PS 1 Problem 3

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$$\min \quad \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$$

$$\text{st} \quad x_1 + x_2 + x_3 \le -3$$

$$\mathcal{L} = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) + \mu(x_1 + x_2 + x_3 + 3)$$

By KKT NOC, if x^* is a local minima

$\Rightarrow \nabla_x \mathcal{L} = 0$ & $\exists\ \mu_j^* \geq 0 \quad \forall j \in A(x^*)$

$\mu_j^* = 0 \quad \forall j \notin A(x^*)$

<u>Case 1</u> $x_1 + x_2 + x_3 + 3 \le 0$ active

$\Rightarrow \nabla_x \mathcal{L} = 0$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$\Rightarrow x_1 = -\mu = x_2 = x_3$

$x_1 + x_2 + x_3 + 3 = 0$

$\Rightarrow 3 - 3\mu = 0$

$\Rightarrow \mu = 1 \qquad \geq 0$ ✓

$$\therefore \left.\begin{aligned} x_1 &= -1 \\ x_2 &= -1 \\ x_3 &= -1. \end{aligned}\right\} \text{ is a candidate for optimality}$$

<u>Case 2</u> constraint inactive $\Rightarrow \mu = 0$

$\Rightarrow \nabla_x \mathcal{L} = 0$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow \left.\begin{aligned} x_1 &= 0 \\ x_2 &= 0 \\ x_3 &= 0 \end{aligned}\right\} \text{ also a candidate}$$

$$\boxed{\text{Candidates: } \begin{array}{l} x_1=-1 \\ x_2=-1 \\ x_3=-1 \end{array} \;\&\; \begin{array}{l} x_1=0 \\ x_2=0 \\ x_3=0 \end{array}}$$

We see that $\frac{1}{2}(x_1^2+x_2^2+x_3^2)$ minimized at $x_1=0=x_2=x_3$. So $(0,0,0)$ is local min^m.