# ECON 202 Intermediate Macroeconomics: Production, Labor Demand, Investment and Endogenous Labor Supply Lec 23-25

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# Overview

i. This lecture corresponds to chapter 12 from your main textbook.
ii. In this lecture, we dig deeper into the *microeconomic underpinnings* of the firm problem by deriving expressions for labor and investment demand. We also enrich the household side to include an endogenous labor choice.
iii. Precursor and sets the foundation for Neo-Classical Model and also the New Keynesian model to follow.

# Production Function

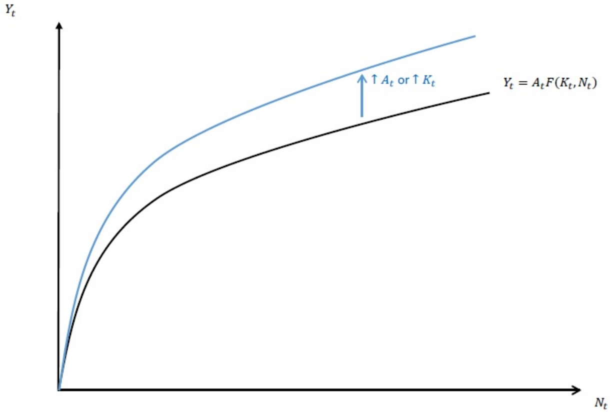
i. In this chapter, we work with $Y_t = A_t F(K_t, N_t)$.

ii. $F$ satisfies the same regularity conditions on first and second order partial derivatives as it did in the chapter on Solow Model. Both labor and capital goods also continue to be essential in the production process.

iii. We now allow for both transitory but unanticipated and permanent but anticipated changes in $A_t$ over time.

iv. As usual, we worth with Cobb Doughlas function: $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$.

# Shift in Production Function



# Firms

i. There is a representative household who owns the representative firm, but management is separated from ownership (i.e. the household and firm are separate decision-making entities). Both entities live for 2 periods.

ii. The *firm is endowed* with existing or predetermined capital $K_t$ and hires labor in the labor market.

iii. The law of motion for capital is the same as in Solow Model: $K_{t+1} = (1 - \delta)K_t + I_t$.

iv. Firms must borrow from a financial intermediary to fund investment. The cost of borrowing is $r_t$ and $B_t^I = I_t$, so that all investment is financed by borrowing.

# Dividends

i. In period $t$, the firm has revenue of $Y_t$ and pays wage bill $w_t N_t$. There already exists endowment stock of capital $K_t$ and new investment $I_t$ is derived from borrowing on which no interest has to paid yet $\implies D_t = Y_t - w_t N_t$.

ii. In period $t+1$, the firm will die so that $K_{t+2} = 0 \implies I_{t+1} = -(1-\delta)K_{t+1}$ (liquidation sale).

iii. The borrowing for investment in first period has to be repaid now with interest, which is $(1+r_t)B_t^I$.

iv. Thus, dividends for households in $t+1$ are $D_{t+1} = Y_{t+1} - w_{t+1}N_{t+1} + (1-\delta)K_{t+1} - (1+r_t)B_t^I$.

# Firm Optimization

i. The present discounted value of firm is $V_t = D_t + \frac{1}{1+r_t} D_{t+1}$.

ii. The firm optimization problem can therefore be written as
$$\max_{N_t, I_t} V_t = A_t F(K_t, N_t) - w_t N_t + \frac{1}{1+r_t}\left(A_{t+1} F(K_{t+1}, N_{t+1}) - w_{t+1} N_{t+1} + (1-\delta) K_{t+1} - (1+r_t) I_t\right).$$
subject to $K_{t+1} = (1-\delta)K_t + I_t$ and $B_t^I = I_t$.

iii. The two constraints can be combined to get $B_t^I = K_{t+1} - (1-\delta)K_t$.

