DESIGN AND ANALYSIS OF EXPERIMENTATION

Project Report by Sonai Pandiyan and Charlotte Vermeiren

1. INTRODUCTION

The goal of our experiment was to find the best set-up to drain 0.5 L of liquid from a plastic bottle as fast as possible. In order to find this set-up, four possible influencing factors were selected and their effect on the drainage time was tested. These factors were: (a) the diameter of the mouth of the bottle, (b) the viscosity of the liquid, (c) the angle under which the water bottle is drained, and (d) the total volume of liquid present in the bottle. For all these factors, at least two levels were tested. In this report, a statistical analysis of the data is performed in order to determine which conditions allow for the fastest drainage of the liquid.

2. EXPERIMENTAL SET-UP

In order to find the best set-up to drain 0.5 L of liquid in the least amount of time, a factorial analysis was performed in which the volume of water was drained under different conditions from a 1.5 L plastic bottle attached to a support (Fig. 1, Appendix A). The response variable of this analysis was *t* (s), the drainage time, which was measured with a chronometer. For all four variables that were tested for a possible influence on this drainage time, at least two levels were chosen. This resulted in sixteen possible set-ups for the full factorial design, each for which four measurements were performed (four replicates). Exception on this was the base set-up (all factors set at their “standard” settings, see Table 1), for which we used ten replicates in order to be able to check for the normality of the data distribution.

The diameter of the mouth of the bottle (variable A) was adjusted by either screwing on the cap with a small hole in it, or removing the cap. To alter the viscosity of the water (variable C), grenadine was added in grenadine: water ratio of 3:1. The angle under which the water bottle was drained (variable C), was varied between 45° and 90° by changing the position of the water bottle holder on the support(Fig. 1, Appendix A), and the total starting volume (variable D) was either set at 0.5 L and 1 L. In order to be able to perform the response surface method and find the optimal set-up, two additional levels were tested for the angle (45° and 90°), although not in a full factorial design.

3. RESULTS

The resulting drainage time’s *t* for all sixteen set-ups of the full factorial experiment (2^4) is shown in Table 1 (Appendix B). The different set-ups were tested in a random order, but for convenience of reading the results are grouped per factor and shown in a logical order of adjustments. The diameter of the mouth of the bottle clearly had a large effect on the drainage time, and this for all possible combinations. The other factors seemed to have a less pronounced effect on *t*, but conclusions on the size of these effects can only be drawn after statistical analysis of the data.

In order to be able to find the best possible set-up, the response surface method will be performed on the data. For this method, two levels per factor do not suffice. Therefore, the experiments was performed for two additional levels of variable C, the angle (i.e. at 60° and 75°), and this for both levels of the bottle mouth diameter. The results of these additional runs are shown in Table 2 of Appendix B. Also the total volume of fluid inside the water bottle is increased to 0.750 L and experiments are conducted with two different diameter and two different angles and their combinations. The results of these additional runs are shown in Table 2a.

4. STATISTICAL ANALYSIS

All statistical analyses reported in this project report were performed with the statistical software Minitab.

4.1 PROBABILITY PLOTS

Probability plotting is a graphical technique that is widely used to determine whether it is valid to assume a certain hypothesized distribution of the data. Initially, such a hypothesis is often made based on a simple visual examination of the data. Common distributions that are easy to recognise visually are the normal distribution, lognormal distribution and the Weibull distribution.

Our data were tested for all three previously mentioned distributions. Resulting plots made in Minitab are shown in Appendix C (Fig. 3). Both the normal and lognormal distribution seems to be appropriate to describe the data, as in both cases the P-value is well above 0.05 (the chosen level of significance). However, since the P-value is higher for the normal probability plot, it can be concluded that the normal distribution is a good model to describe the distribution of our data.

4.2 PLOTS OF DRAINAGE TIME T (S) AS A FUNCTION OF THE FACTORS

In Fig. 4 (Appendix C), the time *t* needed to drain 500 mL of water is shown for four different angles, i.e. 45°, 60°, 75° and 90° relative to the table top. The plot of the mean drainage times with standard deviations gives us a quick and easy to compare overview of the differences, while the box plot gives us more information on the distribution of the data and possible outliers. From a first look at these plots, it seems that changing the factors affect the response variable (draining time) differently.

4.3 DECISION MAKING FOR TWO AND MULTIPLE SAMPLES

Two samples

To determine the best set-up for draining 500 mL of water from a plastic bottle, we would like to know first whether the selected factors has a significant influence on the drainage time and these variations are not due to the inherent variance involed in the process. In order to determine this, we perform a two sample t-test to compare the mean drainage times of treatment A (larger mouth diameter), B (higher viscosity), C (different angle) and D (increased total volume) with the base set-up (-1). The null hypothesis of these tests is that μ_1-μ_2 = 0, in which the μ's are the means of the base set-up and the considered treatment respectively. Because the property of our interest is the lowering of the drainage time, we chose to take μ_1-μ_2 > 0 as the alternative hypothesis.

1. **Parameter of interest:** We want to test if the mean drainage times of the different treatments (only one factor altered, i.e. treatments 2-5) differ from the mean drainage time for the base set-up. For this test, we assume that the data are normally distributed.
2. **Null Hypothesis:** H_0: $\mu_1-\mu_2 = 0$
3. **Alternative hypothesis:** H_1: $\mu_1-\mu_2 > 0$
4. **Test statistic:** Since we cannot be sure that the variances of the tested treatments are equal, the T_0^* test statistics was used.

$$t_0^* = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5. **Reject H_0 if:** The significance level α to reject H_0 was chosen to be 0.05(reject if p<0.05).

6. **Computations:** Because we performed the t-test on all four treatments (2-5 in Table 1) the means, sample sizes and according degrees of freedom are presented in the Table below. All means are always compared to the base set-up. See Fig. X for the results from Minitab.

(s)	**mean** $\bar{x}$	**st. dev.** ***s***	**n**	**degrees of freedom**	t_0^*	**P-value**
(-1)	8.140	1.080	10	-	-	
A	2.215	0.219	4	10	16.52	0.000
B	7.495	0.207	4	10	1.80	0.102
C	7.000	0.179	4	10	3.22	0.009
D	6.438	0.210	4	10	4.76	0.000

7. **Conclusions:** From the P-values in the table, we can draw conclusions on whether or not to accept the H_0 hypothesis. If the P-value is below our chosen significance level α=0.05, the null hypothesis can be rejected and it can be concluded that the investigated means are significantly different. This is the case for three of the selected factors, i.e. the diameter of the bottle mouth, the angle under which the water bottle is drained and the total liquid volume initially present in the bottle. For these factors, we can state that they lower the time needed to train 500 mL of water from the bottle.

Furthermore, we would also like to investigate if the variances differ between treatments. This is done with an f-test. This test is being done because for performing factorial experiment it is required that the variance between different treatments should remain same. In this case, we only performed the test for one factor, i.e. the angle. Both means μ and variances σ^2 are unknown.

1. **Parameter of interest:** We want to test if the variance in drainage times of the angle treatment differs from the variance in drainage times for the base set-up. For this test, we assume that the data are normally distributed.
2. **Null Hypothesis:** H_0: $\sigma_1/\sigma_2 = 1$
3. **Alternative hypothesis:** H_1: $\sigma_1/\sigma_2 \neq 1$
4. **Test statistic:** The F-statistic is used

$$f_0 = \frac{s_1^2}{s_2^2}$$

5. **Reject H_0 if:** The significance level α to reject H_0 was chosen to be 0.05.
6. **Computations:** Because s_1 = 1.080 (with n=10) and s_2 = 0.179 (with n=4), f_0 is calculated to have a value of 36.44. A complete view of the results is given in Fig. X in Appendix C.
7. **Conclusions:** The P-value calculated by Minitab is 0.051 for Levene's method (which is advised for sample sizes of less than 20 samples). This is larger than our chosen significance level of 0.05 and therefore the null hypothesis cannot be rejected; the variances are equal.

