TTK4115 - Boatlab report

Finn Matras (768661) and Sondre N. Andersen (768641)
Group 15

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Introduction

In this lab assignment, a simple autopilot for a boat was designed. This was done by discretizing, modeling and simulating parts of a continuous system. The system was influenced by stochastic signals, and thus Kalman filtering was implemented to filter the measurements based on statistical properties of the disturbances, giving more accurate estimates of the system states. Some parameters regarding both the system, and the nature of the disturbances were not explicitly given, and thus had to be identified.

Assignments

Problem 1

a)

To find the transfer function from δ to ψ the Laplace transform was taken of equations (13 d) and (13 c) from the lab assignment, giving

$$sr = -\frac{r}{T} + \frac{K}{T}\delta \tag{1}$$
$$r = s\psi \tag{2}$$

.

Substituting (2) into (1), and rearranging, gives

$$s^2\psi = -\frac{1}{T}s\psi + \frac{K}{T}\delta \tag{3}$$
$$\frac{\psi}{\delta}(s) = \frac{K}{Ts^2 + s} = \frac{1}{s}\frac{\frac{K}{T}}{s + \frac{1}{T}} \tag{4}$$

From (3) it can be seen that the transfer function includes a pure integrator. In the physical system, this reflects the fact that a constant rudder angle will cause the ship to move on a circular path, constantly increasing the angle. The rest of the transfer function corresponds to a low pass filter, that will filter out quick movements of the rudder.

b)

The boat parameters T and K were then found by using Simulink diagram "boat.mdl" from the lab assignment. The disturbances were turned off, a Sine-wave block connected to the input of the model, and the output connected to a data-out block, as seen in Figure 1. This was then run

ω	A
$0.005\,\mathrm{rad\,s^{-1}}$	29.35
$0.05\,\mathrm{rad\,s^{-1}}$	0.83

Table 1: Results from examining the gain to determine K and T.

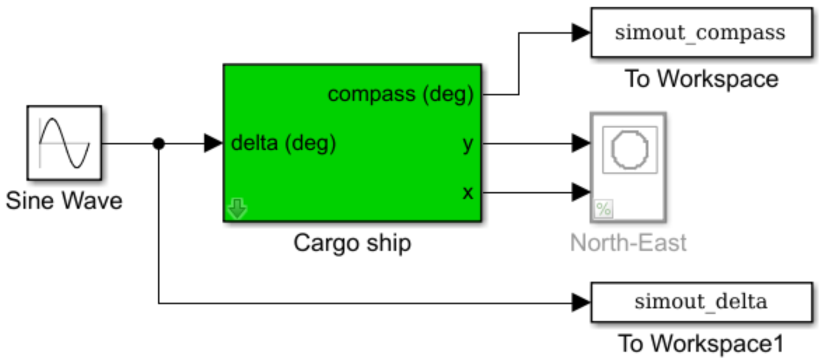


Figure 1: Simulink diagram for Part 1 b and c.

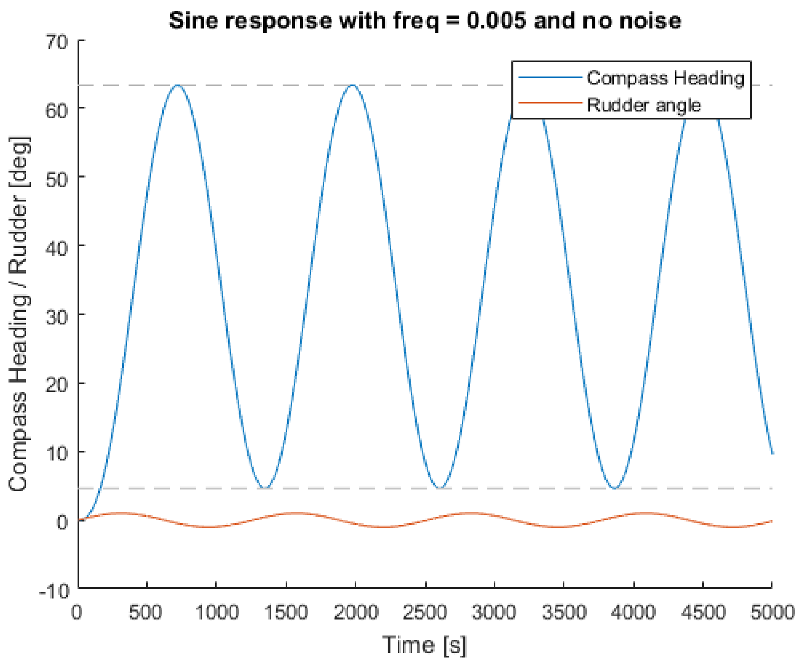


Figure 2: Response with $\omega_1 = 0.005$.

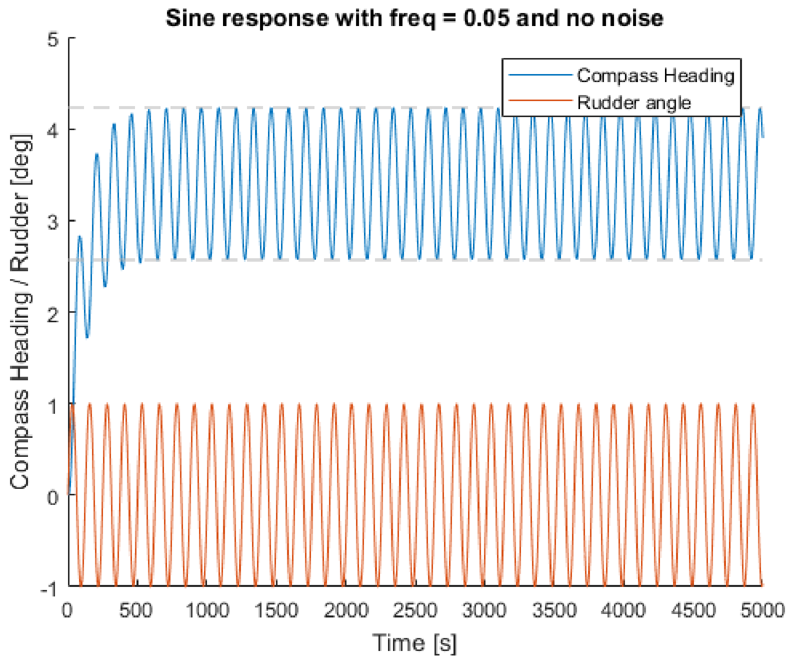


Figure 3: Response with $\omega_2 = 0.05$.

for $\omega_1 = 0.005$ and $\omega_2 = 0.05$. The resulting plotted data can be seen in Figure 2 and Figure 3. (Note the different scaling of the y-axis on these plots) The results are summarized in Table 1.

