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Reliability of Mechanical Systems - Assignment Report (H04Y2A)

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Contents

List of Figures

1 Introduction

The Main aim of the exercise is to design a wing made of composite materials with Reliability Based Design Optimization method(RBDO) and test it using Monte Carlo technique.

The details of the provided wing structure can be found here. The exercise is done with the help of Noesis Optimus software.

2 Response Surface Method

In this section some of the Response Surface Methods were tested and the best among them is selected for the future analysis. Three different model were analyzed here

- Linear RSM model based on 3 level Full Factorial DOE method.
- Quadratic RSM model based on 3 level Full Factorial DOE method.
- Third order RSM model based on Latin Hypercube sampling strategy.

2.1 Linear RSM using Full Factorial DOE

A 3 level full factorial described as 3^3 FF has 3 input factors (t1, t2 and ρ) and each of them sampled at 3 points in each variable axis (low, high, middle) and in these the sampled points(27 sample points) the output (absTzmax) is found.Figure(1(a))shows the sampling points in 3 level Full Factorial Design of Experiments. A linear regression model is formed between the input and output.

$$abs(Tzmax) = f(t1, t2, \rho); \tag{1}$$

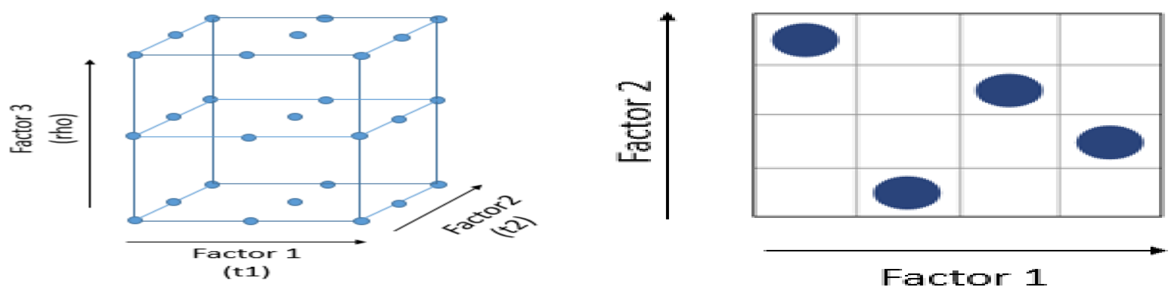
figure(2(a)) shows the contribution of each factors in the linear regression model, and figure(3(a)) shows the residual

2.2 Quadratic RSM using Full Factorial DOE

Like a similar procedure used to create a linear regression model a Quadratic RSM is created

$$abs(Tzmax) = f(t1, t2, \rho, t1^2, t2^2, \rho^2, t1 * t2, t1 * \rho, t2 * \rho); \tag{2}$$

figure(2(b))shows the contribution of the each factors in the quadratic regression model formed and figure(3(b)) shows the residual



(a) (b)

Figure 1: Sampling methods (a):3 level Full Factorial (b): Latin Hypercube

2.3 Linear and Quadratic RSM using Latin Hypercube sampling

Unlike Full factorial experiment which sampled at end point and middle value of each factor in Hypercube it is sampled at 50 point in this case for each variable such that these points doesn't meet other points if straight line is drawn vertically or horizontally.Figure(1(b)) shows such sampling for 2 factor with 4 sampling points. Figure(2(c)&2(d)) shows the contribution of factors in Linear and Quadratic RSM model and figure(3(c)&3(d)) shows the residual for linear and quadratic model formed using Latin Hypercube sampling.

