Lk*xbar))*(Lk*xbar)*(Lk*xbar)';
            Qknext = xbar*x(:,i)' + Qk;
            LQnext = (Lknext*Qknext)';
            L_vals(:,:,i) = Lknext;
            Q_vals(:,:,i) = Qknext;
            A_vals(:,:,i) = LQnext(:,1:3);
            B_vals(:,:,i) = LQnext(:,4:6);
        end
        [K,~,~] = dlqr(A_vals(:,:,i),B_vals(:,:,i),Q,R);
        u(:,i) = -K*x(:,i);
        cost = cost + x(:,i)'*Q*x(:,i) + u(:,i)'*R*u(:,i);

        % Store frobenius norms
        A_fro_vals(i) = norm(A_vals(:,:,i)-A,'fro');
        B_fro_vals(i) = norm(B_vals(:,:,i)-B,'fro');
    end

    fprintf('Cost for N = %d is %.2f \n',N, cost)

    figure
    hold on
    plot([1:N], x(1,:),'r')
    plot([1:N], x(2,:),'g')
    plot([1:N], x(3,:),'b')
    titl = sprintf('State vs. time for N = %d',N);
    legend({'$$x_1$$','$$x_2$$','$$x_3$$'},'Interpreter','latex','FontSize',20,'Location','east');
    xlabel('Time','Interpreter','latex','FontSize',20);
    ylabel('State','Interpreter','latex','FontSize',20);
    title(titl,'Interpreter','latex','FontSize',20);
    grid on

    figure
    hold on
    plot([1:N], u(1,:),'r')
    plot([1:N], u(2,:),'g')
    plot([1:N], u(3,:),'b')
    titl = sprintf('Control vs. time for N = %d',N);
    legend({'$$u_1$$','$$u_2$$','$$u_3$$'},'Interpreter','latex','FontSize',20,'Location','east');
    xlabel('Time','Interpreter','latex','FontSize',20);
