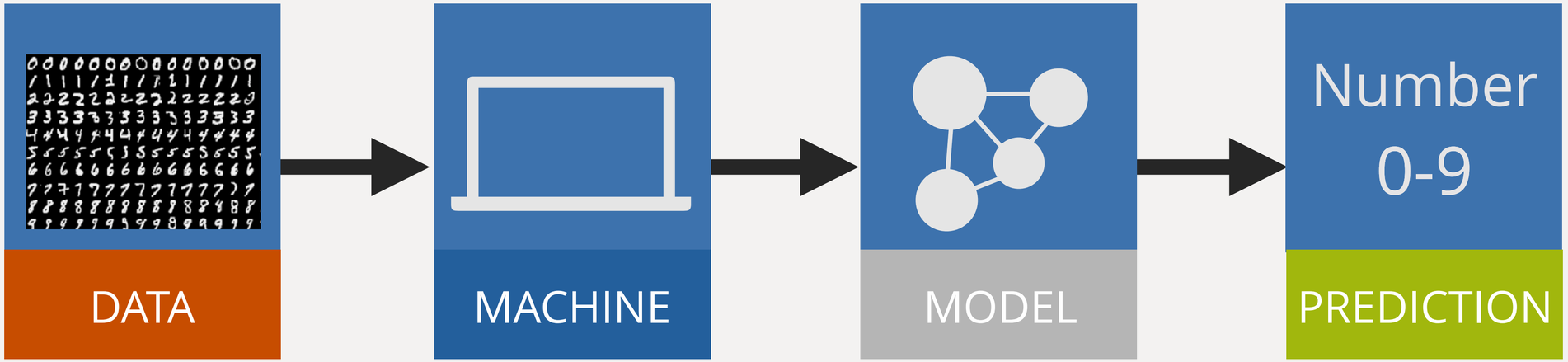
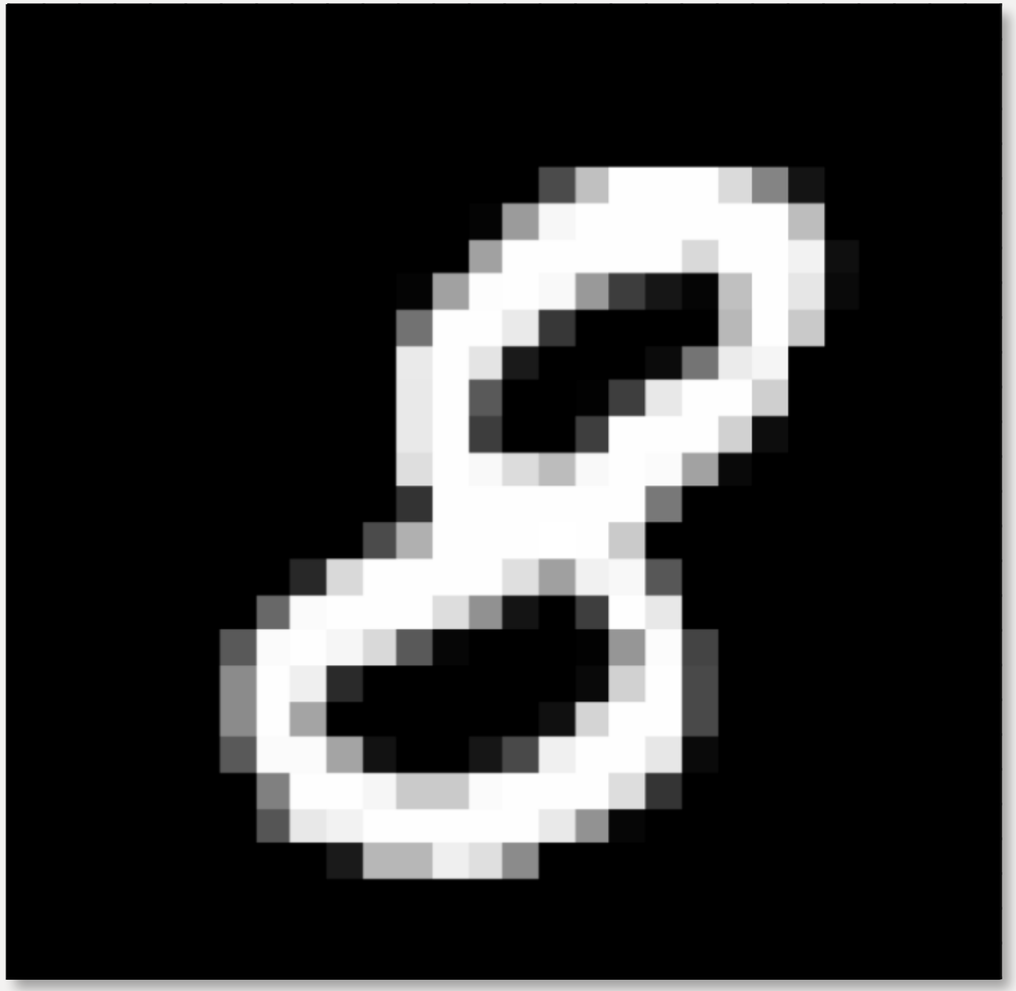
Interpretation of Logistic Regression

