Exercise 4 – 1:

Solve the following equations (assume all denominators are non-zero):

1. $2y - 3 = 7$
2. $2c = c - 8$
3. $3 = 1 - 2c$
4. $4b + 5 = -7$
5. $-3y = 0$
6. $16y + 4 = -10$
7. $12y + 0 = 144$
8. $7 + 5y = 62$
9. $55 = 5x + \frac{3}{4}$
10. $5x = 2x + 45$
11. $23x - 12 = 6 + 3x$
12. $12 - 6x + 34x = 2x - 24 - 64$
13. $6x + 3x = 4 - 5(2x - 3)$
14. $18 - 2p = p + 9$
15. $\dfrac{4}{p} = \dfrac{16}{24}$
16. $-(-16 - p) = 13p - 1$
17. $3f - 10 = 10$
18. $3f + 16 = 4f - 10$
19. $10f + 5 = -2f - 3f + 80$
20. $8(f - 4) = 5(f - 4)$
21. $6 = 6(f + 7) + 5f$
22. $-7x = 8(1 - x)$
23. $5 - \dfrac{7}{b} = \dfrac{2(b+4)}{b}$
24. $\dfrac{x+2}{4} - \dfrac{x-6}{3} = \dfrac{1}{2}$
25. $1 = \dfrac{3a-4}{2a+6}$
26. $\dfrac{2-5a}{3} - 6 = \dfrac{4a}{3} + 2 - a$
27. $2 - \dfrac{4}{b+5} = \dfrac{3b}{b+5}$
28. $3 - \dfrac{y-2}{4} = 4$
29. $1{,}5x + 3{,}125 = 1{,}25x$
30. $1{,}3(2{,}7x + 1) = 4{,}1 - x$
31. $6{,}5x - 4{,}15 = 7 + 4{,}25x$
32. $\frac{1}{3}P + \frac{1}{2}P - 10 = 0$
33. $1\frac{1}{4}(x - 1) - 1\frac{1}{2}(3x + 2) = 0$
34. $\frac{1}{5}(x - 1) = \frac{1}{3}(x - 2) + 3$
35. $\dfrac{5}{2a} + \dfrac{1}{6a} - \dfrac{3}{a} = 2$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2F9C	2. 2F9D	3. 2F9F	4. 2F9G	5. 2F9H	6. 2F9J	7. 2F9K
8. 2F9M	9. 2F9N	10. 2F9P	11. 2F9Q	12. 2F9R	13. 2F9S	14. 2F9T
15. 2F9V	16. 2F9W	17. 2F9X	18. 2F9Y	19. 2F9Z	20. 2FB2	21. 2FB3
22. 2FB4	23. 2FB5	24. 2FB6	25. 2FB7	26. 2FB8	27. 2FB9	28. 2FBB
29. 2FBC	30. 2FBD	31. 2FBF	32. 2FBG	33. 2FBH	34. 2FBJ	35. 2FBK

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4.3 Solving quadratic equations EMA36

A quadratic equation is an equation where the exponent of the variable is at most 2. The following are examples of quadratic equations:

$$2x^2 + 2x = 1$$
$$3x^2 + 2x - 1 = 0$$
$$0 = -2x^2 + 4x - 2$$

Quadratic equations differ from linear equations in that a linear equation has only one solution, while a quadratic equation has at most two solutions. There are some special situations, however, in which a quadratic equation has either one solution or no solutions.

We solve quadratic equations using factorisation. For example, in order to solve $2x^2 - x - 3 = 0$, we need to write it in its equivalent factorised form as $(x + 1)(2x - 3) = 0$. Note that if $a \times b = 0$ then $a = 0$ or $b = 0$.

VISIT:
The following video shows an example of solving a quadratic equation by factorisation.
See video: 2FBM at www.everythingmaths.co.za

Method for solving quadratic equations EMA37

1. Rewrite the equation in the required form, $ax^2 + bx + c = 0$.
2. Divide the entire equation by any common factor of the coefficients to obtain an equation of the form $ax^2 + bx + c = 0$, where a, b and c have no common factors. For example $2x^2 + 4x + 2 = 0$ can be written as $x^2 + 2x + 1 = 0$.
3. Factorise $ax^2 + bx + c = 0$ to be of the form $(rx + s)(ux + v) = 0$.
4. The two solutions are $(rx + s) = 0$ or $(ux + v) = 0$, so $x = -\frac{s}{r}$ or $x = -\frac{v}{u}$, respectively.
5. Check the answer by substituting it back into the original equation.

Worked example 4: Solving quadratic equations

QUESTION

Solve for x:

$$3x^2 + 2x - 1 = 0$$

SOLUTION

Step 1: The equation is already in the required form, $ax^2 + bx + c = 0$
Step 2: Factorise

$$(x + 1)(3x - 1) = 0$$

Step 3: Solve for both factors
We have:

$$x + 1 = 0 \qquad \text{OR} \qquad 3x - 1 = 0$$
$$\therefore x = -1 \qquad \qquad \therefore x = \frac{1}{3}$$

Step 4: Check both answers by substituting back into the original equation
Step 5: Write the final answer
The solution to $3x^2 + 2x - 1 = 0$ is $x = -1$ or $x = \frac{1}{3}$.

