
ESCHER: Eschewing Importance Sampling in Games by Computing a History Value Function to Estimate Regret

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Abstract

Recent techniques for approximating Nash equilibria in very large games leverage neural networks to learn approximately optimal policies (strategies). One promising line of research uses neural networks to approximate counterfactual regret minimization (CFR) or its modern variants. DREAM, the only current CFR-based neural method that is model free and therefore scalable to very large games, trains a neural network on an estimated regret target that can have extremely high variance due to an importance sampling term inherited from Monte Carlo CFR (MCCFR). In this paper we propose an unbiased model-free method that does not require any importance sampling. Our method, ESCHER, is principled and is guaranteed to converge to an approximate Nash equilibrium with high probability in the tabular case. We show that the variance of the estimated regret of a tabular version of ESCHER with an oracle value function is significantly lower than that of outcome sampling MCCFR and tabular DREAM with an oracle value function. We then show that a deep learning version of ESCHER outperforms the prior state of the art—DREAM and neural fictitious self play (NFSP)—and the difference becomes dramatic as game size increases.

1 Introduction

A core challenge in computational game theory is the problem of learning strategies that approximate Nash equilibrium in very large imperfect-information games such as Starcraft (Vinyals et al., 2019), dark chess (Zhang & Sandholm, 2021), and Stratego (McAleer et al., 2020). Due to the size of these games, tabular game-solving algorithms such as counterfactual regret minimization (CFR) are unable to produce such equilibrium strategies. To sidestep the issue, in the past *stochastic* methods such as *Monte-Carlo CFR (MCCFR)* have been proposed. These methods use computationally inexpensive unbiased estimators of the regret (i.e., utility gradient) of each player, trading off speed for convergence guarantees that hold with high probability rather than in the worst case. Several unbiased estimation techniques of utility gradients are known. Some, such as *external sampling*, produce low-variance gradient estimates that are dense, and therefore are prohibitive in the settings mentioned above. Others, such as *outcome sampling*, produce high-variance estimates that are sparse

and can be computed given only the realization of play, and are therefore more appropriate for massive games.

However, even outcome-sampling MCCFR is inapplicable in practice. First, since it is a tabular method, it can only update regret on information sets that it has seen during training. In very large games, only a small fraction of all information sets will be seen during training. Therefore, generalization (via neural networks) is necessary. Second, to achieve unbiasedness of the utility gradient estimates, outcome-sampling MCCFR uses importance sampling (specifically, it divides the utility of each terminal state by a reach probability, which is often tiny), leading to estimates with extremely large magnitudes and high variance. This drawback is especially problematic when MCCFR is implemented using function approximation, as the high variance of the updates can cause instability of the neural network training.

Deep CFR (Brown et al., 2019) addresses the first shortcoming above by training a neural network to estimate the regrets cumulated by outcome-sampling MCCFR, but is vulnerable to the second shortcoming, causing the neural network training procedure to be unstable. DREAM (Steinberger et al., 2020) improves on Deep CFR by *partially* addressing the second shortcoming by using a history-based value function as a baseline (Schmid et al., 2019). This baseline greatly reduces the variance in the updates and is shown to have much better performance than simply regressing on the MCCFR updates. However, DREAM still uses importance sampling to remain unbiased. So, while DREAM was shown to work in small artificial poker variants, it is still vulnerable to the high variance of the utility gradients and indeed we demonstrate that in games with long horizons and/or large action spaces, this importance sampling term causes DREAM to fail.

In this paper, we introduce *Eschewing importance Sampling by Computing a History value function to Estimate Regret (ESCHER)*, a method that is unbiased, low variance, and does not use importance sampling. Similar to DREAM, ESCHER uses a history-dependent value function, but instead of using it as a baseline, ESCHER uses it directly as an estimator of the counterfactual value. To remove the need to weight by the reach to the current information state, ESCHER samples actions from a fixed sampling policy that does not change from one iteration to the next. Since this distribution is static, our fixed sampling policy simply weights certain information sets more than others. When the fixed sampling policy is close to the balanced policy (i.e., one where each leaf is reached with equal probability), these weighting terms minimally affect overall convergence of ESCHER with high probability.

In experiments with a tabular version of ESCHER that uses an oracle value function, we find that ESCHER performs comparably to OS-MCCFR and tabular DREAM with an oracle value function, but has orders of magnitude lower estimated regret variance. In experiments with a deep learning version of ESCHER on the large games of phantom tic tac toe and dark hex, we find that ESCHER outperforms NFSP and DREAM, and that the performance difference increases to be dramatic as the size of the game increases.

2 Background

We consider extensive-form games with perfect recall (Osborne & Rubinstein, 1994; Hansen et al., 2004). An extensive-form game progresses through a sequence of player actions, and has a *world state* $w \in \mathcal{W}$ at each step. In an N -player game, $\mathcal{A} = \mathcal{A}_1 \times \dots \times \mathcal{A}_N$ is the space of joint actions for the players. $\mathcal{A}_i(w) \subseteq \mathcal{A}_i$ denotes the set of legal actions for player $i \in \mathcal{N} = \{1, \dots, N\}$ at world state w and $a = (a_1, \dots, a_N) \in \mathcal{A}$ denotes a joint action. At each world state, after the players choose a joint action, a transition function $\mathcal{T}(w, a) \in \Delta^{\mathcal{W}}$ determines the probability distribution of the next world state w' . Upon transition from world state w to w' via joint action a , player i makes an *observation* $o_i = \mathcal{O}_i(w, a, w')$. In each world state w , player i receives a utility $\mathcal{U}_i(w)$. The game ends when the players reach a terminal world state. In this paper, we consider games that are guaranteed to end in a finite number of actions.

