Assignment 2

Question 1: Back Savers Backpack Production

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

Product Data :

Item	**Collegiate (x)**	**Mini (y)**
Fabric required (sq ft/unit)	3	2
Labor required (minutes/unit)	45	40
Unit profit ($)	32	24
Sales forecast cap (units/week)	1000	1200

Resource Data :

Resource	**Amount available per week**
Fabric (sq ft)	5,000
Labor (minutes)	84,000 (35 workers × 40 hrs × 60 min)

- **a) Decision Variables**

Let x = number of Collegiate backpacks produced per week.

Let y = number of Mini backpacks produced per week.

- **b) Objective Function**

Maximize total profit per week:

$Z = 32x + 24y$

- **c) Constraints**

1. Fabric: $3x + 2y \leq 5000$

2. Labor: $45x + 40y \leq 84{,}000$

3. Sales Limits: $x \leq 1000$, $y \leq 1200$

4. Nonnegativity: $x \geq 0$, $y \geq 0$

- **d) Full Mathematical Formulation**

Maximize $Z = 32x + 24y$
subject to:
$3x + 2y \leq 5000$ (fabric)
$45x + 40y \leq 84{,}000$ (labor)
$x \leq 1000$ (Collegiate sales)
$y \leq 1200$ (Mini sales)
$x, y \geq 0$

Question 2: Weigelt Corporation Production Allocation

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability. The product can be made in three sizes: large, medium, and small, yielding net unit profits of $420, $360, and $300, respectively. Plants 1, 2, and 3 have capacities of 750, 900, and 450 units per day. Storage limits are 13,000, 12,000, and 5,000 square feet, and each large, medium, and small unit requires 20, 15, and 12 square feet. Sales forecasts are 900 large, 1200 medium, and 750 small units per day. To avoid layoffs, management requires that all plants use the same percentage of their capacity.

- **a) Decision Variables**

x1L, x1M, x1S = Large, Medium, Small units produced at Plant 1.

x2L, x2M, x2S = Large, Medium, Small units produced at Plant 2.

x3L, x3M, x3S = Large, Medium, Small units produced at Plant 3.

To model the "same percentage of capacity used at each plant" rule, introduce:

- α = common utilization fraction of each plant's daily excess capacity used for this new product.

- **b) Linear Programming Model**

Objective Function:

Profits per unit: Large $420, Medium $360, Small $300.

Maximize Z = 420(x1L + x2L + x3L) + 360(x1M + x2M + x3M) + 300(x1S + x2S + x3S)

- **Constraints:**

1. **Capacity Utilization:**

Daily excess capacities: Plant 1 = 750 units, Plant 2 = 900 units, Plant 3 = 450 units.
"Same percentage" = each plant's total output equals its capacity times the same α

$x1L + x1M + x1S \leq 750\alpha$

$x2L + x2M + x2S \leq 900\alpha$

$x3L + x3M + x3S \leq 450\alpha$

2. Storage Constraints:

Space per unit: Large 20, Medium 15, Small 12 sq ft.
Available space: Plant 1: 13,000; Plant 2: 12,000; Plant 3: 5,000 sq ft.

$20x1L + 15x1M + 12x1S \leq 13{,}000$

$20x2L + 15x2M + 12x2S \leq 12{,}000$

$20x3L + 15x3M + 12x3S \leq 5{,}000$

3. Sales Demand Constraints:

Sales limits per day: Large 900, Medium 1,200, Small 750

$x1L + x2L + x3L \leq 900$

$x1M + x2M + x3M \leq 1200$

$x1S + x2S + x3S \leq 750$

4. Nonnegativity and Bound on α:

$x_{ij} \geq 0$ for all i,j and $0 \leq \alpha \leq 1$