$A_1 = \frac{58.70}{2} = 29.35$ and $A_2 = \frac{1.66}{2} = 0.83$. Using these measurements, T and K were calculated using

$$|H(j\omega)| = \frac{|\frac{K}{T}|}{\sqrt{\frac{\omega^2}{T^2} + \omega^4}} \ , \tag{5}$$

calculating $|H(j\omega_1)| = A_1$ and $|H(j\omega_2)| = A_2$ and solving with regard to T and K, assuming both K and T positive, yields

$$K = 1.56 \times 10^{-1} \text{ and } T = 7.25 \times 10^1 \text{ s} \ . \tag{6}$$

c)

Now, the previous task was repeated, with wave and measurement noise turned on, to find the corresponding parameters for rough weather. The simulation results can be seen in Figure 4 and Figure 5. As one sees, for $\omega_2 = 0.05$, it is difficult to find an amplitude, but for a slower input signal, it is possible to see that if one would filter out the noise, the amplitude and parameters would be the same as in the previous task.

In this case, it is not easy to get good estimates for the parameters, as there is too much noise. It may be an idea to try with two lower frequencies, so that the noise can be filtered out, and the measurements will be less disturbed. At the current ω_1 and ω_2 it is therefore difficult to get good estimates of the boat parameters in rough weather.

d)

To compare the step input response of the ship and the model, a 1° step, was connected as the input of the ship, as seen in Figure 6. To find the response of the theoretical model, a unit step

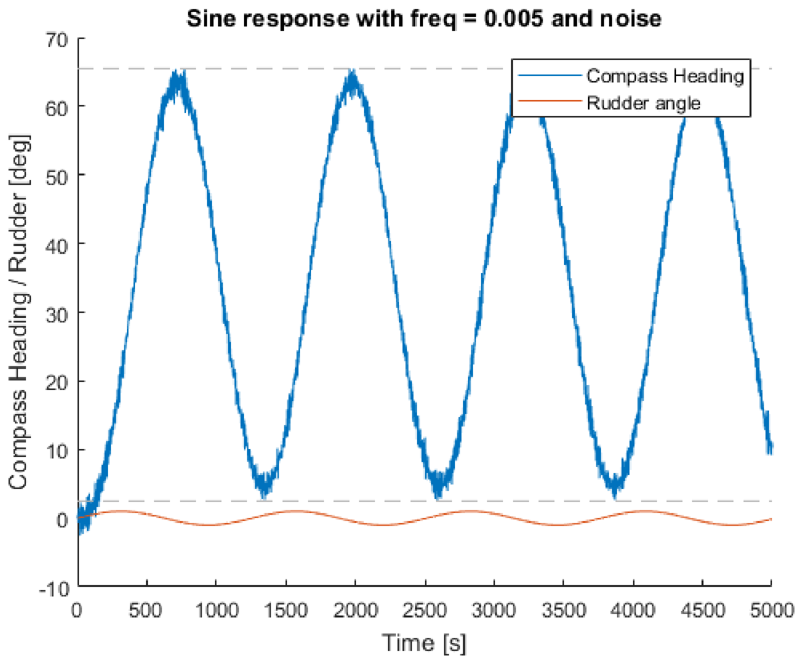


Figure 4: Response with $\omega_1 = 0.005$, waves and measurement noise.

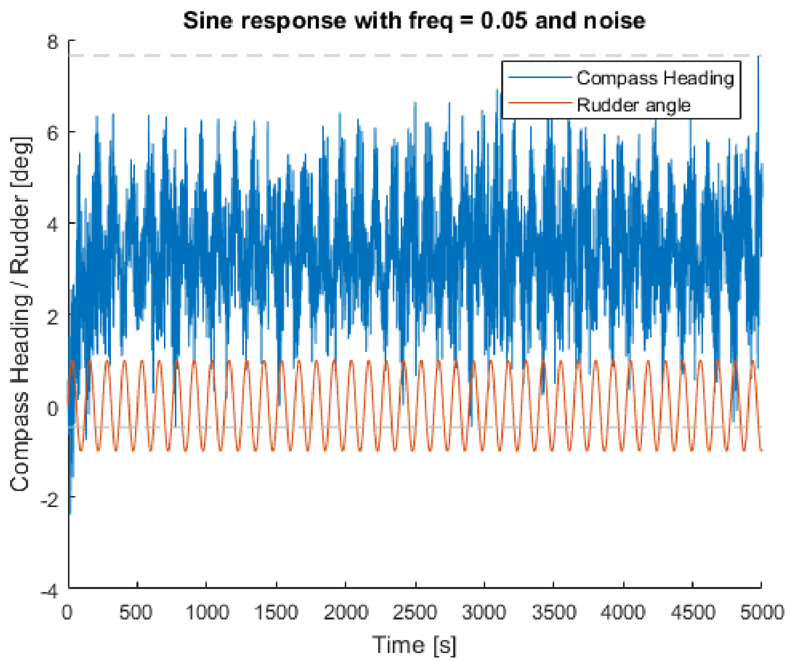


Figure 5: Response with $\omega_2 = 0.05$, waves and measurement noise.

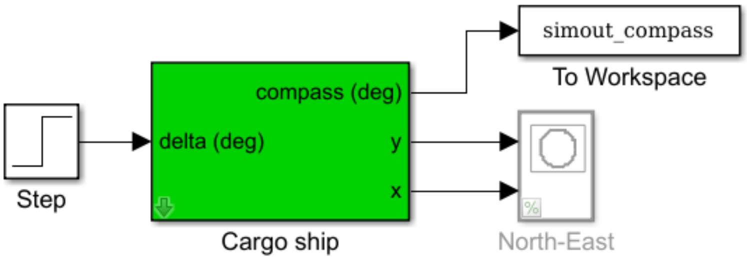


Figure 6: Simulink diagram with unit step as input.

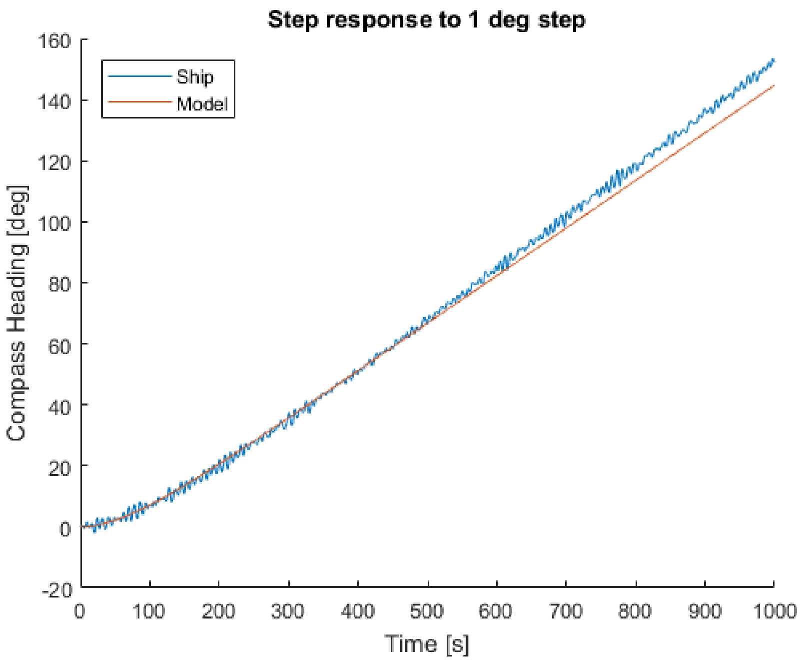


Figure 7: Plot of boat behaviour with unit step as input.