A *history* is a sequence of actions and world states, denoted $h = (w^0, a^0, w^1, a^1, \dots, w^t)$, where w^0 is the known initial world state of the game. $\mathcal{U}_i(h)$ and $\mathcal{A}_i(h)$ are, respectively, the utility and set of legal actions for player i in the last world state of a history h . An *information set* for player i , denoted by s_i , is a sequence of that player's observations and actions up until that time $s_i(h) = (a_i^0, o_i^1, a_i^1, \dots, o_i^t)$. Define the set of all information sets for player i to be \mathcal{I}_i . The set of histories that correspond to an information set s_i is denoted $\mathcal{H}(s_i) = \{h : s_i(h) = s_i\}$, and it is

assumed that they all share the same set of legal actions $\mathcal{A}_i(s_i(h)) = \mathcal{A}_i(h)$. For simplicity we often drop the subscript i for an information set s when the player is implied.

A player's *strategy* π_i is a function mapping from an information set to a probability distribution over actions. A *strategy profile* π is a tuple (π_1, \dots, π_N) . All players other than i are denoted $-i$, and their strategies are jointly denoted π_{-i} . A strategy for a history h is denoted $\pi_i(h) = \pi_i(s_i(h))$ and $\pi(h)$ is the corresponding strategy profile. When a strategy π_i is learned through RL, we refer to the learned strategy as a *policy*.

The *expected value (EV)* $v_i^\pi(h)$ for player i is the expected sum of future utilities for player i in history h , when all players play strategy profile π . The EV for an information set s_i is denoted $v_i^\pi(s_i)$ and the EV for the entire game is denoted $v_i(\pi)$. A *two-player zero-sum* game has $v_1(\pi) + v_2(\pi) = 0$ for all strategy profiles π . The EV for an action in an information set is denoted $v_i^\pi(s_i, a_i)$. A *Nash equilibrium (NE)* is a strategy profile such that, if all players played their NE strategy, no player could achieve higher EV by deviating from it. Formally, π^* is a NE if $v_i(\pi^*) = \max_{\pi_i} v_i(\pi_i, \pi_{-i}^*)$ for each player i .

The *exploitability* $e(\pi)$ of a strategy profile π is defined as $e(\pi) = \sum_{i \in \mathcal{N}} \max_{\pi'_i} v_i(\pi'_i, \pi_{-i})$. A *best response (BR)* strategy $\mathbb{BR}_i(\pi_{-i})$ for player i to a strategy π_{-i} is a strategy that maximally exploits π_{-i} : $\mathbb{BR}_i(\pi_{-i}) = \arg \max_{\pi_i} v_i(\pi_i, \pi_{-i})$. An ϵ -*best response (ϵ -BR)* strategy $\mathbb{BR}_i^\epsilon(\pi_{-i})$ for player i to a strategy π_{-i} is a strategy that is at most ϵ worse for player i than the best response: $v_i(\mathbb{BR}_i^\epsilon(\pi_{-i}), \pi_{-i}) \geq v_i(\mathbb{BR}_i(\pi_{-i}), \pi_{-i}) - \epsilon$. An ϵ -*Nash equilibrium (ϵ -NE)* is a strategy profile π in which, for each player i , π_i is an ϵ -BR to π_{-i} .

2.1 Counterfactual Regret Minimization (CFR)

In this section we review the *counterfactual regret minimization (CFR)* framework. All superhuman poker AIs have used advanced variants of the framework as part of their architectures (Bowling et al., 2015; Brown & Sandholm, 2018, 2019). CFR is also the basis of several reinforcement learning algorithms described in section 5. We will leverage and extend the CFR framework in the rest of the paper. We will start by reviewing the framework.

Define $\eta^\pi(h)$ to be the reach weight of joint policy π to reach history h , and z is a terminal history. Define $Z(s)$ to be the set of terminal histories z that can be reached from information state s and define $z[s]$ to be the unique history $h \in s$ that is a subset of z . Define

$$v_i(\pi, h) = \sum_{z \supset h} \eta^\pi(h, z) u_i(z) \quad (1)$$

to be the expected value under π for player i having reached h . Note that this value function takes as input the full-information history h and not an information set. Define

$$v_i^c(\pi, s) = \sum_{z \in Z(s)} \eta_{-i}^\pi(z[s]) \eta^\pi(z[s], z) u_i(z) = \sum_{h \in s} \eta_{-i}^\pi(h) v_i(\pi, h) \quad (2)$$

to be the *counterfactual value* for player i at state s under the joint strategy π . Define the strategy $\pi_{s \rightarrow a}$ to be a modified version of π where a is played at information set s , and the counterfactual state-action value $q_i^c(\pi, s, a) = v_i^c(\pi_{s \rightarrow a}, s)$. For any state s , strategy π , and action $a \in \mathcal{A}(s)$, one can define a local *counterfactual regret* for not switching to playing a at s as $r^c(\pi, s, a) = q_i^c(\pi, s, a) - v_i^c(\pi, s)$. Counterfactual regret minimization (CFR) (Zinkevich et al., 2008a) is a strategy iteration algorithm that produces a sequence of policies: $\{\pi_1, \pi_2, \dots, \pi_T\}$. Each policy $\pi_{t+1}(s)$ is derived directly from a collection of cumulative regrets $R_T(s, a) = \sum_{t=1}^T r^c(\pi_t, s, a)$, for all $a \in \mathcal{A}(s)$ using regret-matching (Hart & Mas-Colell, 2000). In two-player zero-sum games, the average policy $\bar{\pi}_T$ converges to an approximate Nash equilibrium at a rate of $e(\bar{\pi}_T) \leq O(1/\sqrt{T})$.