Multiple samples

For the angle of the bottle relative to the table top, there are more than two levels for which the drainage time was measured, i.e. 45°, 60°, 75° and 90°. In this case, we cannot simply use a t-test to investigate if the means are different from base set-up. Therefore, we have to use Analysis of Variance (ANOVA).

1. **Parameter of interest:** We want to test if the mean drainage times under different angles differ from each other with a one-way ANOVA. For this test, we assume that the data are

normally distributed and that the variances are equal, which was already proven to be true in the previous test.

2. **Null Hypothesis:** H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
3. **Alternative hypothesis:** H_1: at least one of the μ's is not equal to the others
4. **Test statistic:** The F-statistic is used

$$f_0 = \frac{s_1^2}{s_2^2}$$

5. **Reject H_0 if:** The significance level α to reject H_0 was chosen to be 0.05.
6. **Computations:** Because s_1 = 1.080 (with n=10), s_2 = 0.257 (with n=4), s_3 = 0.203 (with n=4) and s_4 = 0.179 (with n=4), f_0 is calculated to have a value of 5.48. A complete view of the results is given in Fig. X in Appendix C, along with a graph.
7. **Conclusions:** The P-value calculated by Minitab is 0.007. This is smaller than our chosen significance level of 0.05 and therefore the null hypothesis has to be rejected; at least some of the means are not equal.

4.4 Single Regression

To check the relation between one of the control variables (angle) and the response variable (draining time) a regression model is created, using the data of the time measurements for the different angles (i.e. 45°, 60°, 75° and 90°). First look at the plot (shown in figure 3) suggest that there might be a quadratic relationship, but as we do not know what to expect from a theoretical point of view, both a linear and a quadratic regression model were tested. The forms of these models are $Y = \beta_0 + \beta_1 x + \epsilon$ and $Y = \beta_0 + \beta_1 x + \beta_1 x^2 + \epsilon$ respectively.

Minitab was used to calculate the estimates for β_0, β_1 and β_2, i.e. $\widehat{\beta_0}$, $\widehat{\beta_1}$ and $\widehat{\beta_2}$ with the least squares method. The results for this calculation are presented in Fig. 7 of Appendix C. These results agree with our suspicion that there is a quadratic relationship between the angle under which the bottle is drained and the time required for the drainage. The regression model therefore becomes:

$$t(s) = 13.12 - 0.2175x + 0.001803x^2$$

With x the angle expressed in degrees. However, as the R^2 is rather low, i.e. only 42.05%, the model cannot be deemed to be a very good one. A graphical representation of this regression model is shown in Fig. 7.

To determine if the drainage time is really significantly influenced by the angle under which the bottle is drained, we investigate if the coefficient of the variable, β_1, and of the quadratic term of the variable, β_2, are significantly different from 0. For this, we use a single sample t-test.

1. **Parameter of interest:** We want to test if β_1 is significantly different from zero and, as a consequence, if the angle has a significant impact on the drainage time. For this test, we have to assume a normal distribution. Since Minitab shows us that the residuals are normally distributed, this assumption seems valid.
2. **Null Hypothesis:** H_0: $\beta_1 = 0$; $\beta_2 = 0$
3. **Alternative hypothesis:** H_1: $\beta_1 \neq 0$; $\beta_2 \neq 0$
4. **Test statistic:** Since we cannot be sure that the variances of the tested treatments are equal, the T_0^* test statistics was used.

$$t_0 = \frac{\widehat{\beta_1}}{se(\widehat{\beta_1})}$$

5. **Reject H_0 if:** The significance level α to reject H_0 was chosen to be 0.05.

6. **Computations:** The results obtained with Minitab are shown in Fig. X. This shows that the T-value for β_1 is -2.02 and that for β_2 is 2.33.
7. **Conclusion:** The calculated P-values for the coefficients are 0.058 for the β_1 coefficient and 0.031 for the β_2 coefficient. This implies that for the latter $\beta_,$ the null hypothesis cannot be rejected and, therefore, that its value is not significantly different from 0. From this, we can conclude that the quadratic model can be reduced to a linear regression model. This model would be:$t(s) = 5.080 - 0.0319x$.

With the information given by Minitab, the confidence interval of the β_0 and β_1 coefficient of the linear regression model can be determined as well. For a 95% confidence interval and with 4 replicates per treatment, t = 4.303. With the help of Eq. 6-31 and 6-32, we can then calculate that $0.0120 \leq \beta_1 \leq 0.0758$ and $-1.745 \leq \beta_0 \leq 8.415$.

Furthermore, the resulting information can also be used to determine a prediction interval for future observations. Again, for a α-value of 0.05, we find that t = 4.303. With the help of Eq. 6-36, we can therefore calculate the 95% prediction interval for a new measurement, e.g. an angle of 50°, to be $2.645 \leq Y_0 \leq 10.705$.

4.5 DESIGN OF EXPERIMENTS

Full Factorial Design

In our full factorial design, there are four factors (k=4), each of which have two levels: (A) the diameter of the bottle mouth; 22mm or 34 mm, (B) the viscosity of the liquid; low or high, (C) the angle under which the bottle is drained; 45° or 90°, and (D) the total volume of the liquid; 500 mL or 1000 mL. This leads to a 2^4 design, with four replicates per treatment. With the obtained measurement data, this yields the effects as shown in Table 3 (Appendix B).
The values of these effects clearly show the dominant role of the diameter of the bottle mouth in determining the drainage time, followed by the total liquid volume in the bottle. The regression model obtained for this full factorial design is:

$$\begin{aligned} t(s) = 4.4875 - 2.0672A - 0.2687B - 0.0084C - 0.4550D + 0.2428AB + 0.1419AC \\ + 0.2878AD + 0.0816BC - 0.3062BD + 0.2078CD - 0.0806ABC + 0.0953ABD \\ - 0.2519ACD - 0.0647BCD + 0.0756ABCD \end{aligned}$$

(Result of the full factorial experiment from Minitab is shown in fig 8)

However, the P-values found in Table 3 indicate that only (or: no less than) nine of these effects are significant, i.e. A, B, D, AB, AC, AD, BD, CD and ACD (P-value < 0.05). Therefore, the regression model can be simplified by eliminating the factors that has least effect on the draining time.

Residual analysis:
From this simplified regression model, the residuals can be calculated. The normal probability plot of these residuals is shown in Fig. 8a. As they seem to follow the line quite well on the normal probability plot, we can conclude that the data are indeed normally distributed as shown in figure 8a
Furthermore, it has to be checked if the variances are equal for all levels of the four factors. To investigate this, the residuals for every factor separately per level are plotted. As there does not seem to be a big difference between the levels, we can assume that the variances are equal. This is the prerequisite for factorial experiments.

Fractional Factorial Design

For the fractional factorial design, a 2^{4-1} design was performed, which leads to eight treatments being selected to run instead of the sixteen treatments of the full factorial. The selected treatments are (-1), A, B, C, D, AB, AC and AD. With the help of Minitab, we find the following effects for these treatments and their aliases.

