Answers to theory questions LB

May 14, 2016

1 Pros and cons of LB

1.1 Pros

- Explicit update rule, no system of equations to solve.
  - Done by doing a trick. Introduce a new distribution function and see that the sum over states give the same global quantities.
- Easy to parallelize.
  - Each cell only depend locally on the neighbouring cells in the streaming step and only on itself in the collision.
- Simulating mesoscale
  - Unlike NS, LB looks at distributions of "bunches" of particles. May catch behaviour too fine for NS to detect.
- Can handle multiphase flows relatively easy.
  - Usually done by modifying the collision operator.

1.2 Cons

- Explicit update rule
  - Need small timesteps.
  - Finite propagation speed of information.
- Only using a square grid can be limiting.

2 Convective-/viscous term

Consider the boltzmann equation,

$$\frac{\delta f}{\delta t} + \vec{v} \cdot \nabla f = -\Delta(f - f^{eq}), \tag{1}$$

and the BGK approximation

$$\Delta(f - f^{eq}) = -\frac{1}{\tau}(f - f^{eq}) \tag{2}$$

3 Linear LBM to non-linear NSE

In the LBM we have a linear update rule,

$$f_i(\vec{x} + \vec{c}_i dt, t + dt) = f_i(\vec{x}, t) - \frac{1}{\tau}(f_i(\vec{x}, t) - f_i^{eq}), \tag{3}$$

which gives us the updated distribution of particles with velocity $\vec{c}_i$ at position $\vec{x} + \vec{c}_i dt$ and time $t + dt$. $i = 0 : Q - 1$ where Q is the size of our velocity space.

3.1 Heuristic argument

The NSE solves a global system on a macroscale. To get the macroscopic velocities from the LBM velocities we need to integrate the velocity density distribution over the velocity set. We hence introduce nonlinearity.

4 Conservation of mass and momentum in the collision rule

Collision rule:

$$f_i^*(\vec{x},\, t) = f_i(\vec{x},\, t) - \frac{1}{\tau}(f_i(\vec{x},\, t) - f_i^{eq}(\vec{x},\, t)) \tag{4}$$

Approximation to the equilibrium state which holds when the distribution is close to equilibrium:

$$f_i^{eq}(\vec{x},\, t) \approx w_i \rho \left[1 + \frac{\vec{c}_i \cdot \vec{u}}{c_s^2} + \frac{(\vec{c}_i \cdot \vec{u})^2}{c_s^4} - \frac{\vec{u} \cdot \vec{u}}{c_s^2} \right]. \tag{5}$$

Properties on w_i (there are more, but only these are needed here):

$$c_s^2 = \sum_{i=0}^{Q-1} w_i \cdot c_i^2, \tag{6}$$

$$\sum_{i=0}^{Q-1} w_i = 1, \tag{7}$$

$$\sum_{i=0}^{Q-1} w_i \cdot c_i^\alpha = 0 \quad \alpha = x,\, y,\, z. \tag{8}$$

Before the collision step we have the density

$$\rho = \sum_{i=0}^{Q-1} f_i(\vec{x},\, t). \tag{9}$$

After the collision the density is

$$\rho^* = \sum_{i=0}^{Q-1} f_i^*(\vec{x},\, t). \tag{10}$$

writing this out by using eq. 4 and 5 we get

$$\rho^* = \sum_{i=0}^{Q-1} \left(f_i(\vec{x},\, t) - \frac{1}{\tau}\rho w_i \left[1 + \frac{\vec{c}_i \cdot \vec{u}}{c_s^2} + \frac{(\vec{c}_i \cdot \vec{u})^2}{c_s^4} - \frac{\vec{u} \cdot \vec{u}}{c_s^2} \right] \right). \tag{11}$$

5 Conservation of mass and momentum in the streaming rule