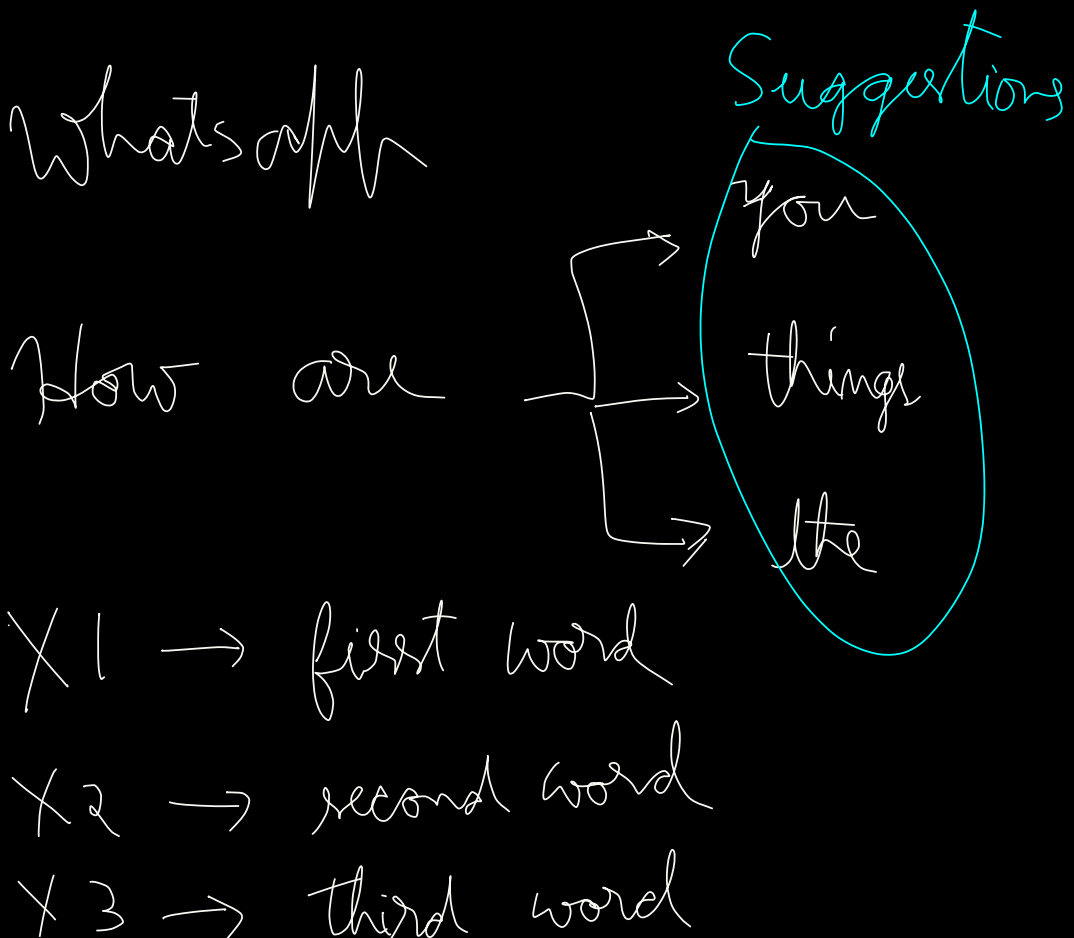
# Conditional Probability




$X1 \rightarrow$ first word

$X2 \rightarrow$ second word

$X3 \rightarrow$ third word

Given that $X_1 =$ "How" and $X_2 =$ "are"

Compute $X_3$ for every word

Conditional Prob

$$P[X_3 = \text{"the"} \mid X_1 = \text{"How"},\ X_2 = \text{"are"}]$$

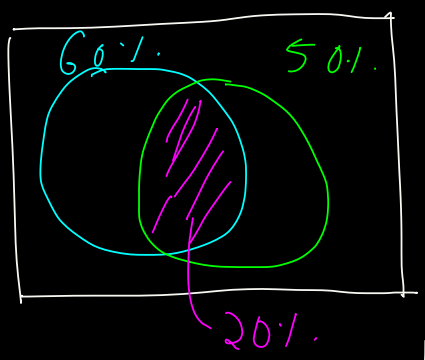
Lots of data, it manages to compute

Choose those words which have max prob given $\{X_1 = \text{How}, X_2 = \text{are}\}$

$X_1 =$ "are" $X_2 =$ "how"

$X_3$ → however
→ how's
→ emoji

It is known that 60% people use Swiggy, 50% use Zomato. 20% people use both.