2.2 Monte Carlo Counterfactual Regret Minimization (MCCFR)

In the standard CFR algorithm, the quantities required to produce new policies in Equations 1 and 2 require full traversals of the game to compute exactly. Monte Carlo CFR (Lanctot et al., 2009) is a *stochastic* version of CFR which instead *estimates* these quantities. In particular, MCCFR uses a sampling approach which specifies a distribution over blocks Z_j of terminal histories such that $\cup_j Z_j = \mathcal{Z}$, the set of terminal histories. Upon sampling a block j , a certain *sampled counterfactual*

value $\hat{v}^c(\pi, s | j)$ (defined in detail later in this section) is computed for all prefix histories that occur in Z_j . Then, estimated regrets are accumulated and new policies derived as in CFR. The main result is that $\mathbb{E}[\hat{v}^c(\pi, s | j)] = v^c(\pi, s)$, so MCCFR is an unbiased approximation of CFR, and inherits its convergence properties albeit under a probabilistic guarantee.

Blocks are sampled via sampling policy $\tilde{\pi}$ which is commonly a function of the players' joint policy π . Two sampling variants were defined in the original MCCFR paper: *outcome sampling* (OS-MCCFR) and *external sampling* (ES-MCCFR). External sampling samples only the opponent (and chance's) choices; hence, it requires a forward model of the game to recursively traverse over all of the subtrees under the player's actions. Outcome sampling is the most extreme sampling variant where blocks consist of a single terminal history: it is the only model-free variant of MCCFR compliant with the standard reinforcement learning loop where the agent learns entirely from experience with the environment. The OS-MCCFR counterfactual value estimator when the opponent samples from their current policy as is commonly done is given as follows:

$$\hat{v}_i(\pi, s|z) = \frac{\eta^{\pi-i}(z[s])\eta^\pi(z[s], z)u_i(z)}{\eta^{\tilde{\pi}}(z)} = \frac{1}{\eta^{\tilde{\pi}_i}(z[s])} \frac{\eta^{\pi_i}(z[s], z)}{\eta^{\tilde{\pi}_i}(z[s], z)} u_i(z) \quad (3)$$

The importance sampling term that is used to satisfy the unbiasedness of the values can have a significant detrimental effect on the convergence rate Gibson et al. (2012). Variance reduction techniques provide some empirical benefit Schmid et al. (2019); Davis et al. (2019), but have not been evaluated on games with long trajectories where the importance corrections have their largest impact.

2.3 Deep Counterfactual Regret Minimization

Deep CFR (Brown et al., 2019; Steinberger, 2019; Li et al., 2019) is a method that uses neural networks to scale MCCFR to large games. Deep CFR performs external sampling MCCFR and trains a regret network $R_i(s, a|\psi)$ on a replay buffer of information sets and estimated cumulative regrets. The regret network is trained to approximate the cumulative regrets seen so far at that information state. The estimated counterfactual regrets are computed the same as in MCCFR, namely using importance sampling or external sampling.

2.4 DREAM

DREAM (Steinberger et al., 2020) builds on Deep CFR and approximates OS-MCCFR with deep neural networks. Like Deep CFR, it trains a regret network $R_i(s, a|\psi)$ on a replay buffer of information sets and estimated cumulative regrets. Additionally, in order to limit the high variance of OS-MCCFR, DREAM uses a learned history value function $q_i(\pi, h, a|\theta)$ and uses it as a baseline (Schmid et al., 2019). While the baseline helps remove variance in the estimation of future utility, in order to remain unbiased DREAM must use importance sampling as in OS-MCCFR. We show empirically that variance of the DREAM estimator of the counterfactual value, although lower than OS-MCCFR, will often be quite high, even in small games and with an oracle history value function. This high variance estimator might make neural network training very difficult. In contrast, ESCHER has no importance sampling term and instead directly uses the learned history value function $q_i(\pi, h, a|\theta)$ to estimate regrets.

3 Tabular ESCHER with Oracle Value Function

In this section we define a tabular version of our proposed algorithm, *ESCHER*, where we assume oracle access to a value function. While in practice this tabular algorithm will not compare well to existing approaches due to the expense of generating an oracle value function, in this section we show that if we assume access to an oracle value function at no cost, then our tabular method is sound and converges to a Nash equilibrium with high probability. In the next section we introduce our main method which is a deep version of this tabular method and learns a neural network value function from data.

As shown in Equation 3, the OS-MCCFR estimator can be seen as containing two separate terms. The first $1/\eta^{\tilde{\pi}_i}(z[s])$ term ensures that each information set is updated equally often in expectation. The second $\eta^{\pi_i}(z[s], z)u_i(z)/\eta^{\tilde{\pi}_i}(z[s], z)$ term is an unbiased estimator of the history value $v_i(\pi, z[s])$. The main idea behind ESCHER is to remove the first importance sampling term by ensuring that the

Algorithm 1 Tabular ESCHER with Oracle Value Function

```
for  $t = 1, \dots, T$  do
  for update player  $i \in \{0, 1\}$  do
    Sample trajectory  $\tau$  using sampling distribution  $\tilde{\pi}^i$  (Equation 4)
    for each state  $s \in \tau$  do
      for each action  $a$  do
        Estimate immediate regret vector  $\hat{r}(\pi, s, a) = q_i(\pi, h, a) - v_i(\pi, h)$ 
        Update total estimated regret of action  $a$  at infostate  $s$ :  $\hat{R}(s, a) = \hat{R}(s, a) + \hat{r}(\pi, s, a)$ 
        Update  $\pi_i(s, a)$  via regret matching on total estimated regret
      return average policy  $\bar{\pi}$ 
```

sampling distribution for the update player remains fixed across iterations, and to replace the second term with a history value function $\hat{v}_i(\pi, z[s])$.