MNIST Data Set

Data Source: MNIST Dataset by LeCun et al. (1999) / CC-by-SA 3.0

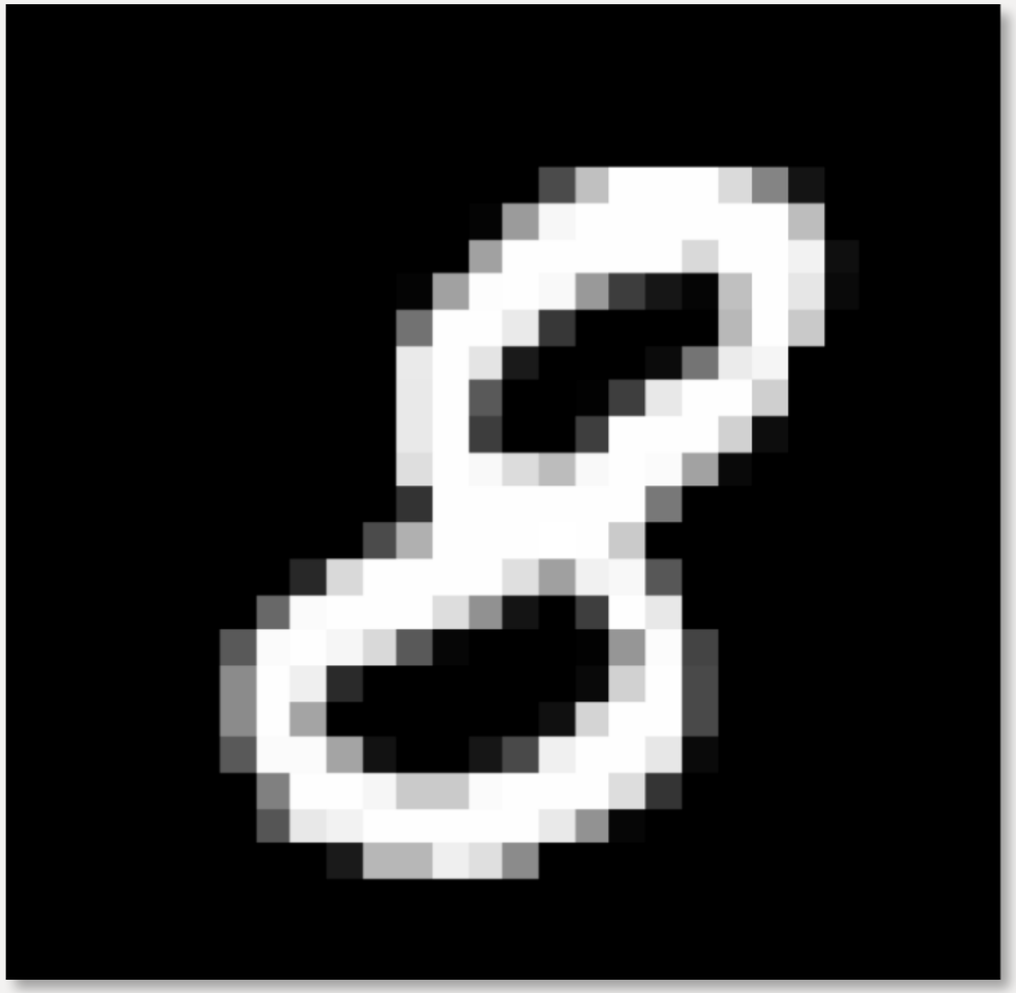

DATA
MACHINE
MODEL
Number
0-9
PREDICTION

Images are Encoded as Numbers



Data Source: MNIST Dataset by LeCun et al. (1999) / CC-by-SA 3.0

Images are Encoded as Numbers



→

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0	0	0	0	9	210	253	163	5	19	49	130	244	253	251	137	4	0	0	0	0	0	0	0	0	0	0	0
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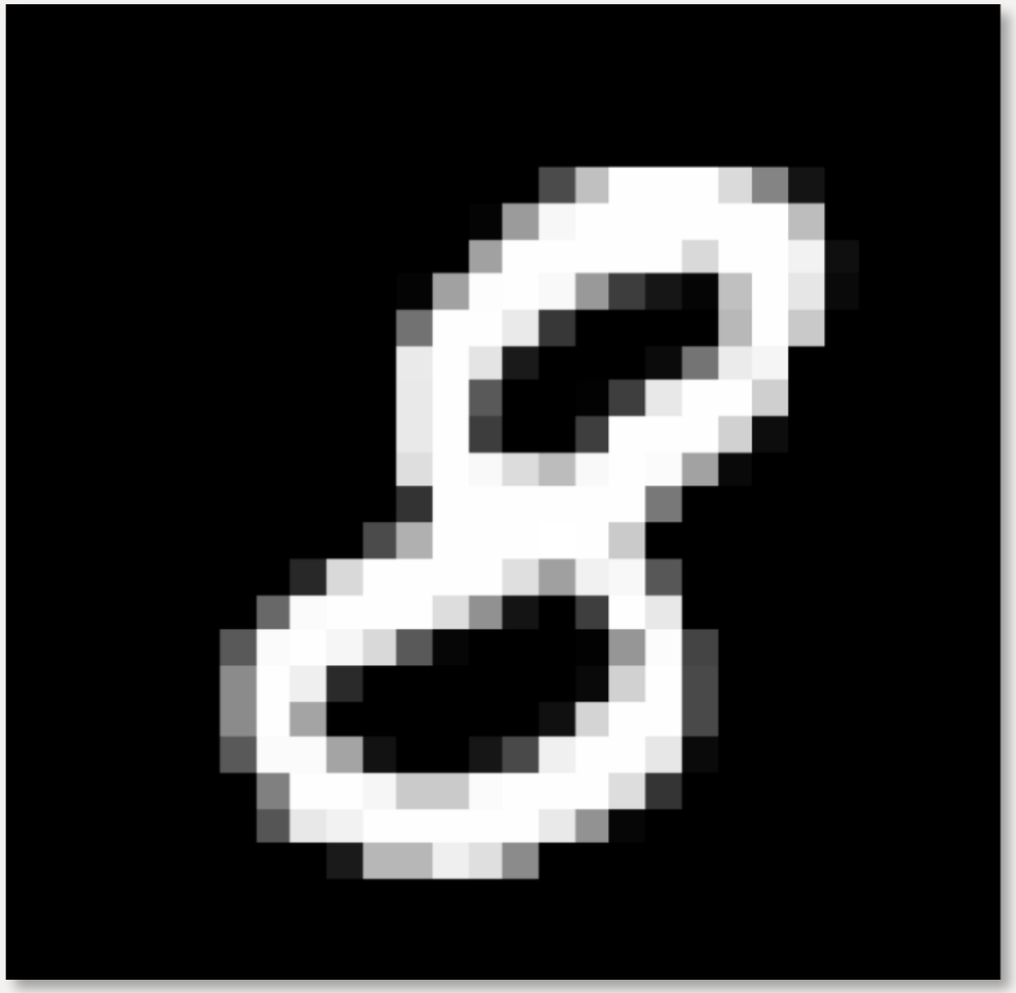
Data Source: MNIST Dataset by LeCun et al. (1999) / CC-by-SA 3.0



Images are Encoded as Numbers

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0	0	0	0	0	0	0	0	0	213	253	50	0	18	123	198	253	254	253	247	85	0	0	0	0	0	0	0
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Data Source: MNIST Dataset by LeCun et al. (1999) / CC-by-SA 3.0