was multiplied with the transfer function in (3),

$$\psi_{step}(s) = H(s) = \frac{\frac{K}{T}}{(\frac{s^2}{T} + s^3)}, \tag{7}$$

and taking the inverse Laplace transform yields

$$\psi_{step}(t) = KT(e^{\frac{-t}{T}} - 1) + Kt. \tag{8}$$

This expression shows the slow transient response where the current angular velocity must be changed, and the constant accumulation of heading caused by a constant rudder angle. A plot of this behaviour can be seen in Figure 7.

As one can see in Figure 7, both models are approximately the same, however, there is a small discrepancy in slope, this may be due to inaccuracy in determining the gain, K, from the amplitude of the response. After 1000 s, the difference is about 5°. Based on this, the model can be seen as a good model for the real system.

Listing 1: Matlab script used to find PSD plot of data from wave.mat file.

```
sample_freq = 10; % Hz
window_size = 4096;
TWOPI = 2 * pi;

% Compute Power Spectral Density
[pxx, f] = pwelch(psi_w(2,:), window_size, sample_freq);

% Plot pxx vs. f in a double log plot
plot(log(f.*TWOPI), log(pxx./TWOPI));
grid on;
title('Power Spectral Density - Wave Influence');
xlabel('Frequency log(rad/s)');
ylabel('Power log(s/rad)');
```

Problem 2

a)

To find an estimate for the Power Spectral Density function of the ship, a Matlab script was written, as shown in Listing 1. The resulting graph of the Power Spectral Density function of ψ_w, $S_{\psi_w}(\omega)$ is shown in blue in Figure 8. The plot shows a clear peak where the resonant frequency of the ship is, and falling response for higher frequencies. A peculiar result from this plot is that the disturbance seems to have a non-zero power for low frequencies. This does not make much physical sense, as the waves are transient events, and should average out over long periods. A further discussion of this is given at the end of problem 2.

b)

Using equation 13a and 13b from the lab-assignment to find the analytical expression for the transfer function of the wave response model, taking the Laplace transforms, and inserting 13a into 13b yields

$$s\psi_w(s) = -\omega_0^2 \frac{\psi_w(s)}{s} - 2\lambda\omega_0\psi_w(s) + K_w w_w(s) \tag{9}$$

rearranging with respect to ψ_w and w_w gives

$$\frac{\psi_w}{w_w} = \frac{K_w s}{s^2 + 2\lambda\omega_0 s + \omega_0^2}. \tag{10}$$

Using (3.2.2) from the textbook [1]

$$P_{\psi_w}(\omega) = \left|\frac{\psi_w(j\omega)}{w_w(j\omega)}\right|^2 S_f(j\omega) \tag{11}$$

with $S_f(j\omega)$ equal to 1 for white noise. Inserting $\frac{\psi_w}{w_w}$ as $G(j\omega)$, yields

$$P_{\psi_w}(\omega) = \frac{K_w^2\omega^2}{\omega_0^4 + \omega^4 + (4\lambda^2 - 2)\omega_0^2\omega^2}. \tag{12}$$

[1] Robert G. Brown, Patrick Y. C. Hwang (2012), *Introduction to Random Signals and Applied Kalman Filtering*, John Wiley & Sons Ltd.

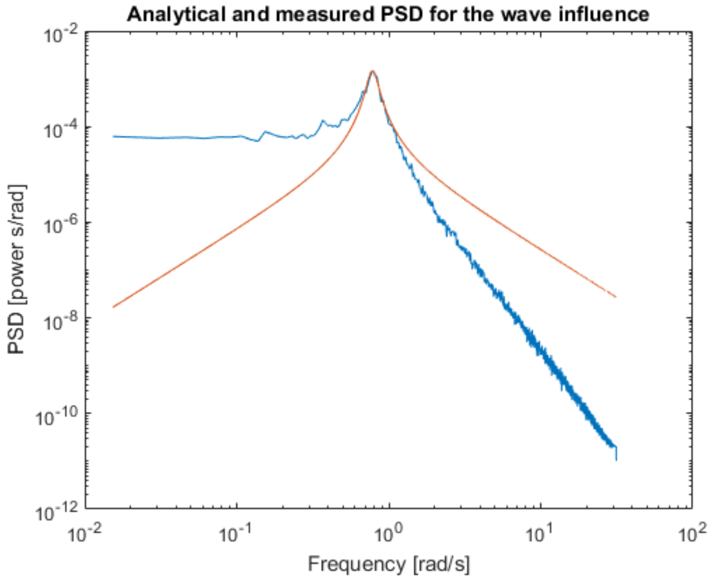


Figure 8: PSD plot of data from wave.mat file. The orange curve is the fitted model, and the blue is the data from wave.mat

c)

The resonant frequency, ω_0, of the system should correspond to the highest peak of the power spectral density plot. This will be the least attenuated frequency, and therefore the one that will take the longest to die out.

After finding the PSD with matlab, the peak can easily be extracted using the `max()` function. Using the syntax `[peak_value, peak_index] = max(ppx)` returns both the peak value, and the index of the peak. ω_0 can then be found by indexing the frequency array returned by the `pwelch` function, with `peak_index`.

ω_0 was found to be equal to $0.78\,\mathrm{rad\,s^{-1}}$. This can also be seen from the location of the peak in Figure 8. This value seems reasonable for a large ship, as it would take a few seconds to swing back and forth.

d)

Defining $K_w = 2\lambda\omega_0\sigma = 5.15 \times 10^{-3}\,\sqrt{(\mathrm{power} \times \mathrm{rad})/\mathrm{s}}$, where σ^2, the peak value of $P_{\psi_w}(\omega)$, was found using the Matlab function `[peak_value, peak_index] = max(pxx)`. The graph of this function can be seen in Figure 8. The peak value is measured to be 1.48×10^{-3} power $\times$ s/rad, resulting in $\sigma = 38.5 \times 10^{-3}\,\sqrt{\mathrm{s/rad}}$.

To find the dampening factor, λ, the script in Listing 2 was run to get the best fitting of the curve, resulting in $\lambda = 8.55 \times 10{-}2$.

