Research on Slam Algorithm of Iterated Extended Kalman Filtering for Multi-sensor Fusion

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ABSTRACT
For the problem of the error of EKF in linearization, an iterated extended Kalman filtering algorithm based on IMU and laser sensor for environmental feature matching is proposed. The K-Nearest Neighbor algorithm is applied to match the data obtained by IMU and laser sensor. In the measurement phase, the nonlinear system is linearized multiple by EKF. The simulation results show that the new method effectively reduces the nonlinear error and inhibits the error accumulation caused by IMU. Compared with traditional extended Kalman filtering, the root mean square error of position and azimuth was reduced by 48%, 16% and 29%, respectively. As the result, the navigation accuracy of system is improved and state tracking performance outperforms better than traditional EKF.

CCS Concepts
• Theory of computation → Design and analysis of algorithms → Algorithm design techniques → Backtracking.

Keywords
Multi-sensor fusion; iterated extend kalman filtering; SLAM algorithm.

1. INTRODUCTION
In recent years, the simultaneous localization and mapping (SLAM) algorithm has been a hot topic in the field of robot research [1], and it is considered to be the key to achieve the real autonomous navigation of robot. SLAM can be described as a process that the procedure of building a map by robot and determining the own location of robot at the same time in the unknown environment.

Kalman filtering can reduce the noise effect and the error effectively, so it is widely used in linear Gaussian systems [2]. However, in the SLAM research, the motion model of the robot and the observation model of the sensor are non-linear, and the noise error is not Gaussian distribution. Kalman filtering can lead to divergence easily, and the estimation performance is poor, so the application of Kalman filter is greatly limited. The SLAM algorithm based on extended Kalman filter (EKF) is the earliest and most common method to solve SLAM problem [3]. By this method, the motion model and observation model of SLAM are carried out in the first-order Taylor expansion. This expansion completes the linearization process of nonlinear system, and then, Kalman filtering is used once again to realize SLAM process.

But for traditional EKF, a large error cannot be avoided in the process of linearization through the first-order Taylor expansion. And the calculation of the Jacobian matrix is often complicated, but the jacobian matrix is not fully utilized, which not only increases the computational burden, but also adversely affects the SLAM process. In order to solve this problem, in this paper, the multi-sensor fusion is applied to receive the environmental feature information, and the state is updated through iterated extended Kalman filter (IEKF) [4-5] during the measurement update phase, so that the process is linearized, and the positioning error is reduced. Meanwhile, the stability of system is improved.

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2. EXTENDED KALMAN FILTERING
Before EKF, the principles and processes of Kalman filtering are introduced firstly, because EKF is an improved algorithm based on Kalman filtering

2.1 Brief of Kalman Filtering
The dynamic system can be described by using the following state space model:

$$X(k+1) = F_t X(k) + G(k)W(k) \tag{1}$$

$$Z(k) = HX(k) + V(k) \tag{2}$$

Define k is the discrete time, $X(k)$ is the state equation of the system at time k and $Z(k)$ is the observation equation of the corresponding state, where $X(k) \in R^n$ and $Z(k) \in R^m$. $W(k)$ is white Gaussian noise of motion model ,and $V(k)$ is white Gaussian noise of observation model. In normal conditions, It is assumed that the mean of $W(k)$ and $V(k)$ is both zero, and the variance matrix is independent of the white noise of Q and R . In addition, F_t is the state transfer matrix, $G(k)$ is the noise driven matrix and H is the observation matrix. The filtering process can be briefly categorized into the following steps.