A+BCD = -4.264
B+ACD = -1.041
C+ABD = 0.174
D+ABC = -1.071
AB+CD = 0.901
AC+BD = -0.329
AD+BC = 0.739

Like for the full factorial design, the effects show the dominant role of the diameter of the bottle mouth in determining the drainage time. This results in the following regression model:

$$t(s) = 4.563 - 2.132A - 0.521B + 0.087C - 0.536D + 0.451AB - 0.164AC + 0.369AD$$

(Result of the fractional factorial experiment from Minitab is shown in fig 9)

However, the condition for accepting this regression model is that the higher order effects (BCD, ACD, ABD, ABC, CD, BD and BC) can be neglected. From the full factorial design, we know this is not true. Again, the residuals can be calculated from this regression model. The normal probability plot of these residuals is shown in Fig. 9a. As they seem to follow the line quite well, we can conclude that the data are indeed normally distributed. Furthermore, it has to be checked if the variances are equal for all levels of the four factors. To investigate this, the residuals for every factor separately per level are plotted. As there does not seem to be a big difference between the levels, we can assume that the variances are equal.

4.6 Response surface method

In order to find the optimal set-up for draining 500 mL of water in the least amount of time, two continuous variables (angle and total Volume) and two categorical variables (hole size and viscosity) we performed some extra runs for both the angle (with small and large hole size) and the total liquid volume. With these extra runs, we could use the response surface method to find the optimum. The data used for surface response method is presented in table4.The results obtained from Minitab are presented in Fig. 10 and figure 11 shows the contour plot and surface plot for trail 4.

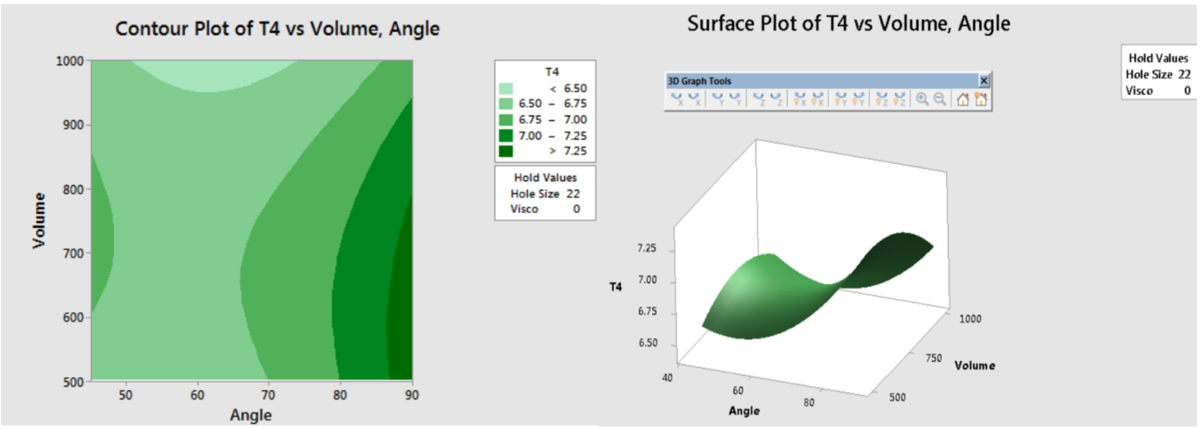


Figure 11: Contour plot and surface plot obtained by Minitab for the response surface method.

The twist in the figure 11 shows interaction between the two factors.From this analysis, we find that the optimal set-up for the shortest drainage time (with the four selected factors as variables) is the large hole size (34mm), the lower viscosity (only water), an angle of 76,8° and a total liquid volume of 1000 mL.

5. Appendix

5.1 Appendix A: Figures

(a) setup view 1

(b) setup view 2

(c) Concentrate used for changing visocity

(d) drain hole

Figure 1: Experimental set-up of the drainage time experiment.

5.2 Appendix B: Tables

Table 1: Results of the drainage time measurements for all 16 set-ups. For the base measurement (all factors set at their "standard" setting), 10 replicates were used to check for normality of the data distribution.

	code	Reps	diameter	viscosity	angle	total volume	drainage time t (s)			
		(#)	(mm)	(low-high)	(°)	(mL)	1	2	3	4
1	(-1)	10	22	Low	90	500	9.60	7.12	9.62	6.94
							8.15	7.84	6.42	8.30
							8.66	8.73	-	-
2	A	4	34	Low	90	500	2.44	1.94	2.33	2.15
3	B	4	22	High	90	500	7.35	7.30	7.59	7.74
4	C	4	22	Low	45	500	7.15	7.16	6.84	6.85
5	D	4	22	Low	90	1000	6.31	6.64	6.59	6.21
6	AB	4	34	High	90	500	2.65	2.84	2.62	2.31
7	AC	4	34	Low	45	500	2.60	2.72	2.53	2.51
8	AD	4	34	Low	90	1000	2.94	2.32	2.20	2.19
9	BC	4	22	High	45	500	7.15	6.84	6.95	6.58
10	BD	4	22	High	90	1000	4.40	4.40	4.42	5.05
11	CD	4	22	Low	45	1000	6.42	6.93	7.03	7.67
12	ABC	4	34	High	45	500	2.79	2.98	2.78	3.21
13	ABD	4	34	High	90	1000	1.88	1.86	1.91	2.01
14	ACD	4	34	Low	45	1000	2.64	2.80	2.55	2.28
15	BCD	4	22	High	45	1000	5.08	5.38	5.56	4.90
16	ABCD	4	34	High	45	1000	1.68	2.42	1.96	2.41

Table 2: Results of the drainage time measurements for all four levels of the angle variable.

	code	Reps	diameter	viscosity	angle	total volume	drainage time t (s)			
		(#)	(mm)	(low-high)	(°)	(mL)	1	2	3	4
1	C_1	10	22	Low	90	500	9.60	7.12	9.62	6.94
							8.15	7.84	6.42	8.30
							8.66	8.73		
2	C_2	4	22	Low	75	500	7.25	7.14	6.98	6.66
3	C_3	4	22	Low	60	500	6.33	6.73	6.60	6.32
4	C_4	4	22	Low	45	500	7.15	7.16	6.84	6.85
5	AC_1	4	34	Low	90	500	2.44	1.94	2.33	2.15
6	AC_2	4	34	Low	75	500	2.26	2.26	2.37	2.10
7	AC_3	4	34	Low	60	500	2.38	2.00	2.26	2.29
8	AC_4	4	34	Low	45	500	2.60	2.72	2.53	2.51

Table 3a: Results of the drainage time measurements with total volume 750 mL.

	code	Reps	diameter	Viscosity	angle	total volume	drainage time t (s)			
		(#)	(mm)	(low-high)	(°)	(mL)	1	2	3	4
1	D_2	4	22	Low	90	750	8.25	7.68	8.49	8.5
2	AD_2	4	34	Low	90	750	1.32	1.64	1.35	1.9
3	CD_2	4	22	Low	45	750	5.75	6.12	6.18	5.92

Table 4: Results obtained by Minitab for the full factorial design.