Correction

As one can see in Figure 8, the PSD-function based on the model, and the one from the handed out data set does not correspond at low frequencies. This difference is not so easily visible in a normal plot, but in a logarithmic plot it can be easily seen. It seems like the model has one differentiator more, than there is in the data-set. To check if this is the case, the transfer function found in (10), was implemented in Simulink, with white noise as input. The corresponding Simulink diagram is shown in Figure 9, and the resulting PSD-plot is shown in Figure 10. This was then repeated, but now without the s in the numerator. The Simulink diagram is shown in Figure 11. This is the same as the wave.mat file that is handed out, which is shown in Figure 8. As one can

Listing 2: Matlab script used to curve fit the PSD function to the data from wave.mat file.

```
% Define function to fit
% x is a placeholder the unknown parameter, lambda
fun = @(x,w)...
    (2*x*w0*sigma)^2.*w.^2./...
    (w.^4+w0^2.*w.^2*(4*x^2-2)+w0^4);
lambda0 = 0.5; % Initial guess
lambda = lsqcurvefit(fun, lambda0, w, pxx);
```

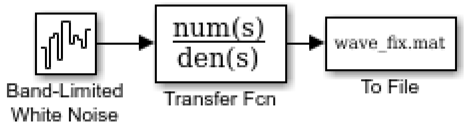


Figure 9: Simulink diagram for generating new wave influence data, according to the correct transfer function for the system.

see, the later PSD is not based on the same model as the equations given in the lab-task. An implementation was also made from equation (13a) and (13b) as a time-space model, and the plot was the same as for the transfer function shown in Figure 10. This shows that the wave.mat file probably was generated with an error in the transfer-function, resulting in incorrect data, which may give incorrect readouts when trying to fit the curves, giving slightly wrong parameters. The Matlab and Simulink files for this part can be found in "p5p2_fix.m" and "p5p2_model _fix.mdl".

With the new PSD-data file, the following values for K, σ and λ were found:

$$K = 3.27 \times 10^{-3}\ \sqrt{\text{power} \times \text{rad/s}} \tag{13}$$

$$\sigma = 3.01 \times 10^{-2}\ \sqrt{\text{s/rad}} \tag{14}$$

$$\lambda = 6.94 \times 10^{-2} \tag{15}$$

Problem 3

a)

To design a PD-controller of the form

$$H_{pd}(s) = K_{pd}\frac{1 + T_d s}{1 + T_f s}, \tag{16}$$

equations (13c) and (13d) from the lab-assignment were used to find the equation for $\ddot{\psi}$

$$\ddot{\psi} = -\frac{1}{T}r + \frac{K}{T}(\delta - b). \tag{17}$$

Taking the Laplace transform, setting all disturbances to zero and rearranging yields:

$$\frac{\psi}{\delta}(s) = \frac{\frac{K}{T}}{s^2 + \frac{s}{T}}. \tag{18}$$

Multiplying $H_{pd}(s)H_{ship}(s)$, and choosing $T_d = T$ to cancel the transfer function time constant, gives an open loop transfer function as shown below.

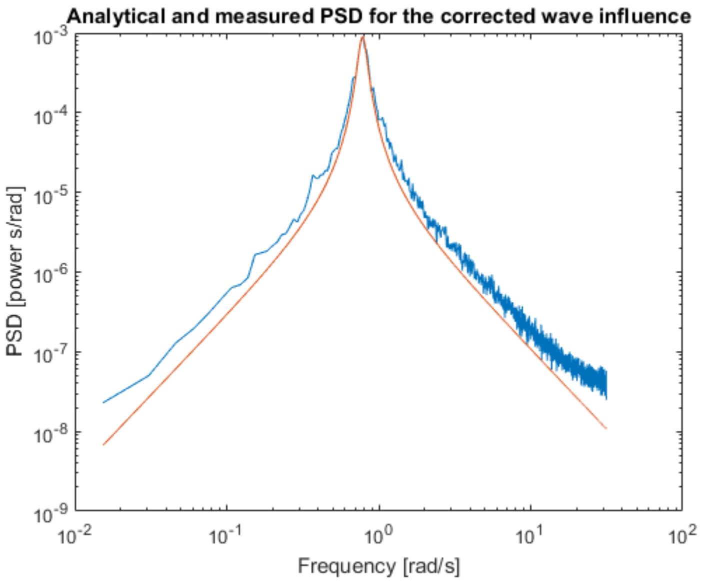


Figure 10: PSD plot of new corrected wave influence file. The orange curve is the fitted model, and the blue is the data from new, fixed wave_fix.mat file.

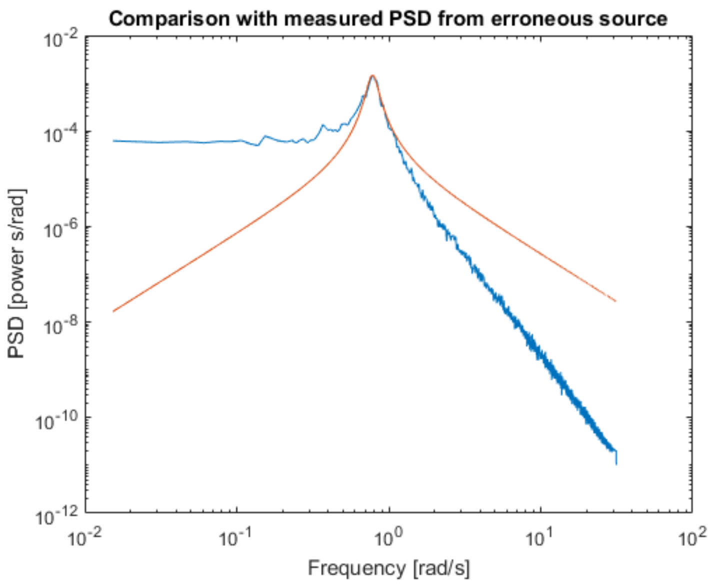


Figure 11: PSD plot of data from wave.mat file. The orange curve is the fitted model, and the blue is the data from the generated data, based on the wrong model.

Listing 3: Matlab script used to find K_{pd} and T_f for the PD regulator, with the wanted phase margin and ω_c.

```
% Nomoto constants
K = 0.1560;

% PD regulator parameters
Tf = 8.402;
Kpd = 0.8370;

H0 = tf(K * Kpd, [Tf 1 0]);

margin(H0);
```

$$H_{pd}(s)H_{ship}(s) = \frac{KK_{pd}}{s + T_f s^2} \tag{19}$$

Using trial and error and the Matlab script shown in Listing 3, the values for K_{pd} and T_f were found, giving a phase margin $= 50°$ and $\omega_c = 0.1\,\text{rads}$

The corresponding Bode diagram is shown in Figure 12.

b)

The transfer function was implemented in a Simulink diagram, as shown in Figure 14, and the output graph showing the ship heading is shown in Figure 13. As one can see from the figures the rudder angle oscillates between about $\pm 1°$, with a steady state standard deviation of 0.337° and mean of $-0.0014°$. During the first few seconds there is a high spike in rudder angle, and the signal gets saturated. However, the PD-regulator manages to keep the ship following the 30° reference, with steady state mean of 30.0° and a standard deviation of 0.047°.

