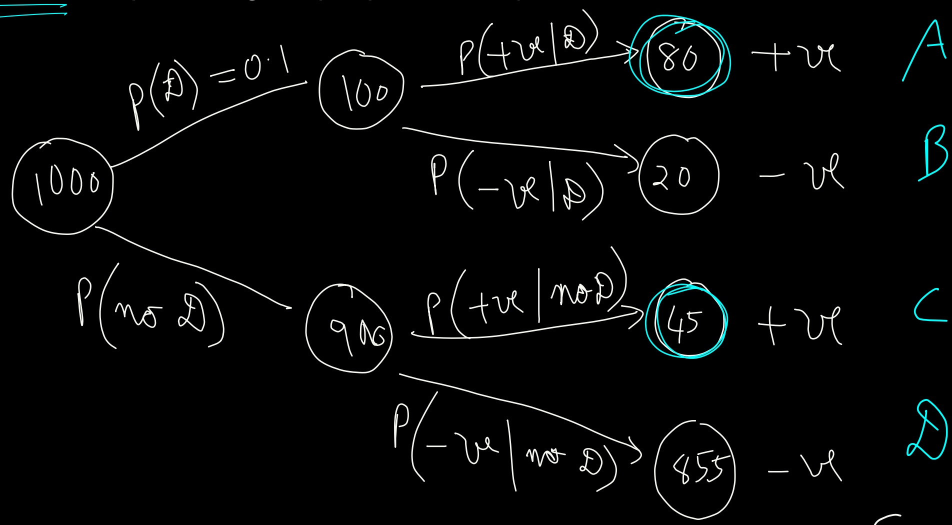
A disease affects 10% of the population.
Among those who have the disease, 80% get “positive” test result
Among those who don’t have the disease, 5% get “positive” test result
Overall, what percentage of people tested “positive”?



$$\frac{80+45}{1000}$$

12.5%

$$P[+ve \cap D] = \frac{80}{1000}$$

$$P[+ve \mid D]\, P[D]$$

$$(0.8)\ (0.1) = 0.08$$

$$P[+ve \cap \text{no } D] = \frac{45}{1000}$$

$$P[+ve \mid \text{no } D]\, P(\text{no } D)$$

$$(0.05)(0.9)$$

Suppose you are tested positive.
What is the probability that you
have the disease?

$$P(D \mid +ve)$$

A) $P(D)$

B) $P(+ve \mid D)$

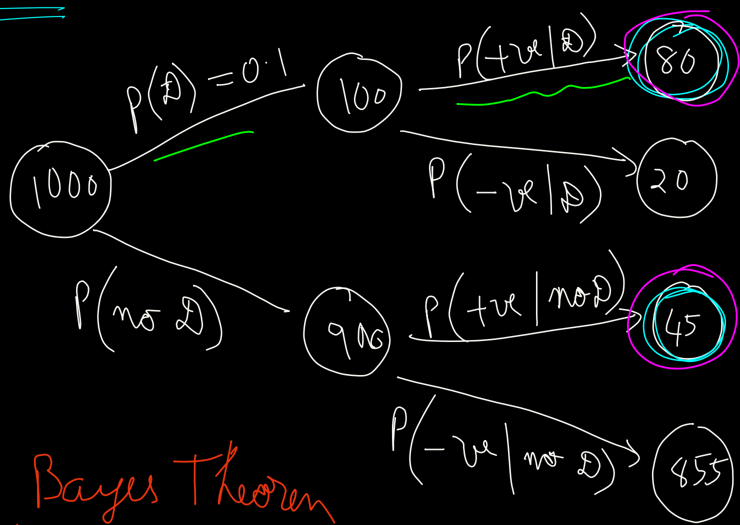
C) $P(D \mid +ve)$

D) $P(D \cap +ve)$

If you are positive, then you belong to (80 + 45) people

Among these, how many have disease? $\rightarrow$ 80

$$P(D \mid +ve) = \frac{80}{80+45} = 0.64$$



Bayes Theorem

$$\boxed{P(D \mid +ve) = \frac{P[D \cap +ve]}{P[+ve]}} = \frac{\overset{(0.8)}{P[+ve \mid D]}\ \overset{(0.1)}{P(D)}}{P[+ve]}$$