    ylabel('Control','Interpreter','latex','FontSize',20);
    title(titl,'Interpreter','latex','FontSize',20);
    grid on

    figure
    hold on
    plot([1:N],A_fro_vals,'r')
    plot([1:N],B_fro_vals,'b')
    titl = sprintf('Frobenius norms vs. time for N = %d',N);
    legend({'$$||\hat{A_k}-A||$$','$$||\hat{B_k}-B||$$'},'Interpreter','latex','FontSize',20,'Location','east');
    xlabel('Time','Interpreter','latex','FontSize',20);
    ylabel('Frobenius norm','Interpreter','latex','FontSize',20);
    title(titl,'Interpreter','latex','FontSize',18);
    grid on
end
```

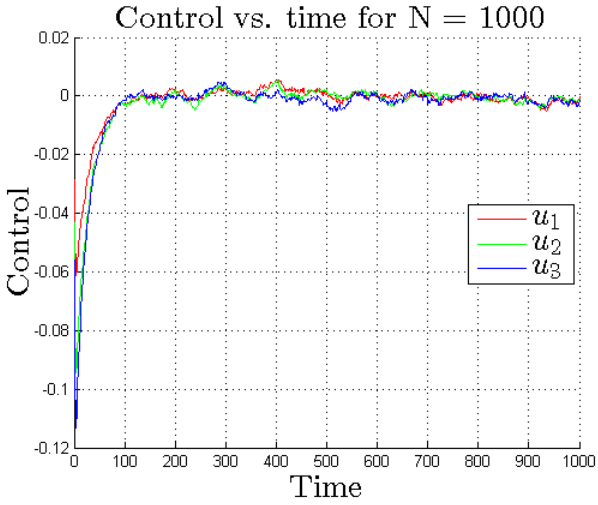
```
Cost for N = 100 is 478.99
Cost for N = 1000 is 499.20
Cost for N = 10000 is 601.38
```

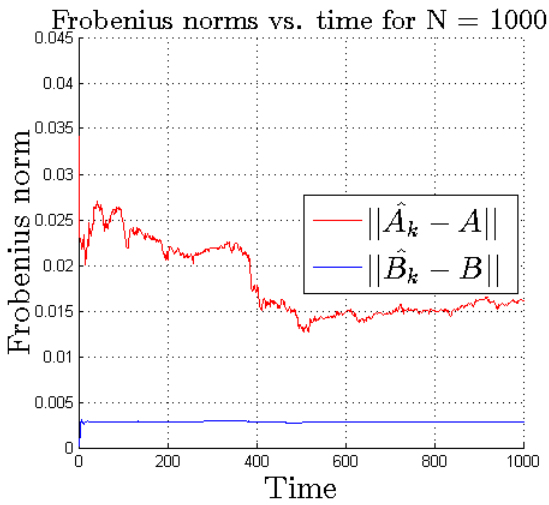

State vs. time for N = 100
State
Time
x_1
x_2
x_3

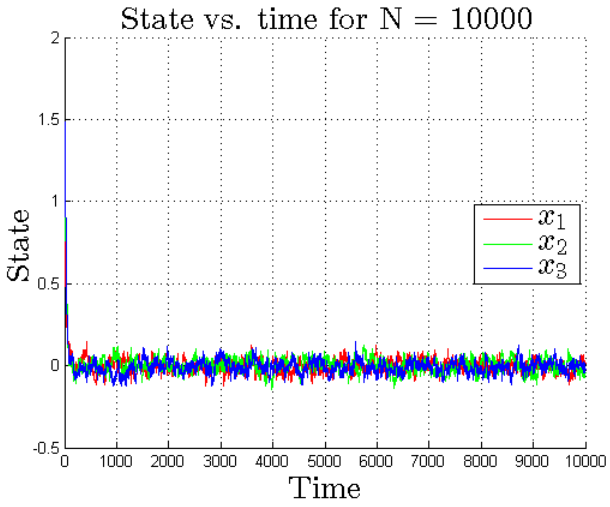

Control vs. time for N = 100
Control
Time
u_1
u_2
u_3

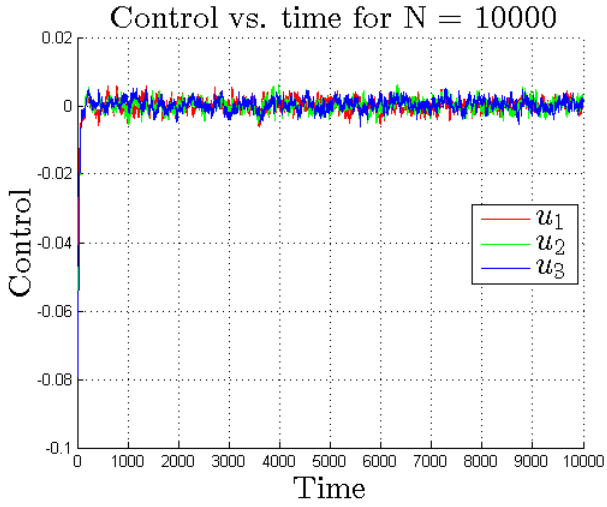

Frobenius norms vs. time for N = 100
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$

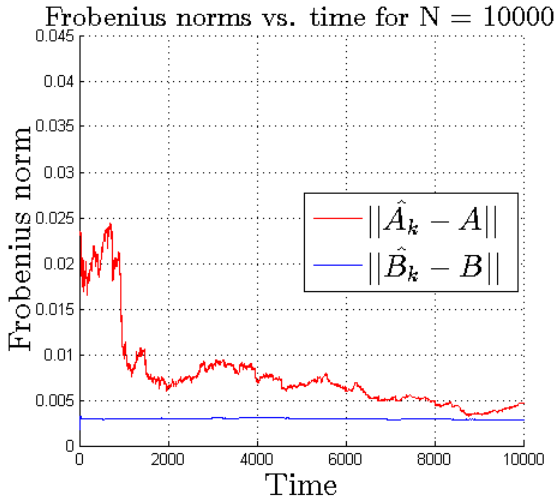

State vs. time for N = 1000
State
Time
x_1
x_2
x_3


Control vs. time for N = 1000
Control
Time
u_1
u_2
u_3


Frobenius norms vs. time for N = 1000
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$





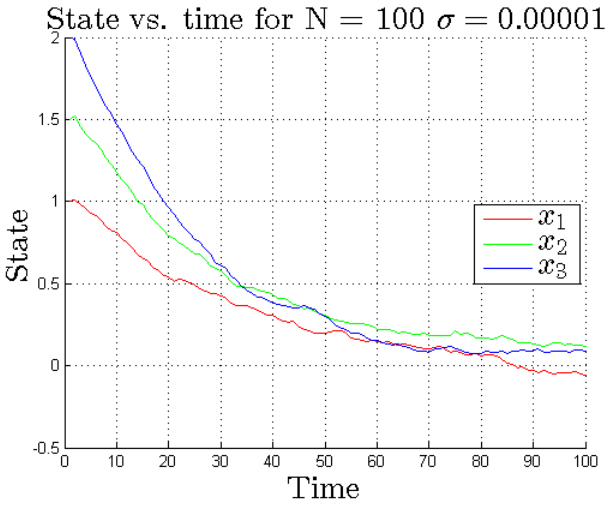


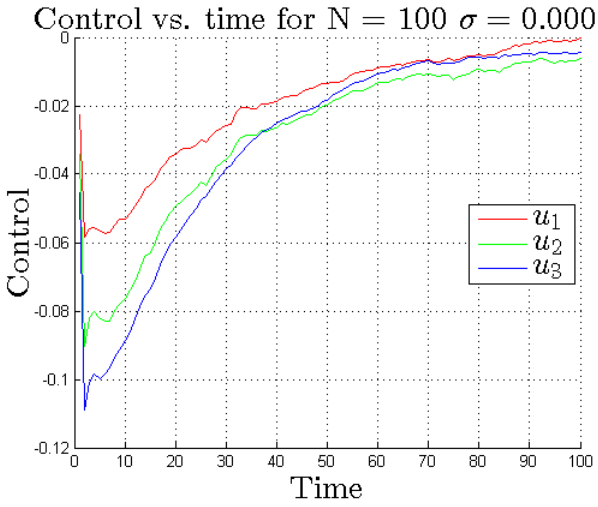
