```matlab
% Problem (OCP)_2 from Pset 6 - Hermite-Simpson Rule

clear all; clf; clc; format long;

% Parameters
global N; N = 6; % Choose here the number of discretization points
global mu; mu = 3.9915e14;
global rE; rE = 6378145;
global h0; h0 = 7500;
global D; D = 5e-3;
global b; b = 1e-3;
global uMax; uMax = 1.2e5;

% Scenario
global T; T = 258.;
global y0; y0 = 0.;
global v0; v0 = 0.;
global m0; m0 = 12000;
global mf; mf = 1000;

% Bound on the state: better conditioning the formulation (see below)
global yMax; yMax = 5e6;
global vMax; vMax = 2000;

% Since this optimal control problem is highly nonlinear, without
% an appropriate intialization direct methods unlikely converge.
% In the following lines, we provide such initialization by recalling the
% solution that we obtained for the simplified Goddard problem in the Pset 5.
% For the height, we just select a stright-line in time connecting y0 to
% 1.5e5 (which is more or less the final height that we found in Pset 5).
% For the velocity, we select the average v(t) = vMax/2 in [0,tf].
% For the mass, we select a straight-line in time between 0 and tSw, the
% switching time computed in Pset 5 (see below).
% Finally, for the control, we select the maximal value u(t) = uMax in [0,tf].

% Finding what index NSw the time tSw corresponds to
global tSw; tSw = (m0 - mf)/(b*uMax);
h = (1.0*T/(1.0*N));
NSw = 0; indexFound = 0; iterator = 0;
while indexFound == 0
    % If iterator*h <= tSw < iteartor*h + h, then we have found the index
    if iterator*h <= tSw && tSw < (iterator + 1)*h
        NSw = iterator + 1;
        indexFound = 1;
    end
    iterator = iterator + 1;
end
uInit = zeros(N+1,1);
yInit = zeros(N+1,1);
vInit = 0.5*vMax*ones(N+1,1);
mInit = mf*ones(N+1,1);
% Initialization exxplained above
for i=1:N+1
    yInit(i) = y0*(1. - (i-1)*1.0/N) + 1.5e5*(i-1)*1.0/N;
    if (i-1) <= NSw
        mInit(i) = m0*(1. - (i-1)*1.0/NSw) + mf*(i-1)*1.0/NSw;
    end
    if i<= N
        if (i-1)*1.0*T/N < tSw
            uInit(i) = uMax;
        end
    end
end
% Initialization for fmincon
varInit = [yInit; vInit; mInit; uInit];

% Lower and upper bounds.
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lb = zeros(4*N+4,1); ub = uMax*ones(4*N+4,1); % For the control: 0 \le u \le uMax
ub(1:N+1) = yMax; % For the state y : 0 \le y \le yMax
ub(N+2:2*N+2) = vMax; % For the state v : 0 \le v \le vMax
lb(2*N+3:3*N+3) = mf; ub(2*N+3:3*N+3) = m0; % For the state m : mf \le v \le m0

% Solving the problme via fmincon
options=optimoptions('fmincon','Display','iter','Algorithm','sqp','MaxFunEvals',100000,'MaxIter',10000);
% options=optimoptions('fmincon','Display','iter','Algorithm','sqp','MaxFunctionEvaluations',100000,'MaxIterations',10000);
[var,Fval,convergence] = fmincon(@cost,varInit,[],[],[],[],lb,ub,@constraint,options); % Solving the problem
convergence % = 1, good

% Collecting the solution. Note that var = [y;v;m;u]
y = var(1:N+1); v = var(N+2:2*N+2); m = var(2*N+3:3*N+3); u = var(3*N+4:4*N+4); % Collecting the solution
tState = zeros(N+1,1);
for i = 1:N
    tState(i+1) = tState(i) + (1.0*T/(1.0*N));
end
t = zeros(N+1,1);
for i = 1:N
    t(i+1) = t(i) + (1.0*T/(1.0*N));
end

% Plotting
% subplot(221); plot(tState,y,'linewidth',3);
% title('\textbf{a) Height}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$h$} \ \textbf{(m)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(222); plot(tState,v,'linewidth',3) ;
% title('\textbf{b) Velocity}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$v$} \ \textbf{(m/s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(223); plot(tState,m,'linewidth',3) ;
% title('\textbf{c) Mass}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$m$} \ \textbf{(kg)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
% subplot(224); plot(t,u,'linewidth',3);
% title('\textbf{d) Optimal Control}','interpreter','latex','FontSize',22,'FontWeight','bold');
% xlabel('\boldmath{$t$} \ \textbf{(s)}','interpreter','latex','FontSize',20,'FontWeight','bold');
