STATS 412

Eighth Class Note

In Son Zeng

03 October, 2018

My Office Hour:

My office hours are on **16:30 - 18:00 Tuesday** and **13:30 - 15:00 Friday**, at **USB 2165**. You may check the campus map to get to my office. I am prepared for your questions, so please feel free to come to my office hours.

Calculus Review:

• Last week, I uploaded a book for probability to the Piazza. Particularly, I recommend you to read (Type the page number in PDF):

(1) Improper integral: page 203 - 210

(2) Double integral: page 214 - 218

(3) Cumulative distribution function: page 224 - 229

(4) Expectation and Variance: page 235 - 244

(5) Joint Distributions: page 299 - 304

(6) Combination (Ignore the proof): page 38 - 42

(7) Binomial Distribution: page 135 - 146

These are great practices to prepare you with essential integration skills for the subsequent homework and the midterm.

Reminders for Assignment 4

• For double integrations, you may encounter trouble if you directly integrate out the $f(x, y)$. In this case, if you integrate y first, you can pull the terms including only x out first and simplify the stuff you have to integrate.

• The concepts "0, otherwise" is always important. You should include this for every probability density/mass function, conditional distribution and marginal distribution.

Key Points during Lecture:

Relationship between Bernoulli and Binomial Distribution: If $X_1, X_2, ..., X_n$ are independent Bernoulli trials with $Ber(p)$, then the random variable $Y = X_1 + X_2 + ... + X_n \sim Bin(n, p)$. Also, since the expectation and variance of Bernoulli distribution are $E(X) = p$ and $Var(X) = p(1-p)$, by independence, we obtain the expectation and variance of $Bin(n, p)$ are

$$\boxed{E(Y) = E(X_1 + X_2 + ... + X_n) = E(X_1) + E(X_2) + ... + E(X_n) = p + p + ... + p = np} \tag{1}$$

$$\boxed{Var(Y) = Var(X_1 + X_2 + ... + X_n) = Var(X_1) + ... + Var(X_n) = p(1-p) + ... + p(1-p) = np(1-p)} \tag{2}$$

Optional Question: Since the probability mass function for binomial distribution is given by,

$$\boxed{P(Y = y) = \begin{cases} \binom{n}{y} p^y (1-p)^{n-y}, y = 0, 1, 2, ..., n-1, n \\ 0, otherwise \end{cases}} \tag{3}$$

Could you compute $E(X)$ and $V(X)$ by applying the formula (summation of possible points)? This will give the formal proof of binomial distribution.

Test Reminder: When you take a square root of variance to obtain the standard deviation, be sure to include the absolute value. It is because standard deviation is the positive square root of the variance. For example,

$$\boxed{SD(\hat{p}) = SD\Big(\frac{X}{n}\Big) = \Big|\frac{1}{n}\Big| SD(X) = \frac{1}{n}\sqrt{np(1-p)} = \sqrt{\frac{p(1-p)}{n}}} \tag{4}$$

As we do not know p, we use $\hat{p}$ to estimate p with uncertainty $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

Optional Question: When does the uncertainty achieve its maximum?

Last Comment:

Please inform me to fix the typos and grammatical mistakes if they exist. It is a great practice of writing and I appreciate your help!