Part c- adaptive LQR controller with white noise

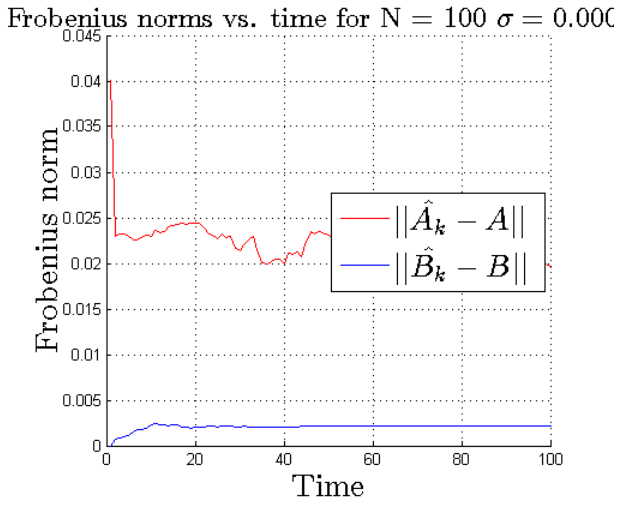
```
N_vals = [100 1000 10000];
white_noise_stdev_vals = [0.00001 0.001 0.01 1];
for white_noise_stdev = white_noise_stdev_vals
    for N = N_vals
        x = zeros(3, N);
        u = zeros(3, N);
        A_vals = zeros(3,3,N);
        B_vals = zeros(3,3,N);
        L_vals = zeros(6,6,N);
        Q_vals = zeros(6,3,N);
        A_fro_vals = zeros(1,N);
        B_fro_vals = zeros(1,N);
        cost = 0;
        for i = 1:N
            if i == 1
                x(:,i) = xInit;
                A_vals(:,:,i) = A0;
                B_vals(:,:,i) = B0;
                L_vals(:,:,i) = eye(6);
                Q_vals(:,:,i) = [A0' B0']';
            else
                noise = normrnd(0, noiseStdDev, [3,1]);
                xk = x(:,i-1);
                uk = u(:,i-1);
                Ak = A_vals(:,:,i-1);
                Bk = B_vals(:,:,i-1);
                x(:,i) = A*xk + B*uk + noise;

                % Update L, Q, A, B vals for ith
                xbar = [xk' uk']';
                Lk = L_vals(:,:,i-1);
                Qk = Q_vals(:,:,i-1);
                Lknext = Lk - (1/(1+xbar'*Lk*xbar))*(Lk*xbar)*(Lk*xbar)';
                Qknext = xbar*x(:,i)' + Qk;
                LQnext = (Lknext*Qknext)';
                L_vals(:,:,i) = Lknext;
