Red–Black Trees

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Outline

Background

Binary Search Trees are widely used due to their efficient searching ability. However, their performance depends entirely on the height of the tree.

- Balanced trees give fast performance.
- Unbalanced trees degrade to linear structures.
- Red–Black Trees guarantee balance automatically.

Why Red–Black Trees?

Real-world applications require guaranteed performance.

- Databases require predictable access time.
- Operating systems cannot afford worst-case slowdowns.
- Red–Black Trees provide guaranteed $O(\log n)$ time.

BST Characteristics

A Binary Search Tree maintains an ordered structure.

- Left subtree contains smaller keys.
- Right subtree contains larger keys.
- Efficient search when tree is balanced.

However, BSTs do not enforce balance.

Skewed Tree Formation

If elements are inserted in sorted order, BST becomes skewed.

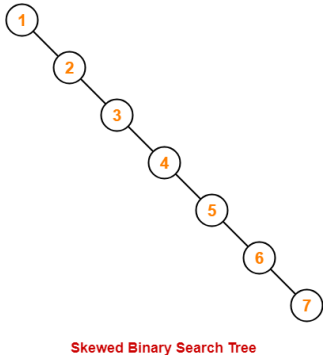


Figure: Red–Black Tree Structure

Search time becomes $O(n)$.

Key Insight

To maintain efficiency, tree height must be controlled.

- Height directly impacts performance.
- Self-balancing trees restructure automatically.
- Balance is preserved after insertions and deletions.

Self-Balancing Trees

Common self-balancing tree structures include:

- AVL Trees – strictly balanced
- Red–Black Trees – loosely balanced
- B-Trees – disk-optimized

Red–Black Trees require fewer rotations than AVL Trees.

Definition and Core Idea

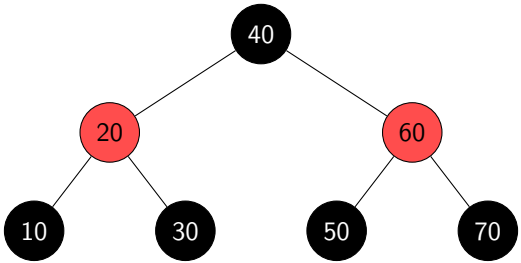
- A Red–Black Tree is a self-balancing Binary Search Tree that maintains height automatically.
- It follows all standard BST ordering rules for storing and searching keys.
- Each node contains an additional color attribute, either red or black.
- The color information is used to impose structural constraints on the tree.
- These constraints ensure that the tree never becomes highly skewed.

How Coloring Maintains Balance

- Coloring rules restrict how red and black nodes can appear along any root-to-leaf path.
- Black nodes act as anchors that regulate the overall height of the tree.
- Red nodes allow limited imbalance without immediately violating tree properties.
- The coloring strategy guarantees that no path is more than twice as long as another.
- This approach achieves balance with fewer rotations compared to strictly balanced trees.

Example of a Red–Black Tree

Balanced Structure with Coloring



- The root node is black, satisfying the root property.
- Red nodes have only black children, preventing adjacent red nodes.
- Every root-to-leaf path contains the same number of black nodes.
- The tree remains balanced while preserving BST ordering.

Fundamental Rules

- Every node in the tree is assigned exactly one color, either red or black.
- The root node of the tree is always colored black to maintain uniform structure.
- All leaf nodes (NIL or null children) are considered black by definition.
- A red node cannot have a red parent or red child, preventing consecutive red nodes.
- Every path from a node to its descendant NIL leaves contains the same number of black nodes.

Balancing Principle

- Black height is the number of black nodes on any path from a node to a NIL leaf.
- All root-to-leaf paths in a Red–Black Tree have the same black height.
- This constraint prevents the tree from becoming highly unbalanced.
- Black nodes provide the structural balance of the tree.

Why Height Remains Logarithmic

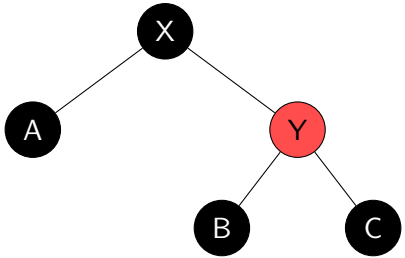
$$h \leq 2 \log_2(n+1)$$

- The black-height property ensures a minimum number of black nodes on every path.
- Red nodes cannot appear consecutively, limiting how tall any path can grow.
- Every longest path contains alternating red and black nodes at most.
- As a result, the height of a Red–Black Tree is bounded logarithmically.