		effect	coefficient	standard error (coeff)	T-value	P-value
1	A	-4.1344	-2.0672	0.0569	-36.31	0.000
2	B	-0.5375	-0.2687	0.0569	-4.72	0.000
3	C	-0.0169	-0.0084	0.0569	-0.15	0.883
4	D	-0.9100	-0.4550	0.0569	-7.99	0.000
5	AB	0.4856	0.2428	0.0569	4.26	0.000
6	AC	0.2837	0.1419	0.0569	2.49	0.016
7	AD	0.5756	0.2878	0.0569	5.05	0.000
8	BC	0.1631	0.0816	0.0569	1.43	0.158
9	BD	-0.6125	-0.3062	0.0569	-5.38	0.000
10	CD	0.4156	0.2078	0.0569	3.65	0.001
11	ABC	-0.1613	-0.0806	0.0569	-1.42	0.163
12	ABD	0.1906	0.0953	0.0569	1.67	0.101
13	ACD	-0.5038	-0.2519	0.0569	-4.42	0.000
14	BCD	-0.1294	-0.0647	0.0569	-1.14	0.262
15	ABCD	0.1513	0.0756	0.0569	1.33	0.190

Table4: showing data used for surface response method

Data used for surface response method											
Hole Size	**Viscosity**	**Angle**	**Volume**	**Trail 1**	**Trail 2**	**Trail 3**	**Trail 4**	**StdOrder**	**RunOrder**	**Blocks**	**PtType**
22	0	90	500	9.60	7.12	9.62	6.94	1	1	1	1
34	0	90	500	2.44	1.94	2.33	2.15	2	2	1	1
22	375	90	500	7.35	7.3	7.59	7.74	3	3	1	1
22	0	45	500	7.15	7.16	6.84	6.85	4	4	1	1
22	0	90	1000	6.31	6.64	6.59	6.21	5	5	1	1
34	375	90	500	2.65	2.84	2.62	2.31	6	6	1	1
22	375	45	500	7.15	6.84	6.95	6.58	7	7	1	1
22	0	45	1000	6.42	6.93	7.03	7.67	8	8	1	1
34	0	45	500	2.60	2.72	2.53	2.51	9	9	1	1
34	0	90	1000	2.94	2.32	2.2	2.19	10	10	1	1
22	375	90	1000	4.40	4.4	4.42	5.05	11	11	1	1
34	375	45	500	2.79	2.98	2.78	3.21	12	12	1	1
22	375	45	1000	5.08	5.38	5.56	4.9	13	13	1	1
34	375	90	1000	1.88	1.86	1.91	2.01	14	14	1	1
34	0	45	1000	2.64	2.8	2.55	2.28	15	15	1	1
34	375	45	1000	1.68	2.42	1.96	2.41	16	16	1	1
22	0	75	500	7.25	7.14	6.98	6.66	17	17	1	1
34	0	75	500	2.26	2.26	2.37	2.1	18	18	1	1
22	0	60	500	6.33	6.73	6.6	6.32	19	19	1	1
34	0	60	500	2.38	2	2.26	2.29	20	20	1	1
22	0	90	750	8.25	7.68	8.49	8.5	21	21	1	1
34	0	90	750	1.32	1.64	1.35	1.9	22	22	1	1
22	0	45	750	5.75	6.12	6.18	5.92	23	23	1	1

5.3 GRAPHS

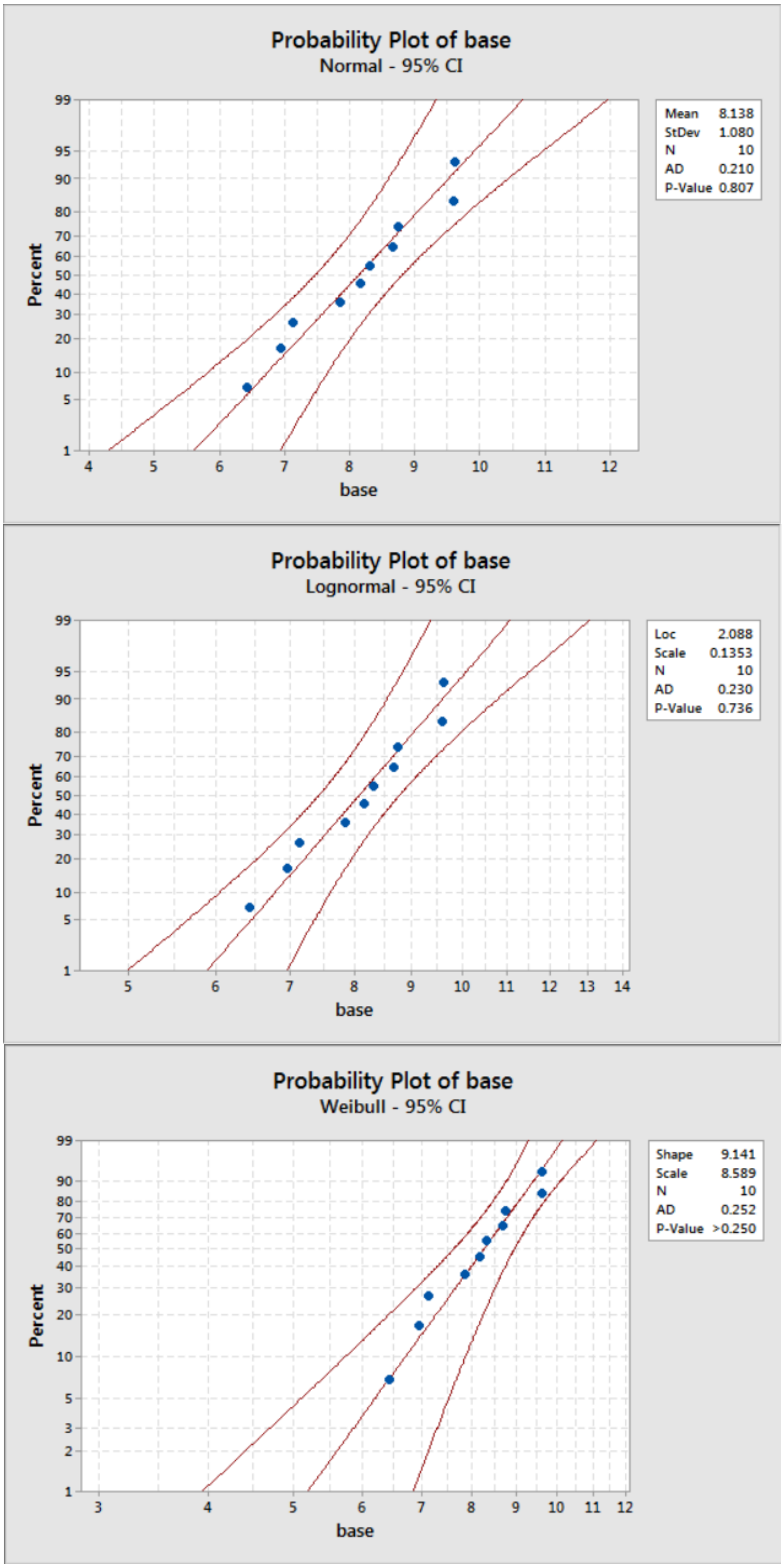


Figure 2: Probability plots for a normal (upper graph), lognormal (middle graph) and Weibull (lower graph) distribution.

