

# ADA

# Assignment

Q:1 Derive T.C. for  $\text{int} i=1; j \leq n; i++$

for  $\text{int } j=1; j \leq i; j++$

$\text{int } k=1; k \leq j; k++$

Value of  $i$  increases by 1  
Value of  $j$  increases by 1

$$\begin{aligned} T.C. &= O(n \cdot n) \\ &\approx O(n^2) \end{aligned}$$

Q:2 Explain Space complexity in

-ii Recursive Fibonacci

The recursive fibonacci formula is defined as:

$\text{int fib(int } n\})$

if  $n \leq 1$  return  $n$

return  $\text{fib}(n-1) + \text{fib}(n-2)$

Space Complexity

Due to recursive calls, the function creates a call stack.

The max depth of recursive call stack is  $n$

$$\Rightarrow S.C. \approx O(n)$$

### - (iii) Matrix chain Multiplication (MCM) :

Definition:

We try to find the min # of multiplications needed to multiply a chain of matrices. The DP sol<sup>n</sup> uses a table to store intermediate results.

S.C.

If there are n matrices, we

typical use:

A 2D DP table  $dpc[i][j]$  where

$dpc[i][j]$  stores the min cost of multiplying matrices i to j.

S.C.  $\Theta(n^2)$

→ Optimaliy, a separate table cost to store subproblem positions for parenthesization.

Q: 3 Linear Search overview:

Linear search checks each element one by one from the beginning until finds the target.

Best case:

- When the element is at final position
- Best case comparisons: 1

- Worst case:

When the element is at the last position  
or not present at all

worst case Comparisons: 100

- Average case:

Two scenarios:

1. if element is present, the average  
number of comparisons

$$1 + \frac{2 + 3 + \dots + 100}{100} = 50.5$$

2. If element is not present, 100 comparisons need.

If we assume 50-50 probability of being  
present or absent

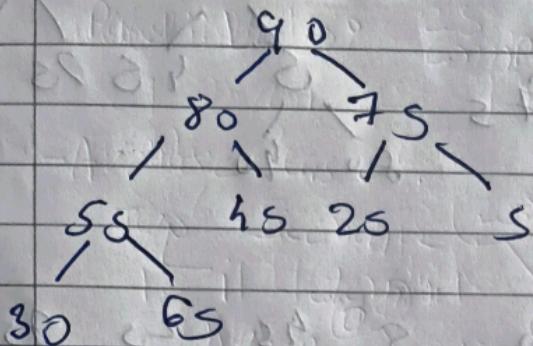
$$\text{Avg. Comparisons} = 0.5 \cdot 50.5 + 0.5 \cdot 100 = 75.25$$

case	Comparisons
Best	1
Worst	100
Avg. case (Present)	50.5
Avg. case (Absent + equally)	75.25

Ort Sort [55, 25, 90, 65, 45, 75, 5, 30, 80]