The autopilot works, but the rudder input has some high frequency components, this would wear out the actuator over time, so a better controller would not try to compensate for these high frequencies, knowing that their effect will average out to zero.

c)

To find the step response of the regulator with current disturbance, the current disturbance was turned on in the Simulink diagram, and the response to the 30° reference is shown in Figure 15. Now the steady state mean and standard deviation for the rudder angle are 0.337° and 3.00° respectively. For the ship heading, the corresponding standard deviation and mean are 0.047° and 26.41°. There is steady state deviation from the reference in the heading. This is because the PD-regulator can not take into account the current disturbance, as it is close to constant.

This autopilot is not able to follow the reference. The rudder input still has some high frequency components that would wear out the actuator over time. In addition the boat does not manage to track the reference, but follows an erroneous course as a result of the rudder bias not being compensated for.

d)

To find the behaviour with both wave disturbance and measurement noise, they were turned on in the ship model. The ship behaves as shown in Figure 16. Now, the ship follows the reference with a stationary standard deviation of 0.802° and a mean of 30.0°. The regulator now has very high and frequent spikes on the output, that would wear out the actuator even faster than before, if it is able to keep up with the control signal at all. Based on this, the system does not work

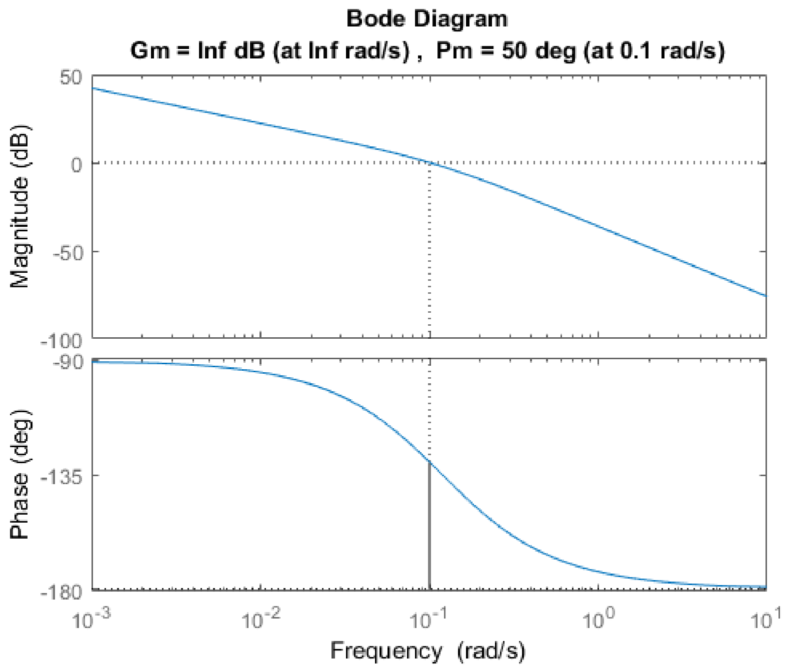


Figure 12: Bode diagram of PD-regulator with calculated values.

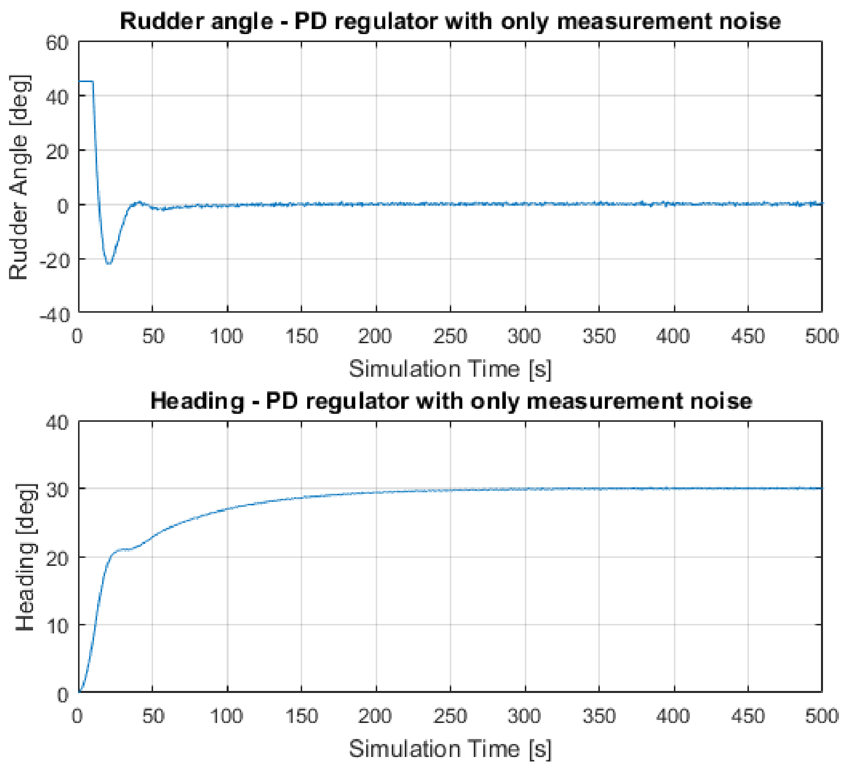


Figure 13: Ship rudder angle and heading with 30° step input, measurement noise, and no other disturbances.

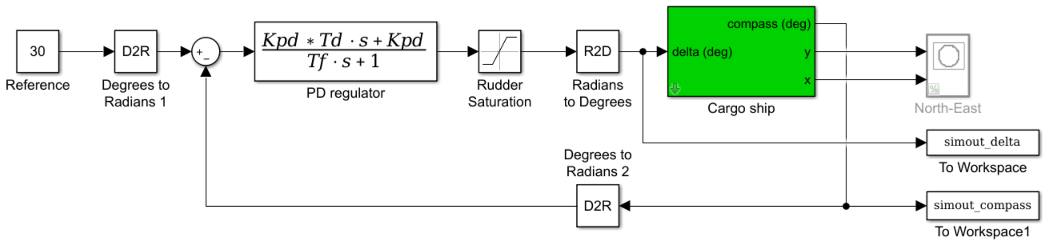


Figure 14: Simulink diagram of boat system and regulator with 30° step input.

