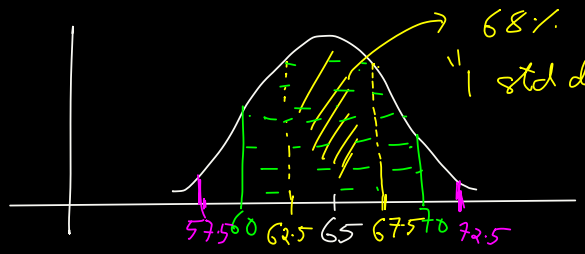
Gaussian Distribution
("Normal" distribution)

- 68/95/99 rule (Empirical rule)
- Z-score
- Computing Probability
- Retail example

Heights



68%
"1 std dev"

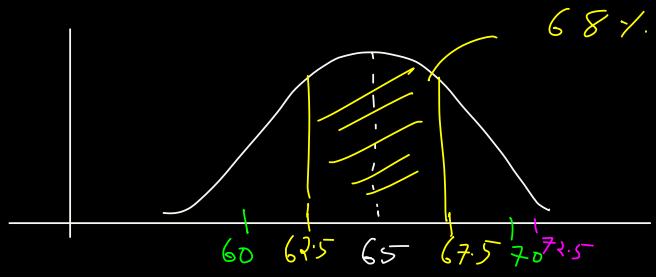
"mean" $\mu = 65$

"std dev" $\sigma = 2.5$

Between 60 & 70 $\longrightarrow$ 95% "2 std dev"

Between 57.5 & 72.5 $\longrightarrow$ 99.7% "3 std dev"

The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches.
What fraction of people are shorter than 67.5?

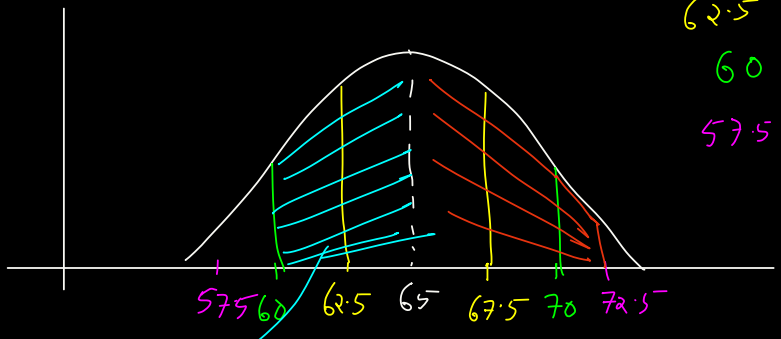


① What fraction is shorter than 65 ? $\longrightarrow 0.5$

② What fraction is b/w 65 & 67.5 ? $\longrightarrow \frac{0.68}{2} = 0.34$

fraction below 67.5 $\longrightarrow 0.5 + 0.34 = 0.84$

The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches.
What is the fraction of people whose height is between 60 and 72.5?



62.5 & 67.5 → 68%.

60 & 70 → 95%.

57.5 & 72.5 → 99.7%.

60 & 65 → $\frac{95}{2}$ %.

65 & 72.5 → $\frac{99.7}{2}$ %.

60 & 72.5 → $\left(\frac{95}{2} + \frac{99.7}{2}\right)$ %. 97.35%.

The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches

Height in inches

$62.5 = 65 + (-1)(2.5)$
$60 = 65 + (-2)(2.5)$

$67.5 = 65 + (1)(2.5)$
$70 = 65 + (2)(2.5)$
$72.5 = 65 + (3)(2.5)$

How many std dev away from mean?

69.1 is "3" std dev away from mean

$69.1 = 65 + 3(2.5)$

$3 = \frac{69.1 - 65}{2.5} = 1.64$ "Z-Score"

70.8

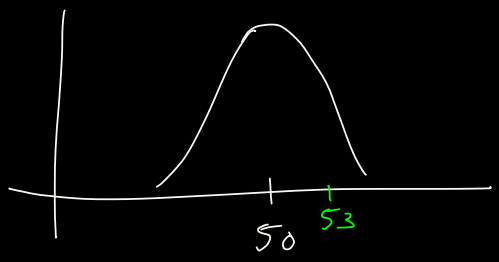
$3 = \frac{70.8 - 65}{2.5} =$

What fraction are shorter than 69.1 inches?