# Firm Optimization

i. Substitute $B_t^I = K_{t+1} - (1-\delta)K_t$ instead of $I_t$ in the maximand to get the equivalent unconstrained optimization problem with $K_{t+1}$ and $N_t$ as choice variables.
$\max_{N_t, K_{t+1}} V_t = A_t F(K_t, N_t) - w_t N_t$
$+\frac{1}{1+r_t}\left(A_{t+1}F(K_{t+1}, N_{t+1}) - w_{t+1}N_{t+1} + (1-\delta)K_{t+1}\right) +$
$\frac{1}{1+r_t}\left(-(1+r_t)\left(K_{t+1} - (1-\delta)K_t\right)\right)$.

ii. The FOC's with respect to $N_t$ and $K_{t+1}$ are
$\frac{\partial V}{\partial N_t} = A_t F_N(K_t, N_t) - w_t = 0$ and
$\frac{\partial V}{\partial K_{t+1}} = \frac{1}{1+r_t}\left(A_{t+1}F_K(K_{t+1}, N_{t+1}) + 1 - \delta - (1+r_t)\right) = 0$.

# Firm Optimization

i. Simplify the FOC's from last slide to get $w_t = A_t F_N\left(K_t, N_t\right)$ and $1 + r_t = A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1 - \delta)$.

ii. These are the firm optimality conditions for choice of labor and capital.

iii. The first equation implicitly defines firm demand for labor, suggesting that labor should be hired up to a point when marginal product of labor equals the real wage.

iv. The second condition for capital implies that capital must be hired up to the point, where marginal cost of capital investment: $1 + r_t$ equals the marginal benefit of capital.

# Labor Demand and Investment Demand

i. The labor demand function is of form $N_t = N^d\left(\underset{-}{w_t}, \underset{+}{A_t}, \underset{+}{K_t}\right)$ and capital demand function is of form $K_{t+1} = K^d\left(\underset{-}{r_t}, \underset{+}{A_{t+1}}\right)$.

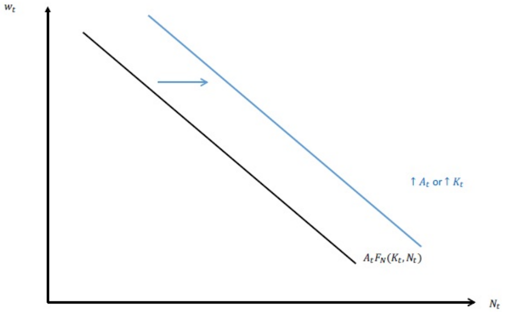
ii. Investment demand function is $I_t = I^d\left(\underset{-}{r_t}, \underset{+}{A_{t+1}}, \underset{-}{K_t}\right)$.

iii. A higher level of $K_t$ lowers investment needed to achieve any given level of $K_{t+1}$.

iv. The positive or negative sign under the arguments indicate the sign of partial derivative of the function with respect to the particular argument.

v. Capital is forward looking since it depends on future technology/TFP, rather than current TFP.

# Labor Demand Function



# Household

i. The household maximizes $U = u(C_t, 1 - N_t) + \beta u(C_{t+1}, 1 - N_{t+1})$, where $L_t = 1 - N_t$ is leisure consumption.

ii. The flow budget constraints are: $C_t + S_t \leq w_t N_t + D_t$ when young and $C_{t+1} + S_{t+1} \leq w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1} + (1 + r_t) S_t$ when old.

iii. Just as in previous lecture, we can work work equality versions of both constraints and combine them to derive the intertemporal budget constraint, which is: $C_t + \frac{C_{t+1}}{1+r_t} = w_t N_t + D_t + \frac{w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1}}{1+r_t}$.