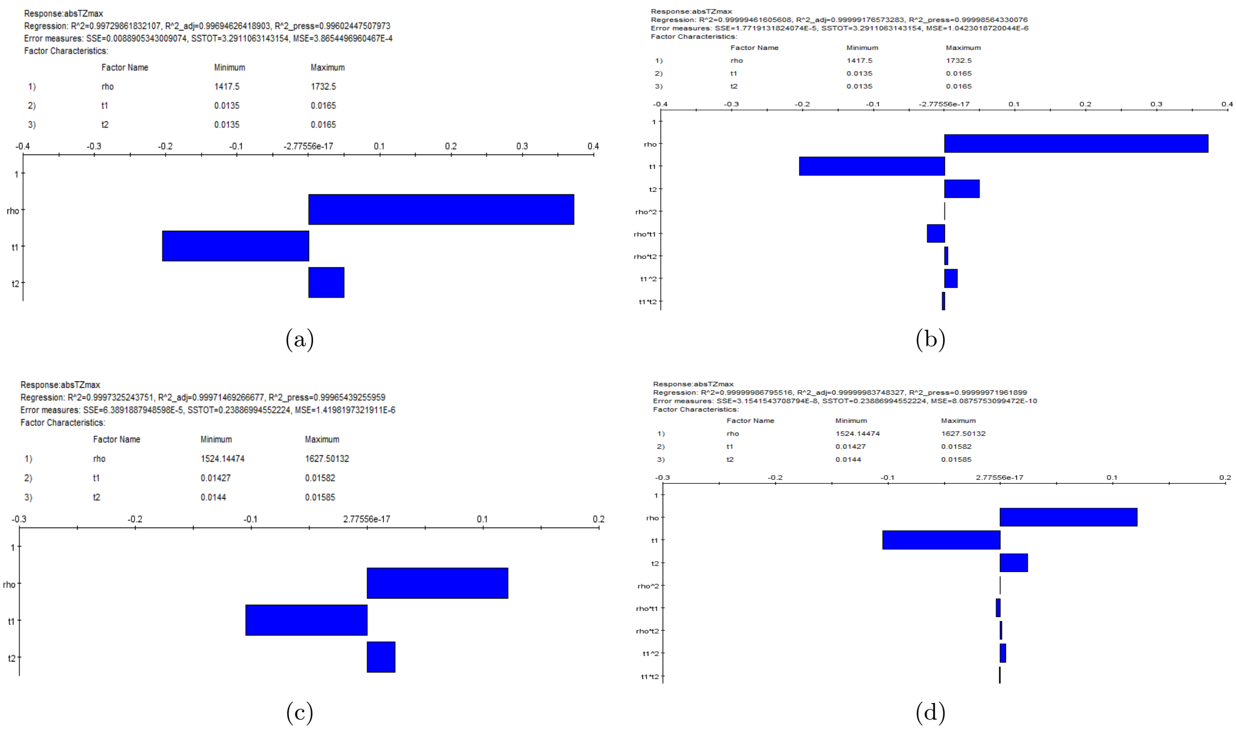
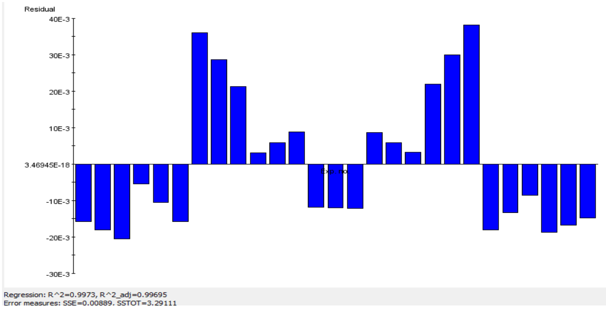
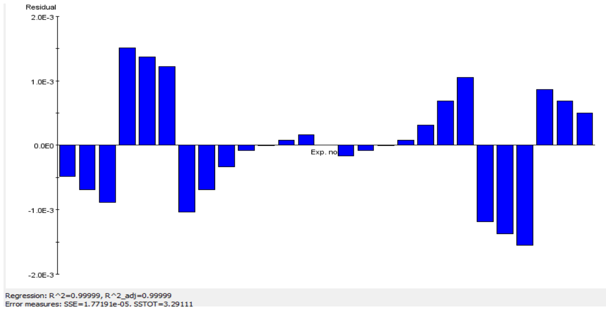