                Q_vals(:,:,i) = Qknext;
                A_vals(:,:,i) = LQnext(:,1:3);
                B_vals(:,:,i) = LQnext(:,4:6);
            end
            white_noise = normrnd(0, white_noise_stdev, [3,1]);
            [K,~,~] = dlqr(A_vals(:,:,i),B_vals(:,:,i),Q,R);
            u(:,i) = -K*x(:,i) + white_noise;
            cost = cost + x(:,i)'*Q*x(:,i) + u(:,i)'*R*u(:,i);
```

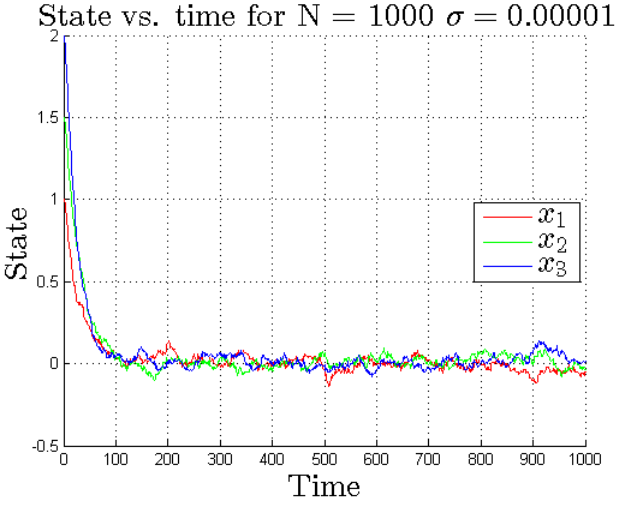
```
            % Store frobenius norms
            A_fro_vals(i) = norm(A_vals(:,:,i)-A,'fro');
            B_fro_vals(i) = norm(B_vals(:,:,i)-B,'fro');
        end

        fprintf('Cost for N = %d, white noise std dev = %.5f is %.2f \n',N, white_noise_stdev, cost)

        figure
        hold on
        plot([1:N], x(1,:),'r')
        plot([1:N], x(2,:),'g')
        plot([1:N], x(3,:),'b')
        titl = sprintf('State vs. time for N = %d $\\sigma = %.5f$',N, white_noise_stdev);
        legend({'$$x_1$$','$$x_2$$','$$x_3$$'},'Interpreter','latex','FontSize',20,'Location','east');
        xlabel('Time','Interpreter','latex','FontSize',20);
        ylabel('State','Interpreter','latex','FontSize',20);
        title(titl,'Interpreter','latex','FontSize',20);
        grid on

        figure
        hold on
        plot([1:N], u(1,:),'r')
        plot([1:N], u(2,:),'g')
        plot([1:N], u(3,:),'b')