Images are Encoded as Numbers

$X =$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	9	212	253	253	224	58	19	0	0	16	139	247	247	71	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	69	241	227	136	200	253	253	253	254	192	34	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	175	254	256	254	254	254	149	59	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	9	210	253	163	5	19	49	130	244	253	251	137	4	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	13	229	253	254	192	253	253	253	254	253	137	0	0	0	0	0	0	0	0	0	0	0	0	0
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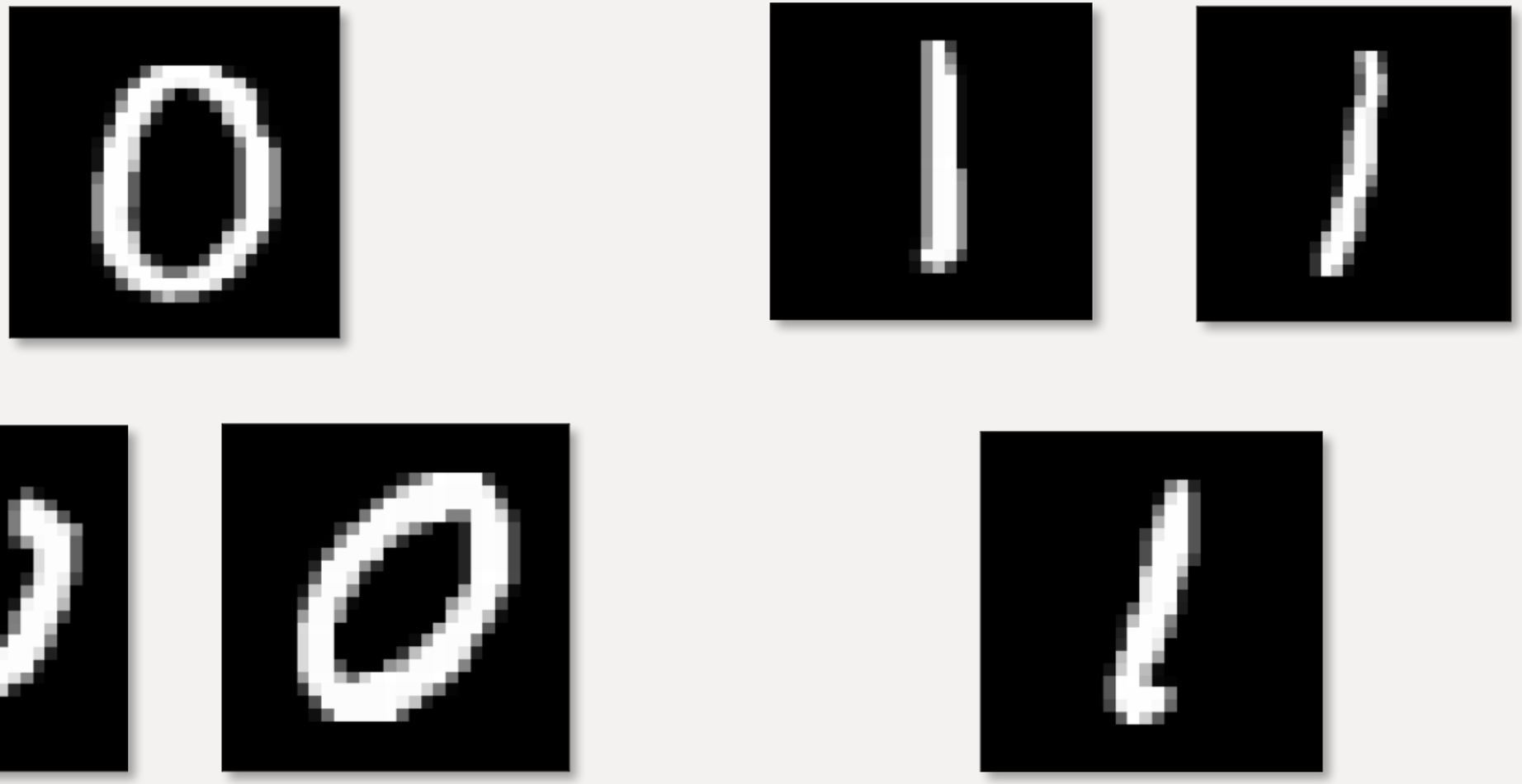
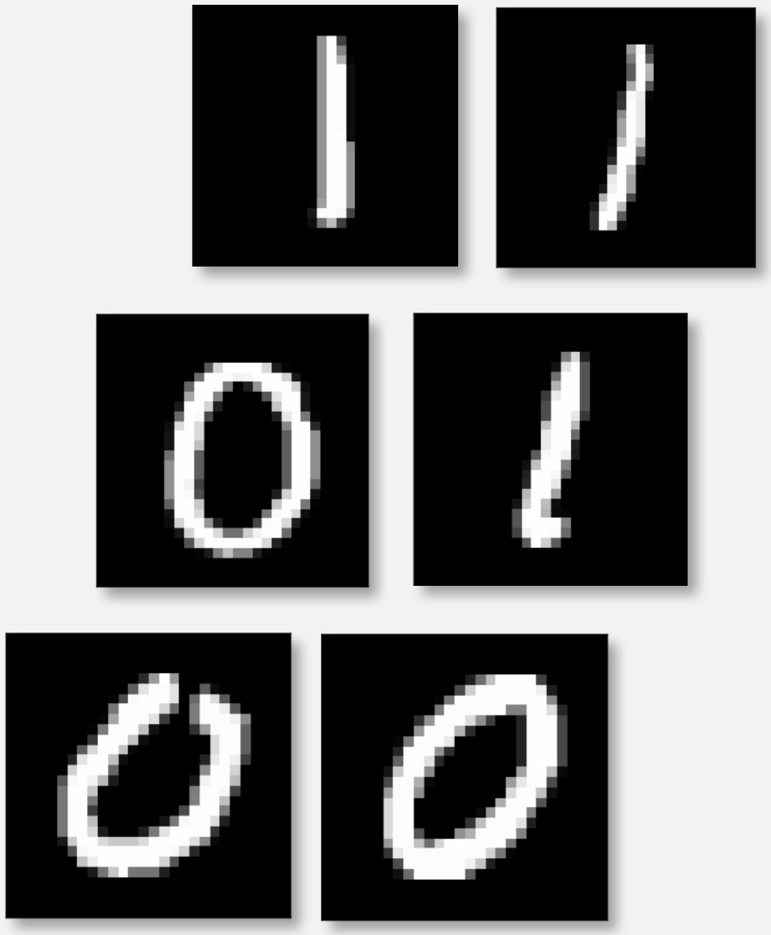
Output is Binary



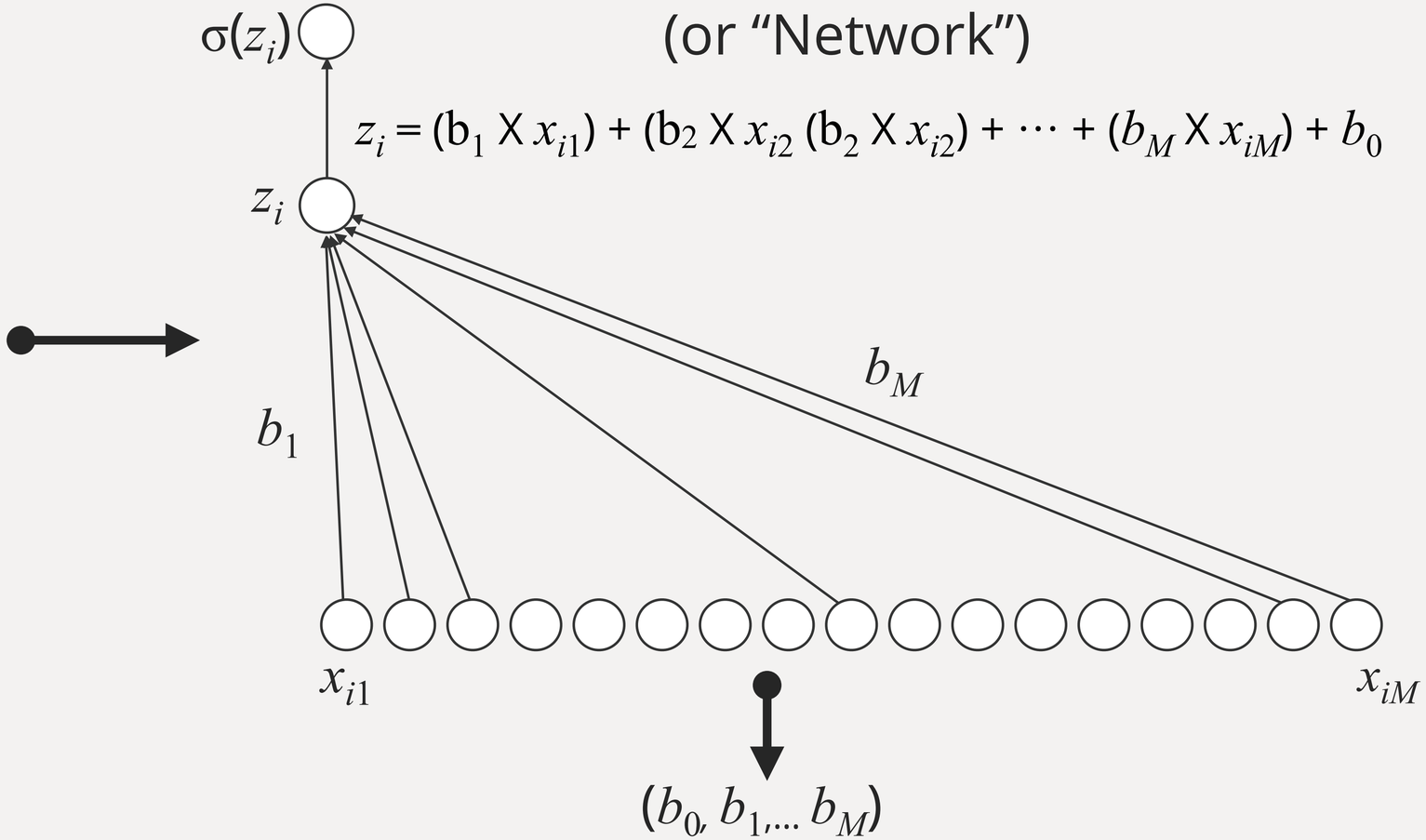
Data Source: MNIST Dataset by LeCun et al. (1999) / CC-by-SA 3.0