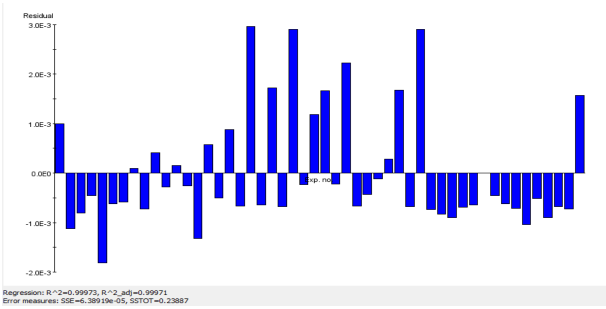
Figure 2: Input factor contribution in (a): Linear model by FF DOE method. (b): Quadratic model by FF DOE method. (c):Linear model by Latin Hypercube sampling method. (d):Quadratic model by Latin Hypercube Sampling method.



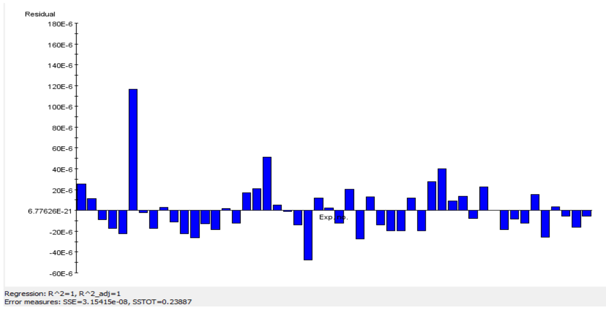
(a)



(b)



(c)



(d)

Figure 3: Residual Analysis on (a): Linear FF DOE model. (b): Quadratic FF DOE model. (c): Linear model made using Latin Hypercube sampling. (d):Quadratic model made using Latin Hypercube sampling.

It is clearly seen that the MSE of the **Quadratic Model made with Latin Hyper cube** is in order of 10^{-10} it is more accurate than other model created with Full Factorial DOE sampling methods. So this model will be used in the future for analysis.

3 Monte Carlo Simulation

In Monte Carlo simulation the input variables are sampled randomly. Here 100 sampling points are taken randomly and the output (abs(TZmax)) is found out using the Analysis method and Response surface method.

Pearson (Spearman	rho	t1	t2	mass	absTZmax
rho	1.000 (1.000)	-0.088 (-0.122)	-0.090 (-0.090)	0.964 (0.965)	0.815 (0.803)
t1	-0.088 (-0.122)	1.000 (1.000)	0.069 (0.080)	0.174 (0.098)	-0.636 (-0.643)
t2	-0.090 (-0.090)	0.069 (0.080)	1.000 (1.000)	-0.007 (-0.010)	0.013 (-0.001)
mass	0.964 (0.965)	0.174 (0.098)	-0.007 (-0.010)	1.000 (1.000)	0.646 (0.664)
absTZmax	0.815 (0.803)	-0.636 (-0.643)	0.013 (-0.001)	0.646 (0.664)	1.000 (1.000)

(a)

Pearson (Spearman	rho	t1	t2	mass	absTZmax
rho	1.000 (1.000)	0.035 (0.059)	0.140 (0.114)	0.967 (0.967)	0.778 (0.747)
t1	0.035 (0.059)	1.000 (1.000)	-0.116 (-0.086)	0.278 (0.282)	-0.585 (-0.559)
t2	0.140 (0.114)	-0.116 (-0.086)	1.000 (1.000)	0.174 (0.133)	0.319 (0.280)
mass	0.967 (0.967)	0.278 (0.282)	0.174 (0.133)	1.000 (1.000)	0.612 (0.587)
absTZmax	0.778 (0.747)	-0.585 (-0.559)	0.319 (0.280)	0.612 (0.587)	1.000 (1.000)

(b)

Figure 4: Correlation Matrix from (a):FEA Model (b):Surrogate model

Figure(4(a)&4(b)) shows the correlation matrix calculated form FEA model and Surrogate Model respectively. Even though there are some difference in value of the correlation between the two models following observations are noted