Step 1: Implement the next prediction for the state $\hat{X}(k+1|k) = F_t \hat{X}(k|k)$

Where $\hat{X}(k+1|k)$ is the state of the predicted $k+1$ time for the robot through the state information and state transfer matrix in k time. At the same time, the prediction covariance matrix is calculated as:

$$P(k+1|k) = F_t P(k|k) F_t^T + G(k) B_t Q B_t^T \tag{3}$$

Step 2: Update of status

The variance matrix of covariance P , observation matrix H and $V(k)$ of the motion state are obtained by Kalman filter gain matrix $K(k+1)$. The formula is shown as follows:

$$K(k+1) = P(k+1|k) H^T \left[HP(k+1|k) H^T + R \right]^{-1} \tag{4}$$

And then the updated status is obtained based on the gain matrix $K(k+1)$:

$$\hat{X}(k+1|k+1) = \hat{X}(k+1|k) + K(k+1)\left[Z(k+1) - H\hat{X}(k+1|k) \right] \tag{5}$$

$\hat{X}(k+1|k+1)$ is the position of $k+1$ time and the new covariance matrix is achieved as:

$$P(k+1|k+1) = \left[I_n - K(k+1)H \right] P(k+1|k) \tag{6}$$

In summary, in KF filtering process, the gain matrix k is associated the predicted values with the observed values, so that the status of next moment is updated.

2.2 Structure of Extend Kalman Filtering

The Kalman filter described above is optimal for the general linear Gaussian system, while for nonlinear systems, the estimated effects are not good enough [6]. In order to deal with the general nonlinear situation, the classical extended Kalman filtering is proposed based on linearization, which is widely used in nonlinear problems such as SLAM. On the basis of traditional Kalman filter, the state equation and observation equation of the system can be expressed as

$$X(k+1) = f[k, X(k)] + G(k)W(k) \tag{7}$$

$$Z(k) = h[k, X(k)] + V(k) \tag{8}$$

Define f and h respectively as nonlinear state transfer functions and nonlinear observation functions. EKF algorithm can be described as:

Step 1: Linearization of nonlinear systems.

The system state equation is linearized by using the nonlinear function $f\left[k, X(k)\right]$, which is developed by the first step Taylor formula, is as follows:

$$X(k+1) \approx f\left[k, \hat{X}(k)\right] + \frac{\partial f}{\partial \hat{X}(k)}\left[X(k) - \hat{X}(k)\right] + G(k)W(k) \tag{9}$$

The observation equation is handled in the same way:

$$Z(k+1) \approx h\left[k, \hat{X}(k|k-1)\right] + \frac{\partial h}{\partial \hat{X}(k)}\left[X(k) - \hat{X}(k|k-1)\right] + V(k) \tag{10}$$

For convenience, the jacobian matrix for f and h is define as

$$\frac{\partial f}{\partial \hat{X}(k)} = \phi(k+1|k), \quad \frac{\partial h}{\partial \hat{X}(k)} = H(k+1|k)$$

Step 2: In order to get the recursive equation extended Kalman filter, the linearized model is applied to Kalman filter. Equations being involved are as follows:

$$\hat{X}(k+1|k) = f\left[\hat{X}(k|k)\right] \tag{11}$$

$$P(k+1|k) = \phi(k+1|k) P(k|k) \phi^T(k+1|k) + Q(k+1) \tag{12}$$

$$K(k+1) = P(k+1|k) H^T(k+1) \left[H(k+1) P(k+1|k) H^T(k+1) + R(k+1) \right]^{-1} \tag{13}$$

$$\hat{X}(k+1|k+1) = \hat{X}(k+1|k) + K(k+1)\left[Z(k+1) - h\hat{X}(k+1|k) \right] \tag{14}$$

$$P(k+1) = \left[I_n - K(k+1) H(k+1) \right] P(k+1|k) \tag{15}$$

Finally, the filtering process is completed based on the above equation.

3. SLAM ALGORITHM DESIGN

In order to solve the limitation of EKF algorithm linearization in the SLAM process, the method of iterated extended Kalman filtering based on multi-sensor combined navigation is used to complete the SLAM process in this paper. The concrete fusion location block diagram is shown below.