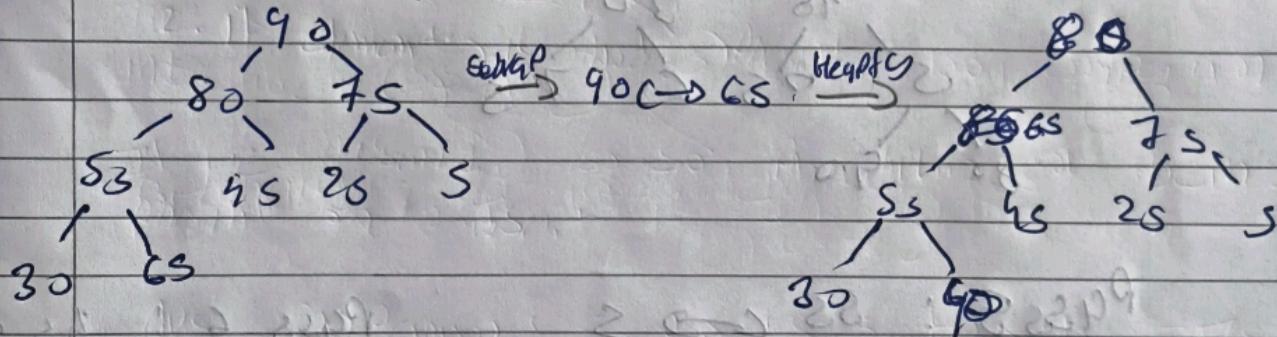
II Heap sort

Step 1: Build Max Heap



Step 2: Heap sort

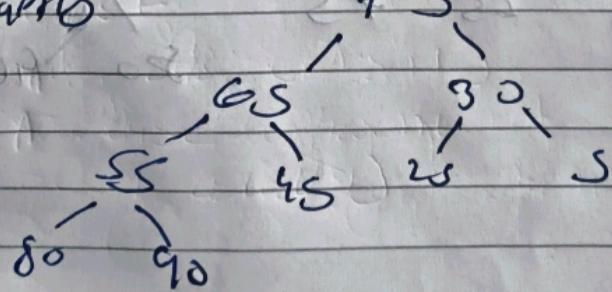
Pass 1: 90 80 75 55 45 25 5 30 65



Pass 2: 80 65 75 55 45 25 5 30 90

swap 80  $\leftrightarrow$  30

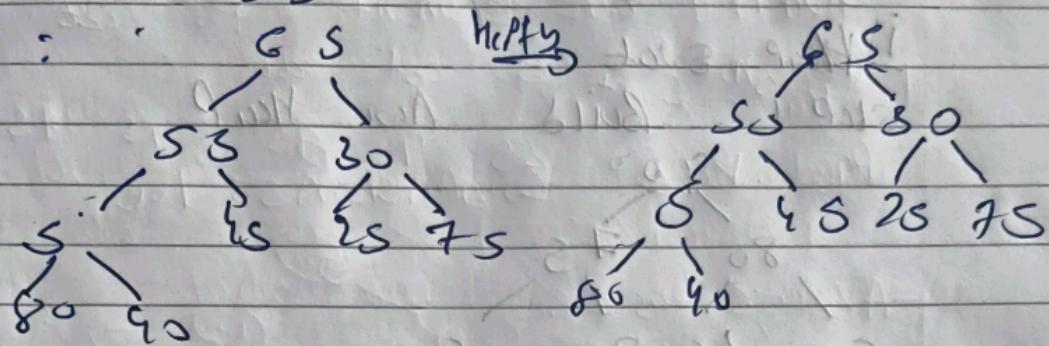
heaps



Pass 3 : 7S GS 30 SS 4S 2S 5S 8S 90

swap : 4S  $\leftrightarrow$  S

Hefty : GS Hefty

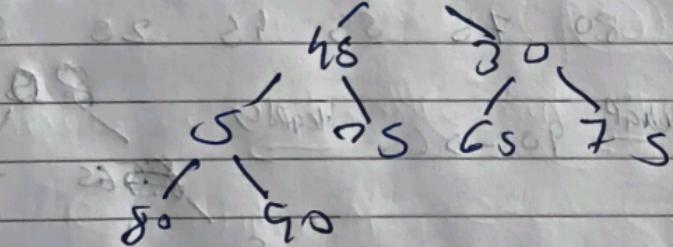


Pass 4 : GS SS 30 S 4S 2S 7S 8S 90

swap : GS  $\leftrightarrow$  2S

Hefty:

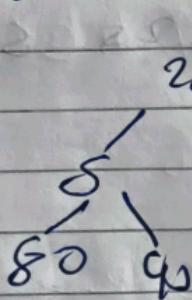
SS



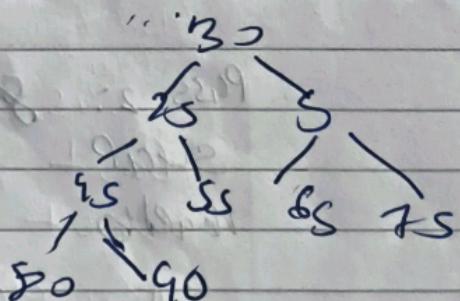
Pass 5 : SS  $\leftrightarrow$  S

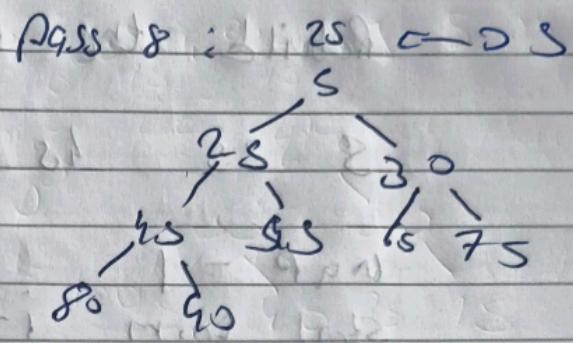
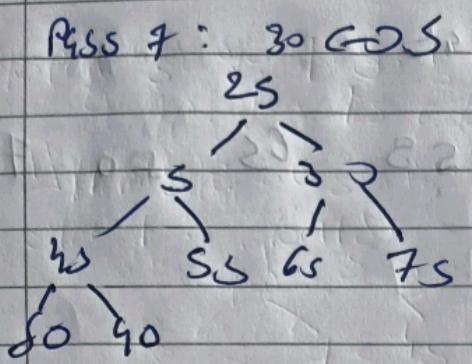
Hefty:

SS



Pass 6 : 4S  $\leftrightarrow$  S





Sorted stages: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90

### 2) Shell Sort

SS, 2S, 4S, 6S, 8S, 10S, 12S, 14S, 16S, 18S, 20S

$\rightarrow \text{Dgap} = 4$

- compare SS & 4S  $\rightarrow$  no change
- compare 2S & 7S  $\rightarrow$  no change
- compare 9S & S  $\rightarrow$  swap
- SS, 2S, 5, 30, 4S, 7S, 90, 6S, 8S
- compare 6S & 3S  $\rightarrow$  swap
- SS, 2S, 5, 30, 4S, 7S, 90, 6S, 8S
- compare 4S & 8S  $\rightarrow$  no change

$\rightarrow \text{Dgap} = 2$

- SS 2S 5 30 4S 7S 90 6S 8S
- Group 1: SS 2S 4S 90 8S
  - Group 2: 5 30 7S 6S

Sort these mini-sets?