# Household Problem

i. $$\max_{C_t, C_{t+1}, N_t, N_{t+1}} U = u(C_t, 1 - N_t) + \beta u(C_{t+1}, 1 - N_{t+1})$$
subject to $C_t + \frac{C_{t+1}}{1+r_t} = w_t N_t + D_t + \frac{w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1}}{1+r_t}$.

ii. $$\mathcal{L} = u(C_t, 1 - N_t) + \beta u(C_{t+1}, 1 - N_{t+1}) + \lambda \left( w_t N_t + D_t + \frac{w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1}}{1+r_t} - C_t - \frac{C_{t+1}}{1+r_t} \right).$$

iii. Take the 4 FOC's: $\mathcal{L}_{C_t}$, $\mathcal{L}_{C_{t+1}}$, $\mathcal{L}_{N_t}$ and $\mathcal{L}_{N_{t+1}}$.

iv. $\mathcal{L}_{C_t} = u_C(C_t, 1 - N_t) = \lambda$,
$\mathcal{L}_{C_{t+1}} = \beta u_C(C_{t+1}, 1 - N_{t+1}) = \frac{\lambda}{1+r_t}$,
$\mathcal{L}_{N_t} = u_L(C_t, 1 - N_t) = \lambda w_t$ and
$\mathcal{L}_{N_{t+1}} = \beta u_L(C_{t+1}, 1 - N_{t+1}) = \frac{\lambda w_{t+1}}{1+r_t}$

# Household Problem

i. Combine the first two FOC's for consumption (last slide) and equate $\lambda$ to get: $u_C(C_t, 1-N_t) = \beta u_C(C_{t+1}, 1-N_{t+1})(1+r_t)$ (Euler Equation).

ii. Combine the FOC for $N_t$ with the FOC for $C_t$ to get: $\lambda w_t = u_L(C_t, 1-N_t) = w_t u_C(C_t, 1-N_t)$.

iii. Combine FOC for $N_{t+1}$ with FOC for $C_{t+1}$ to get: $\beta u_L(C_{t+1}, 1-N_{t+1}) = \frac{\beta u_C(C_{t+1}, 1-N_{t+1})(1+r_t) w_{t+1}}{1+r_t} \implies u_L(C_{t+1}, 1-N_{t+1}) = u_C(C_{t+1}, 1-N_{t+1}) w_{t+1}$.

# Interpretation of Household Optimality Conditions

i. $u_C(C_t, 1 - N_t) = \beta u_C(C_{t+1}, 1 - N_{t+1})(1 + r_t)$ is the Euler equation and has same interpretation as in last lecture.

ii. $u_L(C_t, 1 - N_t) = w_t u_C(C_t, 1 - N_t)$ is the **static** labor supply equation for $N_t$, illustrating the consumption leisure trade off.

iii. Interpretation: If the household decides to consume one more unit of leisure today, it means supplying one less unit of labor supply, $u_L(C_t, 1 - N_t)$ is marginal utility of doing so. However, the opportunity cost is lower wages $w_t$, which translates into lost marginal utility of consumption of $u_C(C_t, 1 - N_t)$ today.

iv. $u_L(C_{t+1}, 1 - N_{t+1}) = u_C(C_{t+1}, 1 - N_{t+1})w_{t+1}$ is the static, labor supply equation for $N_{t+1}$ has the same interpretation as the one for $N_t$.

# Examples of Utility Functions

i. $u(C_t, 1 - N_t) = \ln(C_t) + \theta_t \ln(1 - N_t)$ is additively separable in consumption and leisure.

ii. $u(C_t, 1 - N_t) = \ln\left(C_t + \theta_t \ln(1 - N_t)\right)$ is not additively separable in consumption and leisure.

iii. $\theta_t$ is a *preference shock* and higher $\theta_t$ means increased preference for leisure, relative to consumption.

iv. What does separability between consumption and leisure mean?: $u_{CL} = 0$.

v. As an exercise, derive the Euler equation and the two consumption-leisure equations for $N_t$ and $N_{t+1}$ for each of the above utility functions. Check your answers against the expressions given in textbook.