Similar to OS-MCCFR, the tabular version of ESCHER iteratively updates each player's policy by sampling a single trajectory. When updating a player's strategy, we sample from a fixed distribution (for example the uniform distribution) for that player and sample from the opponent's current policy for the other player. As in MCCFR, we update the estimated regret for all actions in each information state reached in the trajectory. However, unlike in MCCFR, we do not use the terminal utility to estimate regret but instead use the oracle history value function. This reduces variance in the update because the oracle value function at a given history action pair will always be the same if the policies are the same.

ESCHER samples from the opponent's current strategy when it is their turn but samples from a fixed strategy that roughly visits every information set equally likely when it is the update player's turn. As a result, the expected value of the history value is equal to the counterfactual value scaled by a term that weights certain information sets up or down based on the fixed sampling policy. Formally, define the fixed sampling policy $b_i(s, a)$ to be any policy that remains constant across iterations and puts positive probability on every action. This fixed sampling policy can be one of many distributions such as one that samples uniformly over available actions at every information set. An interesting open research direction is finding good fixed sampling policy. In this paper, our fixed sampling policy uniformly samples over actions, which is somewhat similar to the robust sampling technique introduced in Li et al. (2019). When updating player i , we construct a joint fixed sampling policy $\tilde{\pi}^i(s, a)$ to be

$$\tilde{\pi}^i(s, a) = \begin{cases} b_i(s, a), & \text{if it's the update player } i\text{'s turn} \\ \pi_{-i}(s, a), & \text{otherwise} \end{cases} \quad (4)$$

We use a fixed sampling policy because it allows us to remove any importance sampling in our estimator. Unlike OS-MCCFR which must divide by the current player's reach probability to remain unbiased, our method simply weights the regrets of certain information states more than others, but total average regret is still guaranteed to converge to zero.

To remove the importance sampling term that arises from estimating the future expected value from the terminal utility, we substitute this estimate with the oracle value function. Formally, we define our estimator for the immediate counterfactual regret to simply be the oracle advantage:

$$\hat{r}_i(\pi, s, a|h) = q_i(\pi, h, a) - v_i(\pi, h) \quad (5)$$

If we sample from $\tilde{\pi}^i$ when updating player i , then the expected value of our counterfactual regret estimator is:

$$\begin{aligned} \mathbf{E}_{h \sim \tilde{\pi}^i}[\hat{r}_i(\pi, s, a|h)] &= \mathbf{E}_{h \sim \tilde{\pi}^i}[q_i(\pi, h, a) - v_i(\pi, h)] \\ &= \sum_{h \in s} \eta^{\tilde{\pi}^i}(h)[q_i(\pi, h, a) - v_i(\pi, h)] \\ &= \eta_i^{\tilde{\pi}^i}(s) \sum_{h \in s} \eta_{-i}^{\pi}(h)[q_i(\pi, h, a) - v_i(\pi, h)] \\ &= w(s)[v_i^c(\pi, s, a) - v_i^c(\pi, s)] \\ &= w(s)r(s, a) \end{aligned} \quad (6)$$

Where for $h, h' \in s$, $\eta_i^{\pi'}(h) = \eta_i^{\pi'}(h') = \eta_i^{\pi'}(s) =: w(s)$ is the reach probability for reaching that infostate for player i via the fixed sampling distribution. Unlike the MCCFR estimator, our estimator has no importance sampling terms, and as a result has much lower variance. Furthermore, our estimator is only biased to the extent that it weights different information sets by different values depending on the fixed distribution. The correctness of our method is established by the next theorem, whose proof can be found in Appendix A.

Theorem 1. *Assume a fixed sampling policy that puts positive probability on every action. For any $p \in (0, 1)$, with probability at least $1 - p$, the regret accumulated by each agent learning using the tabular algorithm **ESCHER** (Algorithm 1) is upper bounded by $O(\sqrt{T} \cdot \text{poly} \log(1/p))$, where the $O(\cdot)$ notation hides game-dependent and sampling-policy-dependent constants.*

4 ESCHER

As shown in the previous section, when using an oracle value function, our method minimizes regret in the single-agent case and converges to a Nash equilibrium in the two-player case. In this section we describe our main method where we learn a history-dependent value function via a neural network. Similar to Deep CFR and DREAM, we train a regret network $R_i(s, a|\psi)$ over a buffer of information states and targets where the targets come directly from the learned history value function $q_i(\pi, h, a|\theta)$.

Our method is built on Deep CFR. In particular, like Deep CFR we traverse the game tree and add this experience into replay buffers. The first replay buffer stores information states and instantaneous regret estimates is used to train a regret network $R_\psi(s, a)$ that is trained to estimate the cumulative regret at a given information set. Unlike Deep CFR and DREAM, which use the terminal utility and sampling probabilities from the current trajectory to estimate the value, in **ESCHER** the instantaneous regret estimates are estimated using the current history value function alone

$$\hat{r}_i(\pi, s, a|h) = q_i(\pi, h, a|\theta) - \sum_a \pi_i(s, a) q_i(\pi, h, a|\theta) \quad (7)$$

Similar to Deep CFR, each player’s current policy π_i is given by performing regret matching on the output of the current regret network $R_i(s, a|\psi)$.

The second replay buffer stores histories and terminal utilities and is used to train the value network Q_θ to estimate the expected utility for both players when both players are at that history and play from their current policies. Lastly, the third replay buffer stores information states and actions taken by the policy π and uses that data to train an average policy network $\bar{\pi}_\phi$ that approximates the average policy across all iterations. It is this average policy that has no regret and converges to an approximate Nash equilibrium in self play.

