K.K.Wagh Institute of Engineering Education & Research, Nashik
Subject : Engineering Mathematics-III

UNIT-1 : Linear Differential Equations

Type I(a): Complementary function (2 marks)
Type I(b) : Particular Integral by General Methods, MVP Methods (2Marks)

Q.N0	Questions
1	The solution of differential equation $4\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=0$ is a) $e^{-x}(c_1\cos 2x+c_2\sin 2x)$ b) $e^{-x/2}(c_1\cos x+c_2\sin x)$ c) $e^{-2x}(c_1\cos x+c_2\sin x)$ d) $c_1e^{-4x}+c_2e^{-5x}$
2	The solution of differential equation $\frac{d^3y}{dx^3}+6\frac{d^2y}{dx^2}+11\frac{dy}{dx}+6y=0$ is a) $c_1e^{x}+c_2e^{2x}+c_3e^{3x}$ b) $c_1e^{-x}+c_2e^{2x}+c_3e^{-3x}$ c) $c_1e^{-x}+c_2e^{-2x}+c_3e^{-3x}$ d) $c_1e^{x}+c_2e^{-2x}+c_3e^{3x}$
3	The solution of differential equation $\frac{d^3y}{dx^3}-7\frac{dy}{dx}-6y=0$ is a) $c_1e^{x}+c_2e^{2x}+c_3e^{3x}$ b) $c_1e^{-x}+c_2e^{-2x}+c_3e^{6x}$ c) $c_1e^{-x}+c_2e^{2x}+c_3e^{x}$ d) $c_1e^{-x}+c_2e^{-2x}+c_3e^{3x}$
4	The solution of differential equation $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$ is a) $c_1+e^{-x}(c_2x+c_3)$ b) $c_1+e^{x}(c_2x+c_3)$ c) $e^{-x}(c_2x+c_3)$ d) $c_1+c_2e^{x}+c_3e^{-x}$
5	The solution of differential equation $\frac{d^3y}{dx^3}-5\frac{d^2y}{dx^2}+8\frac{dy}{dx}-4y=0$ is a) $c_1e^{x}+e^{2x}(c_2x+c_3)$ b) $c_1e^{x}+c_2e^{2x}+c_3e^{3x}$ c) $e^{2x}(c_2x+c_3)$ d) $c_1e^{-x}+(c_2x+c_3)e^{-2x}$
6	The solution of differential equation $\frac{d^3y}{dx^3}-4\frac{dy}{dx}=0$ is a) $c_1e^{2x}+c_2e^{-2x}$ b) $c_1+c_2\cos 2x+c_3\sin 2x$ c) $c_1e^{x}+c_2e^{-2x}+c_3e^{-3x}$ d) $c_1+c_2e^{2x}+c_3e^{-2x}$
7	The solution of differential equation $\frac{d^3y}{dx^3}+y=0$ is a) $c_1e^{x}+e^{x}(C_2\cos\frac{\sqrt{3}}{2}x+C_3\sin\frac{\sqrt{3}}{2}x)$ b) $c_1e^{-x}+e^{\frac{1}{2}x}(C_2\cos\frac{1}{2}x+C_3\sin\frac{1}{2}x)$ c) $c_1e^{-x}+e^{\frac{1}{2}x}(C_2\cos\frac{\sqrt{3}}{2}x+C_3\sin\frac{\sqrt{3}}{2}x)$ d) $(c_1+c_2x+c_3x^2)e^{-x}$
8	The solution of differential equation $\frac{d^3y}{dx^3}+3\frac{dy}{dx}=0$ is a) $c_1+c_2\cos x+c_3\sin x$ b) $c_1+c_2\cos\sqrt{3}x+c_3\sin\sqrt{3}x$ c) $c_1+c_2e^{\sqrt{3}x}+c_3e^{-\sqrt{3}x}$ d) $c_1\cos x+c_2\sin x$
9	The solution of differential equation $\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}-2\frac{dy}{dx}+12y=0$ is a) $c_1e^{-3x}+e^{x}(C_2\cos\sqrt{3}x+C_3\sin\sqrt{3}x)$ b) $c_1e^{-3x}+(C_2\cos 3x+C_3\sin 3x)$

<u>Type I(C) : Cauchy's & Legendre's D.E., Simultaneous & Symmetrical simultaneous DE</u>(2 Marks)