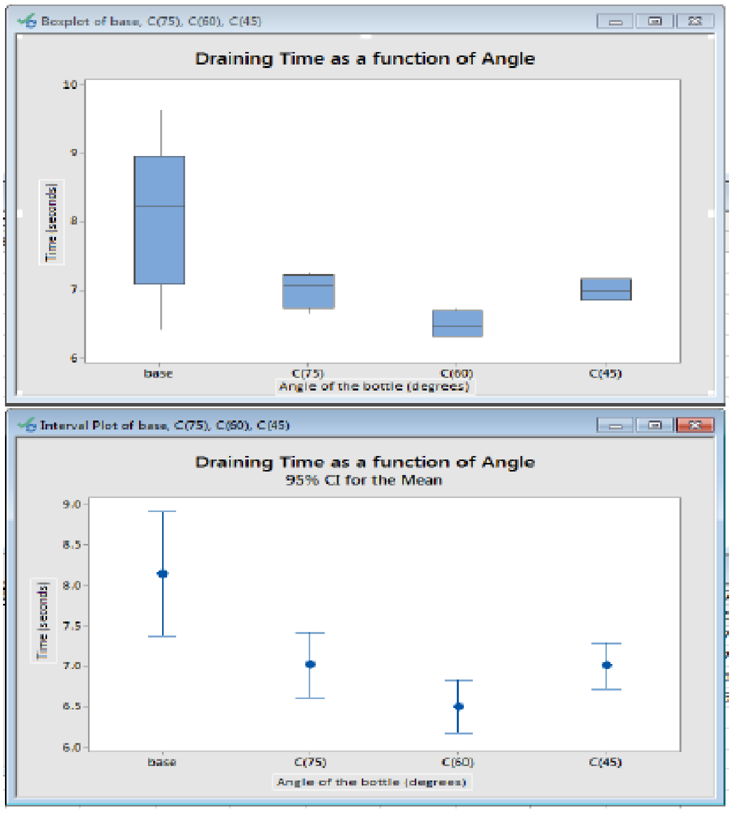
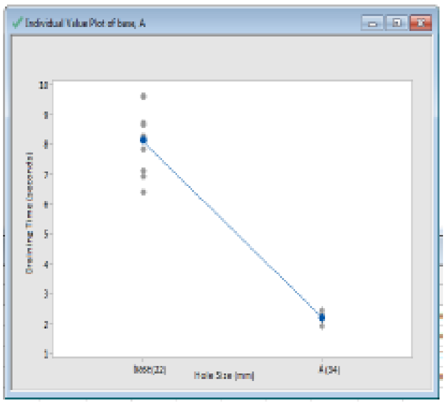
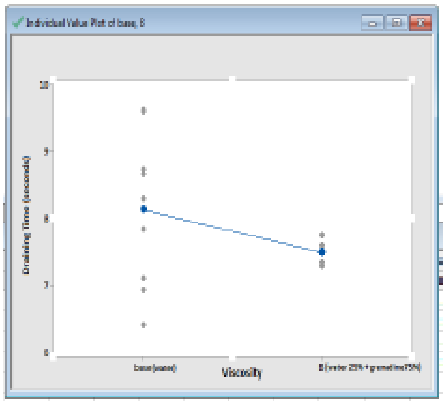


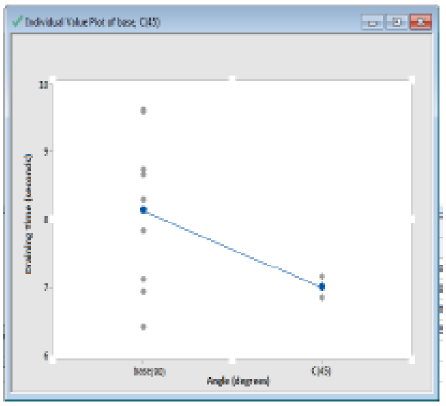
Figure 3: Box plot (upper) and plot of the average with standard deviations (b) of the response variable t(s) as a function of one of the variables, the angle of the bottle compared to the table top.



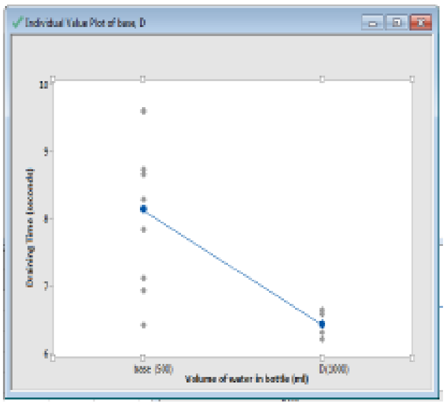
(a) Drain Time vs Hole size



(b) Drain Time vs Viscosity



(c) Drain Time vs Angle



(d) Drain Time Vs Volume

Figure 4a: Effect of control variable's on the response variable

Two-Sample T-Test and CI: base, A

Method

μ_1: mean of base
μ_2: mean of A
Difference: $\mu_1 - \mu_2$

Equal variances are not assumed for this analysis.

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
base	10	8.14	1.08	0.34
A	4	2.215	0.219	0.11

Estimation for Difference

Difference	95% Lower Bound for Difference
5.923	5.273

Test

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis $H_1: \mu_1 - \mu_2 > 0$

T-Value	DF	P-Value
16.52	10	0.000

Two-Sample T-Test and CI: base, B

Method

μ_1: mean of base
μ_2: mean of B
Difference: $\mu_1 - \mu_2$

Equal variances are not assumed for this analysis.

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
base	10	8.14	1.08	0.34
B	4	7.495	0.207	0.10

Estimation for Difference

Difference	95% CI for Difference
0.643	(-0.152, 1.438)

Test

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis $H_1: \mu_1 - \mu_2 \neq 0$

T-Value	DF	P-Value
1.80	10	0.102

Two-Sample T-Test and CI: base, C(45)

Method

μ_1: mean of base
μ_2: mean of C(45)
Difference: $\mu_1 - \mu_2$

Equal variances are not assumed for this analysis.

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
base	10	8.14	1.08	0.34
C(45)	4	7.000	0.179	0.090

Estimation for Difference

Difference	95% CI for Difference
1.138	(0.351, 1.925)

Test

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis $H_1: \mu_1 - \mu_2 \neq 0$

T-Value	DF	P-Value
3.22	10	0.009

Two-Sample T-Test and CI: base, D

Method

μ_1: mean of base
μ_2: mean of D
Difference: $\mu_1 - \mu_2$

Equal variances are not assumed for this analysis.

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
base	10	8.14	1.08	0.34
D	4	6.438	0.210	0.10

Estimation for Difference

Difference	95% Lower Bound for Difference
1.700	1.053

Test

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis $H_1: \mu_1 - \mu_2 > 0$

T-Value	DF	P-Value
4.76	10	0.000

Figure 5: Results from two sample t-tests performed in Minitab for the four different factors: (A) mouth diameter, (B) viscosity, (C) angle and (D) total liquid volume.

One-way ANOVA: base, C(75), C(60), C(45)

Method

Null hypothesis	All means are equal
Alternative hypothesis	Not all means are equal
Significance level	α = 0.05

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
Factor	4	base, C(75), C(60), C(45)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	3	9.963	3.3210	5.48	0.007
Error	18	10.913	0.6063		
Total	21	20.876			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.778643	47.72%	39.01%	34.37%

Means

Factor	N	Mean	StDev	95% CI
base	10	8.138	1.080	(7.621, 8.655)
C(75)	4	7.008	0.257	(6.190, 7.825)
C(60)	4	6.495	0.203	(5.677, 7.313)
C(45)	4	7.0000	0.1791	(6.1821, 7.8179)

Pooled StDev = 0.778643

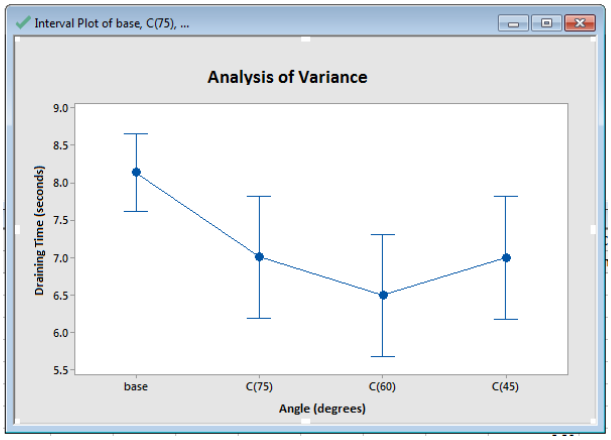


Figure 6: Results obtained with Minitab for the ANOVA performed on the angle treatments.