Group 1: S 1 S 5S 80 90

Group 2: 2S 30 CS 7S

Rebuild the full array

5 25 45 30 55 65 80 75 90

unsorted  $\rightarrow$  1

start from i = 1

- 25 > 5 → ok

- 45 > 25 → ok

- 30 < 45  $\rightarrow$  Shift  $\rightarrow$  5, 25, 30, 45, 55, 65, 80, 75, 90

- 55 > 45  $\rightarrow$  ok

- 65 > 55  $\rightarrow$  ok

- 80 > 65  $\rightarrow$  ok

- 75 < 80  $\rightarrow$  Shift  $\rightarrow$  5, 25, 30, 45, 55, 65, 75, 80, 90

- 90 > 80  $\rightarrow$  ok

final sorted array

5 25 30 45 55 65 75 80 90

Q: 5 Use Counting Sort to sort:

6, 0, 2, 0, 1, 3, 3, 6, 1, 3, 2

Freq: 0 1 2 3 4 5 6  
2 2 2 3 0 0 2  
 $j = 0$

Pass 1:  $i = 0$

array: 0 0 2 0 1 3 3 6 1 3 2  
 $j = 2$   
 $i = 1$

Pass 2:  $i = 1$

array: 0 0 1 1 1 3 3 6 1 3 2  
 $j = 2$   
 $i = 2$

Pass 3:  $i = 2$

array: 0 0 1 1 2 2 3 6 1 3 2  
 $j = 6$   
 $i = 3$

Pass 4:  $i = 3$

array: 0 0 1 1 2 2 3 3 3 3 2

$j = 9$

$i = 4$

Pass 5:  $i = 4$

array  $\rightarrow$  no change  
 $j = 9$   
 $i = 5$

Pass 6:  $i = 5$

array  $\rightarrow$  no change  
 $j = 9$   
 $i = 6$

Pass 1:  $i=0$

array: 0 0 1 1 2 2 3 3 3 6 6  
 $j=1$   
 $i=7$

Sorted array: 0, 0, 1, 1, 2, 2, 3, 3, 3, 6, 6

Q: b sort [121, 235, 55, 971, 321, 176]  
using radix sort +

Pass 1: Sort by units place

1  $\rightarrow$  121, 971

25  $\rightarrow$  235, 55

6  $\rightarrow$  176

0, 2, 3, 4, 7, 8, 9  $\rightarrow$  none

Combined: 121 971 235 55, 176 821

Pass 2: Sort by tens place

2  $\rightarrow$  121, 176, 321

3  $\rightarrow$  235

5  $\rightarrow$  55

7  $\rightarrow$  971

121 176 321 235 55, 971

Pass 3: Sort by hundreds place

0  $\rightarrow$  55

1  $\rightarrow$  121, 176

2  $\rightarrow$  235

3  $\rightarrow$  321

9  $\rightarrow$  971

final sorted array

55 121 170 235 321 971

When is Radix sort better than  
comparison sorts?

Condition

All numbers are  
fixed len

You need stable  
sort

You've large arrays  
or small ints

Why

Radix sort in linear  
time

Radix sort preserves  
relative order

Efficient for datasets  
like [0-999] strings

Out for an array of size 1024:

(a) How many recursive calls are made in Binary Search?  
formula :  $\log_2(n) + 1$

$$\# \text{ calls} = \log_2(1024) + 1 \\ = 11$$

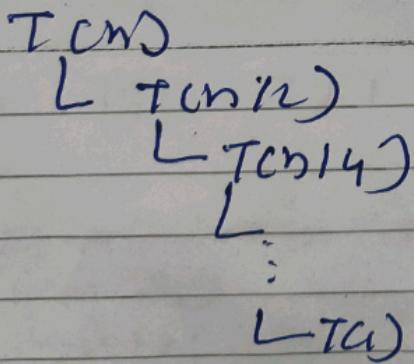
(b) Prove  $T.C. = O(\log n)$

→ At each level to the problem size becomes half.

$$\therefore \text{Total levels} = \log_2 n$$

There is only 1 recursive call, so  
 $T(n) = T(n/2) + O(1)$

Recursion tree :



$$\text{So, } T(n) = O(1) + O(1) + O(1) + \dots + O(1) \\ = O(\log n)$$

Q. 8 find max and min in  $S_1$  and  $P_1$   
 $S_1 = 85, 23, 67, 12, 90, 34, 55$

- Divide & conquer

left  $\rightarrow$   $45, 85, 23, 67$   
left  $\rightarrow$   $45, 85$   
 $\hookrightarrow \max = 85, \min = 45$  (1 comparison)  
right  $\rightarrow 23, 67$   
 $\hookrightarrow \max = 67, \min = 23$  (1 comparison)  
 $\max(85, 67) \Rightarrow \max = 85$  (2 comparisons)  
 $\min(45, 23) \Rightarrow \min = 23$

right  $\rightarrow 12, 90, 34, 55$   
left  $\rightarrow 12, 90$   
 $\hookrightarrow \max = 90, \min = 12$   
right  $\rightarrow 34, 55$   
 $\hookrightarrow \max = 55, \min = 34$   
 $\max(90, 55) \Rightarrow \max = 90$   
 $\min(12, 34) \Rightarrow \min = 12$

$\hookrightarrow \max(90, 85) \Rightarrow \max = 90$   
 $\min(34, 12) \Rightarrow \min = 12$

Each pair  $\Rightarrow 1$  comparison

Each merge  $\Rightarrow 2$  comparisons

$$\text{Total} = 4 \cdot 1 + 3 \cdot 2$$

$$= 10 \text{ comparisons}$$

0.4 Sort 12, 25, 5, 31, 10, 66, 90

### Merge Sort

12 25 5 31 10 66 90

→ left: 12 25 5 31 ← right

→ left: 12 25 ← right

left: 12

right: 25 ← right

merge: 12 < 25 = 12 25

→ right: 31 ← right

→ left: 5

right: 31 ← right

merge: 5 < 31 = 5 31

5 ← right GP scanned

merge: 25 < 12 ← right

12 < 31 ← right 5 12

merge: 25 < 31 ← right 12 25 31

→ right: 10 66 90 ← right

→ left: 10 66

left: 10 ← right 66 90

right: 66 ← right 90

merge: 10 66

right: 66 ← right 10

merge: 10 < 66 ← right 10

66 < 90 10 66 90

→ merge: 5 < 10 + 5, 10 < 12 + 5 10

12 < 66 + 5 10 12

25 < 66 5 10 12 25

31 < 66 5 10 12 25 31 66 90

Q:- Multiply two  $2 \times 2$  matrix using divide and conquer method.