Purpose of Rotations

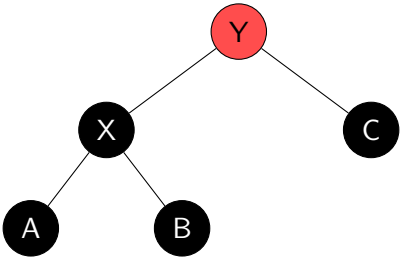
- Tree rotations are local restructuring operations used to restore balance.
- Rotations modify parent–child links without changing BST key order.
- Only a small portion of the tree is affected during a rotation.
- Rotations are applied when Red–Black Tree properties are violated.
- Both insertion and deletion algorithms rely on rotations.

Before Rotation (Right-Heavy Structure)



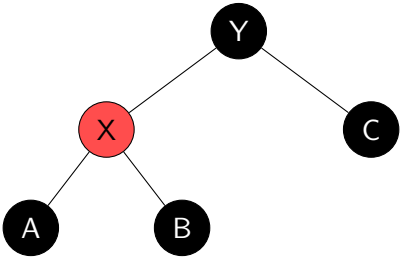
- Node X has a heavier right subtree rooted at Y.
- The imbalance violates height constraints after insertion.
- A left rotation is required at node X.

After Rotation



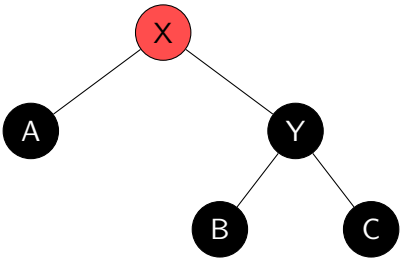
- Node Y moves up and becomes the new subtree root.
- Node X becomes the left child of Y.
- Subtree B is repositioned to preserve BST ordering.

Before Rotation (Left-Heavy Structure)



- Node Y has a heavier left subtree rooted at X.
- The imbalance occurs due to recent insertion or deletion.
- A right rotation is required at node Y.

After Rotation



- Node X moves up to become the new subtree root.
- Node Y becomes the right child of X.
- Subtree B is repositioned without violating BST rules.

High-Level Procedure

- Insertion starts by placing the new key using standard BST insertion rules.
- The newly inserted node is always colored red initially.
- If the parent node is black, no property is violated.
- If the parent node is red, Red–Black Tree properties are violated.
- Violations are fixed using recoloring and rotations.

Why Insert the New Node as Red?

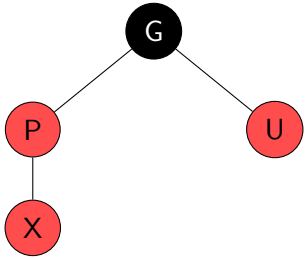
Design Choice

- Inserting a black node would increase the black height of some paths.
- A red node does not affect the black height of the tree.
- Initial red coloring makes it easier to restore balance.
- Recoloring and rotations are applied only if required.

Possible Violations

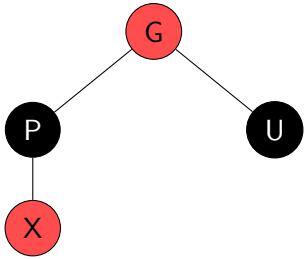
- Case 1: Parent and Uncle are both red.
- Case 2: Parent is red, Uncle is black, and nodes form a triangle.
- Case 3: Parent is red, Uncle is black, and nodes form a straight line.
- Each case applies specific recoloring and rotation rules.

Red Parent and Red Uncle



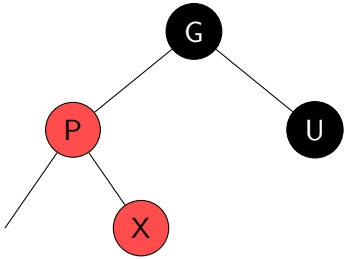
- The newly inserted node X has a red parent.
- The uncle node U is also red.
- This violates the rule that red nodes cannot have red children.

Recoloring to Restore Properties



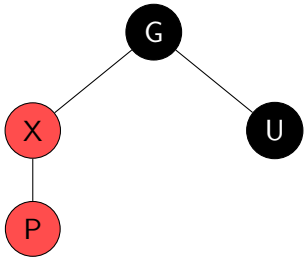
- Parent and uncle are recolored black.
- Grandparent is recolored red.
- The violation may propagate upward.

Triangle Configuration



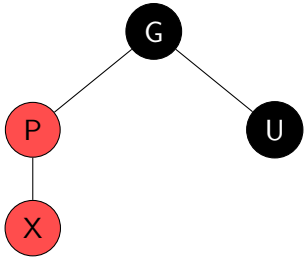
- The parent is red and the uncle is black.
- The newly inserted node forms a triangle shape.
- Direct rotation at the grandparent is not possible.