1	For the D.E. $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x^2+x^{-2}$, complimentary function given by a) c_1x+c_2 b) $c_1\log x+c_2$ c) $c_1\cos x+c_2\sin x$ d) $c_1\cos(\log x)+c_2\sin(\log x)$
2	For the D.E. $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=A+B\log x$, complimentary function given by a) c_1x+c_2 b) $c_1x^2+c_2$ c) $c_1\log x+c_2$ d) $\frac{c_1}{x}+c_2$
3	For the D.E. $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=x^5$, complimentary function given by a) $c_1x^2+c_2x^3$ b) $c_1x^2+c_2x$ c) $c_1x^{-2}+c_2x^{-3}$ d) $c_1x^5+c_2x$
4	For the D.E. $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=\cos(\log x)+x\sin(\log x)$, complimentary function given by a)$[c_1\cos\sqrt{3}(\log x)+c_2\sin\sqrt{3}(\log x)]$ b) $x[c_1\cos\sqrt{2}(\log x)+c_2\sin\sqrt{2}(\log x)]$ c)$x[c_1\cos(\log x)+c_2\sin(\log x)]$ d) $x[c_1\cos\sqrt{3}(\log x)+c_2\sin\sqrt{3}(\log x)]$
5	For the D.E. $r^2\frac{d^2u}{dr^2}+r\frac{du}{dr}-u=-kr^3$, complimentary function given by a)$(c_1\log r+c_2)r$ b) $c_1r+\frac{c_2}{r}$ c) $[c_1\cos(\log r)+c_2\sin(\log r)$ d) $c_1r^2+\frac{c_2}{r^2}$
6	For the D.E. $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x$ Particular integral is given by a) x b) $\frac{x}{2}$ c) $\frac{x}{3}$ d) $2x$
7	For the D.E. $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=x^5$ Particular integral is given by a) $\frac{x^5}{6}$ b) $\frac{x^5}{56}$ c) $\frac{x^4}{6}$ d) $-\frac{x^5}{44}$
8	Solution of D.E. $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=x$ is a) $(c_1x+c_2)-\frac{x^2}{4}$ b) $(c_1x^2+c_2)+\frac{x^2}{4}$ c) $(c_1\log x+c_2)-\frac{x^2}{4}$ d)$(c_1\log x+c_2)+\frac{x^2}{4}$
9	Solution of D.E. $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=\frac{1}{x^2}$ is a) $(c_1x+c_2)-\frac{x^2}{4}$ b) $(c_1x^2+c_2)+\frac{x^2}{4}$ c) $c_1+c_2\frac{1}{x}+\frac{1}{2x^2}$ d)$(c_1\log x+c_2)+\frac{x^2}{4}$
10	For the D.E. $(x+1)^2\frac{d^2y}{dx^2}+(x+1)\frac{dy}{dx}+y=2\sin[\log(x+1)]$

Type I(d): Complementary Functions (1 mark)