% ylabel('\boldmath{$u$}','interpreter','latex','FontSize',20,'FontWeight','bold');
% xlim([-inf inf]);
% ylim([-inf inf]);
% grid on;
```

```
                                                          Norm of First-order
 Iter F-count            f(x) Feasibility  Steplength        step  optimality
    0      29   -1.500000e+05   2.195e+04                           1.000e+00
    1      58   -1.300265e+05   1.408e+02   1.000e+00   6.641e+04   1.483e+06
    2      87   -1.302402e+05   5.410e-01   1.000e+00   1.256e+03   1.517e+04
    3     116   -1.302413e+05   1.637e-05   1.000e+00   3.926e+00   2.542e+01
    4     145   -1.302419e+05   3.305e-07   1.000e+00   8.138e-01   4.686e+00
    5     174   -1.302449e+05   6.053e-06   1.000e+00   4.069e+00   4.686e+00
    6     203   -1.302600e+05   1.458e-04   1.000e+00   2.034e+01   4.686e+00
    7     232   -1.303357e+05   3.643e-03   1.000e+00   1.017e+02   4.686e+00
    8     261   -1.305159e+05   2.088e-02   1.000e+00   2.422e+02   4.685e+00
    9     290   -1.305188e+05   1.050e-05   1.000e+00   4.154e+00   4.685e+00
   10     319   -1.305325e+05   2.386e-04   1.000e+00   2.021e+01   4.685e+00
   11     348   -1.306013e+05   6.002e-03   1.000e+00   1.011e+02   4.685e+00
   12     377   -1.309454e+05   1.512e-01   1.000e+00   5.056e+02   4.685e+00
   13     406   -1.310556e+05   1.555e-02   1.000e+00   1.618e+02   4.685e+00
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   14     435   -1.310562e+05   3.920e-07   1.000e+00   1.055e+00   4.685e+00  
   15     464   -1.310589e+05   1.172e-05   1.000e+00   4.982e+00   4.686e+00  
   16     493   -1.310720e+05   2.956e-04   1.000e+00   2.491e+01   4.686e+00  
   17     522   -1.311378e+05   7.465e-03   1.000e+00   1.246e+02   4.691e+00  
   18     551   -1.314671e+05   1.878e-01   1.000e+00   6.232e+02   4.714e+00  
   19     580   -1.324321e+05   1.631e+00   1.000e+00   1.825e+03   4.781e+00  
   20     609   -1.324546e+05   6.150e-04   1.000e+00   4.081e+01   4.782e+00  
   21     638   -1.325160e+05   7.243e-03   1.000e+00   1.159e+02   4.787e+00  
   22     667   -1.328238e+05   1.825e-01   1.000e+00   5.813e+02   4.808e+00  
   23     696   -1.338126e+05   1.903e+00   1.000e+00   1.866e+03   4.879e+00  
   24     725   -1.338182e+05   7.383e-08   1.000e+00   9.419e+00   4.865e+00  
   25     754   -1.338186e+05   1.038e-07   1.000e+00   8.835e-01   4.865e+00  
   26     783   -1.338207e+05   2.724e-06   1.000e+00   4.418e+00   4.867e+00  
   27     812   -1.338308e+05   6.745e-05   1.000e+00   2.209e+01   4.876e+00  
   28     841   -1.338816e+05   1.686e-03   1.000e+00   1.105e+02   4.921e+00  
   29     870   -1.341357e+05   4.229e-02   1.000e+00   5.527e+02   5.148e+00  
   30     899   -1.354107e+05   1.075e+00   1.000e+00   2.771e+03   6.304e+00  

                                                          Norm of First-order
 Iter F-count            f(x) Feasibility  Steplength        step  optimality
   31     928   -1.413389e+05   2.438e+01   1.000e+00   1.283e+04   1.211e+01  
   32     957   -1.420058e+05   1.624e-01   1.000e+00   1.316e+03   1.260e+01  
   33     986   -1.420188e+05   4.979e-05   1.000e+00   3.210e+01   1.261e+01  
   34    1015   -1.420192e+05   2.789e-07   1.000e+00   8.408e-01   1.261e+01  
   35    1044   -1.420213e+05   7.602e-06   1.000e+00   4.199e+00   1.261e+01  
   36    1073   -1.420319e+05   1.884e-04   1.000e+00   2.100e+01   1.263e+01  
   37    1102   -1.420849e+05   4.593e-03   1.000e+00   1.050e+02   1.271e+01  
   38    1131   -1.423506e+05   1.157e-01   1.000e+00   5.262e+02   1.313e+01  
   39    1160   -1.432112e+05   1.219e+00   1.000e+00   1.701e+03   5.343e+00  
   40    1189   -1.432192e+05   1.178e-07   1.000e+00   1.221e+01   5.042e+00  
   41    1218   -1.432192e+05   1.819e-11   1.000e+00   8.017e-07   5.684e-14  

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in 
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.




convergence =

     1
```