- The output *(abs(TZmax))* is in poisitive correlation with density (ρ) and thickness *(t2)* whereas the thickness *(t1)* has negative correlation with *abs(TZmax)*.
- The output *mass* is positively correlated with density (ρ) and thickness *(t1)* whereas the thickness *(t2)* has negative correlation with *mass*.
- The correlation coefficient of density (ρ) and of thickness *(t1)* is higher than thickness *(t2)*.
- An increase in ρ causes an increase with mass as expected. The increase in mass would thus increase the load on the wing and thus would increase the maximum deflection of the wing. This explains the higher correlation value between ρ and mass and the *abs(TZmax)*.
- An increase in the thickness *(t1)* would cause an increase in the clamping area along. Thus, this would cause a reduction in the deflection seen. This is seen in the correlation matrix as negative correlation.
- An increase in *(t2)* will increase the weight of the blade at the the same time the stiffness of the fin also increases because of this opposite effects it is found that the correlation coefficient is very small.
- The difference in correlation values between analysis and surrogate model is because of the approximation error in surface response method.

4 Design Optimization

Here a cost function which has Manufacturing cost and the mass of the taken.

$$C = M + \frac{1}{5{t_1}^2} + \frac{1}{4{t_2}^2} \tag{3}$$

and also the constraint on max and min value of ρ, *t1*, *t2 & abs(TZmax)* values are given. To predict the cost function value for different input parameter two different models are possible one from the Analysis and other form the surrogate model created before (explained in section (2))

To find the optimal point using optimization program (Sequential Quadratic Programming) is done and the results are shown in figure(5(a)&5(b))

Optimization method = Sequential Quadratic Programming

Objective = Minimize Cost

	Start	End [35] (9.0.0.2)	Low	High
Inputs				
rho	1575	1450	1450	1700
t1	0.015	0.01397	0.01	0.025
t2	0.015	0.01897	0.01	0.025
Outputs				
TZmax	-3.23138	-3.19788		
mass	18861.79	17516.82		
absTZmax	3.23138	3.19788		3.2
Cost	20861.79	19236.96088		
GOAL	20861.79	19236.96088		

(a)

Optimization method = Sequential Quadratic Programming

Objective = Minimize Cost

	Start	End [24] (6.0.0.3)	Low	High
Inputs				
rho	1575	1450	1450	1700
t1	0.015	0.01477	0.01	0.025
t2	0.015	0.01977	0.01	0.025
Outputs				
TZmax	-3.23138	-3.11213		
mass	18861.79	17726.8		
absTZmax	3.23138	3.11213		3.2
Cost	21087.90851	19309.08164		
GOAL	21087.90851	19309.08164		

(b)

Figure 5: Optimum values from (a): Analysis (b):Surrogate model

It is seen that the optimum value's of the input variables differs between the results obtained by the analysis and the surrogate model. It is seen that the surrogate model over estimates the cost than the actual analysis. ie at the start of optimization eventhough the input parameters *(ρ , t1 , t2)* are same in both analysis and surrogate model, the cost function of surrogate is greater than at analysis. Since surrogate model is an approximation to the actual model it is seen that after optimization eventhough the final output of t1 and t2 are greater for surrogate model its cost is greater than the cost from analysis. The reason for *abs(Tzmax)* of surrogate model lesser than analysis is because of the correlation explained in the previous section (the thickness *t1* is greater in surrogate model so more clamping resulting in less deflection). But since the difference of t1 and t2 is not much this surrogate model can be used as approximation.

5 Reliability Assessment

Here reliability index is calculated based on FORM analysis. The optimum values of the input variables obtained from the previous step(shown in figure(5(a))) is used.

Exp. Num.	Label	Annot	rho	t1	t2	TZmax	mass	absTZmax	Cost	SIGMA(ab...	FORMH(ab...
1	1	-	1450	0.01397	0.01897	-3.19773	17517.5	3.19773	19237.00831	0.08008	0.02834

Figure 6: Result of FORM Analysis

the β value is found to be 0.02834.

6 Reliability Based Design Optimization(RBDO)

In this section the cost function is solved for minimum with an extra constraint of β value of ≥ 3. The figure(7(a)&7(b)) shows the optimum values of the variables solved using analysis method and solved using surrogate modeling.