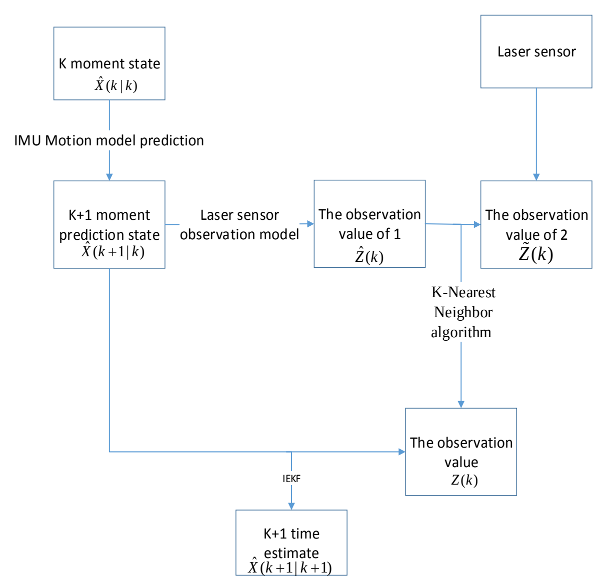

Figure 1. Fusion location block diagram

In the robot localization process, $X(k)$ is the attitude of robot at time k . Define $X(k)=[x_k, y_k, \theta_k]^T$, (x_k, y_k) represents the position information of the robot, and θ_k represents the direction of the robot. Assuming that the positive direction of the X axis is zero, turn counterclockwise to positive and clockwise rotation to negative. The IEKF algorithm is based on EKF and can be divided into several steps: state prediction, observation, iteration and update. Assuming that the initial attitude of the robot is known, and the state estimate obtained at time k is $\hat{X}(k|k)$. IEKF algorithm can be described as:

Step 1: State prediction process

The predicted value of $k+1$ time is obtained by using formula (11), then the IMU motion model is used as predict and update equation of system. Thus, the status of each time can be obtained. In practice, a simplified motion model is usually used to approximate the IMU motion model [7]:

$$X(k+1)=\begin{bmatrix} x(k) \\ y(k) \\ \theta(k) \end{bmatrix}=\begin{bmatrix} x(k)+\Delta T v(k)\cos(\theta(k)+\Delta\theta \\ y(k)+\Delta T v(k)\sin(\theta(k)+\Delta\theta \\ \theta(k)+\dfrac{\Delta T v(k+1)\sin(\Delta\theta)}{M} \end{bmatrix} \tag{16}$$

v is the movement speed of the robot, ΔT is the sensor sampling time, M is the wheelbase of the two driving wheels. The jacobian matrix $\phi(k+1|k)$ can be calculated using the motion model and the covariance matrix $P(k+1|k)$ is computed using formula (11).

Step 2: Calculate the observations

The observation equation of the laser sensor is as follows:

$$Z(k)=\begin{bmatrix} d \\ g \end{bmatrix}=\begin{bmatrix} \sqrt{(x_i-x(k))^2-(y_i-y(k))^2} \\ \arctan\dfrac{y_i-x(k)}{x_i-x(k)}-\theta(k) \end{bmatrix} \tag{17}$$

Defining (x_i, y_i) as the position coordinates of landmark point, d is the distance of the landmark relative to the robot and g is the angle of the landmark relative to the robot. The observation value of time k $\hat{Z}(k)$ is obtained by using the predicted value of time k $\hat{X}(k+1|k)$ and the laser radar sensor as the system observation model. At the same time, another observation $\tilde{Z}(k)$ is obtained through the laser sensor. The two values are matched by K-Nearest Neighbor algorithm [8] to get the final observed values of time k $Z(k)$. The K-Nearest Neighbor algorithm is a common method for data correlation in SLAM process, which can effectively match the two observation values and get better observation values.

Step 3: *Iteration and update*

The jacobian matrix of the observed model can be defined as $\dfrac{\partial h}{\partial \hat{X}(k)}=H(k+1|k)$ .The Kalman gain $K(k+1)$, the status estimate for the first update $\hat{X}_1(k+1)$ and updated covariance matrix $P(k+1)$ are obtained by extending the recursive formula of Kalman filter.

The updated status estimate $\hat{X}_1(k+1)$ is defined as the new predictive value of this point. Once again, the EKF is used to obtain the new estimate $\hat{X}_2(k+1)$, which is the position of the moment obtained after iteration. As a result, the desired location information is calculated after multiple times of iterations.