→ Let two matrices

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

and result  $C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$

→ Divide & Conquer

We treat  $2 \times 2$  matrices as blocks of 4 submatrices

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j}$$

$$\text{so, } c_{11} = a_{11}b_{11} + a_{12}b_{21} \quad c_{21} = a_{21}b_{11} + a_{22}b_{21}$$

$$c_{12} = a_{11}b_{12} + a_{12}b_{22} \quad c_{22} = a_{21}b_{12} + a_{22}b_{22}$$

→ Matrix multiply ( $A, B$ )

If size of  $A = 1 \times 1$ :

return  $[A \times B] \times [B]$

else

divide  $A$  into four parts:  $A_{11}, A_{12}, A_{21}, A_{22}$

divide  $B$  into four parts:  $B_{11}, B_{12}, B_{21}, B_{22}$

$c_{11} = \text{matrix multiply}(A_{11}, B_{11}) + \text{matrix multiply}(A_{12}, B_{21})$

$c_{12} = \text{matrix multiply}(A_{11}, B_{12}) + \text{matrix multiply}(A_{12}, B_{22})$

$c_{21} = \text{matrix multiply}(A_{21}, B_{11}) + \text{matrix multiply}(A_{22}, B_{21})$

$c_{22} = \text{matrix multiply}(A_{21}, B_{12}) + \text{matrix multiply}(A_{22}, B_{22})$

Combine  $c_{11}, c_{12}, c_{21}, c_{22}$  into result matrix  $C$

return  $C$

Q11

Implement recursive carbon function  $a^b$  using divide-and-conquer and analyze time complexity.

Computes  $a^b = a + a + \dots + a$   $b$  times

Recursive  $O(b)$

→ Divide and Conquer

If  $b$  is even:  $a^b = a^{b/2} \times a^{b/2}$

If  $b$  is odd:  $a^b = a \times a^{b/2} \times a^{b/2}$

ALGO:

Power(a, b)

if  $b == 0$

return 1

if  $b == 1$

return a

int res = power(a, b/2)

if  $b/2 == 0$

return complex term

else

return a \* term & term

T.C.  $\rightarrow$  work per level  $= O(1)$

$T(b) = T(b/2) + O(1)$

$\therefore T(b) = O(\log b)$

On a vending machine in your college's electronics lab  
 only accepts coins of denominations £1. Is and 50p.  
 You want to buy component worth £4.  
 $\rightarrow$  Recurrence relation ( $x \geq 1$ ):  
 $dPE[0] = 1$  min ( $dPE[8-1]$ ,  
 $dPE[8-5]$ ) if  $x \geq 5$ ,  
 $dPE[8-10]$  if  $x \geq 10$ .

Base  $dPE[0] = 0$

$$\begin{aligned} dPE[1] &= 1 \text{ term}, dPE[2] = 1 + 1(2+1), dPE[3] = 1 + 2(2+1) \\ dPE[4] &= 1 + 3(4+1), dPE[5] = 1 + 0(5+1) \\ dPE[6] &= 1 + 1(5+1, 1+1), dPE[7] = 1 + 2(5+1, 1+2) \\ dPE[8] &= 1 + 3(5+1, 1+3), dPE[9] = 1 + 9(5+1, 1+4) \\ dPE[10] &= 1 + 8(10+1), dPE[11] = 1 + 1(6+1, 1+1) \\ dPE[12] &= 1 + 2(10+1, 1+2), dPE[13] = 1 + 3(10+1, 8+1) \\ dPE[14] &= 1 + 4(10+1, 1+4) \end{aligned}$$

min coins needed: 5

Combination: £10 + £1 + £1 + £1 + £1

Q13 Explain and apply the principle of optimality

$\hookrightarrow$  Matrix chain multiplication

$\rightarrow$  Given dimensions of matrices  $A_1, A_2, \dots, A_n$  and the minimum numbers of scalar multiplications needed to compute  $A_1 \cdot A_2 \cdot A_3 \cdots A_n$

$\rightarrow$  Applying principle of optimality:

Suppose we slice the chain at position  $k$ :  
 $(A_1 \cdot A_2 \cdot A_3 \cdots A_k) (A_{k+1} \cdots A_n)$

- To get optimal solution for the whole chair
- The augmentation of let  $(A_1, \dots, A_n)$  must also be optimal
- The same is for downside ( $A_{n+1}, \dots, A_m$ ) also optimal
- Recursions  $M(i, j) = \min_{k=i}^{j-1} M(i, k) + M(k+1, j) + p_i - p_{j+1}$
- $$M(i, j) = \min_{k=i}^{j-1} C(i, k) + M(k+1, j) + p_i - p_{j+1}$$

### 2) Knapsack Problem

Given items with widths  $w_i$  & values  $v_i$ ,  
maximize total value in  $\leq k$  knapsack of  
capacity  $w$ .