Learning on MNIST

Training Set

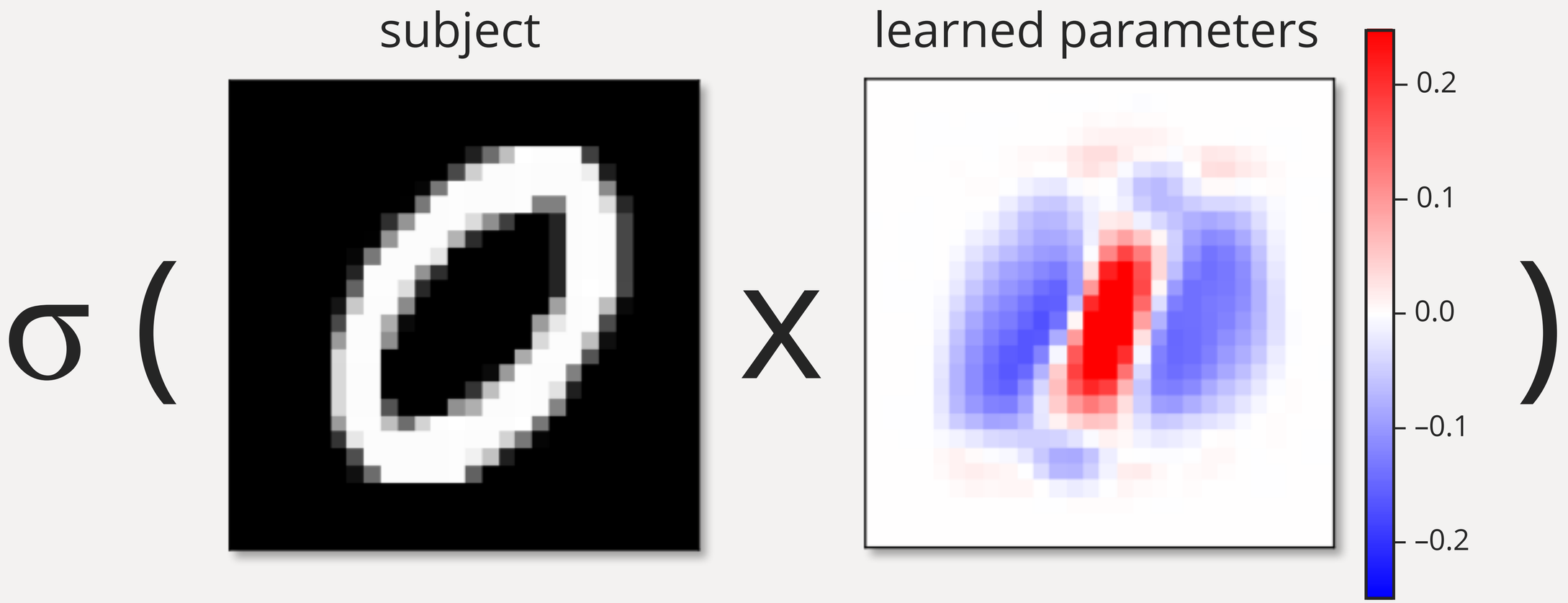


Logistic Regression Model
(or “Network”)



