PS 4 Problem 5

Tuesday, April 30, 2019 1:44 AM

$J = \int_0^1 u^2(t)\,dt$ $\qquad \dot{x}(t) = -2x(t) + u(t)$, $x(0) = 2$, $x(1) = 0$

$H = g + p^T f$

$\Rightarrow H = u^2 + p(-2x + u)$

$\dot{p} = -\frac{\partial H}{\partial x} = 2p$ $\qquad p = e^{rt}$, $r = 2$

$\Rightarrow p = c_1 e^{2t}$

$\frac{\partial H}{\partial u} = 0$ (unbounded control)

$\Rightarrow 2u + p = 0 \Rightarrow u = -p/2$

$\Rightarrow u = -\frac{c_1}{2} e^{2t}$

$\dot{x}(t) = -2x(t) - \frac{c_1}{2} e^{2t}$

$\dot{x}(t) + 2x(t) = -\frac{c_1}{2} e^{2t}$

$\mu = e^{\int 2\,dt} = e^{2t}$

$e^{2t}\dot{x} + 2x e^{2t} = -\frac{c_1}{2} e^{4t}$

$\Rightarrow (x e^{2t})' = -\frac{c_1}{2} e^{4t}$

$\Rightarrow x e^{2t} = \int -\frac{c_1}{2} e^{4t}\,dt$

$\Rightarrow x e^{2t} = -\frac{c_1 e^{4t}}{8} + c_2$

$\Rightarrow x = -\frac{c_1 e^{2t}}{8} + c_2 e^{-2t}$

$x(0) = 2 \Rightarrow -\frac{c_1}{8} + c_2 = 2 \qquad \Rightarrow c_2 = 2 + \frac{c_1}{8}$

$x(1) = 0 \Rightarrow -c_1 \frac{e^2}{8} + c_2 e^{-2} = 0$

$\Rightarrow c_2 = c_1 \frac{e^4}{8}$

$c_1 \frac{e^4}{8} = \frac{16 + c_1}{8}$

$\Rightarrow c_1 (e^4 - 1) = 16$

$\Rightarrow c_1 = \frac{16}{e^4 - 1}$

$c_2 = \frac{2e^4}{e^4 - 1}$

$u^* = -\frac{c_1}{2} e^{2t}$

$x^* = -\frac{c_1 e^{2t}}{8} + c_2 e^{-2t}$

$$u^*(t) = -\frac{8}{e^4 - 1} e^{2t}$$

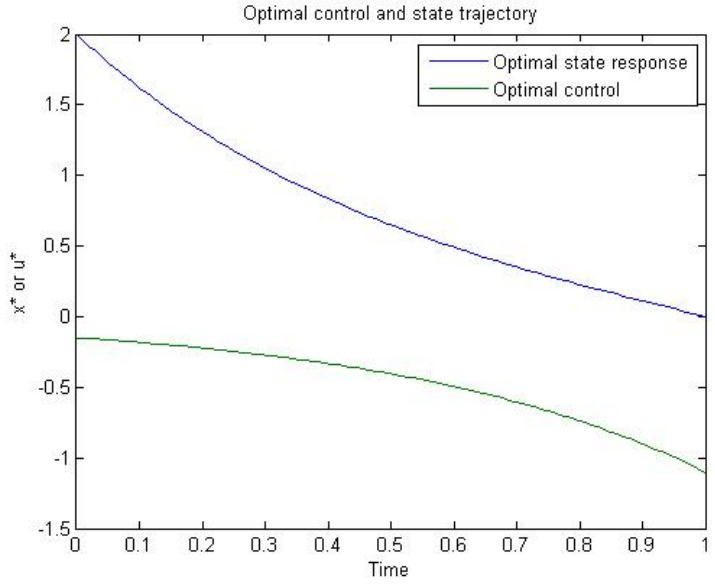
$$x^*(t) = \frac{-2}{e^4 - 1} e^{2t} + \frac{2e^4}{e^4 - 1} e^{-2t}$$

Check: $\dot{x}^*(t) = \frac{-4}{e^4-1} e^{2t} - 4\frac{e^4}{e^4-1} e^{-2t}$

$-2x^* + u^* = \frac{4}{e^4-1} e^{2t} - \frac{4e^4}{e^4-1} e^{-2t} - \frac{8}{e^4-1} e^{2t}$

equal! ✓

$x^*(0) = 2 \quad x^*(1) = 0$




```
%% AA 203 Homework 4
% Somrita Banerjee
clc
clear all
close all
t = linspace(0,1,100);
uStar = (-8/(exp(4)-1)).*exp(2.*t);
a = -2/(exp(4)-1);
b = 2*exp(4)/(exp(4)-1);
xStar = a.*exp(2.*t) + b.*exp(-2.*t);
figure
plot(t, xStar, t, uStar)
legend('Optimal state response','Optimal control')
xlabel('Time')
ylabel('x* or u*')
title('Optimal control and state trajectory');
```