Regression Analysis: Draining Time versus Angle

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	9.9300	4.96502	8.62	0.002
Angle	1	2.3534	2.35342	4.09	0.058
Angle*Angle	1	3.1165	3.11654	5.41	0.031
Error	19	10.9460	0.57611		
Lack-of-Fit	1	0.0329	0.03291	0.05	0.818
Pure Error	18	10.9131	0.60629		
Total	21	20.8761			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.759017	47.57%	42.05%	35.16%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	13.12	3.53	3.72	0.001	
Angle	-0.218	0.108	-2.02	0.058	134.91
Angle*Angle	0.001803	0.000775	2.33	0.031	134.91

Regression Equation

Draining Time = 13.12 - 0.218 Angle + 0.001803 Angle*Angle

Fits and Diagnostics for Unusual Observations

Obs	Draining Time	Fit	Resid	Std Resid	
1	9.600	8.146	1.454	2.02	R
3	9.620	8.146	1.474	2.04	R
7	6.420	8.146	-1.726	-2.39	R

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	6.813	6.8135	9.69	0.005
Angle	1	6.813	6.8135	9.69	0.005
Error	20	14.063	0.7031		
Lack-of-Fit	2	3.149	1.5747	2.60	0.102
Pure Error	18	10.913	0.6063		
Total	21	20.876			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.838528	32.64%	29.27%	18.90%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	5.080	0.775	6.56	0.000	
Angle	0.0319	0.0102	3.11	0.005	1.00

Regression Equation

Draining Time = 5.080 + 0.0319 Angle

Fits and Diagnostics for Unusual Observations

Obs	Draining Time	Fit	Resid	Std Resid	
1	9.600	7.948	1.652	2.06	R
3	9.620	7.948	1.672	2.08	R

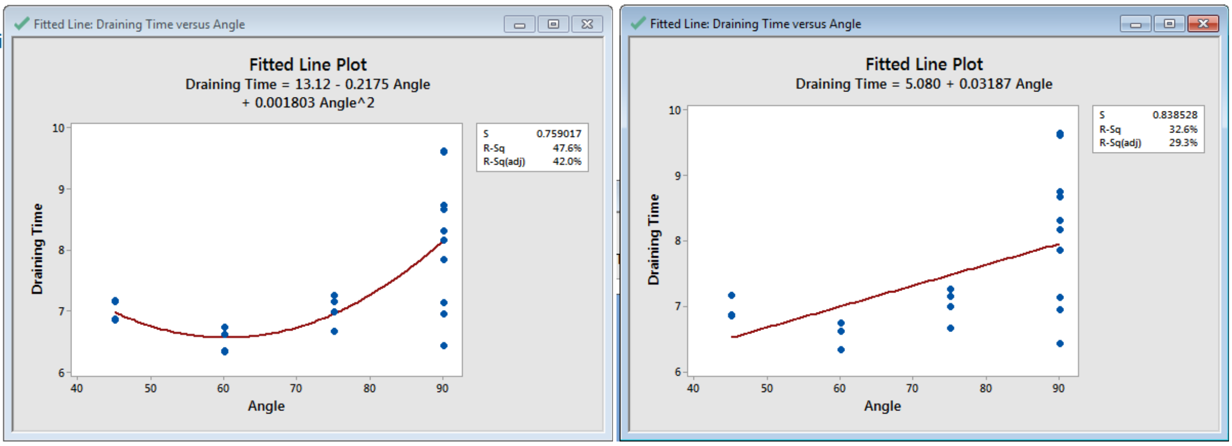


Figure 7: Results obtained with Minitab for the regression analysis: (a) quadratic regression model, (b) linear regression model

Factorial Regression: Draining Time versus Hole size, ... ngle, Volume

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	15	316.612	21.107	101.74	0.000
Linear	4	291.366	72.841	351.09	0.000
Hole size	1	273.489	273.489	1318.18	0.000
Viscosity	1	4.623	4.623	22.28	0.000
Angle	1	0.005	0.005	0.02	0.883
Volume	1	13.250	13.250	63.86	0.000
2-Way Interactions	6	19.555	3.259	15.71	0.000
Hole size*Viscosity	1	3.773	3.773	18.19	0.000
Hole size*Angle	1	1.288	1.288	6.21	0.016
Hole size*Volume	1	5.302	5.302	25.55	0.000
Viscosity*Angle	1	0.426	0.426	2.05	0.158
Viscosity*Volume	1	6.002	6.002	28.93	0.000
Angle*Volume	1	2.764	2.764	13.32	0.001
3-Way Interactions	4	5.325	1.331	6.42	0.000
Hole size*Viscosity*Angle	1	0.416	0.416	2.01	0.163
Hole size*Viscosity*Volume	1	0.581	0.581	2.80	0.101
Hole size*Angle*Volume	1	4.060	4.060	19.57	0.000
Viscosity*Angle*Volume	1	0.268	0.268	1.29	0.262
4-Way Interactions	1	0.366	0.366	1.76	0.190
Hole size*Viscosity*Angle*Volume	1	0.366	0.366	1.76	0.190
Error	48	9.959	0.207		
Total	63	326.571			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.455493	96.95%	96.00%	94.58%

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.455493	96.95%	96.00%	94.58%

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		4.4875	0.0569	78.82	0.000	
Hole size	-4.1344	-2.0672	0.0569	-36.31	0.000	1.00
Viscosity	-0.5375	-0.2687	0.0569	-4.72	0.000	1.00
Angle	-0.0169	-0.0084	0.0569	-0.15	0.883	1.00
Volume	-0.9100	-0.4550	0.0569	-7.99	0.000	1.00
Hole size*Viscosity	0.4856	0.2428	0.0569	4.26	0.000	1.00
Hole size*Angle	0.2837	0.1419	0.0569	2.49	0.016	1.00
Hole size*Volume	0.5756	0.2878	0.0569	5.05	0.000	1.00
Viscosity*Angle	0.1631	0.0816	0.0569	1.43	0.158	1.00
Viscosity*Volume	-0.6125	-0.3062	0.0569	-5.38	0.000	1.00
Angle*Volume	0.4156	0.2078	0.0569	3.65	0.001	1.00
Hole size*Viscosity*Angle	-0.1613	-0.0806	0.0569	-1.42	0.163	1.00
Hole size*Viscosity*Volume	0.1906	0.0953	0.0569	1.67	0.101	1.00
Hole size*Angle*Volume	-0.5038	-0.2519	0.0569	-4.42	0.000	1.00
Viscosity*Angle*Volume	-0.1294	-0.0647	0.0569	-1.14	0.262	1.00
Hole size*Viscosity*Angle*Volume	0.1513	0.0756	0.0569	1.33	0.190	1.00

