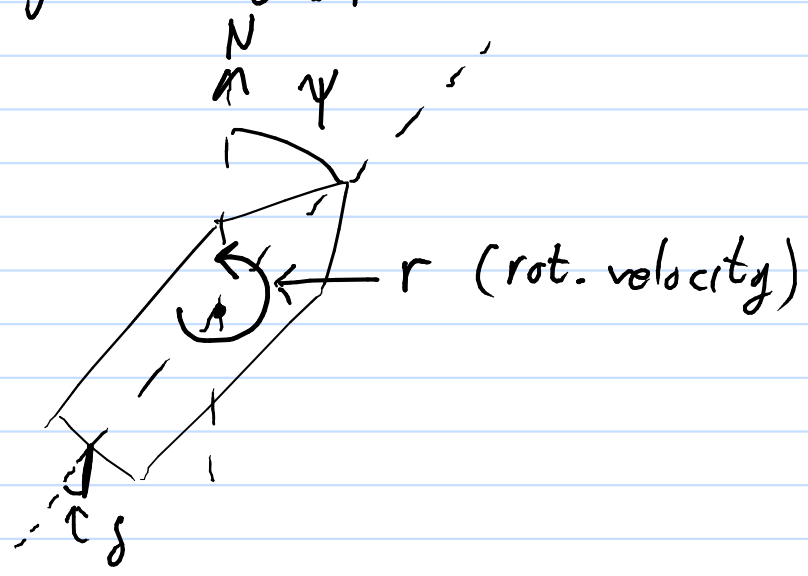
5.1

a) $H(s) = \frac{\Psi}{\delta}(s)$ $\quad \Psi$: Avg. heading in NED, δ: Rudder angle in BODY



(13 d) $\dot{r} = -\frac{1}{T}r + \frac{K}{T}\delta - \frac{K}{T}b$

(13 c) $\dot{\psi} = r$

$\boxed{b = 0}$

$sr = -\frac{1}{T}r + \frac{K}{T}\delta$

$r = s\Psi$

$s^2\Psi = -\frac{1}{T}s\Psi + \frac{K}{T}\delta$

$\left(s^2 + \frac{s}{T}\right)\Psi = \frac{K}{T}\delta \qquad \frac{\Psi}{\delta}(s) = \frac{K}{Ts^2 + s} = \underline{\underline{\frac{K/T}{s^2 + s/T}}}$

b) $|H(j\omega_1)| = \frac{58.66}{2} = 29.33, \quad \omega_1 = 0.005 \ rad/s$

$|H(j\omega_2)| = \frac{1.66}{2} = 0.83, \quad \omega_2 = 0.05 \ rad/s$

$|H(j\omega)| = \left|\frac{K/T}{-\omega^2 + j\omega/T}\right| = \frac{|K/T|}{\left|\frac{\omega}{T}j - \omega^2\right|} = \frac{|K/T|}{\sqrt{\frac{\omega^2}{T^2} + \omega^4}}$

Assuming $K > 0$ and $T > 0$

$K/T = 29.33\sqrt{\frac{0.005^2}{T^2} + 0.005^4} \rightarrow K = 29.33\sqrt{0.005^2 + 0.005^4 T^2}$

$K/T = 0.83\sqrt{\frac{0.05^2}{T^2} + 0.05^4} \rightarrow K = 0.83\sqrt{0.05^2 + 0.05^4 T^2}$

$K^2 = 29.33^2\left(0.005^2 + 0.005^4 T^2\right) \approx 2.151\times10^{-2} + 5.377\times10^{-7}T^2$

$K^2 = 0.83^2\left(0.05^2 + 0.05^4 T^2\right) \approx 1.722\times10^{-3} + 4.306\times10^{-6}T^2$

$\Rightarrow K^2 = 2.433\times10^{-2} \Rightarrow K = 1.560\times10^{-1}$ [1]

$T^2 = 5.251\times10^{3} \qquad T = 7.247\times10^{1}$ [sekund]

c) Step-response $= \mathcal{L}^{-1}\{\mathcal{L}\{u\}H(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\frac{K/T}{s^2 + s/T}\frac{\pi}{180}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{K/T}{s^3 + s^2/T}\frac{\pi}{180}\right\} = \frac{K\pi}{180T}\left(T^2 e^{-t/T} + tT - T^2\right) = K\left(Te^{-t/T} + t - T\right)\frac{\pi}{180}$

$= \left(KT\left(e^{-t/T} - 1\right) + Kt\right)\frac{\pi}{180}$

2b

$s\Psi_w = -\omega_0^2\frac{\Psi_w}{s} - 2\lambda\omega_0\Psi_w + K_w W_w$

$\Psi_w\left(s + \frac{\omega_0^2}{s} + 2\lambda\omega_0\right) = K_w W_w \Rightarrow \frac{\Psi_w}{W_w} = \underline{\underline{\frac{K_w s}{s^2 + 2\lambda\omega_0 s + \omega_0^2}}}$

$P_{\Psi_w}(\omega) = \underbrace{|H(j\omega)|^2}_{=1} P_{W_w}(\omega)$