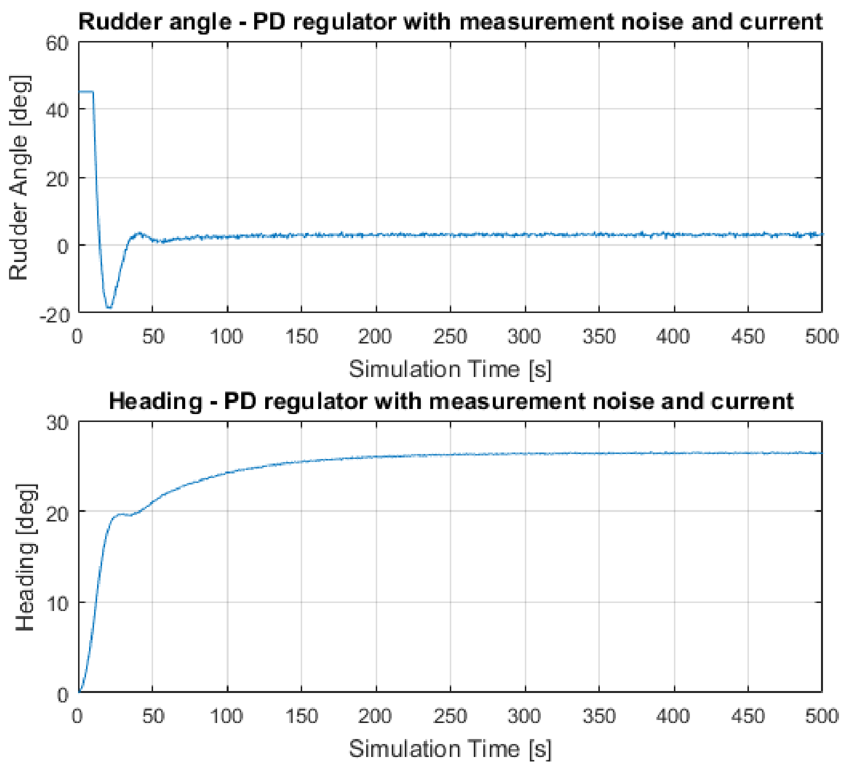


Figure 15: Ship rudder angle and heading with 30° step input, measurement noise, and current disturbances.

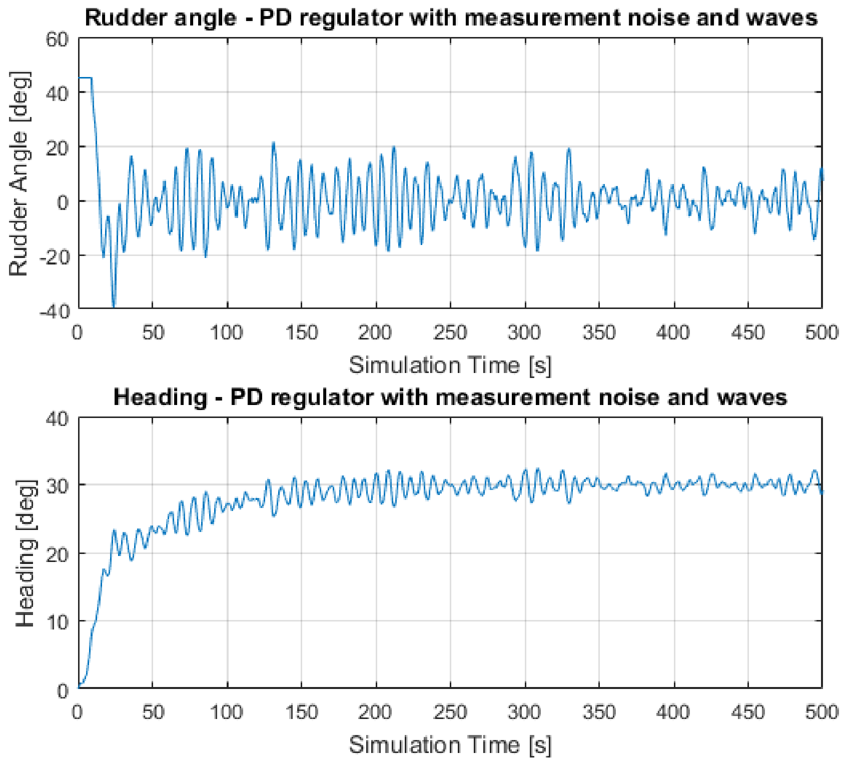


Figure 16: Ship rudder angle and heading with 30° step input, measurement noise, and wave disturbance.

satisfactory, as the regulator would probably destroy the rudder actuator in a short period of time.

Problem 4

a)

The matrices **A**, **B**, **C** and **E** in the state space model of the system can be found using equations (13a) through (13f), and are as shown in (20).

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega_0^2 & -2\lambda\omega_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{T} & -\frac{K}{T} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K}{T} \\ 0 \end{bmatrix} \mathbf{C} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \end{bmatrix} \mathbf{E} = \begin{bmatrix} 0 & 0 \\ K_w & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{20}$$

b)

To check if the system is observable without disturbances, matrices **A** and **C** were recalculated using equations (13a) through (13f) without disturbances gives

$$\mathbf{A_{4b}} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T} \end{bmatrix} \mathbf{C_{4b}} = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{21}$$

Using the command `rank(obsv(A,C))`, calculating the rank of the observability-matrix, the rank equals 2, so it is observable.

c)

Again, recalculating the matrices $\mathbf{A}$ and $\mathbf{C}$, now with current disturbance gives

$$\mathbf{A_{4c}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{1}{T} & -\frac{K}{T} \\ 0 & 0 & 0 \end{bmatrix} \mathbf{C_{4c}} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{22}$$

Using the command `rank(obsv(A,C))`, calculating the rank of the observability-matrix, the rank equals 3, so it is observable.

d)

Again, recalculating the matrices $\mathbf{A}$ and $\mathbf{C}$, this time with wave disturbance yields

$$\mathbf{A_{4d}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_0^2 & -2\lambda\omega_0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{T} \end{bmatrix} \mathbf{C_{4d}} = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} \tag{23}$$

Using the command `rank(obsv(A,C))`, calculating the rank of the observability-matrix, the rank equals 4, so it is observable.

e)

Using the command `rank(obsv(A,C))` on the matrices in (20) calculating the rank of the observability-matrix, the result equals 4. Thus the observability matrix has full rank, and is observable.

As shown, the system is observable for all combinations of measurement noise and disturbances. This makes sense, as all the states ultimately affects the heading, which is the measured variable.