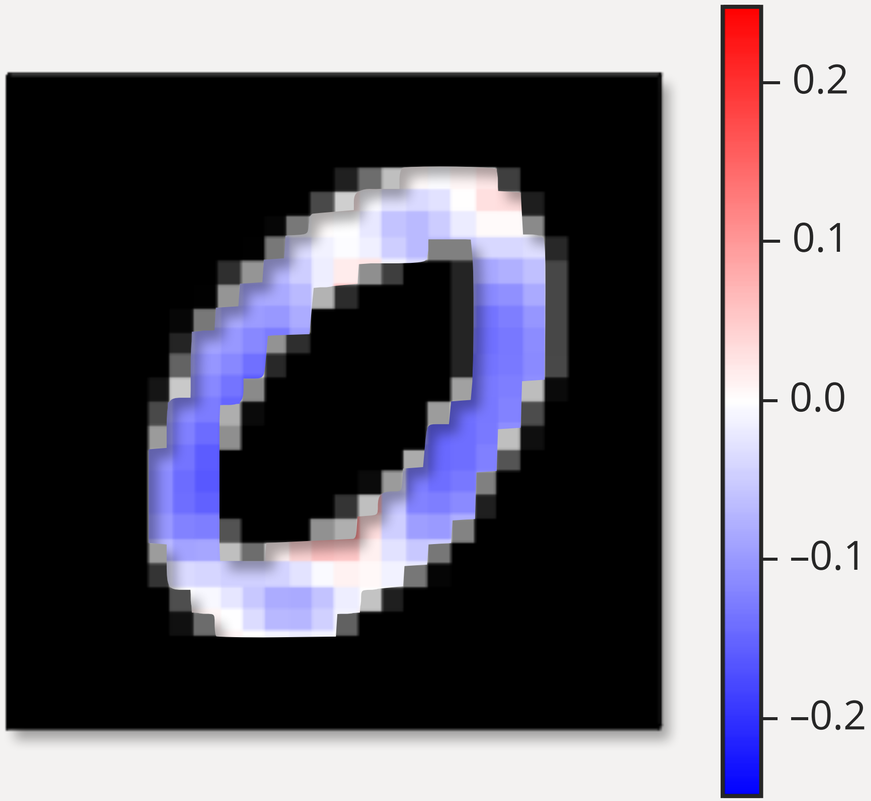
Learned Parameters

Zooming in on 0/1

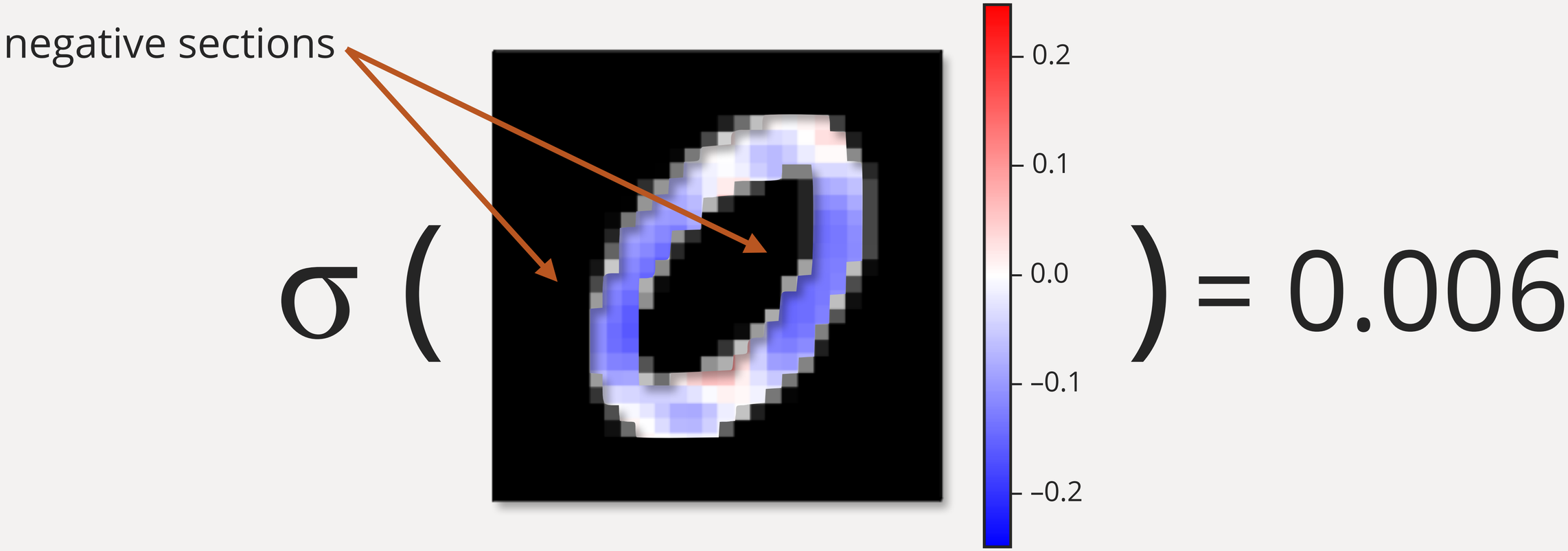


Zooming in on 0/1

$$\sigma\left(\right) = 0.006$$

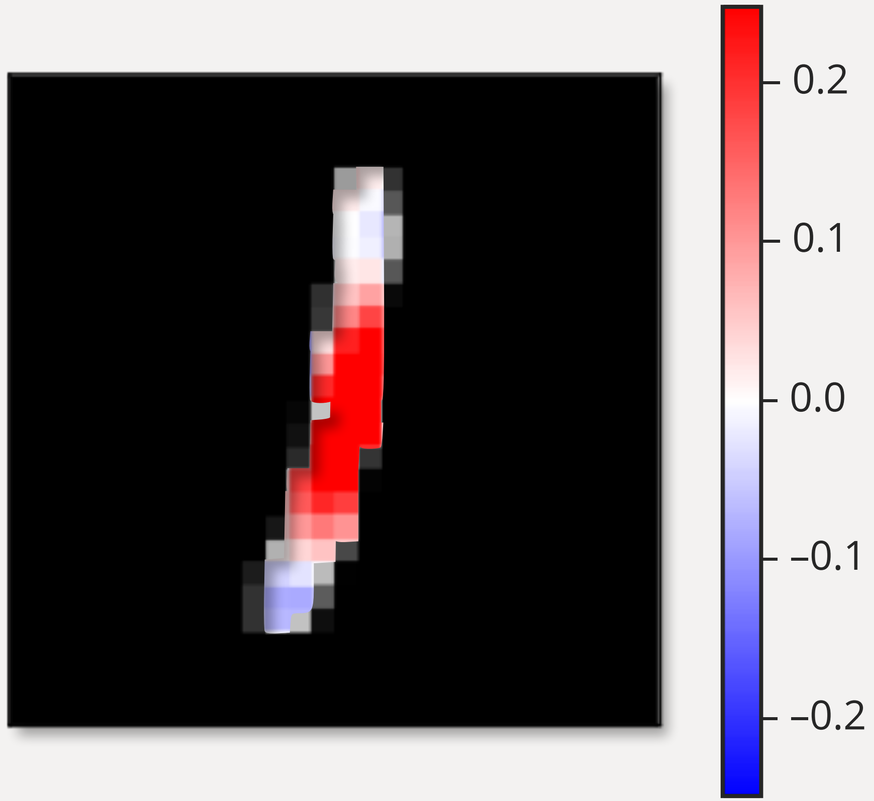


Zooming in on 0/1



Zooming in on 0/1

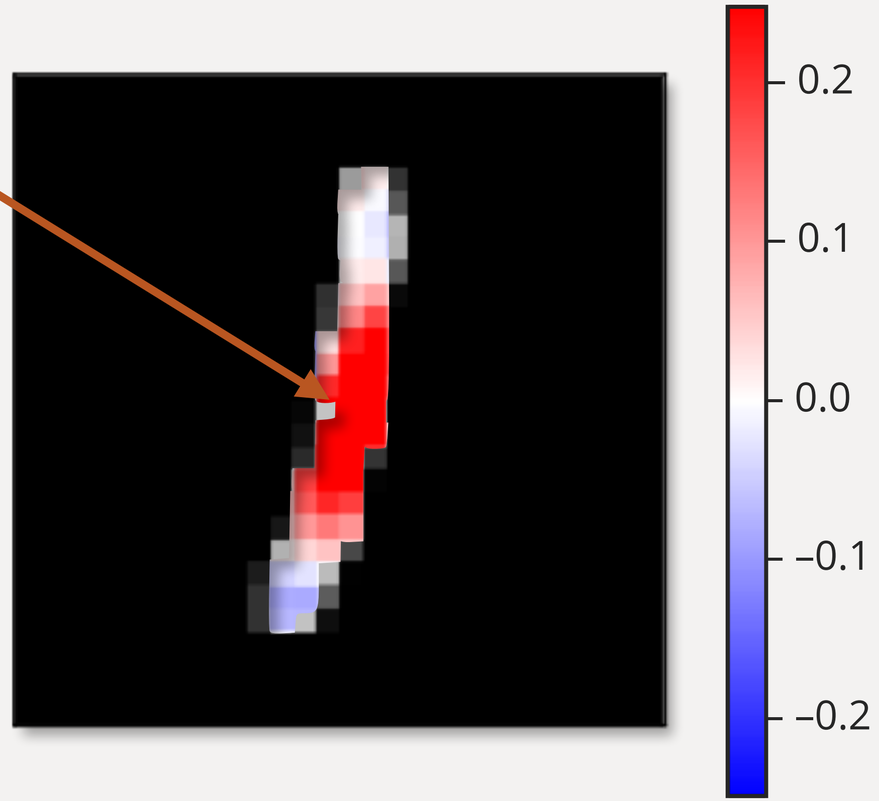
$$\sigma\left(\right) = 0.991$$



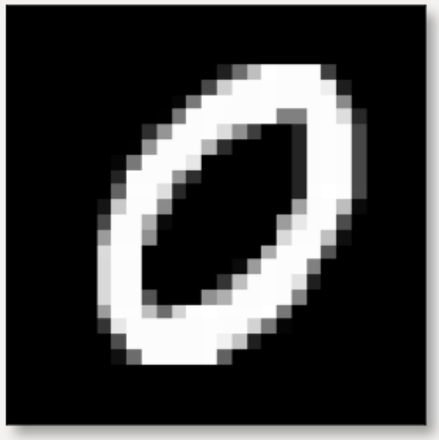
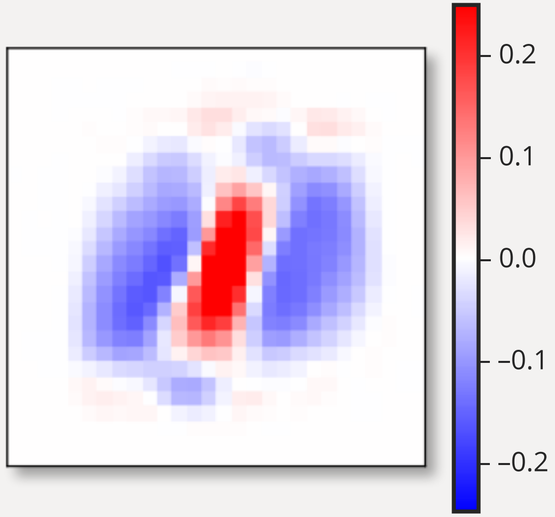
Zooming in on 0/1

positive sections

$$\sigma\left(\right) = 0.991$$

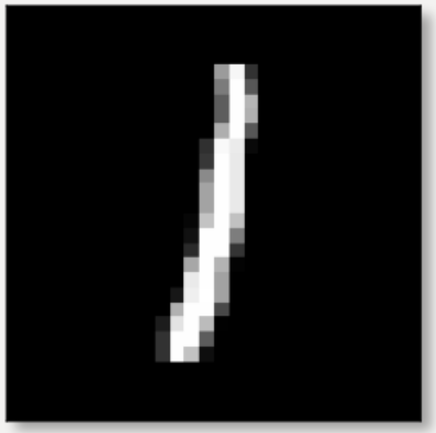
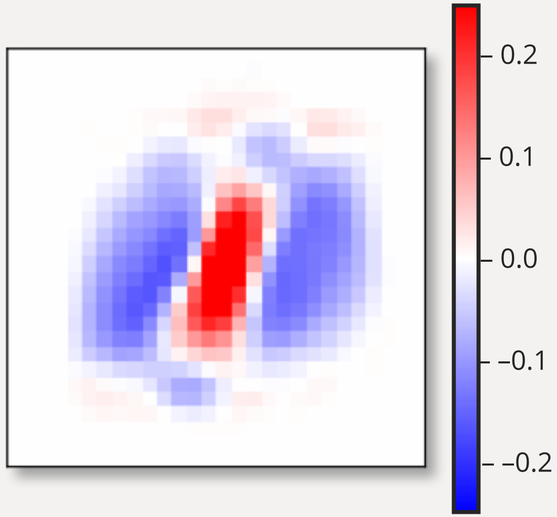