Rotation to Convert into Case 3



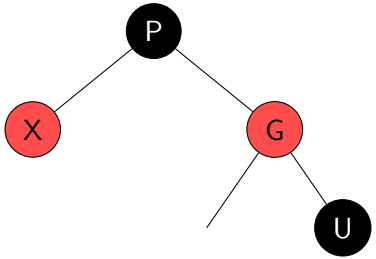
- A rotation is performed at the parent node.
- The structure is converted into a straight-line configuration.
- The problem now reduces to Case 3.

Straight-Line Configuration



- Parent is red and uncle is black.
- Nodes form a straight-line configuration.
- This configuration allows direct rotation at the grandparent.

Rotation and Recoloring



- A rotation is performed at the grandparent node.
- Colors of the parent and grandparent are swapped.
- All Red–Black Tree properties are restored.

Root Recoloring Rule

- After all insertion fix-up cases are applied, the root node is examined.
- If the root node is red, it is recolored black.
- This step ensures that the root property of Red–Black Trees is satisfied.
- Root recoloring does not affect black height consistency.
- With this step, all Red–Black Tree properties are fully restored.

Reasoning and Pseudocode

The logarithmic performance of Red–Black Trees is guaranteed by the height bound:

$$h \leq 2\log_2(n+1)$$

Search Operation (Pseudocode):

```
RB-SEARCH(node, key):
    if node == NIL or node.key == key:
        return node
    if key < node.key:
        return RB-SEARCH(node.left, key)
    else:
        return RB-SEARCH(node.right, key)
```

- Each recursive call moves one level down the tree.
- Maximum recursion depth equals the height of the tree.
- Since height is $O(\log n)$, the operation is logarithmic.

Insertion Operation

- Red–Black Tree insertion begins with a standard Binary Search Tree insertion.
- The BST insertion step takes $O(h)$ time, where h is the height of the tree.
- The newly inserted node is colored red, which does not affect black height.
- The fix-up procedure uses recoloring and at most two rotations.
- Each fix-up step moves upward toward the root and runs at most $O(h)$ times.
- Since the tree height is bounded by $O(\log n)$, insertion runs in $O(\log n)$ time.

Deletion Operation

- Deletion starts with a standard BST deletion, which takes $O(h)$ time.
- If the deleted node is red, no Red–Black Tree properties are violated.
- If a black node is deleted, the black-height property may be violated.
- The delete-fixup procedure restores balance using recoloring and rotations.
- Each fix-up iteration moves upward and executes at most $O(h)$ times.
- As the tree height is always $O(\log n)$, deletion runs in $O(\log n)$ time.

Best, Average, and Worst Case Analysis

The performance of Red–Black Trees depends on the height of the tree, which is always bounded logarithmically.

Operation	Best Case	Average Case	Worst Case
Search	$O(1)$	$O(\log n)$	$O(\log n)$
Insertion	$O(\log n)$	$O(\log n)$	$O(\log n)$
Deletion	$O(\log n)$	$O(\log n)$	$O(\log n)$

- Best case for search occurs when the key is found at the root.
- Average case assumes random key distribution in the tree.
- Worst case remains logarithmic due to strict balancing rules.
- Unlike ordinary BSTs, Red–Black Trees never degrade to linear time.

Where Red–Black Trees Are Used

- Used in Java TreeMap and TreeSet for ordered data storage.
- Implemented in C++ STL map and set containers.
- Employed inside the Linux kernel for scheduling and memory management.
- Used in databases and file systems for indexing structures.

Why Red–Black Trees Are Preferred

- Guaranteed logarithmic performance for all dynamic operations.
- Requires fewer rotations compared to strictly balanced AVL Trees.
- Efficient for applications involving frequent insertions and deletions.
- Widely adopted and well-tested in real-world systems.

Practical Constraints

- Deletion algorithm is more complex compared to insertion.
- Slightly less balanced than AVL Trees in some scenarios.
- Requires additional memory for storing color information.

Key Takeaways

- Red–Black Trees are self-balancing Binary Search Trees using color constraints.
- Structural rules and black-height property keep the tree height logarithmic.
- Rotations and recoloring restore balance efficiently after insertions.
- All core operations run in $O(\log n)$ time in the worst case.

Final Remarks

- Red–Black Trees offer a strong balance between efficiency and simplicity.
- They are suitable for large, dynamic, and performance-critical datasets.
- Their practical design makes them a standard choice in system libraries.

References

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Thank You