1	If the roots $m_1, m_2, m_3, \ldots m_n$ of auxiliary equation $\phi(D)=0$ are real and distinct , then solution of $\phi(D)y=0$ is a) $c_1 e^{m_1x} + c_2 e^{m_2x} + \ldots\ldots + c_n e^{m_nx}$ b) $c_1\cos m_1x + c_2\cos m_2x + \ldots\ldots + c_n\cos m_nx$ c) $m_1 e^{c_1x} + m_2 e^{c_2x} + \ldots\ldots + m_n e^{c_nx}$ d) $c_1\sin m_1x + c_2 \sin m_2x + \ldots\ldots + c_n \sin m_nx$
2	The roots $m_1, m_2, m_3, \ldots m_n$ of auxiliary equation $\phi(D)=0$ are real .If two of these roots are repeated say $m_1 = m_2$ and remaining roots $m_3, m_4, \ldots m_n$ are distinct , then solution of $\phi(D)y=0$ is a) $c_1 e^{m_1x} + c_2 e^{m_2x} + \ldots\ldots + c_n e^{m_nx}$ b) $(c_1 x + c_2)\cos m_1x + c_3\cos m_3x + \ldots\ldots + c_n\cos m_nx$ c) $(c_1 x + c_2) e^{m_1x} + c_3 e^{m_3x} + \ldots\ldots + c_n e^{m_nx}$ d) $(c_1 x + c_2)\sin m_1x + c_3 \sin m_3x + \ldots\ldots + c_n \sin m_nx$
3	The roots $m_1, m_2, m_3, \ldots m_n$ of auxiliary equation $\phi(D)=0$ are real .If three of these roots are repeated say $m_1 = m_2 = m_3$ and remaining roots $m_4, m_5, \ldots m_n$ are distinct , then solution of $\phi(D)y=0$ is a) $c_1 e^{m_1x} + c_2 e^{m_2x} + \ldots\ldots + c_n e^{m_nx}$ b) $(c_1 x^2 + c_2 x + c_3) e^{m_1x} + c_4 e^{m_4x} + \ldots\ldots + c_n e^{m_nx}$ c) $(c_1 x^2 + c_2 x + c_3)\cos m_1x + c_4\cos m_4x + \ldots\ldots + c_n \cos m_nx$ d) $(c_1 x^2 + c_2 x + c_3)\sin m_1x + c_4\sin m_4x + \ldots\ldots + c_n \sin m_nx$
4	If $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$ are two complex roots of auxiliary equation of second order D.E. $\phi(D)y=0$ then its solution is a) $e^{\beta x}[c_1\cos\alpha x + c_2\sin\alpha x]$ b) $e^{\alpha x}[(c_1x+c_2)\cos\beta x + (c_3x+c_4)\sin\beta x]$ c) $c_1 e^{\alpha x} + c_2 e^{\beta x}$ d) $e^{\alpha x}[c_1\cos\beta x + c_2\sin\beta x]$
5	If the complex roots $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$ of auxiliary equation of fourth order D.E. $\phi(D)y=0$ repeated twice then its solution is a) $e^{\beta x}[c_1\cos\alpha x + c_2\sin\alpha x]$ b) $e^{\alpha x}[(c_1x+c_2)\cos\beta x + (c_3x+c_4)\sin\beta x]$ c) $(c_1 x + c_2) e^{\alpha x} + (c_3 x + c_4) e^{\beta x}$ d) $e^{\alpha x}[c_1\cos\beta x + c_2\sin\beta x]$
6	The solution of differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ is a) $c_1 e^{2x} + c_2 e^{-3x}$ b) $c_1 e^{-2x} + c_2 e^{3x}$ c) $c_1 e^{-2x} + c_2 e^{-3x}$ d) $c_1 e^{2x} + c_2 e^{3x}$
7	The solution of differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0$ is a) $c_1 e^{-x} + c_2 e^{6x}$ b) $c_1 e^{-2x} + c_2 e^{-3x}$ c) $c_1 e^{3x} + c_2 e^{2x}$ d) $c_1 e^{-3x} + c_2 e^{-2x}$
8	The solution of differential equation $2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 10y = 0$ is a) $c_1 e^{2x} + c_2 e^{\frac{5}{2}x}$ b) $c_1 e^{-2x} + c_2 e^{-\frac{5}{2}x}$ c) $c_1 e^{-2x} + c_2 e^{\frac{5}{2}x}$ d) $c_1 e^{-2x} + c_2 e^{\frac{3}{2}x}$
9	The solution of differential equation $\frac{d^2y}{dx^2} - 4y = 0$ is

Type I(e): PI by General & Short Methods, MVP, Cauchy's & Legendre's D.E., Simultaneous & Symmetrical simultaneous DE (1 Mark)