→ Applying principles of optimality

At each step of item  $i$

either include item  $i$ ,

or exclude item  $i$ .

If the overall soln is optimal:

If  $i^{\text{th}}$  is included, then the remaining items

must form capacity  $w - w_i$

If  $i^{\text{th}}$  is excluded the remain items

form an optimal soln is capacity  $w$ .

Recurrence relation:

$$k(\text{ch}, w) = \begin{cases} 0 & i < 0 \\ k(\text{ch}-1, w) & w_i > w \\ \max(v_i + k(\text{ch}-1, w-w_i), k(\text{ch}-1, w)) & \text{otherwise} \end{cases}$$

Q: Find  $C(10, 6)$ , using  
1) Recursion

$$C(n, r) = \begin{cases} 1 & \text{if } n=0 \text{ or } r=0 \\ C(n-1, r-1) + C(n-1, r) & \text{otherwise} \end{cases}$$

so for  $C(10, 6)$

$$C(10, 6) = C(9, 5) + C(9, 6)$$

$$C(9, 5) = C(8, 4) + C(8, 5)$$

$$C(8, 4) = C(7, 3) + C(7, 4)$$

⋮

$$\text{base case } C(10, 0) = 1 \text{ and } C(10, 10) = 1$$

final evaluation gives 210.

2) DP

we build Pascal's Triangle w/3s DP										
Row	0	1	2	3	4	5	6	7	8	9
0	1									
1		1								
2	1	2	1							
3	1	3	3	1						
4	1	4	6	4	1					
5	1	5	10	10	5	1				
6	1	6	15	20	15	6	1			
7	1	7	21	35	35	21	7	1		
8	1	8	28	56	70	56	28	8	1	
9	1	9	36	84	126	126	84	36	9	1
10	1	10	45	120	210	252	210	120	45	9

$$\text{for } C(10, 6) = \sum_{i=0}^{10} [10]_3^i = 210$$

Oils	Solve OI	Knapsack Capacity = 10
item	weight	value
1	5	30
2	9	20
3	6	12

dp Table

$$dp[i][w] = \begin{cases} dp[i-1][w] & \text{if weight}[i] > w \\ \max(dp[i-1][w], value[i] + dp[i-1][w - weight[i]]) & \text{otherwise} \end{cases}$$

Table filling:

	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	30	30	30	30	30	30	30	30	30	30
2	0	0	0	0	20	30	30	30	30	30	30
3	0	0	0	0	20	30	30	30	30	30	30

Max value  $\rightarrow 50$

Optimal set: Item 1 and item 2

OIL Matrix dimensions = [40, 20, 30, 0, 10, 30]  
num. of matrix = 4

$$\rightarrow A_1 = 40 \times 20, \quad A_2 = 20 \times 30$$

$$A_3 = 30 \times 10, \quad A_4 = 10 \times 30$$

DP formula

$$M[i][j] = \min_{k \in \{1, 2, 3\}} \{M[i][k] + M[k][j] + P[i-1][k][j]\}$$

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i/j 1 2 3 4

1 0 24000 14000 26000

2 0 0 6000 12000

3 0 0 0 9000

4 0 0 0 0

Chain len = 1

$$MC[1][1] = M[2][2], = M[3][3] = M[4][4] = 1$$

Chain len = 2

$$MC[1][2] = 10 \times 20 \times 30 = 24,000$$

$$M[2][3] = 20 \times 30 \times 10 = 6,000$$

$$M[3][4] = 30 \times 10 \times 30 = 9000$$

Chain len = 3

$$M[1][3] = min C$$

$$(A_1)(A_2 A_3) : 0 + 6000 + 20 \times 20 \times 10 = 14,000$$

$$(A_1 A_2)(A_3) : 24,000 + 0 + 90 \times 30 \times 10 = 36,000$$

$$= 14,000$$

$$M[2][4] = min C$$

$$(A_2)(A_3 A_4) : 0 + 9000 + 20 \times 30 \times 30 = 27,000$$

$$(A_2 A_3)(A_4) : 6000 + 0 + 20 \times 10 \times 30 = 12,000$$

$$\Rightarrow 12,000$$

Chain len = 4

$$MC[1][4] = min C$$

$$(A_1)(A_2 A_3 A_4) : 0 + 12,000 + 40 \times 20 \times 30 = 36,000$$

$$(A_1 A_2)(A_3 A_4) : 24,000 + 9000 + 40 \times 30 \times 30 = 69,000$$

$$(A_1 A_2 A_3)(A_4) : 14,000 + 0 + 40 \times 10 \times 30 = 26,000$$

$$\Rightarrow 26,000$$

Final Ans = 26,000

Optimal Path:  $(A) \times (A_2 \times A_3) \times (A_1)$

If use Ford-Warshall Algo for this array

$$M = \begin{array}{ccc} & A & B \\ A & 0 & 3 & \infty \\ B & \infty & 0 & 1 \\ C & 2 & \infty & 0 \end{array}$$

$$D_0 = M = \begin{bmatrix} 0 & 3 & \infty \\ \infty & 0 & 1 \end{bmatrix}$$

$\rightarrow$  Iteration with  $k = 1 \times 0.5 = 0.5$

$$D_1 = \begin{array}{ccc} & A & B \\ A & 0 & 3 & \infty \\ B & \infty & 0 & 1 \\ C & 2 & 5 & 0 \end{array}$$

