STATS 412

Fifth Class Note

In Son Zeng

24 September, 2018

My Office Hour:

My office hours are on **16:30 - 18:00 Tuesday** and **13:30 - 15:00 Friday**, at **USB 2165**. You may check the campus map to get to my office.

Mid-autumn (Moon) Festival:

Today is the Moon Festival. In Chinese-speaking community, this is a day when people reunion with their family members. One typical food for Moon Festival is mooncake and one famous activity is the Moon Festival Riddle Game. Well, if you have friends who come from Chinese-speaking countries, say **Happy Moon Festival**. They will be impressed! Have a great day.

Reminders for Assignment 3

• The notion that we repeat the same trial or experiment until obtaining the first success/failure/(something) indicates that the random variable X follows geometric distribution. It is a discrete probability distribution with probability mass function:

$$\boxed{P(X=k)=(1-p)^{k-1}\cdot p, k=1,2,3,.....} \tag{1}$$

• We can extend that notion to repeat the same trial or experiment until the r success/failure occurs. In this case, the random variable X follows negative binomial distribution. It is also a discrete probability distribution, with probability mass function:

$$\boxed{P(X=n)=\binom{n-1}{r-1}(1-p)^{n-r}p^{r-1}\cdot p=\binom{n-1}{r-1}(1-p)^{n-r}p^{r}, n=r,r+1,r+2,.....} \tag{2}$$

Key points during lecture:

Solution to Gamma distribution and its Family (from fourth class note):

The density of gamma(α,β) distribution is given by:

$$\boxed{f(x;\alpha,\beta)=\frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)},\ \alpha>0,\ \beta>0,\ 0<x<\infty} \tag{3}$$

Then, its expectation and variance are derived by:

$$\boxed{E(X)=\alpha\cdot\beta} \tag{4}$$

$$\boxed{V(X)=\alpha\cdot\beta^2} \tag{5}$$

To recap, the expectation of Gamma distribution is the product of the shape and scale parameter, while the variance of Gamma distribution is the product of the shape parameter and the square of the scale parameter.

Question: Could any of you try to derive the expected amount of snow on a day that it snows by using the Gamma distribution? This may serve as a more convenient way to compute than doing the integration by part!

Solution: Given $f(x)=9xe^{-3x}$, we can refer to the probability density function of Gamma distribution and obtain $\alpha=2$, $\beta=\frac{1}{3}$ (Check!). Therefore, the expected amount of snow on a day that it snows is

$$\boxed{E(X)=\alpha\cdot\beta=\frac{2}{3}} \tag{6}$$

Okay, now we can even derive the variance and standard deviation. Simply applying the variance formula for Gamma distribution we obtain:

$$\boxed{V(X) = \alpha \cdot \beta^2 = \frac{2}{3^2} = \frac{2}{9}} \tag{7}$$

$$\boxed{SD(X) = \sqrt{V(X)} = \sqrt{\frac{2}{3^2}} = \frac{\sqrt{2}}{3}} \tag{8}$$

General Formulas for Variance and Standard Deviation:

You are told in class that:

$$\boxed{V(aX + b) = a^2 \cdot Var(X)} \tag{9}$$

$$\boxed{SD(aX + b) = |a| \cdot SD(X)} \tag{10}$$

Question: Suppose that salaries at a company have mean \$50,000 and standard deviation \$10,000. Could you derive the variance and standard deviation from above formulas if a) everyone at the company is given a \$500 raise? b) everyone at the company is given a 5% raise?

Test reminder: Unit is important! Do try to include the unit when answering the test problems. Misplace unit can be a serious problem.

Last Comment:

Please inform me to fix the typos and grammatical mistakes if they exist. It is a great practice of writing and I appreciate your help!