1	Particular Integral of linear differential equation with constant coefficient $\phi(D)y = f(x)$ is given by a) $\frac{1}{\phi(D)}$ f(x) b) $\frac{1}{\phi(D)\text{f(x)}}$ c) $\phi(D)\ \frac{1}{\text{f(x)}}$ d) $\frac{1}{\phi(D^2)}$ f(x)
2	$\frac{1}{D-m} f(x),$ where D= $\frac{d}{dx}$ and m is constant, is equal to a) $e^{mx}\int e^{-mx}$ dx b) $\int e^{-mx}$ f(x)dx c) $e^{mx}\int e^{-mx}$ f(x)dx d) $e^{-mx}\int e^{mx}$ f(x)dx
3	$\frac{1}{D+m} f(x),$ where D= $\frac{d}{dx}$ and m is constant, is equal to a) $e^{-mx}\int e^{mx}$ dx b) $\int e^{mx}$ f(x)dx c) $e^{mx}\int e^{-mx}$ f(x)dx d) $e^{-mx}\int e^{mx}$ f(x)dx
4	Particular Integral of $\frac{1}{\phi(D)}\ e^{ax}$, where D= $\frac{d}{dx}$ and $\phi(a) \neq 0$ is a) $\frac{1}{\phi(-a)}\ e^{ax}$ b) $x\frac{1}{\phi(a)}\ e^{ax}$ c) $\frac{1}{\phi(a^2)}\ e^{ax}$ d) $\frac{1}{\phi(a)}\ e^{ax}$
5	Particular Integral of $\frac{1}{(D-a)^r}\ e^{ax}$, where D= $\frac{d}{dx}$ is a) $\frac{1}{r!}\ e^{ax}$ b) $\frac{x^r}{r}\ e^{ax}$ c) $\frac{x^r}{r!}\ e^{ax}$ d) $x^r\ e^{ax}$
6	Particular Integral of $\frac{1}{\phi(D^2)}\sin(ax+b)$, where D= $\frac{d}{dx}$ and $\phi(-a^2) \neq 0$ is a) $\frac{1}{\phi(-a^2)}\cos(ax+b)$ b) $\frac{1}{\phi(-a^2)}\sin(ax+b)$ c) $x\frac{1}{\phi(-a^2)}\sin(ax+b)$ d) $\frac{1}{\phi(a^2)}\sin(ax+b)$
7	Particular Integral of $\frac{1}{\phi(D^2)}\sin(ax+b)$, where D= $\frac{d}{dx}$ and $\phi(-a^2) = 0, \phi'(-a^2) \neq 0$ is a) $x\frac{1}{\phi'(-a^2)}\cos(ax+b)$ b) $x\frac{1}{\phi'(-a^2)}\sin(ax+b)$ c) $\frac{1}{\phi(-a^2)}\sin(ax+b)$ d) $\frac{1}{\phi'(-a^2)}\sin(ax+b)$
8	Particular Integral of $\frac{1}{\phi(D^2)}\cos(ax+b)$, where D= $\frac{d}{dx}$ and $\phi(-a^2) \neq 0$ is a) $\frac{1}{\phi\ (-a^2)}\cos(ax+b)$ b) $\frac{1}{\phi(-a^2)}\sin(ax+b)$

ANSWERS

Group Ia)

1.(b)	2.(c)	3.(d)	4.(b)	5.(a)	6.(d)	7.(c)	8.(b)
9.(a)	10.(a)	11.(c)	12.(d)	13.(b)	14.(b)	15.(d)	16.(a)

Group Ib)

1.(a)	2.(b)	3.(c)	4.(d)	5.(b)	6.(d)	7.(b)	8.(a)
9.(c)	10.(d)	11.(a)	12.(d)	13.(c)	14.(b)	15.(d)	16.(a)
17.(b)	18.(c)	19.(d)	20.(d)	21.(c)	22.(a)	23.(d)	24.(b)
25.(c)	26.(d)	27.(a)	28.(d)	29.(c)	30.(c)	31.(a)	32.(c)
33.(b)	34.(a)	35.(d)	36.(b)	37.(c)	38.(b)	39.(d)	40.(b)
41.(c)	42.(a)	43.(c)	44.(b)	45.(d)	46.(a)		

Group Ic)

1.(d)	2.(c)	3.(a)	4.(d)	5.(b)	6.(b)	7.(a)	8.(d)
9.(d)	10.(b)	11.(a)	12.(c)	13.(d)	14.(a)	15.(d)	16.(b)
17.(c)	18.(a)	19.(c)	20.(b)	21.(d)	22.(b)	23.(a)	24.(d)
25.(a)	26.(c)	27.(a)	28.(c)	29.(c)	30.(b)	31.(b)	32.(d)
33.(a)							

Group Id)

1.(a)	2.(c)	3.(b)	4.(d)	5.(b)	6.(d)	7.(a)	8.(c)
9.(d)	10.(b)	11.(a)	12.(c)	13.(d)	14.(a)	15.(b)	16.(c)

17.(d)	18.(a)	19.(c)					

Group Ie)

1.(a)	2.(c)	3.(d)	4.(d)	5.(c)	6.(b)	7.(b)	8.(a)
9.(d)	10.(c)	11.(a)	12.(c)	13.(d)	14.(c)	15.(a)	16.(d)
17.(b)	18.(b)	19.(c)	20.(a)	21.(b)	22.(d)	23.(a)	24.(c)
25.(d)	26.(a)	27.(c)	28.(b)	29.(c)	30.(d)	31.(b)	32.(d)
33.(a)	34.(d)						