Part d

Monday, June 3, 2019 6:21 PM

$$x(t+1) = c\,x(t)\,x(t-1) + w(t)$$

$$\underbrace{\begin{bmatrix} x(2) \\ \vdots \\ x(N) \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} x(1) & x(0) \\ \vdots & \\ x(N-1) & x(N-2) \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} c \end{bmatrix}}_{\theta} + w, \quad w \sim N(0,\sigma)$$

$$\min_{\theta} \|Y - H\theta\|$$

$$\theta^* = H \backslash Y \quad \text{still applies!}$$

The system is linear in θ and we can compute the matrices Y & H from known data so the linear least squares approach still works.

See code:
N = 10 : Least squares estimate of c is mean 0.07 with std dev 0.44
N = 100 : Least squares estimate of c is mean 0.09 with std dev 0.09
N = 1000 : Least squares estimate of c is mean 0.10 with std dev 0.03

Again, we see that having more data makes our estimates better (closer to true value of c = 0.10 and smaller standard deviation).