Learned Weights for 0/1

σ ( X 

) = 0.006

(6% chance it is a 1)

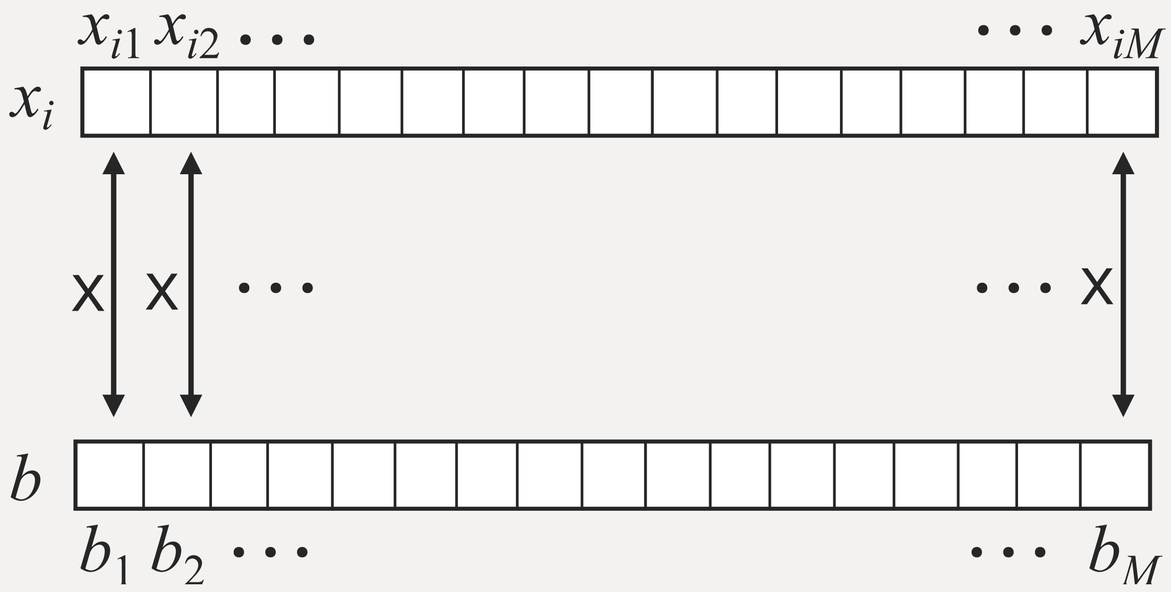
σ ( X 

) = 0.991

(99.1% chance it is a 1)

Interpretation of Logistic Regression

$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$

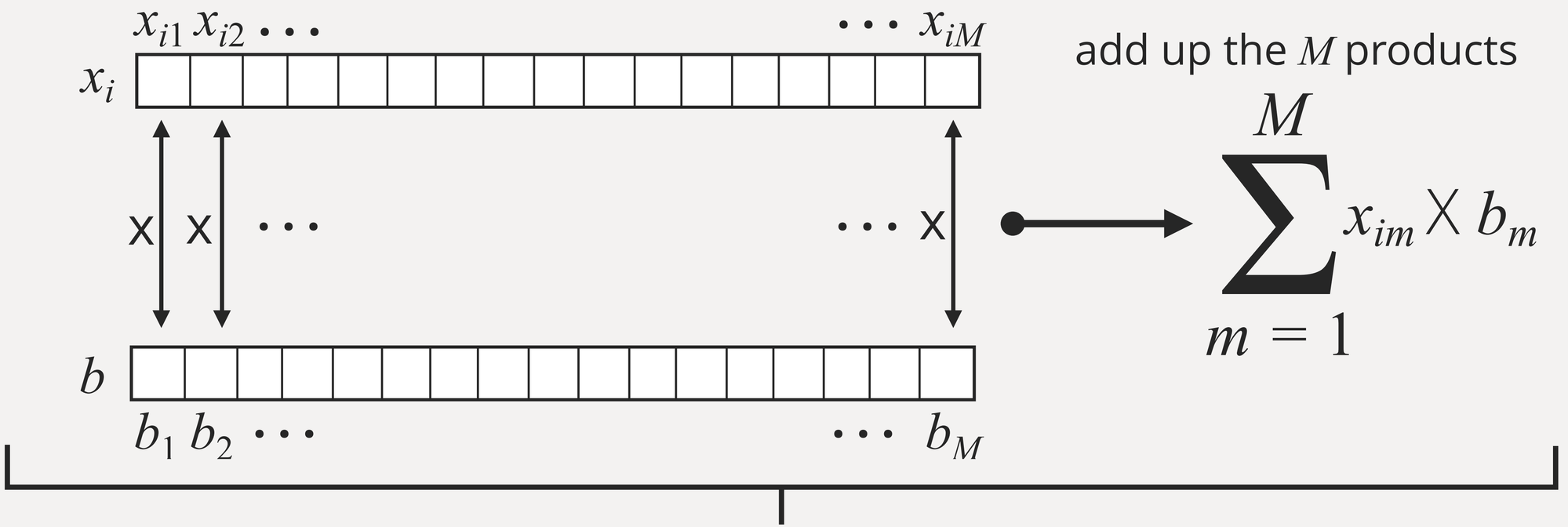


add up the M products

$$\sum_{m=1}^{M} x_{im} \times b_m$$

Interpretation of Logistic Regression

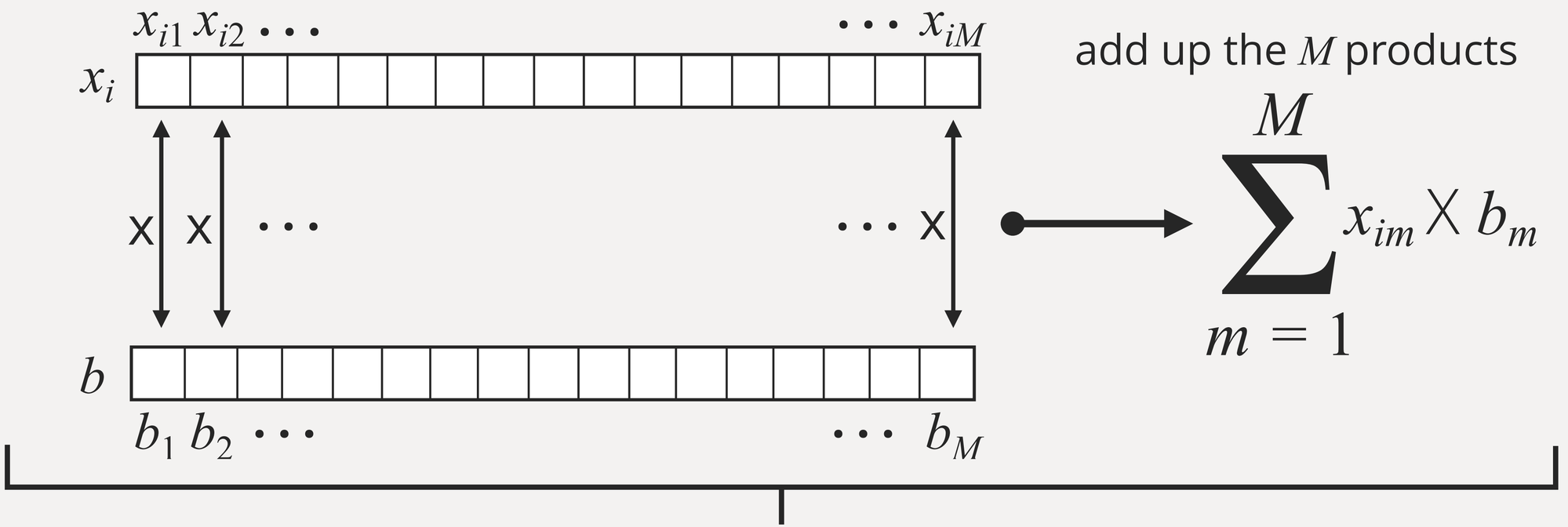
$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$