As described in the previous section, the only difference between our tabular method and MCCFR and our deep method and Deep OS-MCCFR is the estimator for the immediate regret. While Deep OS-MCCFR uses an importance-weighted estimate of the counterfactual value estimated from the utility of the rollout, we instead simply use the value function to estimate the immediate regret. We describe our algorithm in Algorithm 2.

5 Related Work

Superhuman performance in two-player games usually involves two components: the first focuses on finding a model-free blueprint strategy, which is the setting we focus on in this paper. The second component improves this blueprint online via model-based subgame solving and search (Burch et al., 2014; Moravcik et al., 2016; Brown et al., 2018, 2020; Brown & Sandholm, 2017b; Schmid et al., 2021). This combination of blueprint strategies with subgame solving has led to state-of the art performance in Go (Silver et al., 2017), Poker (Brown & Sandholm, 2017a, 2018; Moravčík et al., 2017), Diplomacy (Gray et al., 2020), and The Resistance: Avalon (Serrino et al., 2019). Methods that only use a blueprint have achieved state-of-the-art performance on Starcraft (Vinyals et al., 2019), Gran Turismo (Wurman et al., 2022), DouDizhu (Zha et al., 2021), Mahjong (Li et al., 2020), and Barrage Stratego (McAleer et al., 2020). Because **ESCHER** is a method for finding a blueprint, it can be combined with subgame solving and is complementary to these approaches. In the rest of this section we focus on other model-free methods for finding blueprints.

Algorithm 2 ESCHER

```
Initialize history value function  $q$ 
Initialize policy  $\pi_i$  for both players
for  $t = 1, \dots, T$  do
  Retrain history value function  $q$  on value buffer data
  Reinitialize regret networks  $R_0, R_1$ 
  for update player  $i \in \{0, 1\}$  do
    for  $P$  trajectories do
      Get trajectory  $\tau$  using sampling distribution (Equation 4)
      for each state  $s \in \tau$  do
        for each action  $a$  do
          Estimate immediate cf-regret  $\hat{r}(\pi, s, a) = q_i(\pi, h, a|\theta) - \sum_a \pi_i(s, a)q_i(\pi, h, a|\theta)$ 
          Add  $(s, \hat{r}(\pi, s))$  to cumulative regret buffer
          Add  $(s, a')$  to average policy buffer where  $a'$  is action taken at state  $s$  in trajectory  $\tau$ 
        Train regret network  $R_i$  on cumulative regret buffer
      Train average policy network  $\bar{\pi}_\phi$  on average policy buffer
  return average policy network  $\bar{\pi}_\phi$ 
```

Deep CFR (Brown et al., 2019; Steinberger, 2019) is a general method that trains a neural network on a buffer of counterfactual values. However, Deep CFR uses external sampling, which may be impractical for games with a large branching factor, such as Stratego and Barrage Stratego. DREAM (Steinberger et al., 2020) and ARMAC (Gruslys et al., 2020) are model-free regret-based deep learning approaches. ReCFR (Liu et al., 2022) propose a bootstrap method for estimating cumulative regrets with neural networks that could potentially be combined with our method.

Neural Fictitious Self-Play (NFSP) (Heinrich & Silver, 2016) approximates fictitious play by progressively training a best response against an average of all past opponent policies using reinforcement learning. The average policy converges to an approximate Nash equilibrium in two-player zero-sum games.

Policy Space Response Oracles (PSRO) (Lanctot et al., 2017; Muller et al., 2019; Feng et al., 2021; McAleer et al., 2022) are another promising method for approximately solving very large games. PSRO maintains a population of reinforcement learning policies and iteratively trains a best response to a mixture of the opponent’s population. PSRO is a fundamentally different method than the previously described methods in that in certain games it can be much faster but in other games it can take exponentially long in the worst case. Neural Extensive Form Double Oracle (NXDO) (McAleer et al., 2021) combines PSRO with extensive-form game solvers, and could potentially be combined with our method.

There is an emerging literature connecting reinforcement learning to game theory. QPG (Srinivasan et al., 2018) shows that state-conditioned Q -values are related to counterfactual values by a reach weighted term summed over all histories in an infostate and proposes an actor-critic algorithm that empirically converges to a NE when the learning rate is annealed. NeuRD (Hennes et al., 2020), and F-FoReL (Perolat et al., 2021) approximate replicator dynamics and follow the regularized leader, respectively, with policy gradients. Robust reinforcement learning (Pinto et al., 2017), seeks to train an RL policy to be robust against an adversarial environment. In future work we will look to apply ESCHER to this setting.

Markov games (or stochastic games) are extensive-form games where the world state information is shared among all players at each timestep, but players take simultaneous actions. Recent literature has shown that reinforcement learning algorithms converge to Nash equilibrium in two-player zero-sum Markov games (Brafman & Tennenholtz, 2002; Wei et al., 2017; Perolat et al., 2018; Xie et al., 2020; Daskalakis et al., 2020; Jin et al., 2021) and in multi-player general-sum Markov potential games (Leonardos et al., 2021; Mguni et al., 2021; Fox et al., 2022; Zhang et al., 2021; Ding et al., 2022).

Actor Critic Hedge (ACH) (Fu et al., 2022) is a similar method that does not use importance sampling but instead uses a value function to estimate regrets. Their method differs from ours in two important ways. First, ACH uses an information set based value function. As a result, the expected value of the estimator weights certain information sets with a weight that changes from one iteration to the next.