$\rightarrow$  Iteration with  $k > B$

$$D_2 = \begin{array}{ccc} & A & B & C \\ A & 0 & 3 & 4 \\ B & \infty & 0 & 1 \\ C & 2 & 5 & 0 \end{array}$$

$\rightarrow$  Iteration with  $C$

$$D_3 = \begin{array}{ccc} & 0 & 3 & 4 \\ 3 & 0 & 1 & 1 \\ 2 & 5 & 0 & 2 \end{array}$$

$\rightarrow$  And this is final answer  $A \times (A_2 \times A_3) \times A_1$

Q18 We are given  
 $X = "ACTTAB"$   
 $Y = "ATXAYB"$

→ DP Table formula

$$DP[i][j] = \begin{cases} 0 & \text{if } i=0 \text{ or } j=0 \\ DP[i-1][j-1] + 1 & \text{if } X[i-1] = Y[j-1] \\ \max(DP[i-1][j], DP[i][j-1]) & \text{otherwise} \end{cases}$$

→ DP Table

	X	A	T	X	A	Y	B
0	0	0	0	0	0	0	0
A	0	1	1	1	1	1	1
C	0	1	1	1	1	1	1
DP	0	1	1	1	1	1	1
T	0	1	2	2	2	2	2
A	0	1	2	2	3	3	3
B	0	1	2	2	3	3	4

$$LCS \text{ len} = DP[GJFT] = 4$$

LCS str = "ATAB"

Assembly	Line 1	Line 2	Line 3	Machine 1
S1	2	1	2	2
S2	8	5	2	2
S3	6	4	3	3

→ There are 2 lines and 3 stations  
we may either:

stay on the same line, or  
switch to the other line

Let

$f_1[i] = \min$  time to reach station  $i$  on line 1

$f_2[j] = \min$  time to reach station  $j$  on line 2

Recurrence:

$$f_1[i] = \min(f_1[i-1] + \text{time}[i], f_2[i-1] + \text{transit}[i-1] + \text{time}[i])$$

$$f_2[j] = \min(f_2[j-1] + \text{time}[j], f_1[j-1] + \text{transit}[j-1] + \text{time}[j])$$

$$\rightarrow \text{initializing } f_1[1] + f_1[1] = 7, f_2[1] = 9$$

station 2:

$$f_1[2] = \min(f_1[1] + 8, f_2[1] + 2 + 8) = 15$$

$$f_2[2] = \min(f_1[1] + 5, f_1[1] + 2 + 5) = 14$$

station 3:

$$f_1[3] = \min(f_1[2] + 6, f_2[2] + 1 + 6) = 21$$

$$f_2[3] = \min(f_2[2] + 4, f_1[2] + 1 + 4) = 18$$

$$\text{final answer} = \min(21, 18)$$

$\Rightarrow 18$

Optimal path: Line 2  $\rightarrow$  Line 2  $\rightarrow$  Line 2

Q10 Compare greedy divide and conquer, dynamic programming  
with real world examples

Dynamic

### 1. Greedy Algo:

Idea: Take the best local choice at each step hoping it leads to global optimum.  
Properties: works only if problem has greedy - choice property and optimal substructure.

- Real-world Example: Active selection

- problem: Select max num of non-overlapping meetings/intervals/activities.

- why greedy work: locally choosing the earliest finish guarantees optimal ans.

Other real-world cases: Huffman coding, Dijkstra's shortest path.

### 2. Divide and Conquer

Idea: Break Problem into smaller subproblems solve recursively, then combine such property: subproblems are independent of each other

Real world Example: Merge Sort

Problem: Sorting a large dataset

Approach: Divide and conquers into halves sort each half until the merge

why work: subproblems are independent and give the result

### 3) Dynamic Programming (DP)

Idea: Break problem into overlapping subproblems,  
store soln' quick recompilation  
Properties: optimal substructure + overlap subproblems

Real-world example: 0/1 knapsack problem  
Problem: choose items to maximize value  
without exceeding weight capacity.

Approach: Build a DP table to reuse  
subresult instead of recalculat

$$T.C. = O(n \cdot w)$$

Other real world uses: MCM, flood-control  
shortest path, LCS

Q.21 Given,

start : 1 3 0 5 8 5

end : 4 5 6 7 9 9

Activity A<sub>1</sub> A<sub>2</sub> A<sub>3</sub> A<sub>4</sub> A<sub>5</sub> A<sub>6</sub>

Step 1: sort by finish time

Activities in sorted order so no need to sort

Step 2: Select activities greedily

Pick first activity: A<sub>1</sub> (1 to 4)

A<sub>2</sub> (3 to 5) → overlap → no take

A<sub>5</sub> (6 to 9) → unique → no pick

A<sub>4</sub> (5 to 7) → pick A<sub>4</sub> (5 to 7)