Regression Equation in Uncoded Units

Draining Time = 4.4875 - 2.0672 Hole size - 0.2687 Viscosity - 0.0084 Angle - 0.4550 Volume
+ 0.2428 Hole size*Viscosity + 0.1419 Hole size*Angle
+ 0.2878 Hole size*Volume + 0.0816 Viscosity*Angle - 0.3062 Viscosity*Volume
+ 0.2078 Angle*Volume - 0.0806 Hole size*Viscosity*Angle
+ 0.0953 Hole size*Viscosity*Volume - 0.2519 Hole size*Angle*Volume
- 0.0647 Viscosity*Angle*Volume + 0.0756 Hole size*Viscosity*Angle*Volume

Alias Structure

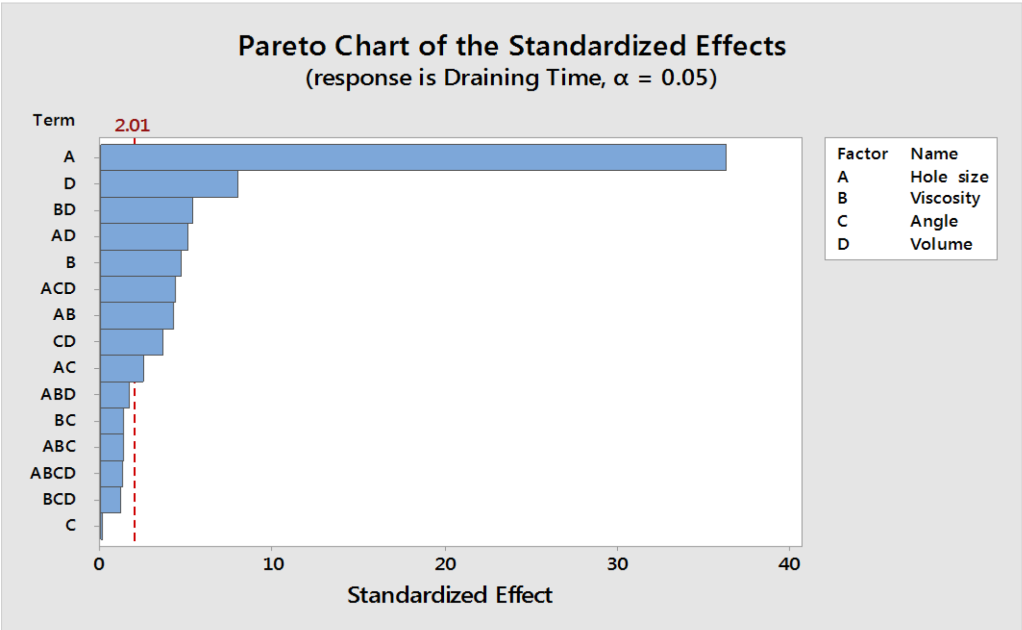
Factor	Name
A	Hole size
B	Viscosity
C	Angle
D	Volume

Aliases
I
A
B
C
D
AB
AC
AD
BC
BD
CD
ABC
ABD
ACD
BCD
ABCD

Fits and Diagnostics for Unusual Observations

Obs	Draining Time	Fit	Resid	Std Resid	
19	9.620	8.320	1.300	3.30	R
50	6.940	8.320	-1.380	-3.50	R
59	9.600	8.320	1.280	3.24	R
61	7.120	8.320	-1.200	-3.04	R

Figure 8: Results obtained with Minitab for the full factorial analysis



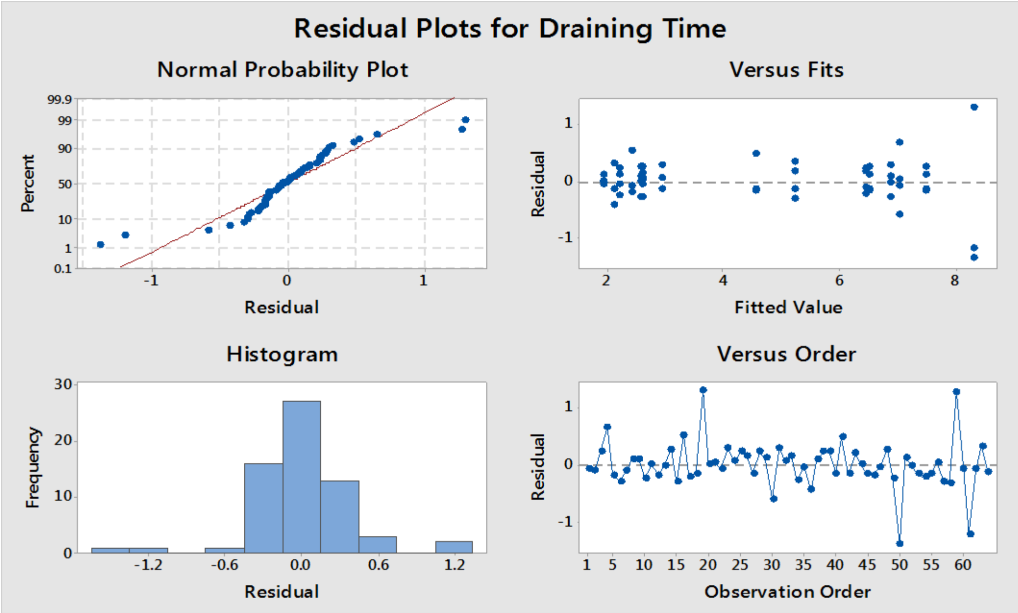


Figure 8a: showing effect of each variable in draining time and Residual plot after simplifying the expression by removing some factors in full factorial experiments

Factorial Regression: Draining Time versus Hole size, ... Angle, Volume

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	7	175.261	25.037	67.60	0.000
Linear	4	163.532	40.883	110.38	0.000
Hole size	1	145.437	145.437	392.65	0.000
Viscosity	1	8.674	8.674	23.42	0.000
Angle	1	0.242	0.242	0.65	0.427
Volume	1	9.181	9.181	24.79	0.000
2-Way Interactions	3	11.729	3.910	10.55	0.000
Hole size*Viscosity	1	6.498	6.498	17.54	0.000
Hole size*Angle	1	0.865	0.865	2.33	0.140
Hole size*Volume	1	4.366	4.366	11.79	0.002
Error	24	8.890	0.370		
Total	31	184.150			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.608605	95.17%	93.76%	91.42%

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		4.563	0.108	42.41	0.000	
Hole size	-4.264	-2.132	0.108	-19.82	0.000	1.00
Viscosity	-1.041	-0.521	0.108	-4.84	0.000	1.00
Angle	0.174	0.087	0.108	0.81	0.427	1.00
Volume	-1.071	-0.536	0.108	-4.98	0.000	1.00
Hole size*Viscosity	0.901	0.451	0.108	4.19	0.000	1.00
Hole size*Angle	-0.329	-0.164	0.108	-1.53	0.140	1.00
Hole size*Volume	0.739	0.369	0.108	3.43	0.002	1.00

Regression Equation in Uncoded Units

Draining Time = 4.563 - 2.132 Hole size - 0.521 Viscosity + 0.087 Angle - 0.536 Volume + 0.451 Hole size*Viscosity - 0.164 Hole size*Angle + 0.369 Hole size*Volume