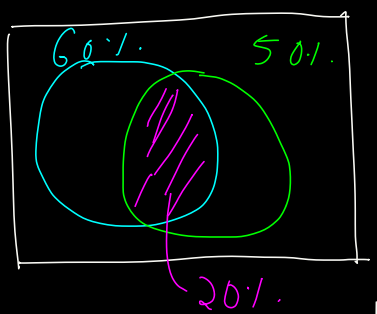

$$P[Sw \mid Z] = \frac{20}{50}$$

Among 60 swiggy users, how many of them Zomato also? $\longrightarrow$ 20

What fraction of Sw users also use Zomato $\frac{20}{60}$ $\frac{1}{3}$ or Swiggy users also use Zomato!

Given that a person is a Sw user

$$P[Zom \mid Swiggy] = \frac{20}{60}$$




how

Swi → 60%.

Zom → 50%.

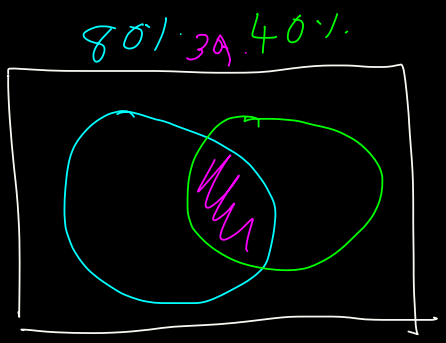
Swiggy ∩ Zomato → 20%.

$$P[Z|S] = \frac{20}{60} = \frac{P[S \cap Z]}{P[S]}$$

Conditional Prob

Given that a person is a Swiggy user, what is the Prob that he is also a Zom user

It is known that 80% people like cappuccino, 40% people like espresso, and 30% like both. Among the people who like cappuccino, what fraction of people like espresso?




$$P[C] = 0.8 \qquad P[E] = 0.4$$

$$P(C \cap E] = 0.3$$

$$P[E|C] = \frac{P[E \cap C]}{P[C]} = \frac{0.3}{0.8} = \frac{3}{8}$$

In a city, 7% of people are on Twitter.
5% of people are on LinkedIn.
4% of people are on both LinkedIn and Twitter.

