Exercise set 8: Image alignment

Bonus submission deadline: 11.05.2021 23:59

The exercises marked with **BONUS** should be returned in Moodle by the submission deadline. Successfully solving it will give you bonus points to boost your final grade. During the exercise session, the solution of the exercises will be discussed.

Exercise 1: RANSAC (RETURN)

The file `points.mat` contains a 2×200 matrix of points which are supposed to be on a circle. You know that there are outliers and zero-mean Gaussian noise with variance $\sigma^2 = 1$. In the script `task1.m`, use RANSAC to estimate the center and radius of the circle. The probability that at least one random sample is outlier free should be $p = 0.999$. Some hints:

- There exists one and only one circle passing through three non-collinear points.
- Brush up your high school mathematics about the equation of a circle and how to extract center and radius from it. To estimate the parameters, you may want to revise last week exercise about least squares.
- If your estimated center and radius are $\mathbf{x}_0$ and r, consider inliers the points $\mathbf{x}$ for which $|\ \|\mathbf{x}-\mathbf{x}_0\|-r| \leq t$ where $t = \sigma\sqrt{3.84}$ is the threshold.
- It was shown in the lecture that the probability of picking N times a sample of size s containing at least one outlier is $(1-(1-e)^s)^N$, hence the probability to pick at least once an outlier free sample is $p = 1-(1-(1-e)^s)^N$, where e is the outlier ratio. Solve for N from this equation to obtain a formula for the maximum number of iterations.
- Here you are not given the outliers ratio. Hence at each iteration, after having computed the number of inliers, you will have to estimate e and update the maximum number of iterations N accordingly.
- After the RANSAC loop has terminated, estimate the circle parameters one more time using least squares on the inliers.

Exercise 2: 2D transformations taxonomy

1. Consider the following transformations: translation, scaling, affine transformation, rotation, similarity, euclidean transformation, projective transformation (homography). Draw a diagram showing their hierarchy (i.e. what transformation is a special case of what).

2. Write the transformation matrix for each of the previous transformations. Indicate also the number of degrees of freedom.

3. Complete the following table, indicating what quantities are *invariant* under each transformation, i.e. the transformation does not change them.

	projective	affine	similarity	euclidean
concurrency				
collinearity				
parallelism				
ratio of areas				
ratio of lengths				
angles				
lengths				

4. Are transformations commutative? Give an example.

5. Prove that each affine transformation $\mathbf{A}$ can be obtained as a composition of rotations, scaling and translation.
Hint: An affine transformation can be written in the form $\mathbf{A} = \begin{bmatrix} \mathbf{L} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}$. Hence, each affine transformation is obtained first applying $\mathbf{L}$ and then the translation $\mathbf{t}$. Can you decompose $\mathbf{L}$ into 2 rotations and one scaling?

Exercise 3: 2D similarity estimation

The 2D similarity maps a point $\mathbf{x} = [x, y]$ to $\mathbf{x}' = [x', y']$ with equation

$$\mathbf{x}' = s\mathbf{R}\mathbf{x} + \mathbf{t} = s \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \tag{1}$$

How many point correspondences do you need at least to estimate the transformation parameters? Give an algorithm to estimate the parameters from the minimum number of correspondences.