# Household Consumption Function

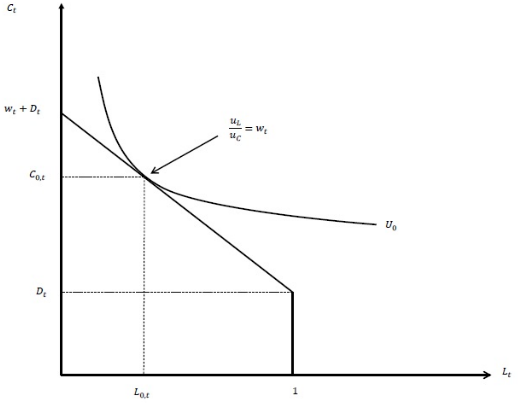
i. Just as in our last lecture, the consumption function has form $C_t = C\left(\underset{-}{r_t}, \underset{+}{Y_t}, \underset{+}{Y_{t+1}}\right)$.

ii. Consumption will increase if current income increases, but by less than the increase in current income ($MPC < 1$).

iii. Consumption will also increase if the household anticipates an increase in future income.

iv. There are competing income and substitution effects at work with regard to the real interest rate and we assume that the substitution effect dominates, so that consumption is decreasing in the real interest rate.

# Consumption Leisure Decision



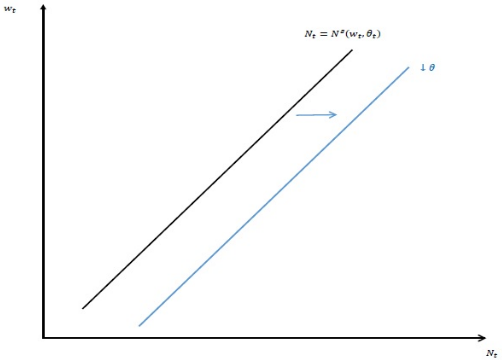
# Consumption-Leisure Choice

i. The FOC for $N_t$ from previous slides yields $\frac{u_L(C_t,1-N_t)}{u_C(C_t,1-N_t)} = w_t$, which is also evident in the graph on previous slide.

ii. Interpretation: The real wage is the slope of budget constraint with endogenous income, which must be equal to the slope of the indifference curve, when utility depends on consumption and leisure (remember inter micro!).

iii. We assume that substitution effect of higher $w_t$ dominates the wealth effect so that labor supply is increasing in $w_t$ and higher $\theta$ causes labor supply to reduce so that:
$N_t = N^s\left(\underset{+}{w_t}, \underset{-}{\theta}\right)$ is the generic form of labor supply function.

# Labor Supply Function



# Equilibrium

i. Equilibrium is defined as the set of prices and allocations under which all agents such as firms, households etc are optimizing their objectives *and all* markets clear *simultaneously*.

ii. The following slide writes down the 6 equilibrium conditions, with 6 endogenous variables.

iii. $Y_{t+1}$ is a future endogenous variable.

# Equilibrium

i. Consumption Function: $C_t = C\left(\underset{-}{r_t}, \underset{+}{Y_t}, \underset{+}{Y_{t+1}}\right)$.

ii. Labor Supply Function: $N_t = N^s\left(\underset{+}{w_t}, \underset{-}{\theta}\right)$.

iii. Labor Demand Function: $N_t = N^d\left(\underset{-}{w_t}, \underset{+}{A_t}, \underset{+}{K_t}\right)$.

iv. Investment Demand Function: $I_t = I^d\left(\underset{-}{r_t}, \underset{+}{A_{t+1}}, \underset{-}{K_t}\right)$.

v. Production Function: $Y_t = A_t F(K_t, N_t)$

vi. Goods Market Clearing Condition: $Y_t = C_t + I_t$.

# Modigilani Miller Theorem

i. In this model, we assumed that firm investment was borrowed or *debt financed* since the firm must borrow from financial intermediary.

ii. Alternatively, the firm could finance capital investment by issuing *equity* or issuing new shares.

iii. *Modigilani Miller Theorem:* Under some conditions, it does not matter whether the firm finances itself via debt or equity since the implied investment demand function is the same.

iv. The theorem assumes no taxes, no bankruptcy cost, and no asymmetric information between borrowers and lenders.

v. We will not cover the mathematical argument for the theorem, but you can read it from the textbook as an optional exercise.

**Thank you**