Compact Notation: $x_i \odot b$

Interpretation of Logistic Regression

$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$



Compact Notation: $x_i \odot b$

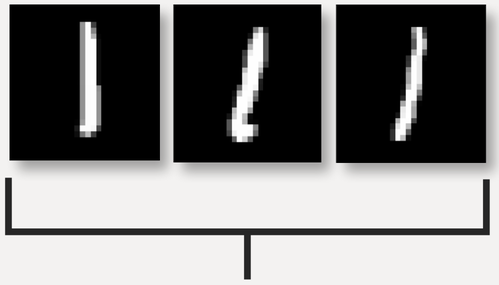
inner product

Interpretation of Model

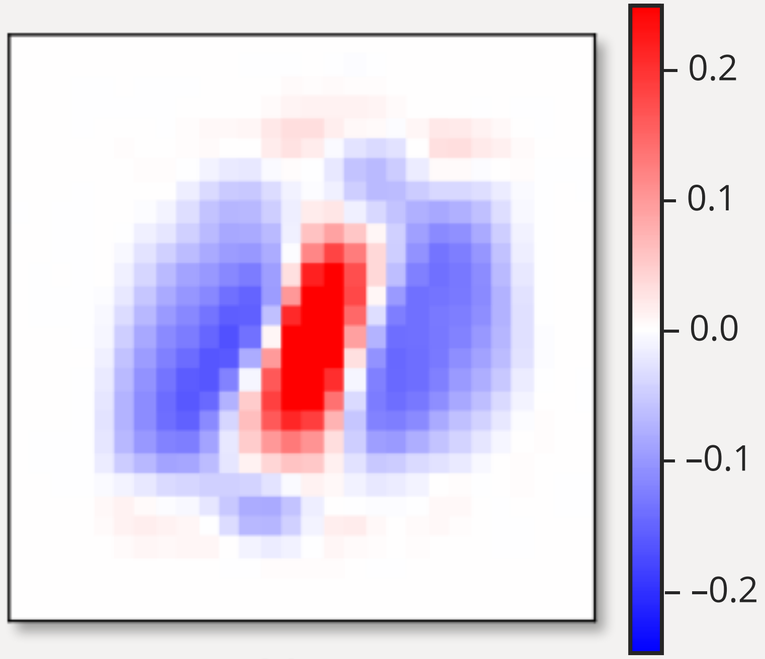
$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$

$$= b_0 + x_i \odot b$$

examples of class y = 0



examples of class y = 1



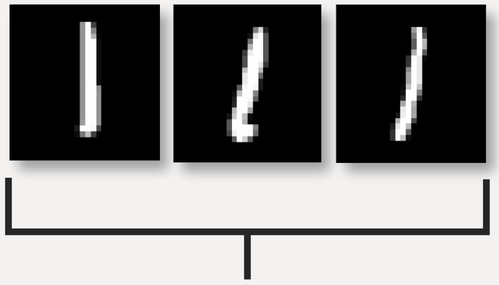
filter

Interpretation of Model

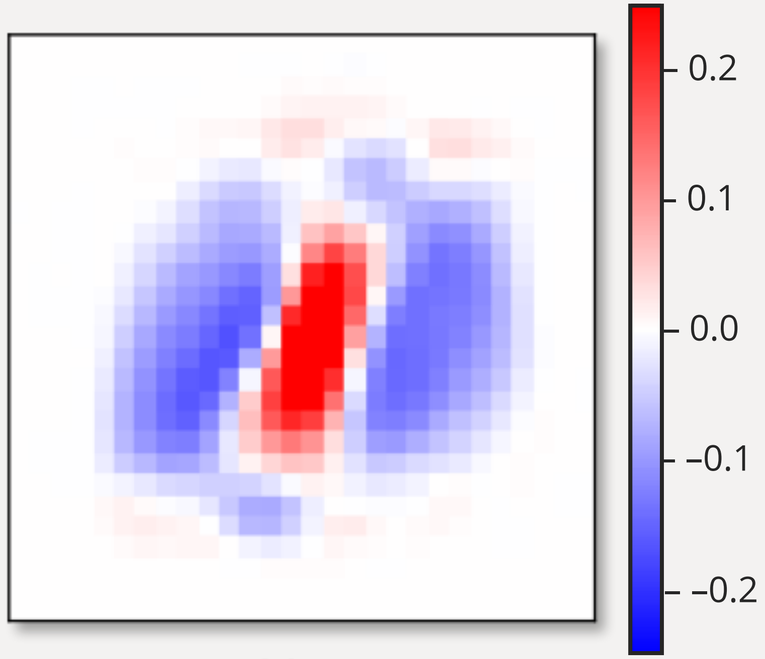
$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$

$$= b_0 + x_i \odot b$$

examples of class y = 0



examples of class y = 1



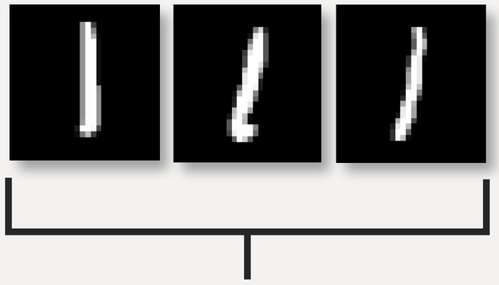
filter

Interpretation of Model

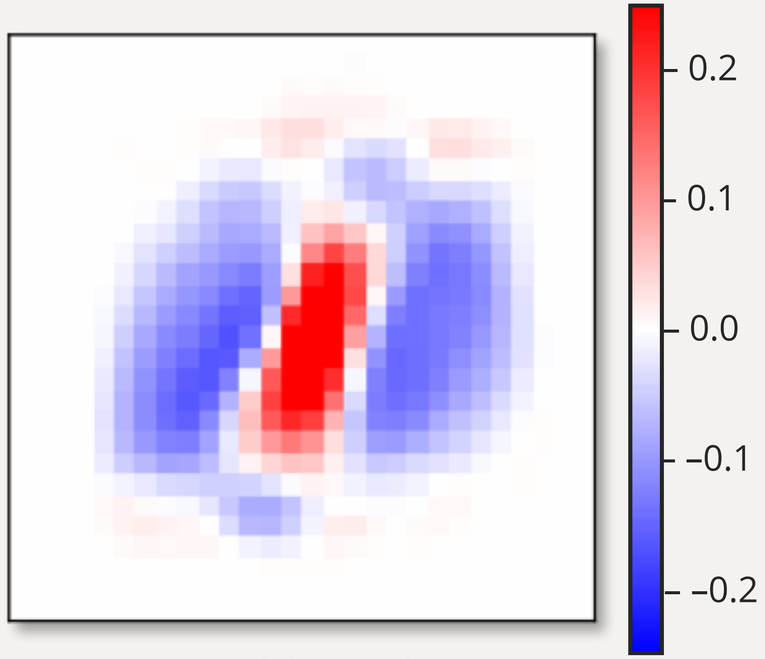
$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$