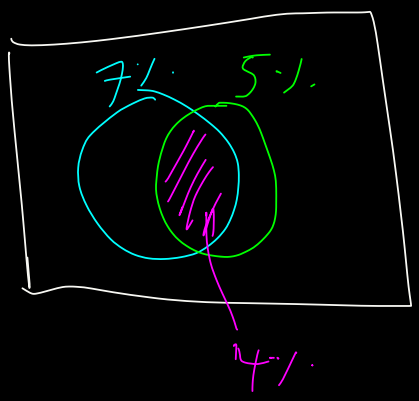
$P[T] = 0.07$

$P[L] = 0.05$

$P[L \mid T] = \frac{4}{7}$

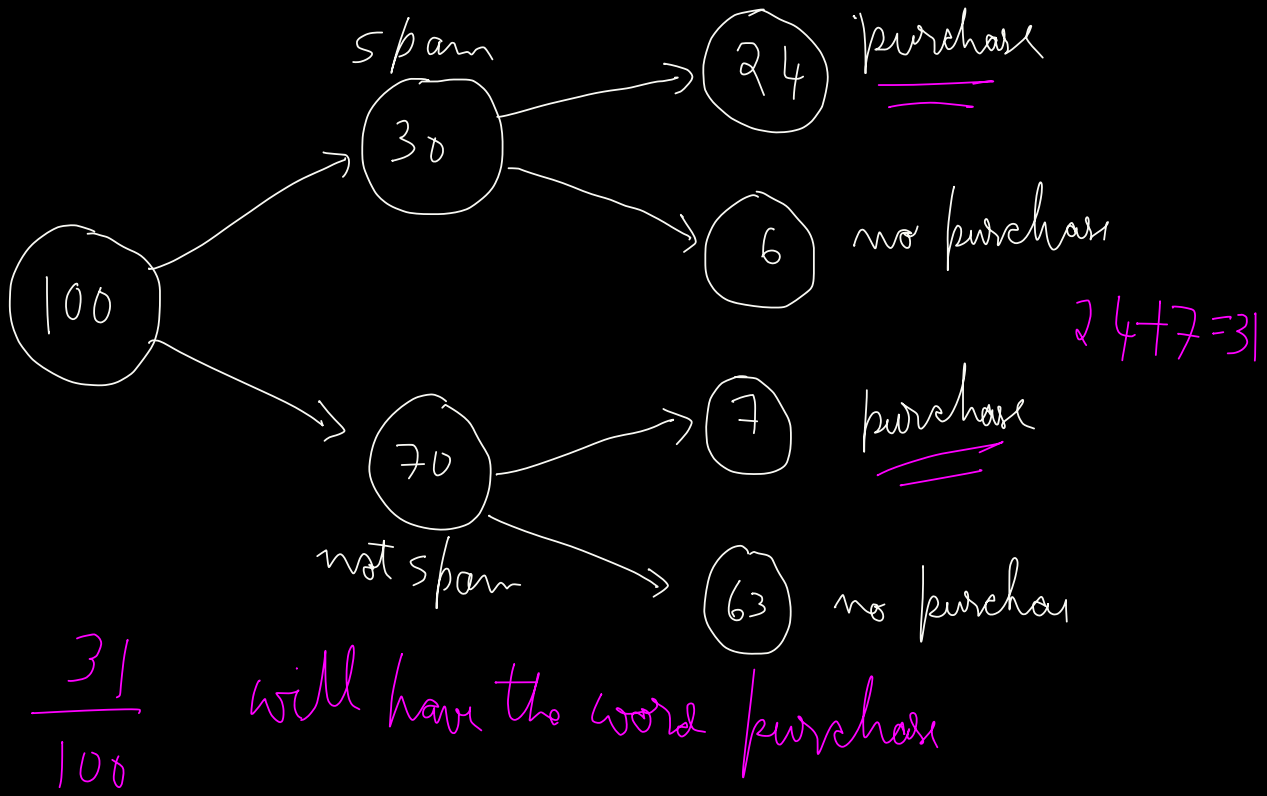
$P[T \mid L] = \frac{4}{5}$

would you bet?
no!



It is known that 30% of emails are spam, and 70% are not spam
The word “purchase” occurs in 80% of spam emails. It also occurs in 10% of non-spam emails

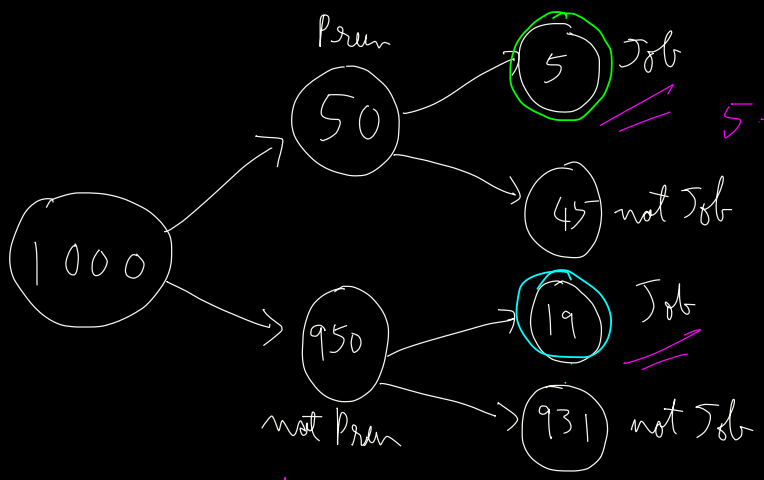
Overall, in what percentage of emails would we see the word “purchase”?