Alias Structure

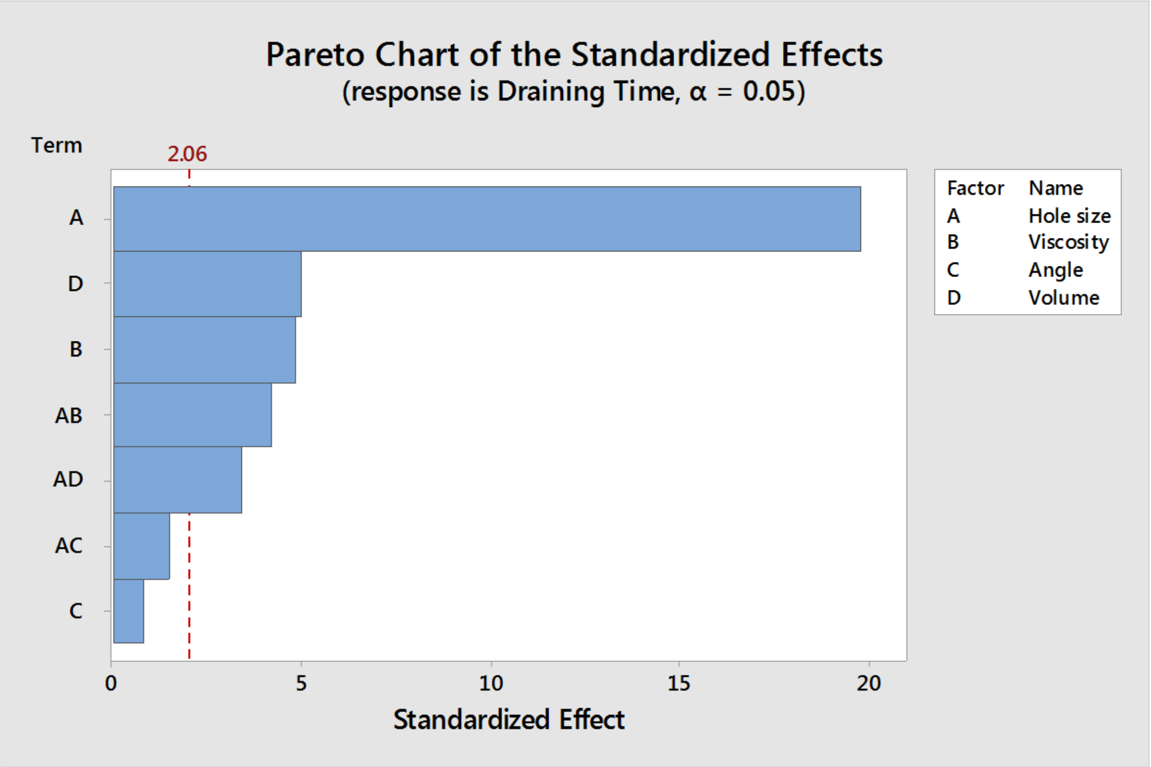
Factor	Name
A	Hole size
B	Viscosity
C	Angle
D	Volume

Aliases
I + ABCD
A + BCD
B + ACD
C + ABD
D + ABC
AB + CD
AC + BD
AD + BC

Fits and Diagnostics for Unusual Observations

Obs	Draining Time	Fit	Resid	Std Resid	
1	9.600	8.320	1.280	2.43	R
9	7.120	8.320	-1.200	-2.28	R
17	9.620	8.320	1.300	2.47	R
25	6.940	8.320	-1.380	-2.62	R

Figure 9: showing result of ½ factorial experiments obtained from Minitab



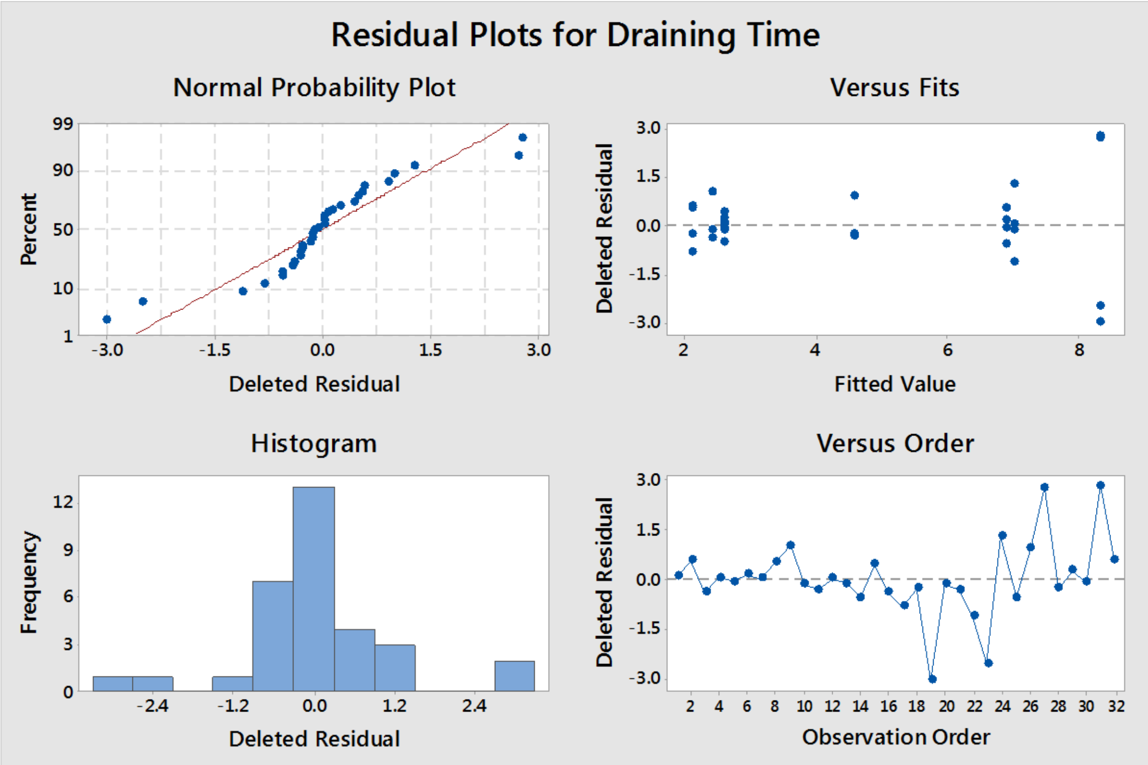


Figure 9a: showing effect of each variable in draining time and Residual plot after simplifying the expression by removing some factors in 1/2 factorial experiments

Response Optimization: T4, T3, T2, T1

Parameters

Response	Goal	Lower	Target	Upper	Weight	Importance
T4	Minimum		1.90	8.50	1	1
T3	Minimum		1.35	9.62	1	1
T2	Minimum		1.64	7.68	1	1
T1	Minimum		1.32	9.60	1	1

Solution

Solution	Angle	Volume	Hole Size	Visco	T4 Fit	T3 Fit	T2 Fit	T1 Fit	Composite Desirability
1	76.8182	1000	34	375	1.52023	0.790398	1.63656	0.730448	1

Multiple Response Prediction

Variable	Setting
Angle	76.8182
Volume	1000
Hole Size	34
Visco	375

Response	Fit	SE Fit	95% CI	95% PI
T4	1.520	0.719	(-0.081, 3.122)	(-0.805, 3.845)
T3	0.790	0.694	(-0.755, 2.336)	(-1.454, 3.035)
T2	1.637	0.436	(0.666, 2.608)	(0.227, 3.046)
T1	0.730	0.691	(-0.810, 2.271)	(-1.507, 2.968)

Figure 10: Results obtained from Minitab for the response surface method.