① Z-table → Z-Score : 1.64 → 0.9495

② Python Scipy

Balls produced by manufacturer have mean 50 mm and std dev 2 mm.
What fraction of balls are smaller than 53 mm?



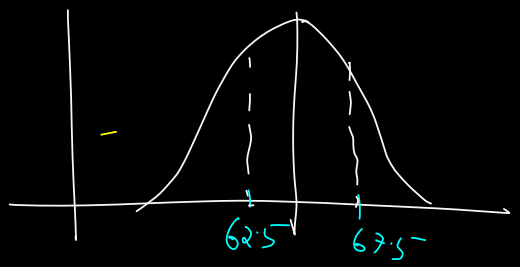
$$3 = \frac{53-50}{2} = 1.5$$

$$\text{normcdf}(1.5) = 0.93$$

Derive the Emhirical rule (68/95/99)

$\mu = 65$

$\sigma = 2.5$



① what fraction shorter than 67.5 → norm. cdf(1)

$z = 1$

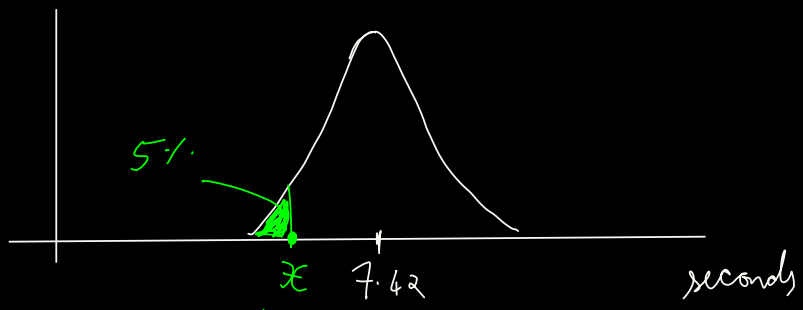
② what fraction shorter than 62.5 → norm. cdf(−1)

$z = -1$

The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches

One person says:
96% people are shorter than me. What is my height?

Skaters take a mean of 7.42 seconds and std dev of 0.34 seconds for 500 meters.
What should his speed be such that he is faster than 95% of his competitors?



Only 5% of people take (time) less time than x seconds

$z = \text{norm. pf}(0.05)$ $\quad x = 7.42 + z(0.34)$

$\text{Speed} = \frac{500}{x}$

Inventory

Analyse sales data & choose optimum inventory

Weekly cycle

Sometimes we need in between replenishment

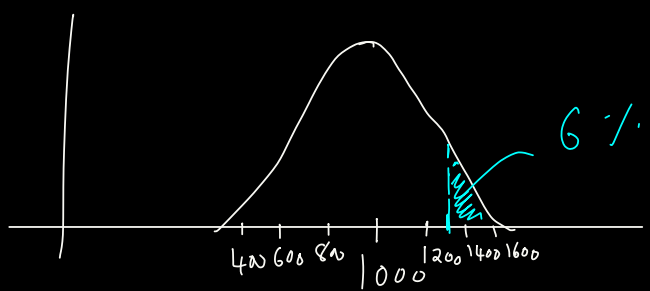
Estimate the demand.

Give probabilistic guarantees

A retail outlet sells around 1000 toothpastes a week, with std dev = 200.

$\mu = 1000$

$\sigma = 200$



Stock 1300 units?

what fraction of weeks will we go out of stock (need replenishment)

$$z = \frac{1300 - 1000}{200} = 1.5$$

1 - norm.cdf(3)

We want at most 3% of time out of stock

What should be the inventory?

$$z = \frac{x - 1000}{200}$$

97% percentile

$x = 1000 + \text{norm.ppf}(0.97)\ 200 = 1376$