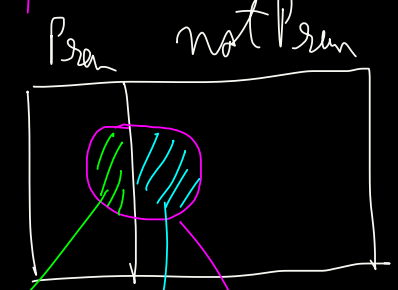

24 + 7 = 31

$\frac{31}{100}$ will have the word purchase

It is known that 5% of all LinkedIn users are premium users
10% of premium users are actively seeking new job opportunities.
Only 2% of non-premium users are actively seeking new job opportunities
Overall, what percentage of people are actively seeking new job opportunities?




$5 + 19 = 24$




$\frac{5}{1000}$ $\frac{19}{1000}$ Job seeker

$\frac{24}{1000}$ are seeking Job

$P[\text{Prem}] = 0.05$ $P[\text{not Prem}] = 0.95$

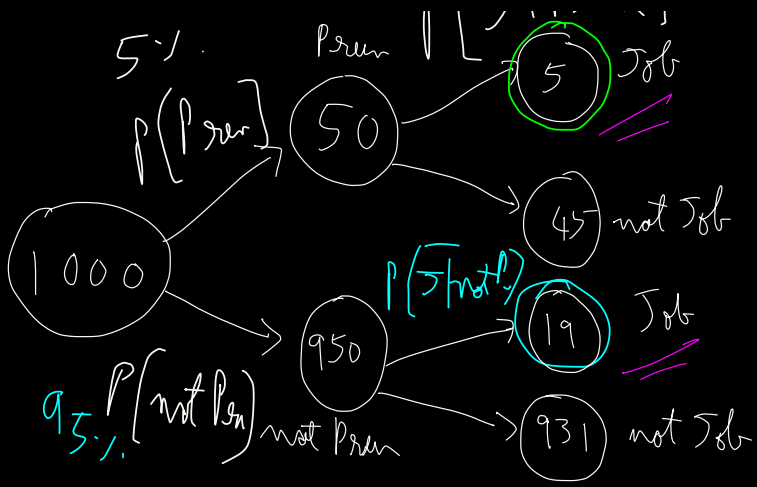
$P[\text{Prem} \cap \text{Job}] = \frac{5}{1000}$ $P[\text{not Prem} \cap \text{Job}] = \frac{19}{1000}$

$P[\text{Prem} \cap \text{not Job}] = \frac{45}{1000}$ $P[\text{not Prem} \cap \text{not Job}] = \frac{931}{1000}$

$P[\text{Job} \mid \text{Prem}] = 0.1$ $P[\text{Job} \mid \text{not Prem}] = 0.02$

$P[\text{Job}] = \frac{24}{1000}$

10%.
$P[\neg \mid \text{Prem}]$




$$P[J \cap Prem] = \frac{5}{1000}$$

$$P(Prem)\,P(J|Prem)$$

$$\left(\frac{50}{1000}\right)\left(\frac{5}{50}\right) = \frac{5}{100}$$

$$P(\text{not Prem}) = \frac{950}{1000}$$

$$P[J \mid \text{not Prem}] = \frac{19}{950}$$

$$P[J \cap \text{not Prem}]$$

$$\left(\frac{950}{1000}\right)\left(\frac{19}{950}\right) = \frac{19}{1000}$$

$$P[\overline{Job}] = P[\overline{Job} \cap Prem] + P[\overline{Job} \cap \text{not } P]$$