Problem 5

a)

Using Matlab, with the script in Listing 4, giving the exact discretization, the new, discretized system from the matrices in (20) becomes

$$\mathbf{x}[k+1] = \mathbf{A_d}\mathbf{x}[k] + \mathbf{B_d}\mathbf{u}[k] + \mathbf{E_d}\bar{\mathbb{w}}[k] = e^{\mathbf{A}T}\mathbf{x}[k] + \left(\int_0^T e^{\mathbf{A}v} dv \right) \mathbf{B}\mathbf{u}[k] + \mathbf{E_d}\bar{\mathbb{w}}[k] \tag{24}$$

$$\mathbf{y}[k] = \mathbf{C_d}\mathbf{x}[k] + \bar{\mathbb{v}}[k] \tag{25}$$

with matrices

$$\mathbf{A_d} = \begin{bmatrix} 0.9970 & 0.0965 & 0 & 0 & 0 \\ -0.0587 & 0.9308 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.0999 & -1.0758 \times 10^{-5} \\ 0 & 0 & 0 & 0.9986 & -2.1511 \times 10^{-4} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{26}$$

$$\mathbf{B_d} = \begin{bmatrix} 0 \\ 0 \\ 1.0758 \times 10^{-5} \\ 0.0002 \\ 0 \end{bmatrix} \tag{27}$$

Listing 4: Matlab script used to discretize the ship model using exact discretization.

```
% Discretisation
[Ad, Bd] = c2d(A, B, Ts);
[~, Ed] = c2d(A, E, Ts);

% Loans method
loan_exponent = [...
   A         E * Q * E';
   zeros(5) -A'
   ];

loan_m = expm(loan_exponent * Ts);
Qd = loan_m(1:5, 6:10) * loan_m(1:5, 1:5)';

Rd = R / Ts;
```

$$\mathbf{E_d} = \begin{bmatrix} 1.3141 \times 10^-4 & 0 \\ 2.5971 \times 10^-3 & 0 \\ 0 & -3.5865 \times 10^-7 \\ 0 & -1.0758 \times 10^-5 \\ 0 & 0.1000 \end{bmatrix} \tag{28}$$

and the matrix $\mathbf{C}$ stays the same.

This might be an oversimplification, as the $\mathbf{E_d}$ matrix is calculated by treating the white noise process, $\bar{\mathtt{w}}$, as an input to the system in the same way as $\mathbf{u}$. Since the derivation of $\mathbf{E_d}$ using the `c2d` function assumes a constant input during each sample period, and the white-noise disturbance is not constant. This method might result in slightly wrong results. Although the white noise disturbance has just as much power in this high part of the spectrum, the model of the ship attenuates frequencies this high very effectively. Thus, even if the signal itself can not be assumed constant during the sample period, its effect on the ship probably can.

b)

To find an estimate of the variance of the measurement noise, the rudder-input, δ, was set to 0, and turning only measurement noise on, similar to the ship behavior in still water with no wind. And taking the variance of the output, using the Matlab-command `var(y)`. The resulting estimate for the variance of the measurement noise is 2.0×10^{-3} square degrees. As one can see, the variance is fairly small and is probably negligible in comparison to the parameters of the ship.

In the discretized system, this noise will be averaged over each sample period, resulting in a lower overall variance. To find this variance, the variance of the continuous noise must be divided by the sample period, giving a variance for the discrete noise of $\frac{2.0 \times 10^{-3}}{0.1} = 2.0 \times 10^{-4}$.

c)

Using the matrices in equation (15) from the corrected lab-assignment, a Kalman Filter was implemented as an S-function in the "Sfunctionshell.m" file using the discretized system. This S-function was then included in the Simulink diagram as shown in Figure 17. The update function is shown in Listing 5.

The estimated states from the Kalman Filter and the actual measurements are shown in Figure 18. As one can see, the Kalman Filter follows the measurements closely, but it does not

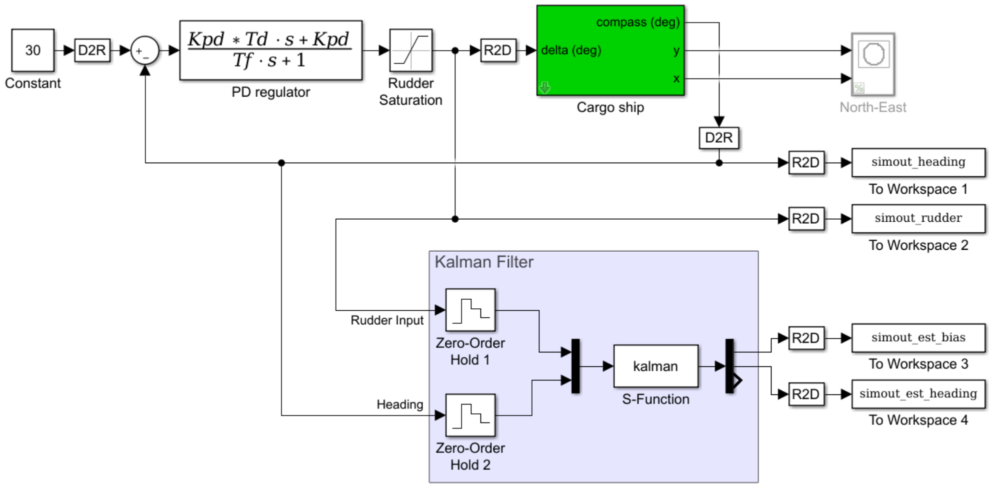


Figure 17: Simulink diagram of ship with PD-regulator and Kalman Filter.

Listing 5: S-function implementation of the Kalman Filter.

```
function sys = mdlUpdate(t,x,u,data)
   % Extract covariance matrix from x
   Pm_vec = x(11:35);
   n = sqrt(length(Pm_vec));
   Pm = reshape(Pm_vec, n, n);

   % Calculate Kalman filter gain, L
   L = Pm * data.C' / (data.C * Pm * data.C' + data.R);

   % Update estimate
   x_priori = x(1:5);
   x_posteriori = x_priori + L * (u(2) - (data.C * x_priori));

   % Update covariance matrix
   P = (eye(5) - (L * data.C)) * Pm * ...
      (eye(5) - (L * data.C))' + ...
      L * data.R * L';

   % Project ahead
   x_priori = data.A * x_posteriori + data.B * u(1);
   Pm = data.A * P * data.A' + data.Q;

   % Return new filter state vector
   sys = [x_priori; x_posteriori; Pm(:)];
end
```

have all the high frequency components that the measurements do. It is worth noticing that there is still a stationary deviation due to no feed forward from the estimated bias or an integral term in the regulator.

d)

To eliminate the bias, a feed forward was connected as shown in the Simulink diagram in Figure 19. The autopilot has a lot less high frequency and lower amplitude actuation's, making it a lot calmer than the autopilot from Part 3c. The actuator is exposed to significantly less levels of wear, and it is probably able to keep up with the actuation signal from the regulator. The stationary deviation seen in the previous task, and Part 3c, is now removed by the feed forward from the estimated bias in the Kalman Filter.

e)

In addition to the feed forward from the estimated bias, the filtered heading was used as input to the PD-regulator. As in the previous task, the autopilot now works a lot better, as the output signal of the regulator has a lot lower frequency and amplitude, making it more efficient and increases the life expectancy of the actuator, while the ship still follows the reference.

The Kalman Filtered estimate and actual wave influence is shown in Figure 23. The estimate does not follow the wave influence perfectly, but the estimate follows the measurement quiet closely. As one can see, the Kalman Filter is both able to remove the high-frequency components from the output of the regulator, as well as the nearly constant disturbance of the current.