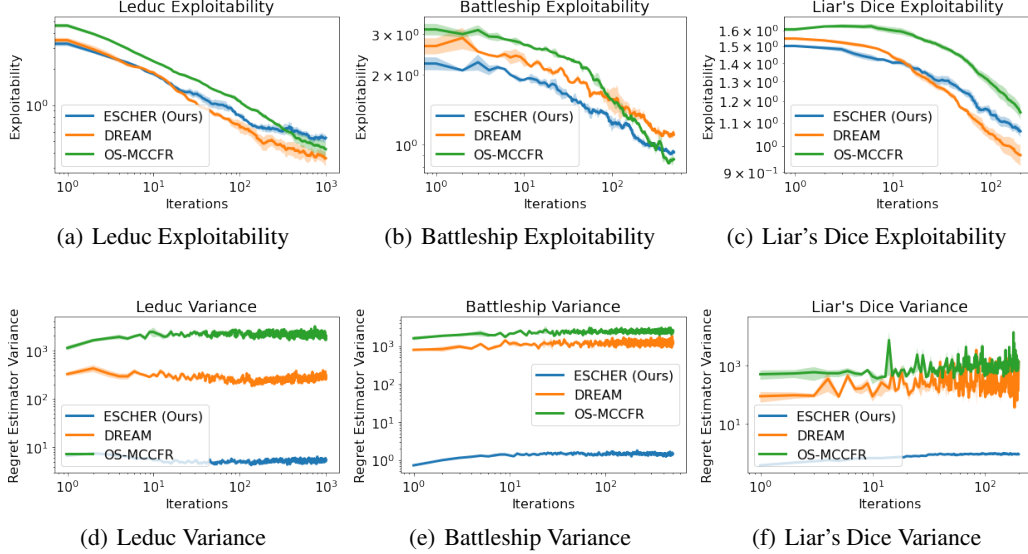


Figure 1: The tabular version of ESCHER with an oracle value function is competitive with the tabular version of DREAM with an oracle value function and with OS-MCCFR in terms of exploitability (top row). The regret estimator in ESCHER has orders of magnitude lower variance than those of DREAM and OS-MCCFR (bottom row).

Because of this, the regret bound of the tabular method can remain constant if the difference between this weighting term remains high across iterations. Second, ACH uses Hedge as its policy instead of regret matching. Hedge is very unstable when combined with MCCFR and is difficult to tune. Third, ACH trains a policy on a buffer of data collected from the most recent policy. This choice removes the downside of training on a large replay buffer of all past experience but might introduce extra instability. This design choice could be combined with our approach as an alternative way of training the policy, but we did not run an ablation study on this. We attempted to replicate a tabular version of ACH but were not able to match the results in their paper.

6 Results

6.1 Tabular Results

We compare a tabular version of ESCHER with oracle value functions to a tabular version of DREAM with oracle value functions and with OS-MCCFR. We run experiments on Leduc poker, Battleship, and Liar’s dice, and use the implementations from OpenSpiel (Lanctot et al., 2019). We see in Figure 1 on the top row that ESCHER remains competitive with DREAM and OS-MCCFR on these games. On the bottom row we plot the average variance of the regret estimators over all information sets visited over an iteration window for each of these algorithms. While DREAM does improve upon OS-MCCFR, it still has orders of magnitude higher variance than ESCHER. Although this does not matter much in tabular experiments, we conjecture that high regret estimator variance makes neural network training unstable without prohibitively large buffer sizes.

6.2 Deep Results

In this section we compare our method to DREAM and NFSP, the most popular baselines that are also open-source, on the medium sized games of Phantom Tic Tac Toe (TTT) and Dark Hex. Phantom TTT and Dark Hex are similar in that they are both imperfect information versions of perfect-information games played on square boards. Phantom TTT is played on a 3×3 board while dark hex 4 is played on a size 4×4 board and dark hex 5 is played on a size 5×5 board. Because these games are large, we are not able to compare exact exploitability so instead we compare performance through head-to-head evaluation. Results are shown in Figure 2, where the x axis tracks the number of information sets

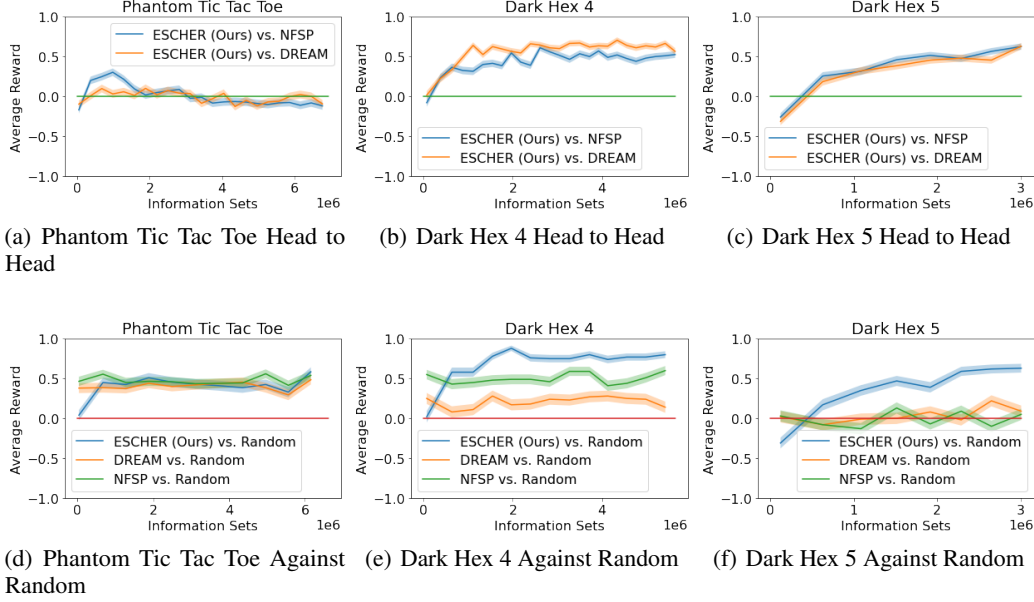


Figure 2: ESCHER is competitive with NFSP and DREAM in Phantom Tic Tac Toe. But as the size of the game increases, ESCHER performs increasingly better than both DREAM and NFSP.

visited during training. We see that our method is competitive with DREAM and NFSP on Phantom TTT. On the larger game of Dark Hex 4, ESCHER beats DREAM and NFSP head to head and also scores higher against a random opponent. Moving to the largest game of Dark Hex 5, we see that ESCHER beats DREAM and NFSP head to head and also is able to beat a random opponent while DREAM and NFSP are no better than random. We do not compare to other methods such as NeuRD (Omidshafiei et al., 2019), F-FoReL (Perolat et al., 2021), and ARMAC (Gruslys et al., 2020) because we could not find open-source implementations of their RL forms.