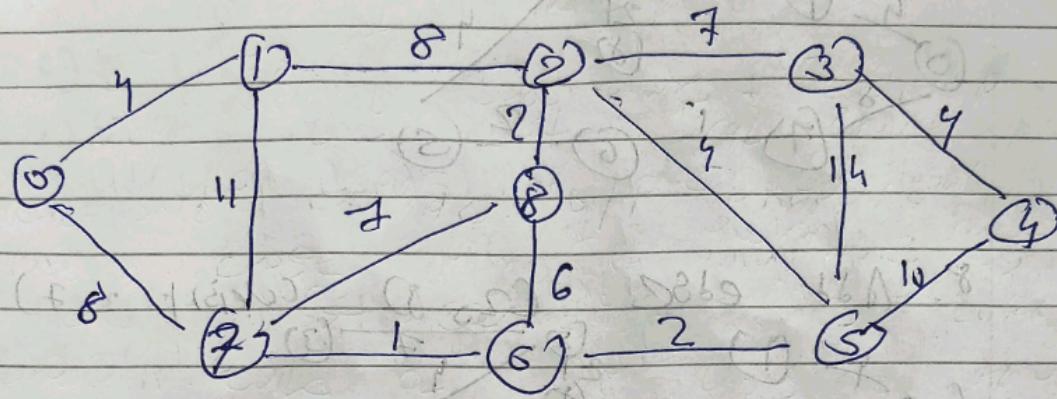
A<sub>3</sub> (8 to 9) → pick A<sub>3</sub> (8 to 9)

A<sub>6</sub> (5 to 9) → overlap → no take

Find Answer

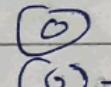
A1C(1-4), A4C5(7), A5C(8-9)

Q.22

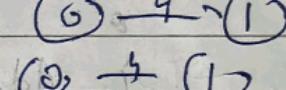


IS Apply Prim's algo to vertex 0.

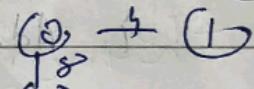
1. start with vertex 0



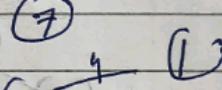
2. Add edge 0-1 (current = 4)



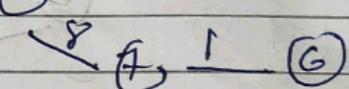
3. Add edge 0-7 (current = 8)



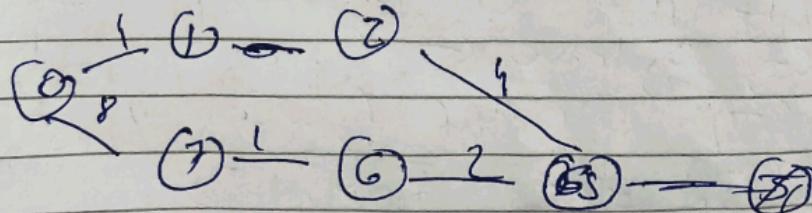
4. Add edge 7-8 (current = 16)



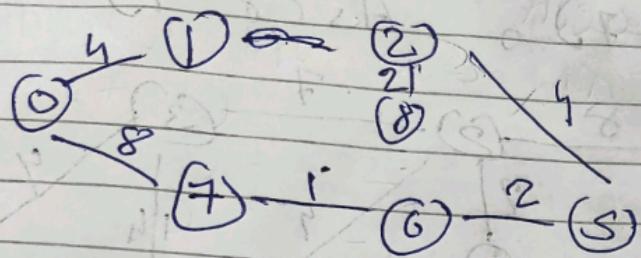
5. Add edge (6-5) (current = 24)



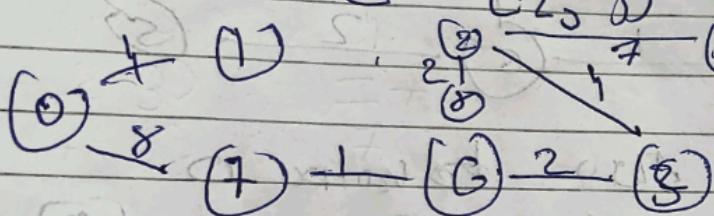
6. Add edge 2-5 (current = 32)



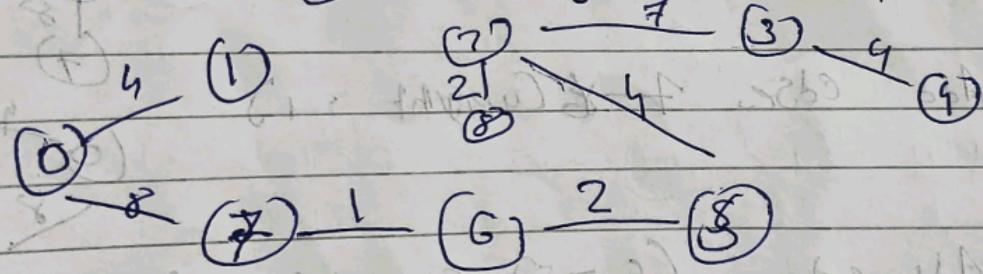
7. Add edge  $(2, 7)$   $(2, 8)$  (weight = 7)



8. Add edge  $(2, 0)$  (weight = 7)



9. Add edge  $(3, 0)$  (weight = 9)



Final MST

$$\begin{aligned} \text{Total weight} &= 4 + 8 + 1 + 2 + 4 + 2 + 7 + 9 \\ &= 37 \end{aligned}$$

→ Kruskal Algo.

