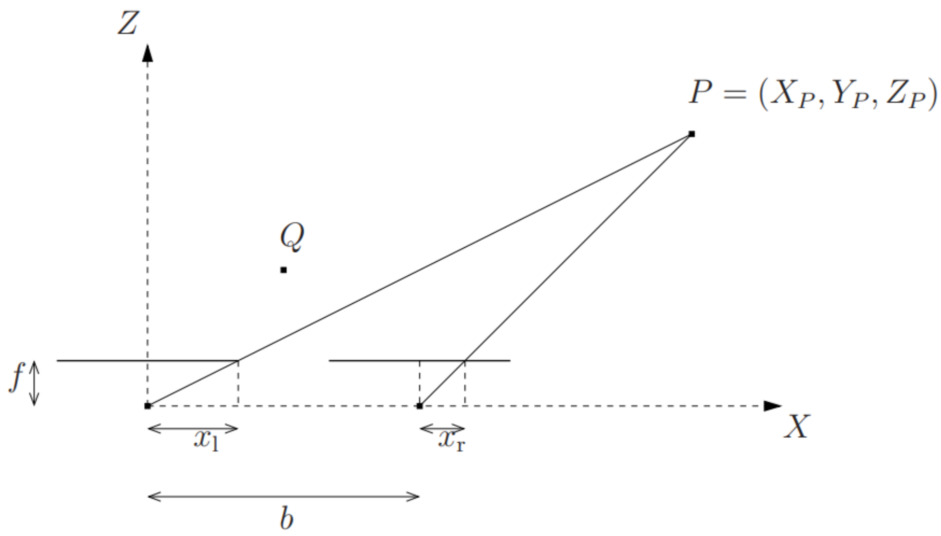
Exercise set 9: Multiple view geometry

Bonus submission deadline: 18.05.2021 23:59

The exercises marked with **BONUS** should be returned in Moodle by the submission deadline. Successfully solving it will give you bonus points to boost your final grade. During the exercise session, the solution of the exercises will be discussed.

Exercise 1: Disparity estimation (BONUS)

The figure below shows a typical stereo configuration, where two similar pinhole cameras are placed side by side. The focal length of the cameras is f and the distance between the camera centers is b. The point P is located in front of the cameras and its disparity d is the distance between the corresponding image points (i.e. $d = |x_l - x_r|$). The disparity depends only on the parameters b and f and the Z-coordinate of P.



Kuva 1: Setup for exercise 1

- Assume that $d = 1\,\text{cm}$, $b = 6\,\text{cm}$ and $f = 1\,\text{cm}$. Compute Z_P.
- Assume that the smallest measurable disparity is 1 pixel and the pixel width is 0.01 mm. What is the range of Z-coordinates for those points for which the disparity is below 1 pixel?
- In the configuration illustrated in the picture the camera matrices are $\mathbf{P}_l = \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}$ and $\mathbf{P}_r = \begin{bmatrix}\mathbf{I} & \mathbf{t}\end{bmatrix}$, where $\mathbf{I}$ is the identity matrix and

$\mathbf{t} = \begin{bmatrix} -6 & 0 & 0 \end{bmatrix}^{\mathsf{T}}$. The point Q has coordinates $(3, 0, 3)$. Compute the image of Q on the image plane of the camera on the left and the corresponding epipolar line on the image plane of the camera on the right. (**Hint**: assume this is the calibrated case, i.e. you can use the essential matrix instead of the fundamental matrix. Review from the slides how to compute epipolar lines.)

Exercise 2: epipolar geometry

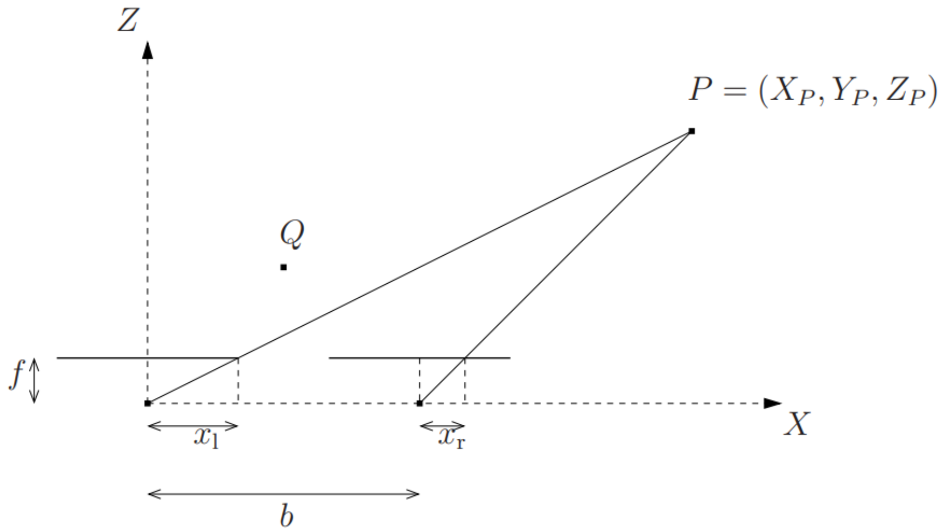
Let's assume that the camera projection matrices of two cameras are $\mathbf{P} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}$ and $\mathbf{P}' = \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}$, where $\mathbf{R}$ is a rotation matrix and $\mathbf{t} = \begin{bmatrix} t_1 & t_2 & t_3 \end{bmatrix}^{\mathsf{T}}$ describes the translation between the cameras. Hence, the cameras have identical internal parameters and the image points are given in the normalized image coordinates (the origin of the image coordinate frame is at the principal point and the focal length is 1). The epipolar constraint is illustrated in Figure 1 below and it implies that if p and p' are corresponding image points then the vectors $\overrightarrow{Op}$, $\overrightarrow{O'p'}$ and $\overrightarrow{O'O}$ are coplanar, i.e

$$\overrightarrow{O'p'} \cdot \left(\overrightarrow{O'O} \times \overrightarrow{Op} \right) = 0 \qquad (1)$$

Let $\mathbf{x} = (x, y, 1)^{\mathsf{T}}$ and $\mathbf{x}' = (x', y', 1)^{\mathsf{T}}$ denote the homoegeneous image coordinate vectors of p and p'. Show that the equation (1) can be written in the form

$$\mathbf{x}'^{\mathsf{T}} \mathbf{E} \mathbf{x} = 0, \qquad (2)$$

where $\mathbf{E}$ is the essential matrix $\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$.



Kuva 2: Setup for exercise 2