7 Discussion: Limitations and Future Research

Our method has a number of ways it can be improved. First, it requires two separate updates in one iteration. Perhaps the method could be more efficient if only on-policy data were used. Second, our method, like Deep CFR, DREAM, and NFSP, trains neural networks on large replay buffers of past experience. Unlike RL algorithms like DQN (Mnih et al., 2015), these replay buffers must record all data ever seen in order to learn an average. This can be a problem when the amount of data required is much larger than the replay buffer memory. Third, we do not use various implementation details that help performance in Deep CFR such as weighting by the sum of the reach probabilities over all iterations. Fourth, our method still has variance that comes from the different values associated with different histories inside the same information set. Perhaps a combination of our method with baselines like in DREAM could further reduce variance. Finally, our method uses separate data to train the value function. Our method could be made much more efficient by also using the data generated for training the policy to also train the value function. We could also use the reward information discovered when training the regret network which we currently do not use.

One direction of future research is finding optimal sampling distributions. In our method we use the uniform distribution over actions as our fixed sampling distribution, but this can be far from optimal. In principle any distribution that remains fixed will guarantee the method to converge with high probability. One possible direction would be to try to estimate the theoretically optimal balanced distribution. Another possible method would be to take the frozen policy at a certain iteration and use that as the sampling distribution for the rest of the game. Other, less principled, methods such as using the average policy might work well in practice as well (Burch et al., 2012).

Another direction is in connecting this work with the reinforcement learning literature. Similar to reinforcement learning, we learn a Q value and a policy, and there are many techniques from

reinforcement learning that are promising to try in this setting. For example, although we learned the value function simply through Monte-Carlo rollouts, one could use bootstrapping-based methods such as TD- λ (Sutton, 1988) and expected SARSA (Rummery & Niranjan, 1994). The policy might be able to be learned via some sort of policy gradient, similar to QPG (Srinivasan et al., 2018), NeuRD (Hennes et al., 2020), and F-FoReL (Perolat et al., 2021).

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A Proofs

We start by recalling a central theorem connecting regret to counterfactual regret (see, *e.g.*, Zinkevich et al. (2008b); Farina et al. (2019b,a)).

Proposition 1. Fix any player i , and let

$$R_s^T := \max_{\hat{a} \in A_s} \sum_{t=1}^T q_i^c(\pi^t, s, \hat{a}) - v_i^c(\pi^t, s) = \max_{\hat{a} \in A_s} r_i(\pi^t, s, \hat{a})$$

be the counterfactual regret cumulated up to time T by the regret minimizer local at each information set s . Then, the regret

$$R_i^T := \max_{\hat{\pi}_i} \sum_{t=1}^T v_i(\hat{\pi}_i, \pi_{-i}^t) - v_i(\pi^t)$$

cumulated by the policies π^t on the overall game tree satisfies

$$R_i^T \leq \sum_s \max\{R_s^T, 0\}.$$

Theorem 1. Assume a fixed sampling policy that puts positive probability on every action. For any $p \in (0, 1)$, with probability at least $1 - p$, the regret accumulated by each agent learning using the tabular algorithm *ESCHER* (Algorithm 1) is upper bounded by $O(\sqrt{T} \cdot \text{poly} \log(1/p))$, where the $O(\cdot)$ notation hides game-dependent and sampling-policy-dependent constants.

Proof. As shown in Section 3, for any information set s the counterfactual regret estimators $\hat{r}_i(\pi, s, a|h)$ are unbiased up to a time-independent multiplicative factor; specifically,

$$\mathbf{E}_{h \sim \tilde{\pi}^i}[\hat{r}_i(\pi, s, a|h)] = w(s)r_i(\pi, s, a)$$

for all actions a available to player i at world states in s . Hence, for each $a \in A_s$ we can construct the martingale difference sequences

$$X_a^t := w(s)r_i(\pi^t, s, a) - \hat{r}_i(\pi^t, s, a|h).$$

Clearly, X_a^t is bounded, with $|X_a^t|$ upper bounded by (twice) the range of payoffs of player i . Hence, from the Azuma-Hoeffding inequality, we obtain that the regret R_s^T cumulated by the local policies produced by *ESCHER* with respect to the correct counterfactuals satisfies, for all $p \in (0, 1)$

$$\mathbf{P} \left[\sum_{t=1}^T X_a^t \leq 2D\sqrt{2T \log \frac{1}{p}} \right] \geq 1 - p,$$

Using a union bound on the actions, we can therefore write

$$\mathbf{P} \left[\max_a \sum_{t=1}^T X_a^t \leq 2D\sqrt{2T \log \frac{|A_s|}{p}} \right] \geq 1 - p,$$