4. SIMULATION AND ANALYSIS

4.1 The Simulation Setup

In the simulation platform of MATLAB, the experimental environment set in the open source SLAM simulator released by Tim Biley [9] is used to specify the waypionts and landmarks of the moving path, and the estimation accuracy of 10-iterations IEKF algorithm and traditional EKF are both been simulated and compared.

The main parameters of the experiment are set initially. The movement speed of robot is $3m/s$, and the maximum turning speed is $\pm 20^\circ/s$. The wheelbase is 5m. The maximum scanning distance of the sensor is $30m$, and scan range is 0^o to 180^o . The sampling period of the two sensors is $50m/s$. The variance of motion noise is set as: $\sigma_v = 0.2m/s$, $\sigma_\beta = 1^\circ$, and observation noise is set as $\sigma_r = 0.2m$, $\sigma_\theta = 1^\circ$.

4.2 Comparison of Paths and Maps

Assuming that the robot starts from the origin position marked as $[0,0,0]^T$ and the EKF and IEKF algorithm are applied respectively .The results of the comparison are shown below, where Figure 3 and Figure 4 are the local enlarged drawing of regions A and B in Figure 2.

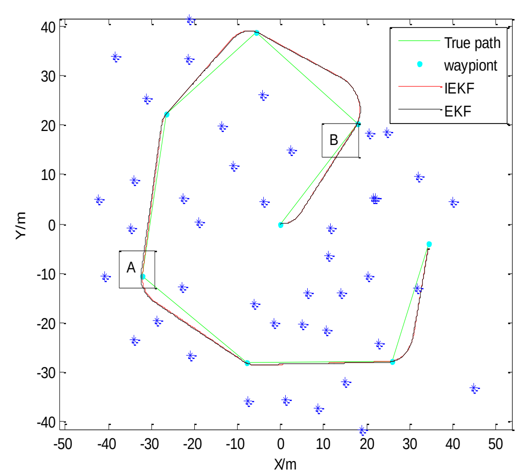

Figure 2. Path Comparison

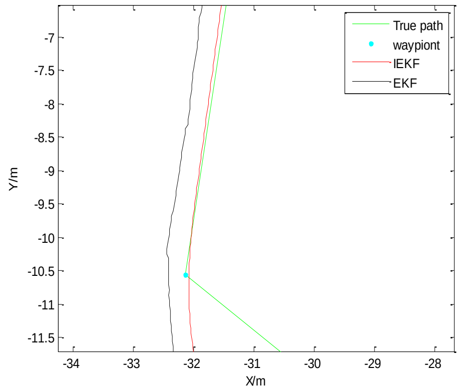

Figure 3. Enlarge the area A

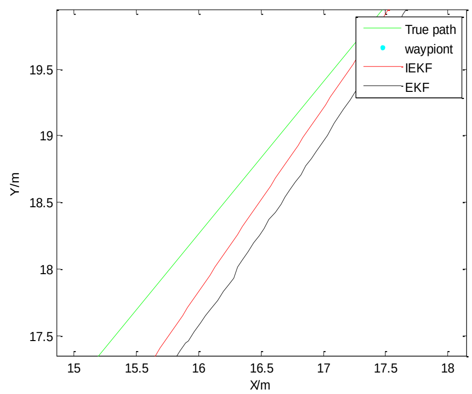

Figure 4. Enlarge the area B

In the picture, the green path is the real path. The red path and the black path are the paths obtained through IEKF and EKF respectively. As can be seen from the local magnification, the estimated path of IEKF algorithm is closer to the real path, especially when passing through waypiont, which is close to waypoint.

4.3 The Error Analysis

Fig. 5 to 7 shows the absolute error of each direction of the robot under two algorithms, green for IEKF and red for EKF.