$$= b_0 + x_i \odot b$$

examples of class y = 0



examples of class y = 1

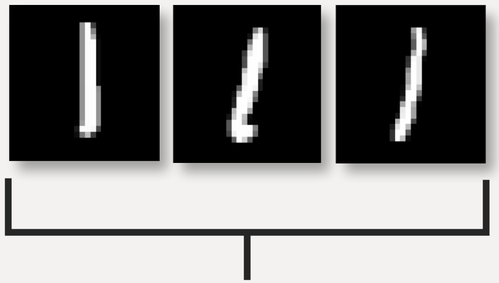


filter b

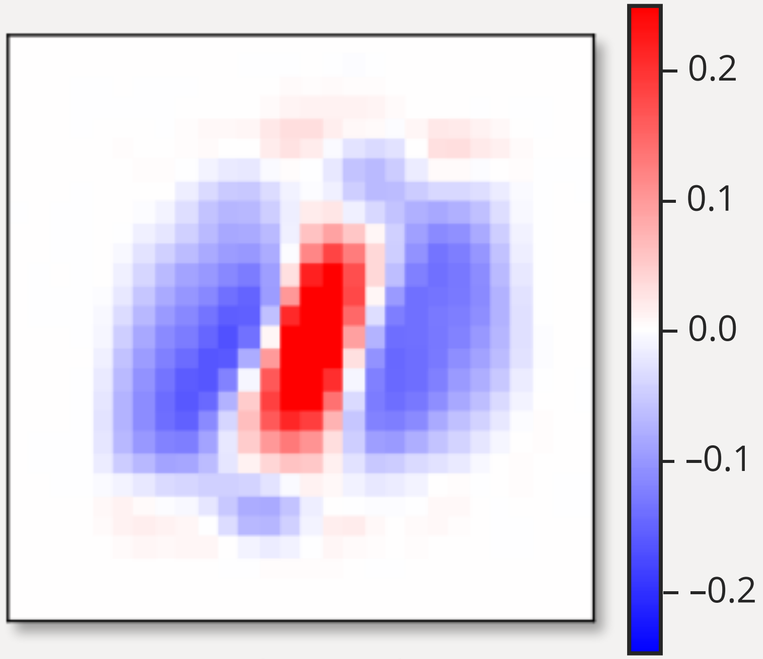
Interpretation of Model

$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$
$$= b_0 + x_i \odot b$$

examples of class $y = 0$



examples of class $y = 1$



filter b

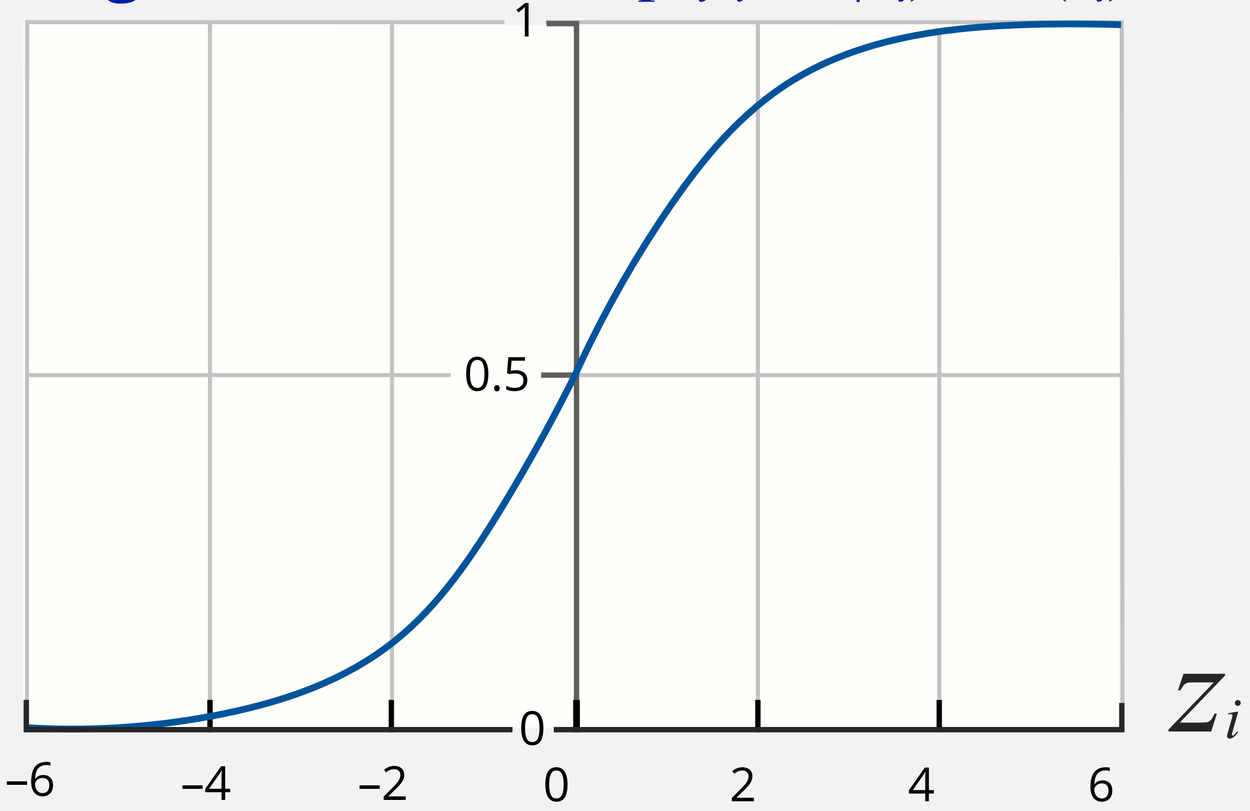
Outcome of y

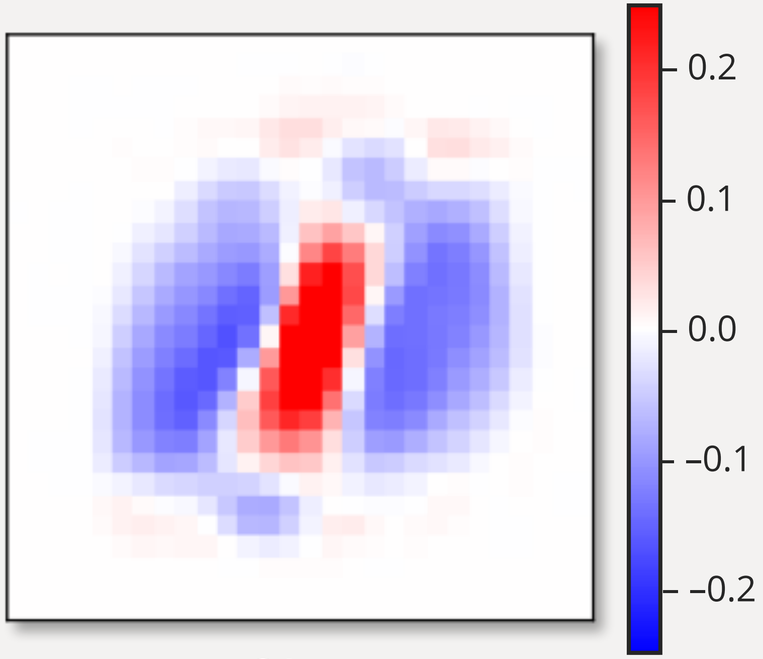
- Strong match between filter and data indicates $p(y_i = 1|x_i)$ is high
- Low match between filter and data indicates $p(y_i = 1|x_i)$ is low

Interpretation of Logistic Regression

$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$

Sigmoid Function $p(y_i = 1|x_i) = \sigma(z_i)$





filter b

Credits

MNIST Dataset of Handwritten Digits (Images)

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