The left-hand side in the probability can be expanded as follows:

$$\begin{aligned} \max_a \sum_{t=1}^T X_a^t &= \max_a \left\{ w(s) \sum_{t=1}^T r_i(\pi^t, s, a) - \sum_{t=1}^T \hat{r}_i(\pi^t, s, a|h) \right\} \\ &\geq \max_a \left\{ w(s) \sum_{t=1}^T r_i(\pi^t, s, a) \right\} - \max_a \left\{ \sum_{t=1}^T \hat{r}_i(\pi^t, s, a|h) \right\} \\ &\geq w(s)R_s^T - |A_s|D\sqrt{T}, \end{aligned}$$

where the last inequality follows from the fact that the regret cumulated by regret matching (which is run on the regret estimates \hat{r}_i) is upper bounded by $|A_s|D\sqrt{T}$. Hence, we can write

$$\begin{aligned} &\mathbf{P} \left[w(s)R_s^T - |A_s|D\sqrt{T} \leq 2D\sqrt{2T \log \frac{|A_s|}{p}} \right] \geq 1 - p \\ \iff &\mathbf{P} \left[R_s^T \leq \frac{D|A_s|}{w(s)}\sqrt{T} + \frac{2D}{w(s)}\sqrt{2T \log \frac{|A_s|}{p}} \right] \geq 1 - p, \end{aligned}$$

where in the second step we used the hypothesis that $w(s) > 0$ for all s . Since the right-hand side inside of the probability is non-negative, we can further write

$$\mathbf{P} \left[\max\{R_s^T, 0\} \leq \frac{D|A_s|}{w(s)}\sqrt{T} + \frac{2D}{w(s)}\sqrt{2T \log \frac{|A_s|}{p}} \right] \geq 1 - p,$$

valid for every information set s .

Now, using the known analysis of CFR (Proposition 1), we obtain that the regret accumulated by the ESCHER iterates satisfies

$$R_i^T \leq \sum_s \max\{0, R_s^T\}.$$

Hence, using a union bound over all information sets $s \in \mathcal{S}_i$ of player i , we find that

$$\begin{aligned} \mathbf{P} \left[R_i^T \leq \left(\sum_s \frac{D|A_s|}{w(s)} + \frac{2D}{w(s)}\sqrt{2 \log \frac{|A_s||\mathcal{S}_i|}{p}} \right) \sqrt{T} \right] &\geq 1 - p \\ \iff \mathbf{P} \left[R_i^T \leq \left(\frac{2D|A_s||\mathcal{S}_i|}{\min_s w(s)}\sqrt{2 \log \frac{|A_s||\mathcal{S}_i|}{p}} \right) \sqrt{T} \right] &\geq 1 - p, \end{aligned}$$

for all $p \in (0, 1)$. Absorbing game-dependent and sampling-policy-dependent constants yields the statements. \square

B Description of Game Instances

We use Openspiel (Lanctot et al., 2019) for all our games. Below we list the parameters used to define each game in Openspiel.

Leduc Game Name: leduc_poker
Parameters: {"players": 2}

Battleship Game Name: battleship
Parameters: {"board_width": 2, "board_height": 2,
"ship_sizes": "[2]", "ship_values": "[2]",
"num_shots": 3, "allow_repeated_shots": False}

Liar's Dice Game Name: liars_dice
Parameters: None

Phantom Tic Tac Toe Game Name: phantom_ttt
Parameters: None

Dark Hex 4 Game Name: dark_hex
Parameters: {"board_size": 4}

Dark Hex 5 Game Name: dark_hex
Parameters: {"board_size": 5}

C Hyperparameters for Deep Experiments

For all deep experiments we first did a hyperparameter search that starts with good hyperparameters for flop hold 'em poker. We report the final hyperparameters used for the deep experiments.

C.1 ESCHER

Parameter	Value
n regret network traversals	1,000
n history value network traversals	1,000
batch size regret network	2048
batch size history value network	2048
train steps regret network	5,000
train steps history value network	5,000
train steps average policy network	10,000

Table 1: ESCHER Hyperparameters for Phantom TTT, Dark Hex 4, Dark Hex 5

When computing the value function, we random noise to the current policy to induce coverage over all information sets. To do this we added 0.01 times a uniform distribution to the current policy and renormalized.

C.2 DREAM

We use the codebase from the original DREAM paper (Steinberger et al., 2020) with a wrapper to integrate with Openspiel (Lanctot et al., 2019) and rllib (Liang et al., 2018). When otherwise specified, we use default parameters from the DREAM codebase.

Parameter	Value
n batches adv training	4,000
n traversals per iter	1,000
n batches per iter baseline	1,000
periodic restart	10
max n las sync simultaneously	12
mini batch size adv	10,000
max buffer size adv	2,000,000
mini batch size avrg	10,000
max buffer size avrg	2,000,000
batch size baseline	2048
n batches avrg training	4000

Table 2: DREAM Hyperparameters for Phantom TTT, Dark Hex 4, Dark Hex 5

C.3 NFSP

We use our own implementation of NFSP that uses RLLib’s (Liang et al., 2018) DQN implementation and outperforms the original paper’s results on Leduc poker.

Parameter	Value
circular buffer size	2e5
total rollout experience gathered each iter	1024 steps
learning rate	0.01
batch size	4096
TD-error loss type	MSE
target network update frequency	every 10,000 steps
RL learner params	DDQN
anticipatory param	0.1
avg policy reservoir buffer size	2e6
avg policy learning starts after	16,000 steps
avg policy learning rate	0.1
avg policy batch size	4096
avg policy optimizer	SGD

Table 3: NFSP Hyperparameters for Phantom TTT, Dark Hex 4, Dark Hex 5

D Code

We have open sourced our code here: <https://github.com/Sandholm-Lab/ESCHER>. Our code is built on top of the OpenSpiel (Lanctot et al., 2019) and Ray (Moritz et al., 2018) frameworks, both of which are open source and available under the Apache-2.0 license.