Sort edges in ascending cost algo.

$$\begin{aligned} (7, 6) : 1, \quad (6, 5) : 2, \quad (0, 1) : 4, \quad (2, 5) : 4 \\ (8, 7) : 7, \quad (2, 3) : 7, \quad (1, 2) : 8, \quad (0, 7) : 8 \\ (3, 4) : 9, \quad (5, 7) : 10, \quad (1, 7) : 11, \quad (3, 5) : 14 \end{aligned}$$

Add  $(7, 6) : 1$ . Sets:  $\{0\}, \{1\}, \{2\}, \{3\}, \{4\},$   
 $\{5\}, \{6, 7\}, \{8\}$

Add  $(6, 5) : 2$ . Sets:  $\{0\}, \{1\}, \{2\}, \{3\}, \{4\},$   
 $\{5\}, \{6, 7\}, \{8\}$

Add  $(2, 5) : 4$ . Sets:  $\{0\}, \{1\}, \{2, 5\}, \{3\}, \{4\},$   
 $\{6, 7\}, \{8\}$

Add  $(0, 1) : 4$ . Sets:  $\{0, 1\}, \{2, 5\}, \{3\}, \{4\},$   
 $\{6, 7\}, \{8\}$

Add  $(2, 3) : 7$ . Sets:  $\{0, 1\}, \{2, 3\}, \{4, 5, 6, 7, 8\}$

Add  $(1, 2) : 8$ . Sets:  $\{0, 1, 2\}, \{3\}, \{4, 5, 6, 7, 8\}$

Add  $(3, 4) : 9$ . Sets:  $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

$$\begin{aligned} \text{MST Total} &= 1 + 2 + 4 + 4 + 7 + 8 + 9 \\ &= 37 \end{aligned}$$

Or23	Job	Profit	Deadline	Sort by
A	100	2		
B	14	1		
C	27	2		
D	25	1		
E	11	3		

→ Sort jobs in descending order wrt profit

Job	Profit	Deadline
A	100	2
C	27	2
D	25	1
B	14	1

→ Identify through the sorted job

Job A (P: 100, D: 2) The latest free slot is available is 2. The slots available so far is 2 so Job A in slot 1.

Job C (P: 27, D: 2) : The latest free slot is available is 1 therefore Job C goes in slot 2.

Job D (P: 25, D: 1) : There is no such slot available anymore so Job D can't schedule

Job B (P: 14, D: 1) can't schedule

Job E (P:15 D:3) Slot 3 is available therefore  
Job E schedule in slot 3.

$$\begin{aligned}\text{Total Profit} &= 100 + 27 + 15 \\ &= 142\end{aligned}$$

Slot 1 : Job C

Slot 2 : Job A

Slot 3 : Job E

Only given : A: 5

B : 9

C : 12

D : 13

E : 16

F : 45

1. Start with two smallest freq.

Merge 'A' (5) and 'B' (9). The new node has a frequency of  $5+9=14$ .

2. Next two smallest freq:

Merge 'C' (12) and 'D' (13). The new node has a frequency of  $12+13=25$ .

3. Next two smallest freq:

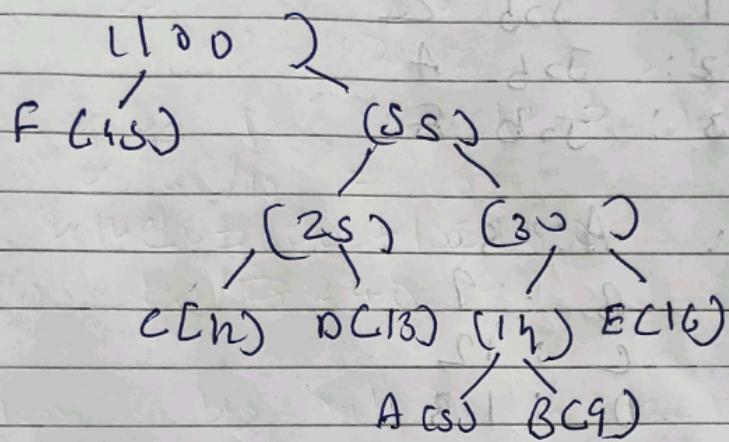
Merge note from step 1 (14) and 'E' (16)  
new note has freq of  $14+16=30$

4. Next two smallest freq:

Merge note from step 2 and step 3.  $25+30=55$

S. Find more merge code  
merge 'F' and 'G' from step 4  
new node 'H' has weight = 100

Final Huffman Tree:



Find bits & encode "ABCDEF"

F: 110 (1 bit), C: 100 (3 bits),  
D: 101 (3 bits), E: 111 (3 bits),  
A: 1100 (4 bits), B: 1101 (4 bits)

Total bits: 1 + 3 + 3 + 3 + 4 + 4  
= 18 bits

Q: Solve fractional knapsack:

Capacity = 50

Item	Weight	Value
1	10	60
2	20	100
3	30	120

Calculate value to weight ratio ( $V/W$ )

$$\text{item 1: } V/W = 60/10 = 6$$

$$\text{item 2: } 100/20 = 5$$

$$\text{item 3: } 120/30 = 4$$

Sort items by ratio in decreasing order

Item	1	2	3
ratio	6	5	4
weight	10	20	30

Breadthily add the items the knapsack based on the sorted order

Add item 1:

Weight added: 10

Value added: 60

Remaining capacity:  $50 - 10 = 40$

Add item 2:

Weight added: 20

Value added: 100

Remaining capacity:  $40 - 20 = 20$

3. Add item 3: ~~remaining capacity = 30~~

fraction to space:  $\frac{\text{Remaining capacity}}{\text{Item 3 unit}} = \frac{2}{3}$

value from fraction  $> \frac{2}{3} \times 120 > 80$

value from fraction  $= \frac{2}{3} \times 30 = 20$

Total value  $> 60 + 100 + 80$   
 $\geq 240$