$$= \frac{5}{1000} + \frac{19}{1000} = \frac{24}{1000}$$

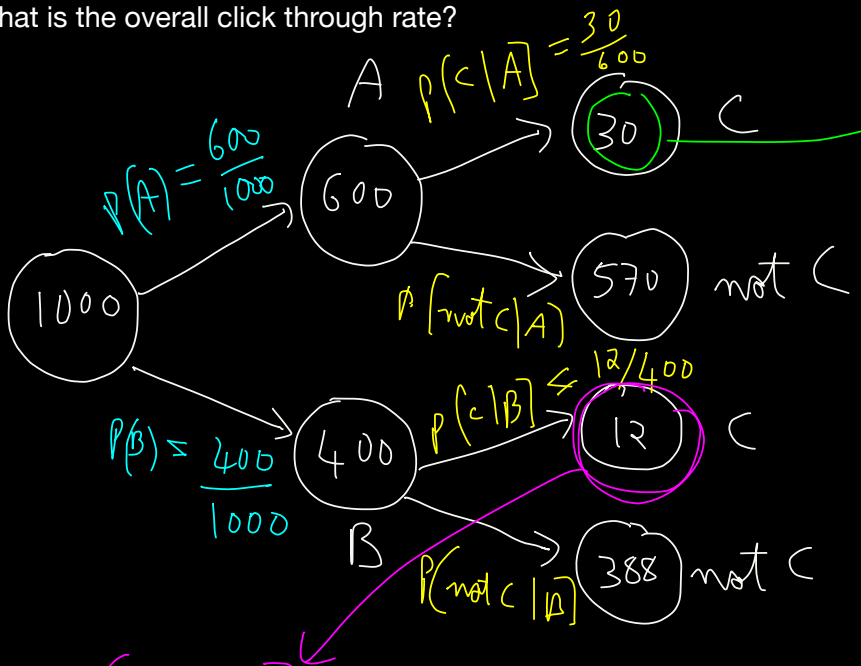
## Law of Total Prob

$$P[\overline{Job}] = P[\overline{Job} \mid Prem]\,P[Prem] + P[\overline{Job} \mid \text{not Prem}]\,P[\text{not}]$$

$$P[A|B] = \frac{P(A \cap B)}{P(B)} \qquad P[A \cap B] = P[A|B]\,P(B)$$

# CTR: click-through rate

An e-commerce website shows two types of ads: Type A and Type B.
60% of the visitors see Type A ads, and 40% visitors see Type B ads
The click-through rate for Type A ads is 5%, while the click-through rate for Type B ads is 3%
What is the overall click through rate?