Optimization method = Sequential Quadratic Programming

Objective = Minimize Cost

	Start	End [198] (5.0.0.2)	Low	High
Inputs				
rho	1575	1450	1450	1700
t1	0.015	0.01516	0.01	0.025
t2	0.015	0.01685	0.01	0.025
Outputs				
absTZmax	3.23138	2.97429		3.2
Cost	20861.79	19383.37308		
FORMH{absTZm...	-0.4211	3.02435	3	
GOAL	20861.79	19383.37308		

(a)

Optimization method = Sequential Quadratic Programming

Objective = Minimize Cost

	Start	End [385] (9.0.0.2)	Low	High
Inputs				
rho	1575	1450	1450	1700
t1	0.015	0.016	0.01	0.025
t2	0.015	0.02041	0.01	0.025
Outputs				
absTZmax	3.23138	2.97956		3.2
Cost	21087.90851	19358.79696		
FORMH{absTZm...	-0.4211	3.02879	3	
GOAL	21087.90851	19358.79696		

(b)

Figure 7: Optimum value after RBDO Design using (a):Analysis (b):Surrogate model

We see that both models yield reasonably same values. The max *absTzmax* for both models are almost same sice we already know that surrogate model overestimates the cost the thickness found by surrogate model is higher than with analysis.

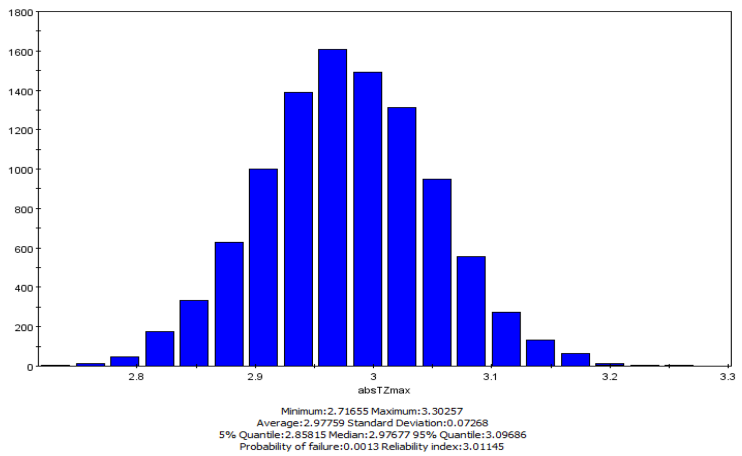


Figure 8: Histogram of the results from Monte Carlo simulation

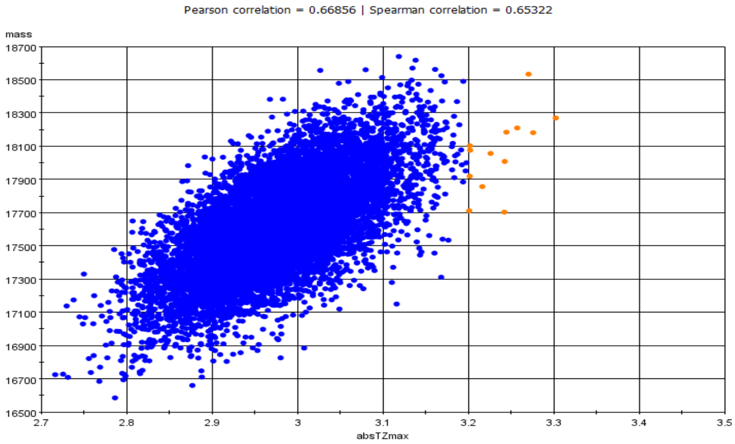


Figure 9: scatter of the results from Monte Carlo simulation

Monte Carlo simulation is run using the the surrogate model for 10000 iterations and the histogram and scatterplot of the results are shown in figure(8&9) it clearly shows that the output $absTzmax$ follows a normal distribution. This may be due to central mean theorem. It is also found that the mean of the distribution is at 2.97759 which is close to the mean found by the above method.