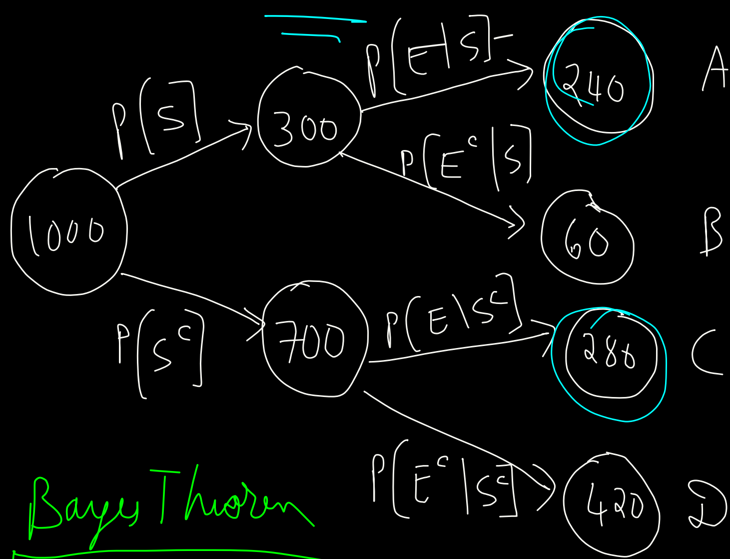
$$P[+ve] = P[+ve \mid D]\,P(D) + P[+ve \mid no\ D]\,P(no\ D)$$

$$= (0.8)(0.1) + (0.05)(0.9) = 0.125$$

$$P(D \mid +ve) = \frac{(0.8)(0.1)}{0.125}$$

For a new cohort in DSML, we have the following information
30% of the people know SQL.
80% of the people who know SQL also know Excel.
40% of the people who do not know SQL, also know Excel.

Among those who know Excel, what percentage know SQL?



Among 240+280 people, what fraction know SQL?

$$\frac{240}{240+280}$$

Bayes Theorem

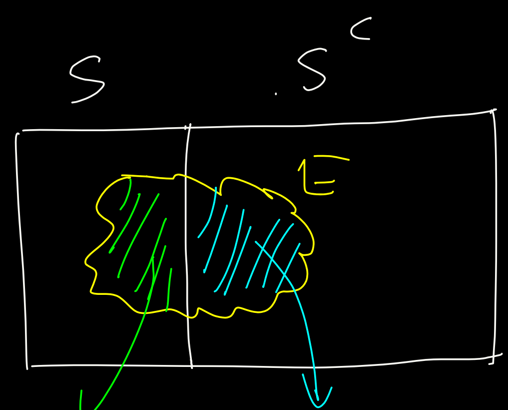
$$\boxed{P[S|E] = \frac{P[S \cap E]}{P[E]}} = \frac{P(E|S)\,P(S)}{P(E)}$$

$$= \frac{(0.8)(0.3)}{(0.8)(0.3)+(0.4)(0.7)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow P(A \cap B) = P(A|B)P(B)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \rightarrow P(A \cap B) = P(B|A)P(A)$$

$$P(E) = P(E|S)P(S) + P(E|S^c)P(S^c)$$



$$E = (S \cap E) \cup (S^c \cap E)$$

$$P(E) = P(S \cap E) + P(S^c \cap E)$$

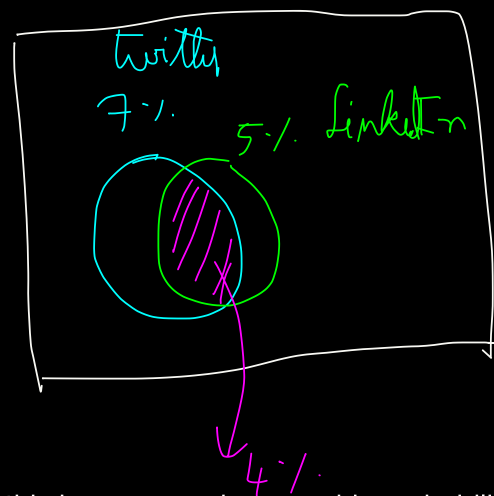
$$\boxed{P(E) = P(E|S)P(S) + P(E|S^c)P(S^c)}$$

Total Prob

In a city, 7% of people are on Twitter.
5% of people are on LinkedIn.
4% of people are on both LinkedIn and Twitter.
A random person is chosen. What is the probability that he is on Twitter?

$$P(T) = 0.07$$

$$P(T \mid L) = \frac{4}{5} = \frac{P(T \cap L)}{P(L)}$$



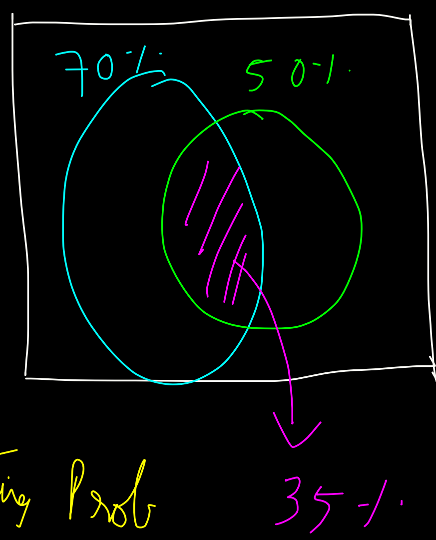
The extra information that a person is on LinkedIn, did this increase or decrease his probability of being on Twitter?
"Dependent" ---> T and L