$\frac{30}{1000} \rightarrow P[A \cap C]$

$P(A)\,P(C|A)$

$\left(\frac{600}{1000}\right)\left(\frac{30}{600}\right) = \frac{30}{1000}$

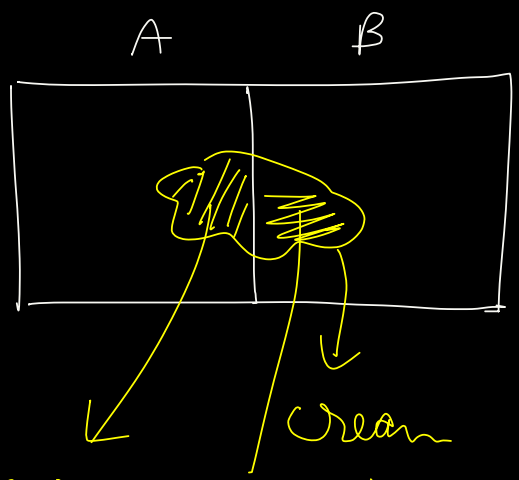
$P(C|A) = \frac{P[A \cap C]}{P[A]}$

$P(C \cap B) = \frac{12}{1000}$

$P(B)\,P(C|B) = \left(\frac{400}{1000}\right)\left(\frac{12}{400}\right)$

$P[C] = \frac{30}{1000} + \frac{12}{1000} = P[A \cap C] + P[B \cap C]$

$P(C) = P(C|A)\,P(A) + P(C|B)\,P(B)$



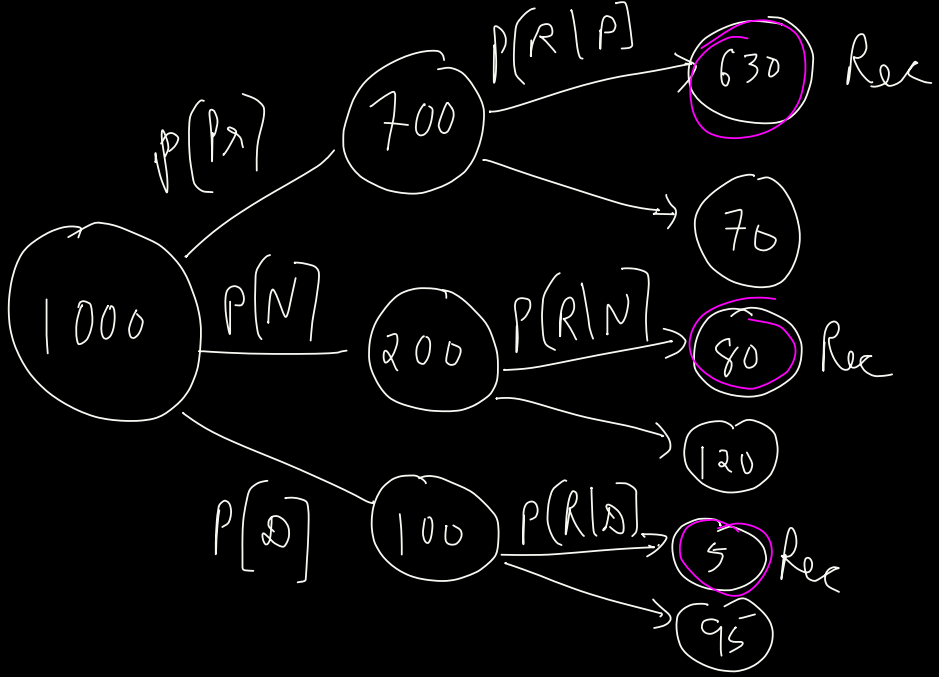

$$C = (C \cap A) \cup (C \cap B)$$

$$P(C) = P(C \cap A) + P(C \cap B)$$

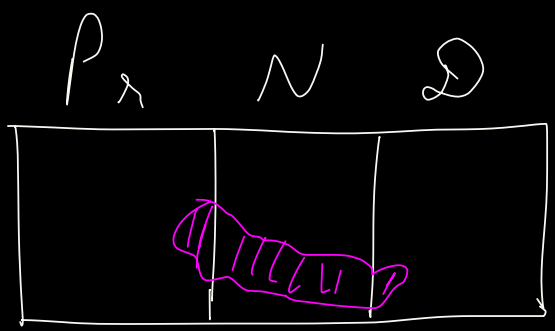
$$P(C) = P(C|A)P(A) + P(C|B)P(B)$$

Law of Total Probability

In an NPS survey, it is seen that 70% are promoters, 20% are neutral, 10% are detractors
90% of promoters recommend the product to a friend
40% of neutral/passive recommend the product to a friend
5% of detractors recommend the product to a friend
Overall, what percentage of people recommend the product to a friend?




$P(Rec) =$
$P(Rec \cap P)$
$+$
$P(Rec \cap N)$
$+$
$P(Rec \cap D)$
$= \frac{630+80+5}{1000}$




Mutually exclusive and exhaustive

$P, N, D \longrightarrow P \cup N \cup D$

has everybody

Mutually exclusive $P_R \cap N = \phi$

$P[P_R \cap N] = 0$ $P_R \cap D = \phi$

etc. $N \cap D = \phi$

Exhaustive: $P_R \cup N \cup D =$ Sample Space

$P[P_R \cup N \cup D] = 1$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$ Conditional Prob

$$P(A \cap B) = P(A|B)\,P(B)$$ Multiplication rule

$$P(C) = P[C \cap A] + P[C \cap A^c]$$

$$P(C) = P(C|A)\,P(A) + P(C|A^c)P(A^c)$$ Total Prob.