Worked example 5: Solving quadratic equations

QUESTION

Find the roots:

$$0 = -2x^2 + 4x - 2$$

SOLUTION

Step 1: The equation is already in the required form, $ax^2 + bx + c = 0$

Step 2: Divide the equation by common factor -2

$$\begin{aligned} -2x^2 + 4x - 2 &= 0 \\ x^2 - 2x + 1 &= 0 \end{aligned}$$

Step 3: Factorise

$$\begin{aligned} (x-1)(x-1) &= 0 \\ (x-1)^2 &= 0 \end{aligned}$$

Step 4: The quadratic is a perfect square

This is an example of a special situation in which there is only one solution to the quadratic equation because both factors are the same.

$$\begin{aligned} x - 1 &= 0 \\ \therefore x &= 1 \end{aligned}$$

Step 5: Check the answer by substituting back into the original equation

Step 6: Write final answer

The solution to $0 = -2x^2 + 4x - 2$ is $x = 1$.

Exercise 4 – 2:

1. Write the following in standard form:
   a) $(r+4)(5r-4) = -16$
   b) $(3r-8)(2r-3) = -15$
   c) $(d+5)(2d+5) = 8$
2. Solve the following equations:
   a) $x^2 + 2x - 15 = 0$
   b) $p^2 - 7p - 18 = 0$
   c) $9x^2 - 6x - 8 = 0$
   d) $5x^2 + 21x - 54 = 0$
   e) $4z^2 + 12z + 8 = 0$
   f) $-b^2 + 7b - 12 = 0$
   g) $-3a^2 + 27a - 54 = 0$
   h) $4y^2 - 9 = 0$

i) $4x^2 + 16x - 9 = 0$

j) $4x^2 - 12x = -9$

k) $20m + 25m^2 = 0$

l) $2x^2 - 5x - 12 = 0$

m) $-75x^2 + 290x = 240$

n) $2x = \frac{1}{3}x^2 - 3x + 14\frac{2}{3}$

o) $x^2 - 4x = -4$

p) $-x^2 + 4x - 6 = 4x^2 - 14x + 3$

q) $t^2 = 3t$

r) $x^2 - 10x = -25$

s) $x^2 = 18$

t) $p^2 - 6p = 7$

u) $4x^2 - 17x - 77 = 0$

v) $14x^2 + 5x = 6$

w) $2x^2 - 2x = 12$

x) $(2a - 3)^2 - 16 = 0$

y) $(x - 6)^2 - 24 = 1$

3. Solve the following equations (note the restrictions that apply):

a) $3y = \dfrac{54}{2y}$

b) $\dfrac{10z}{3} = 1 - \dfrac{1}{3z}$

c) $x + 2 = \dfrac{18}{x} - 1$

d) $y - 3 = \dfrac{5}{4} - \dfrac{1}{y}$

e) $\dfrac{1}{2}(b - 1) = \dfrac{1}{3}\left(\dfrac{2}{b} + 4\right)$

f) $3(y + 1) = \dfrac{4}{y} + 2$

g) $(x + 1)^2 - 2(x + 1) - 15 = 0$

h) $z^4 - 1 = 0$

i) $b^4 - 13b^2 + 36 = 0$

j) $\dfrac{a + 1}{3a - 4} + \dfrac{9}{2a + 5} + \dfrac{2a + 3}{2a + 5} = 0$

k) $\dfrac{x^2 - 2x - 3}{x + 1} = 0$

l) $x + 2 = \dfrac{6x - 12}{x - 2}$

m) $\dfrac{3(a^2 + 1) + 10a}{3a + 1} = 1$

n) $\dfrac{3}{9a^2 - 3a + 1} - \dfrac{3a + 4}{27a^3 + 1} = \dfrac{1}{9a^2 - 1}$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2FBN	1b. 2FBP	1c. 2FBQ	2a. 2FBR	2b. 2FBS	2c. 2FBT	2d. 2FBV	2e. 2FBW
2f. 2FBX	2g. 2FBY	2h. 2FBZ	2i. 2FC2	2j. 2FC3	2k. 2FC4	2l. 2FC5	2m. 2FC6
2n. 2FC7	2o. 2FC8	2p. 2FC9	2q. 2FCB	2r. 2FCC	2s. 2FCD	2t. 2FCF	2u. 2FCG
2v. 2FCH	2w. 2FCJ	2x. 2FCK	2y. 2FCM	3a. 2FCN	3b. 2FCP	3c. 2FCQ	3d. 2FCR
3e. 2FCS	3f. 2FCT	3g. 2FCV	3h. 2FCW	3i. 2FCX	3j. 2FCY	3k. 2FCZ	3l. 2FD2
3m. 2FD3	3n. 2FD4						

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4.4 Solving simultaneous equations EMA38

Up to now we have solved equations with only one unknown variable. When solving for two unknown variables, two equations are required and these equations are known as simultaneous equations. The solutions are the values of the unknown variables which satisfy both equations simultaneously. In general, if there are n unknown variables, then n independent equations are required to obtain a value for each of the n variables.

An example of a system of simultaneous equations is:

$$x + y = -1$$
$$3 = y - 2x$$

We have two independent equations to solve for two unknown variables. We can solve simultaneous equations algebraically using substitution and elimination methods. We will also show that a system of simultaneous equations can be solved graphically.