A website has noticed the following stats.
Among those who saw the ad, 70% saw it on Youtube, 50% saw it on Amazon, 35% saw it on both.
A random person is chosen. What is the probability that he saw the ad on Youtube?

$P(Y) = 0.7$

$P[Y|\underline{A}] = \frac{35}{50} = 0.7$

$\hookrightarrow$ Additional info is not affecting Prob of seeing on Youtube



Y and A are called independent

$P[A] = 0.5$

$P[A|\underline{Y}] = \frac{35}{70} = 0.5$

$\hookrightarrow$ Additional info is not affecting

A and B are independent if

$$P[A|B] = P(A)$$

$$\frac{P(A \cap B)}{P(B)} = P(A)$$

$$P(A \cap B) = P(A)\,P(B)$$

also means independent

Coin & Dice $\longrightarrow$ Sample space

$$S = \left\{ \begin{matrix} H1, & H2, & H3, & H4, & H5, & H6 \\ T1, & T2, & T3, & T4, & T5, & T6 \end{matrix} \right\}$$

A : Coin is heads   B : Dice is 3

$A = \{H1, H2, H3, H4, H5, H6\}$ $B = \{H3, T3\}$

$P(A) = \frac{6}{12}$ $P(B) = \frac{2}{12}$

$A \cap B = \{H3\}$ $P(A \cap B) = \frac{1}{12}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/12}{2/12} = \frac{1}{2} = P(A)$$

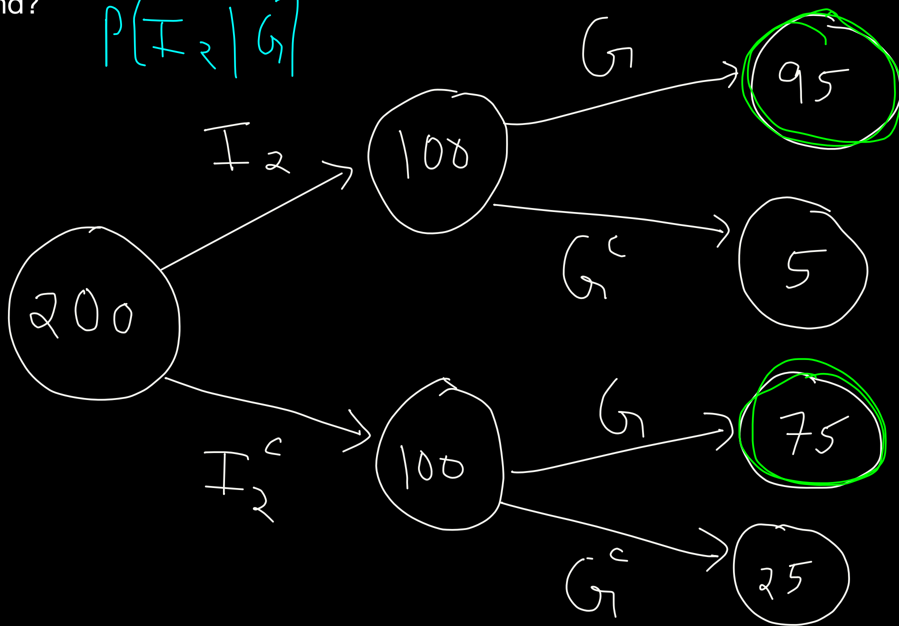
50% of the people who gave the first round we called for the second round

95% of the people who got invited for the second round felt that they had a good first round

75% of the people who did not get invited for the second round also felt that they had a good first round

Given that a person felt good about the first round, what is the probability that he cleared the first round?

$P[I_2 | G]$



$$\frac{95}{95+75}$$

A and B are two independent events, where it is known that P(AUB) = 0.5 and P(A) = 0.3 What is P(B)?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$$

$$0.5 = 0.3 + P(B) - (0.3)\,P(B)$$

$$P(B) = \frac{2}{7}$$

If A and B are mutually exclusive, then A & B are **not** independent

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 0 \neq P(A)$$

not independent

$$\boxed{\begin{matrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{matrix}}$$

$A \rightarrow$ even

$B \rightarrow$ odd