

PART 7

$$u^*(0) = u_{\max}$$

$$\forall t \in [0, t_{sw}] \quad \phi(t) < 0 \Rightarrow u^*(t) = u_{\max}$$

$$\forall t \in (t_{sw}, t_f] \quad \phi(t) > 0 \Rightarrow u^*(t) = 0$$

$$\therefore \forall t \in (t_{sw}, t_f] \quad \dot{m}(t) = -bu^*(t) = 0$$

$$\Rightarrow m(t_{sw}) = m(t_f) = m_f$$

$$\forall t \in [0, t_{sw}] \quad \dot{m}(t) = -bu^*(t) = -bu_{\max} = \text{const}$$

$$\Rightarrow m(t_{sw}) = m(0) + (-bu_{\max})(t_{sw})$$

$$\Rightarrow t_{sw} = \frac{m(t_{sw}) - m_0}{-bu_{\max}}$$

$$\Rightarrow \boxed{t_{sw} = \frac{m_0 - m_f}{bu_{\max}}}$$