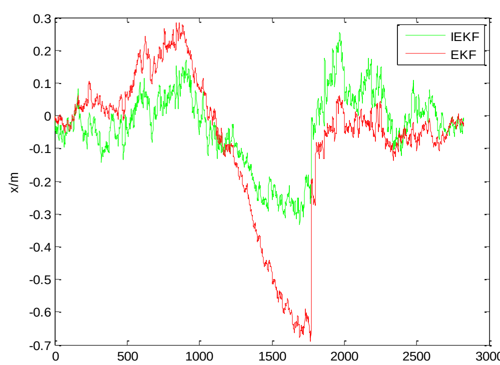

Figure 5. The comparison of absolute error in X direction

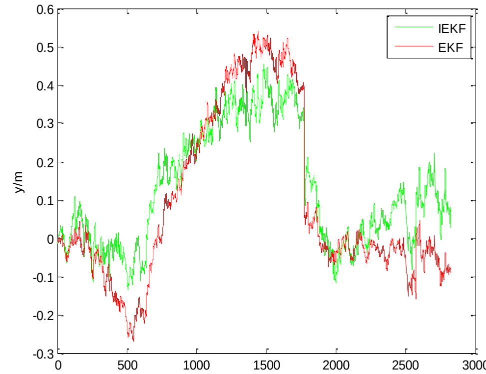

Figure 6. The comparison of absolute error in Y direction

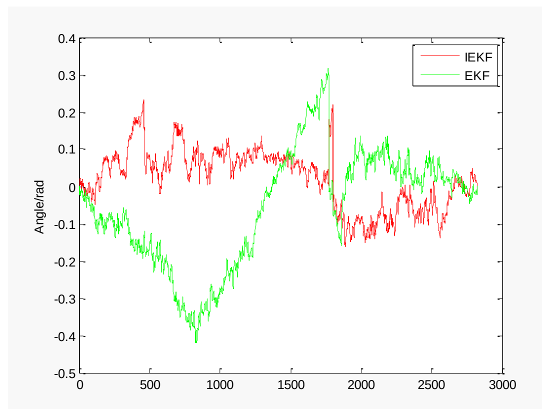

Figure 7. Comparison of Angle absolute error

By comparing the absolute error of two algorithms in each direction, it can be seen that the error resulted from EKF algorithm is relatively large and has obvious fluctuation in all directions, because of the accumulation of errors produced by IMU during the measurement is not effectively inhibited. And IEKF algorithm is significantly more stable and less error is emerged, and effectively inhibits the divergence of error accumulation caused by IMU.

Then, by calculating the mean square root error, the accuracy of the two algorithms is compared, as shown in Table 1

Table 1. MSE of both algorithms

Algorithms	MSE		
	X	Y	Angle
EKF	0.2317	0.2391	0.1671
IEKF	0.1199	0.1994	0.1182

Obviously, the accuracy of IEKF algorithm is better than that of EKF by comparison of root mean square error.

4.4 Iteration Comparison

IEKF algorithm with the traditional EKF algorithm of the difference is in the status update phase .The update status as the predicted value again, multiple iterations and linearization, repeating the entire process of filter to complete the update. Therefore, the number of iterations will also have an impact on the results of the experiment. The root mean square error of different iteration times is compared and the results are shown in Table 2:

Table 2. Influence on accuracy when applying different times of iterations

Number of iterations	MSE			The average time
	X	Y	Angle	
10	0.1199	0.1994	0.1182	13.76s
20	0.0963	0.1255	0.0944	17.56s
50	0.0845	0.1084	0.0751	19.13

It can be seen that, as the number of iterations increases, the positioning accuracy is further improved. However, as the number of iterations increases, the computation and run time will also increase. Therefore, different iteration times can be selected to improve the accuracy according to different hardware conditions. The results showed that the mean square error of the position and azimuth were decreased by 48%, 16% and 29% respectively compared with traditional extended Kalman filtering

4. CONCLUSION

An iterated extended Kalman filtering algorithm based on IMU and laser sensor for environmental feature matching is proposed in this paper. The simulation experiment shows that this new method effectively reduces the error, enhances the system stability and improves the navigation accuracy. After the IEKF being applied in the multi-sensor mobile root navigation system, the mean square error of the position and azimuth were decreased by 48%, 16% and 29% respectively when being compared with the traditional extended Kalman filtering algorithm.

5. ACKNOWLEDGMENT

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