        titl = sprintf('Control vs. time for N = %d $\\sigma = %.5f$',N, white_noise_stdev);
        legend({'$$u_1$$','$$u_2$$','$$u_3$$'},'Interpreter','latex','FontSize',20,'Location','east');
        xlabel('Time','Interpreter','latex','FontSize',20);
        ylabel('Control','Interpreter','latex','FontSize',20);
        title(titl,'Interpreter','latex','FontSize',20);
        grid on

        figure
        hold on
        plot([1:N],A_fro_vals,'r')
        plot([1:N],B_fro_vals,'b')
        titl = sprintf('Frobenius norms vs. time for N = %d $\\sigma = %.5f$',N, white_noise_stdev);
        legend({'$$||\hat{A_k}-A||$$','$$||\hat{B_k}-B||$$'},'Interpreter','latex','FontSize',20,'Location','east');
        xlabel('Time','Interpreter','latex','FontSize',20);
        ylabel('Frobenius norm','Interpreter','latex','FontSize',20);
        title(titl,'Interpreter','latex','FontSize',18);
        grid on
    end
end
```

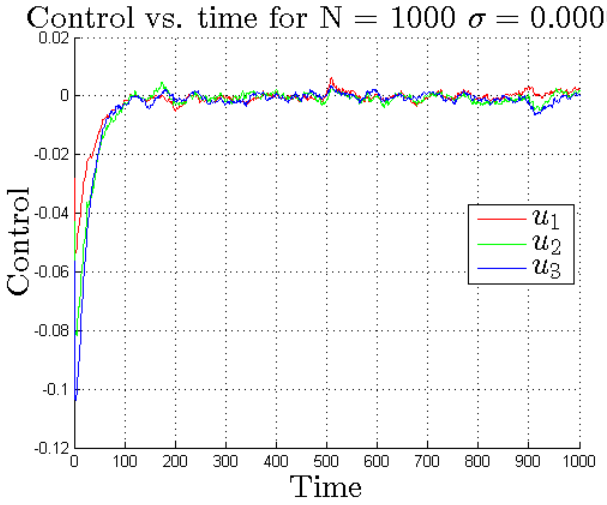
```
Cost for N = 100, white noise std dev = 0.00001 is 492.18
Cost for N = 1000, white noise std dev = 0.00001 is 482.81
Cost for N = 10000, white noise std dev = 0.00001 is 635.98
Cost for N = 100, white noise std dev = 0.00100 is 471.72
Cost for N = 1000, white noise std dev = 0.00100 is 495.86
Cost for N = 10000, white noise std dev = 0.00100 is 620.78
Cost for N = 100, white noise std dev = 0.01000 is 509.27
Cost for N = 1000, white noise std dev = 0.01000 is 777.46
Cost for N = 10000, white noise std dev = 0.01000 is 3776.37
Cost for N = 100, white noise std dev = 1.00000 is 310598.40
Cost for N = 1000, white noise std dev = 1.00000 is 3097063.33
Cost for N = 10000, white noise std dev = 1.00000 is 31337434.89
```

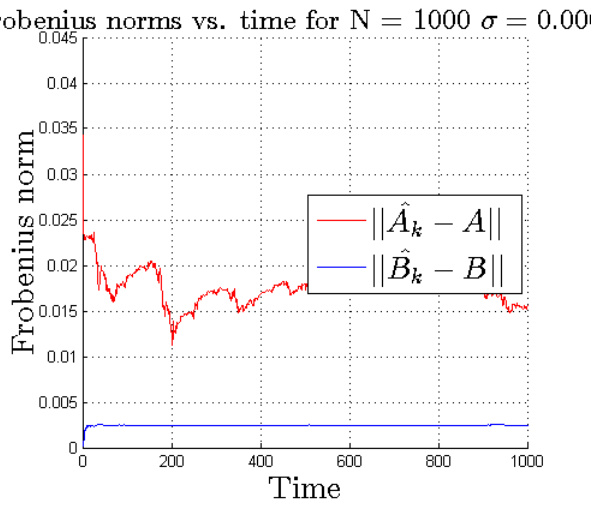

State vs. time for N = 100 $\sigma = 0.00001$
State
Time
x_1
x_2
x_3

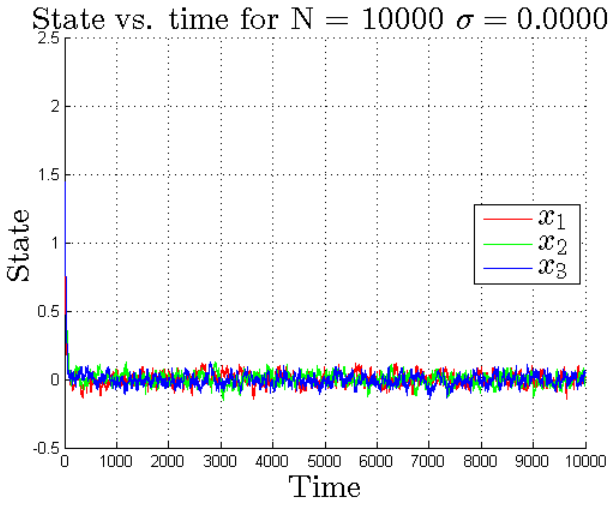

Control vs. time for N = 100 $\sigma = 0.00001$
Control
Time
u_1
u_2
u_3

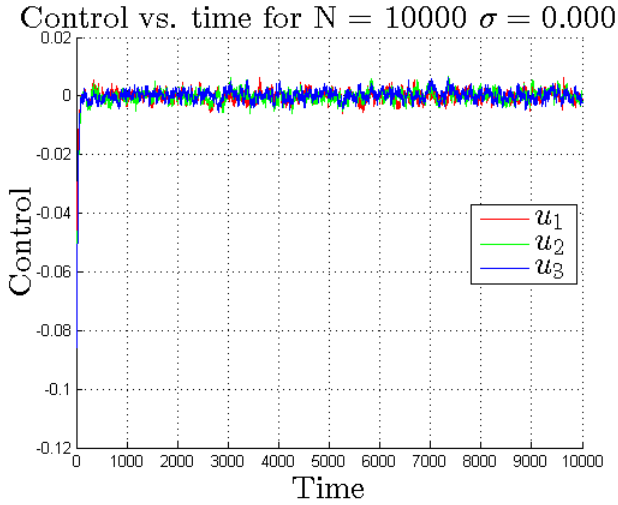

Frobenius norms vs. time for N = 100 $\sigma = 0.000$
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$

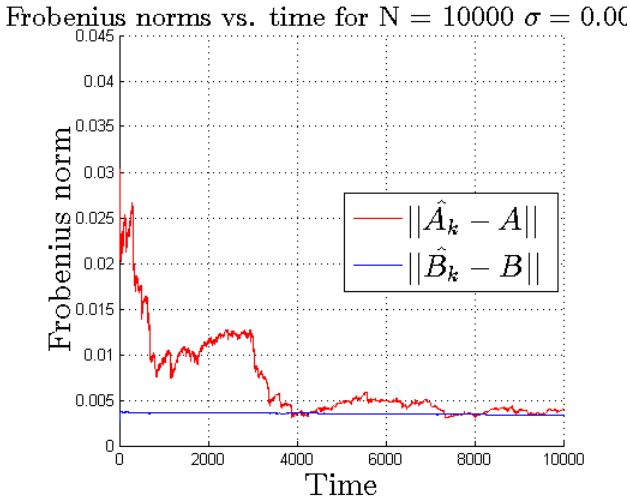

State vs. time for N = 1000 $\sigma = 0.00001$
State
Time
x_1
x_2
x_3

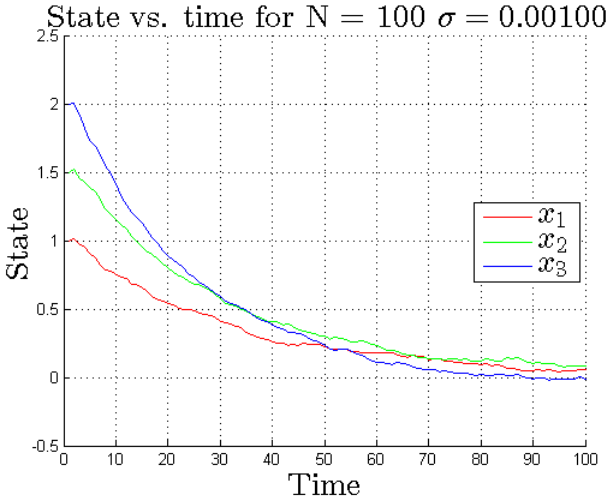

Control vs. time for N = 1000 $\sigma = 0.00001$
Control
Time
u_1
u_2
u_3

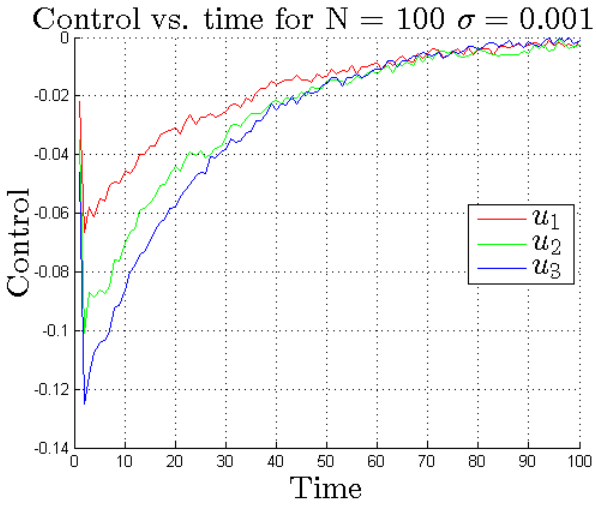

Frobenius norms vs. time for N = 1000 $\sigma = 0.00$
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$

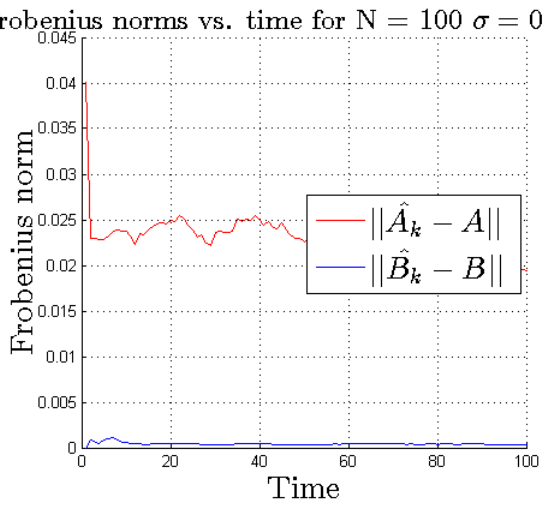

State vs. time for N = 10000 $\sigma = 0.00001$
State
Time
x_1
x_2
x_3

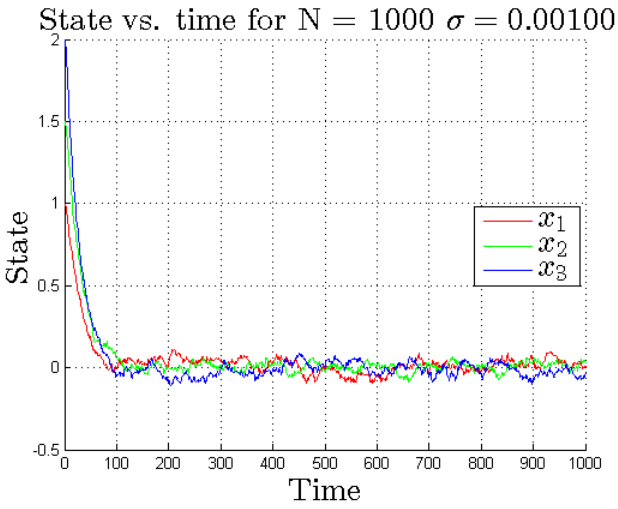

Control vs. time for N = 10000 $\sigma = 0.00001$
Control
Time
u_1
u_2
u_3

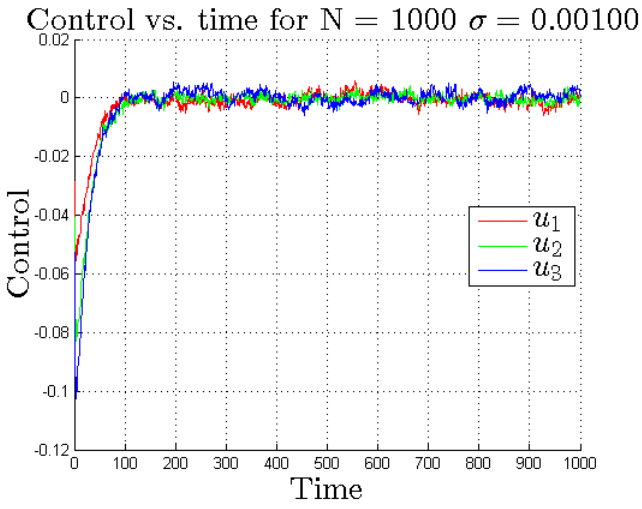

Frobenius norms vs. time for N = 10000 $\sigma = 0.0$
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$

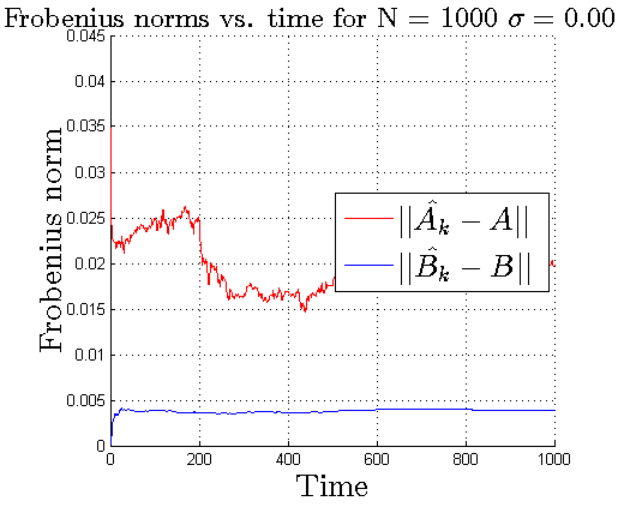

State vs. time for N = 100 $\sigma = 0.00100$
State
Time
x_1
x_2
x_3

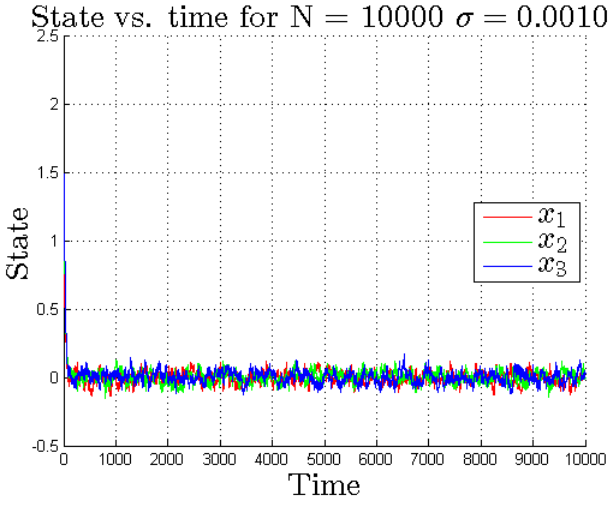

Control vs. time for N = 100 $\sigma = 0.00100$
Control
Time
u_1
u_2
u_3

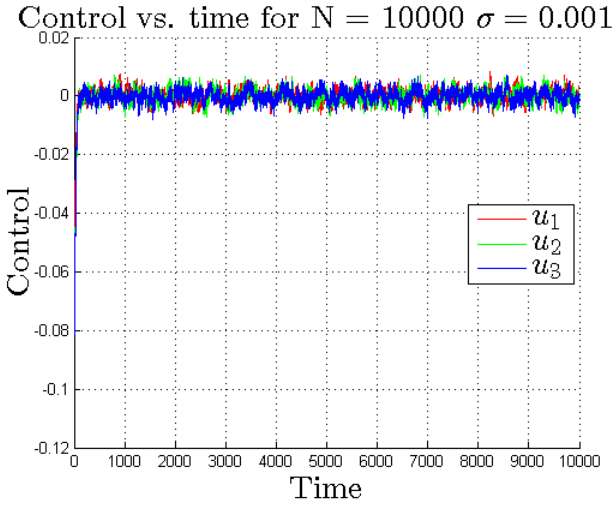

Frobenius norms vs. time for N = 100 $\sigma = 0.001$
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$

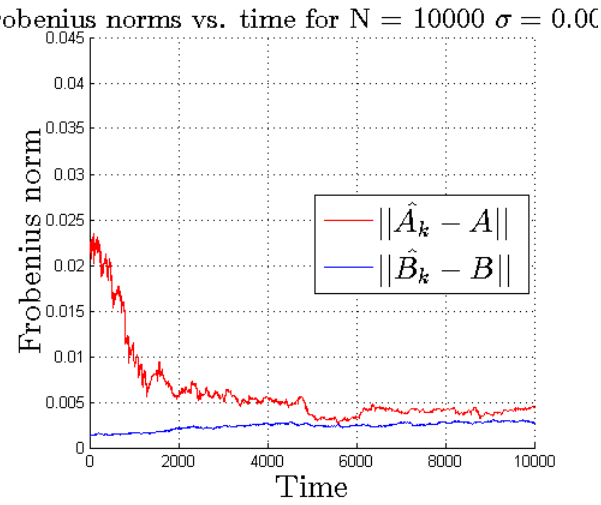

State vs. time for N = 1000 σ = 0.00100
State
Time
x_1
x_2
x_3

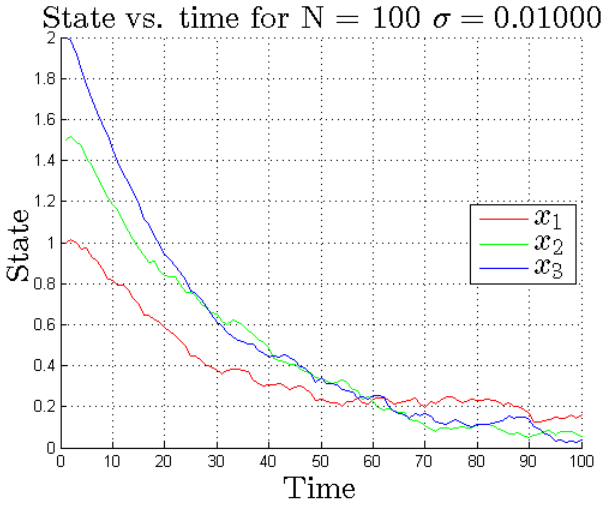

Control vs. time for N = 1000 σ = 0.00100
Control
Time
u_1
u_2
u_3

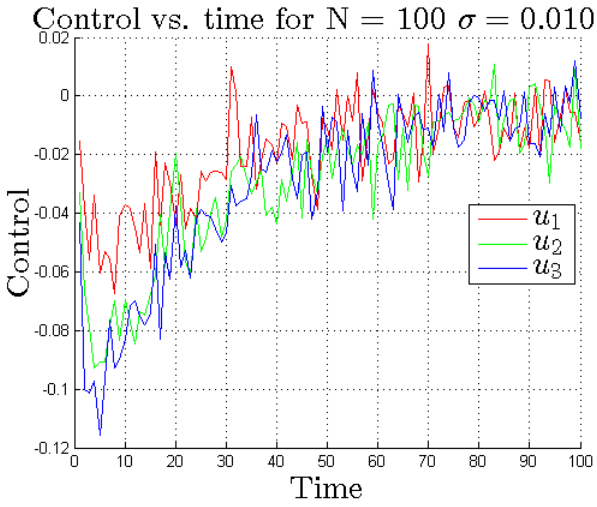

Frobenius norms vs. time for N = 1000 σ = 0.00
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$

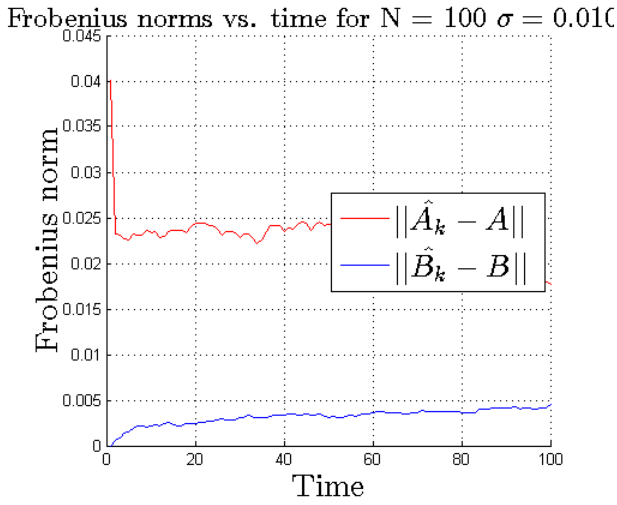

State vs. time for N = 10000 σ = 0.00100
State
Time
x_1
x_2
x_3

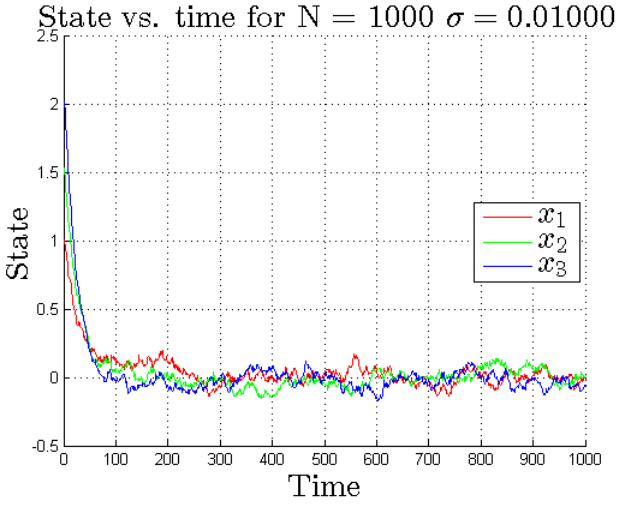

Control vs. time for N = 10000 σ = 0.0010C
Control
Time
u_1
u_2
u_3

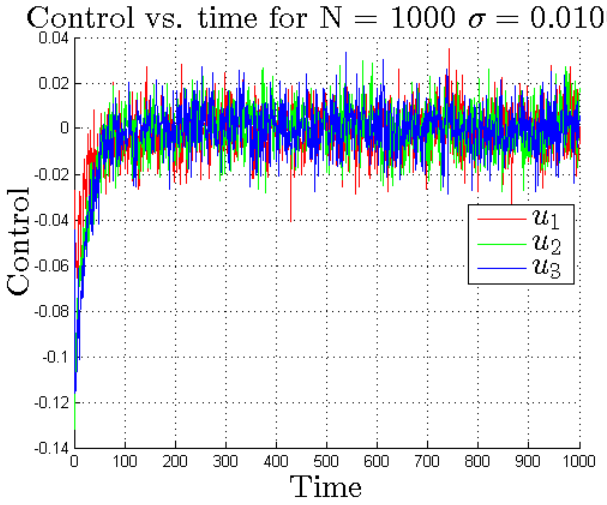

Frobenius norms vs. time for N = 10000 σ = 0.0C
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$

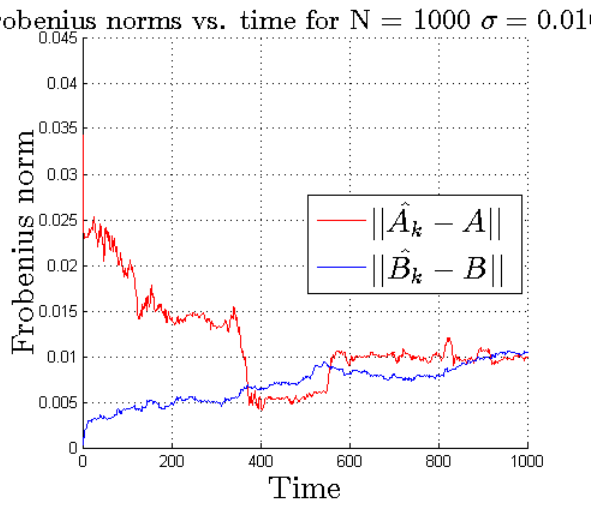

State vs. time for N = 100 $\sigma = 0.01000$
State
Time
x_1
x_2
x_3

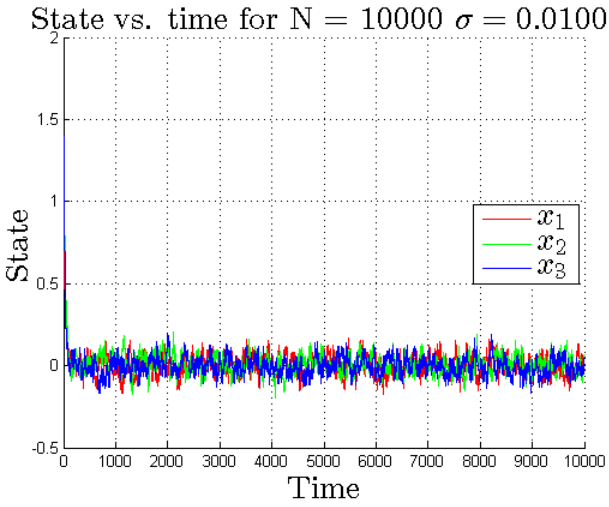

Control vs. time for N = 100 $\sigma = 0.01000$
Control
Time
u_1
u_2
u_3

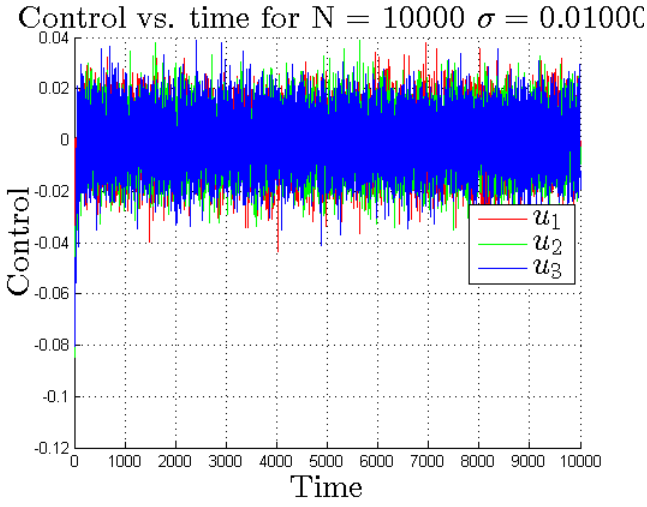

Frobenius norms vs. time for N = 100 $\sigma = 0.010$
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$

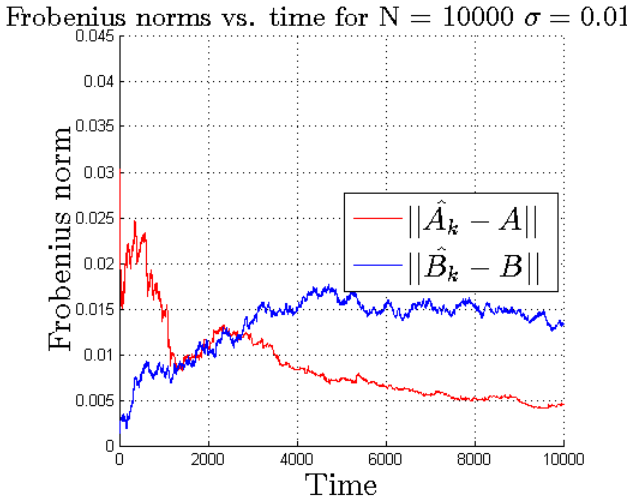

State vs. time for N = 1000 $\sigma = 0.01000$
State
Time
x_1
x_2
x_3

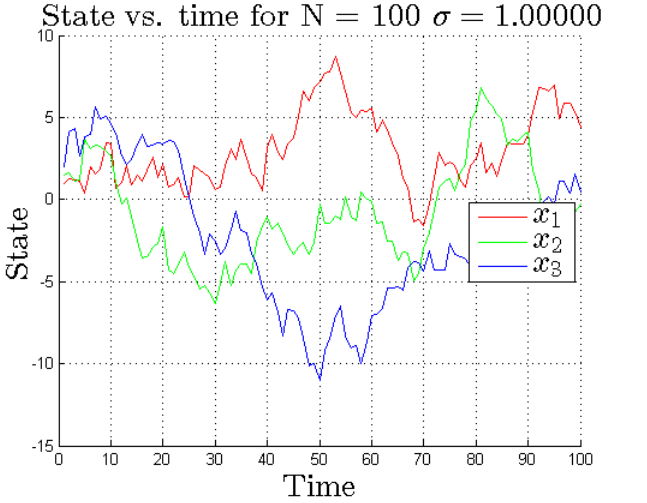

Control vs. time for N = 1000 $\sigma = 0.01000$
Control
Time
u_1
u_2
u_3

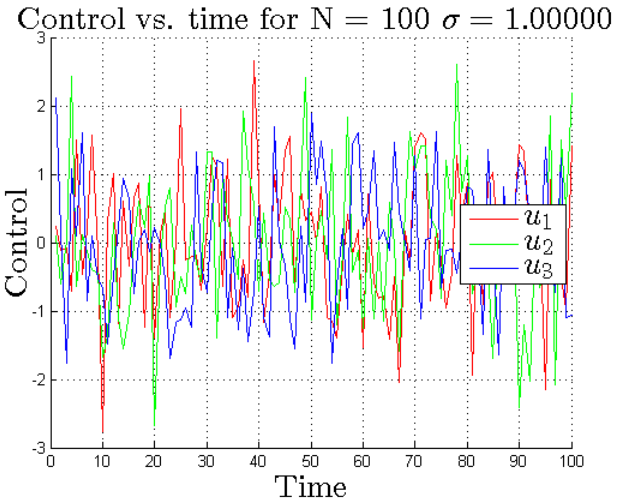

Frobenius norms vs. time for N = 1000 $\sigma = 0.01$
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$

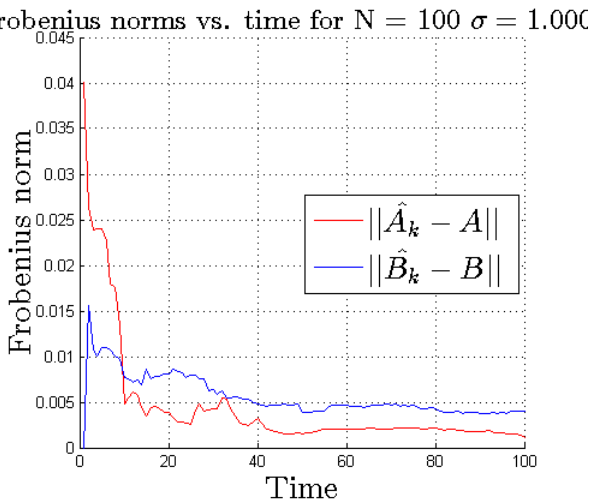

State vs. time for N = 10000 σ = 0.01000
State
Time
x_1
x_2
x_3

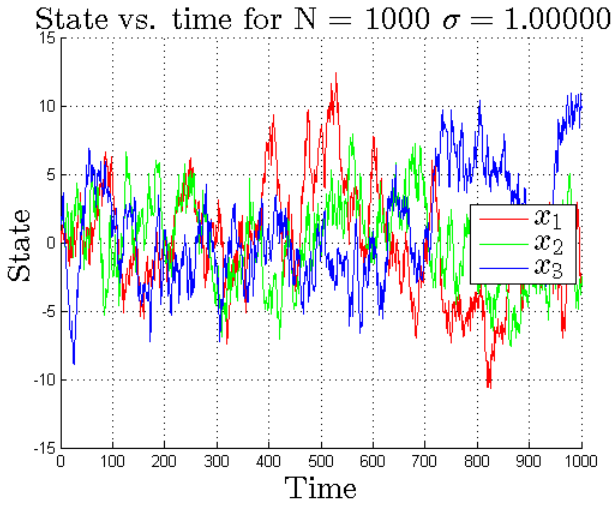

Control vs. time for N = 10000 σ = 0.01000
Control
Time
u_1
u_2
u_3

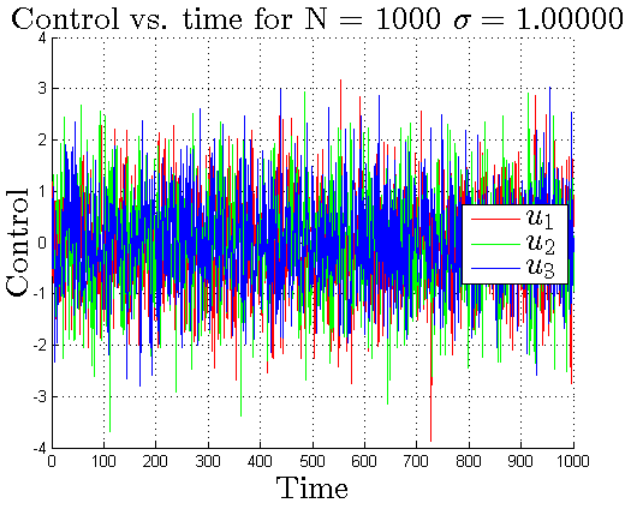

Frobenius norms vs. time for N = 10000 σ = 0.01
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$

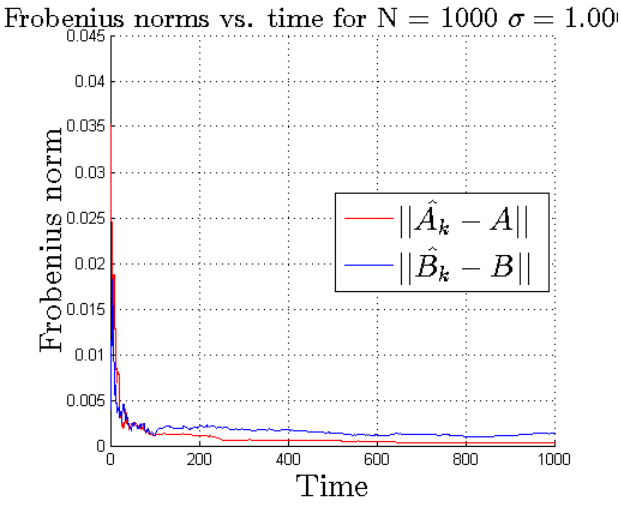

State vs. time for N = 100 σ = 1.00000
State
Time
x_1
x_2
x_3

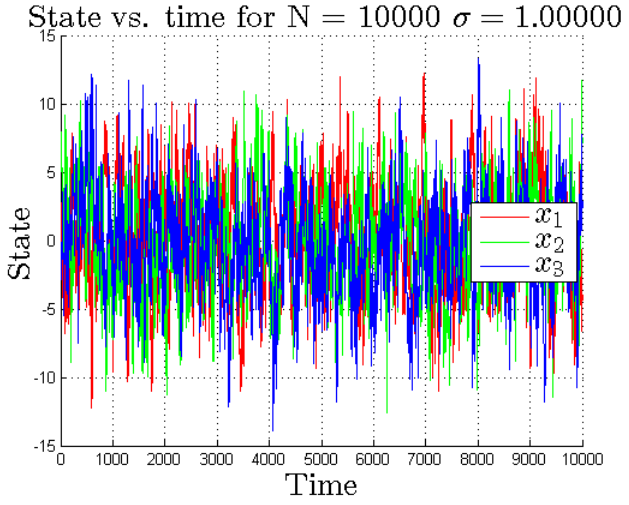

Control vs. time for N = 100 σ = 1.00000
Control
Time
u_1
u_2
u_3

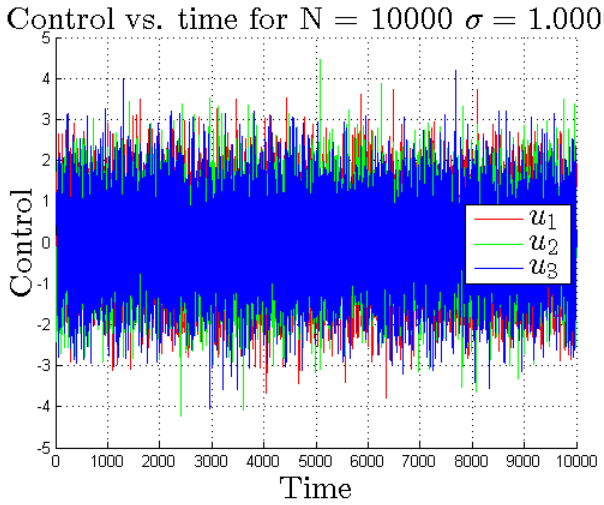

Frobenius norms vs. time for N = 100 σ = 1.000
Frobenius norm
Time
$||\hat{A}_k - A||$
$||\hat{B}_k - B||$

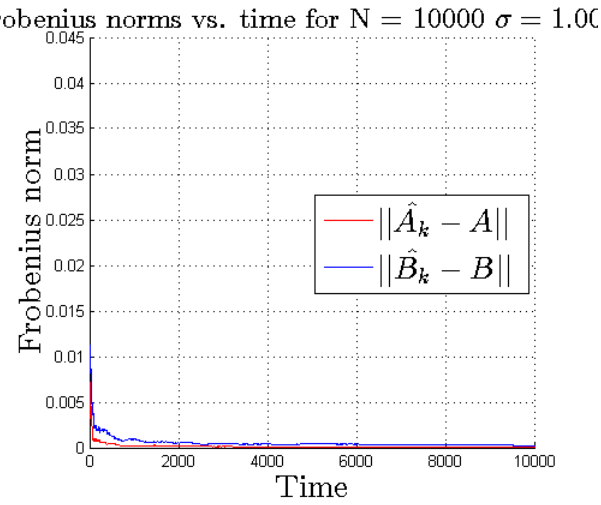












Published with MATLAB® R2014a