With corrected parameters

Now, testing the ship model and Kalman Filter with the parameters found with the corrected data-set, one can see in Figures 25 and 23, that there is more high frequency noise on the measured signal with the corrected parameters. This can also be seen from Figure 11 and 10, as Figure 11 has a lot less gain at higher frequencies than the plot in Figure 10. The Kalman Filter is still able to filter out this increased noise, and the ship follows the reference. The Kalman Filter is robust enough, so that even with slightly wrong parameters based on a wrong model, it does a pretty good job of filtering and estimating the states for both the correct and the wrong parameters.

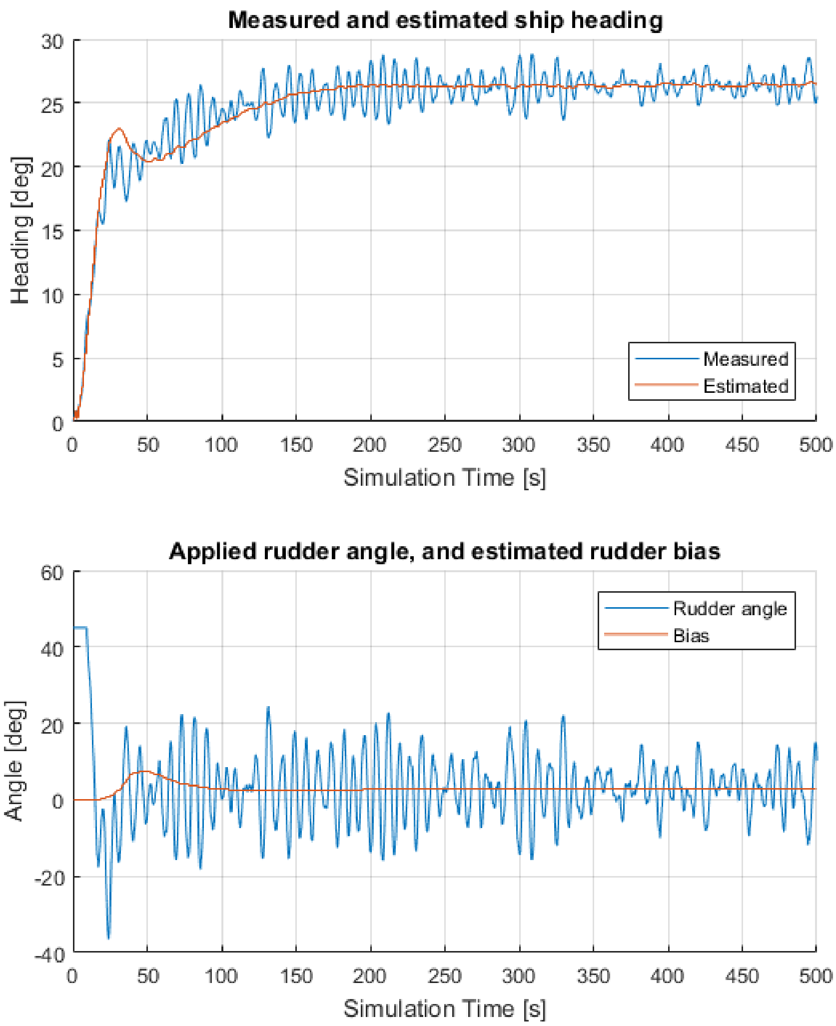


Figure 18: Plot of ship with PD-regulator and Kalman Filter estimation of heading and rudder input.

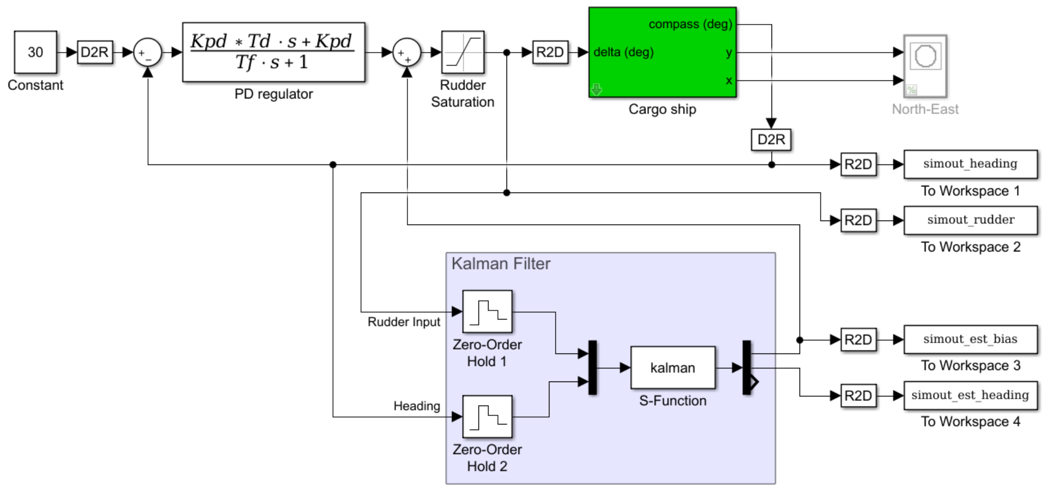


Figure 19: Simulink diagram of ship with PD-regulator and feed forward bias canceling from the Kalman Filter.

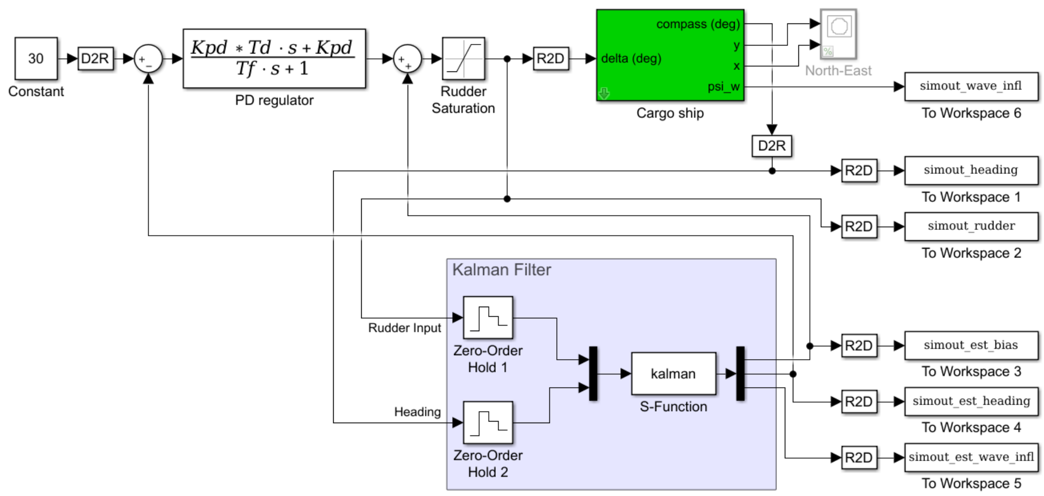


Figure 20: Simulink diagram of ship with PD-regulator and estimated heading and bias from Kalman Filter.

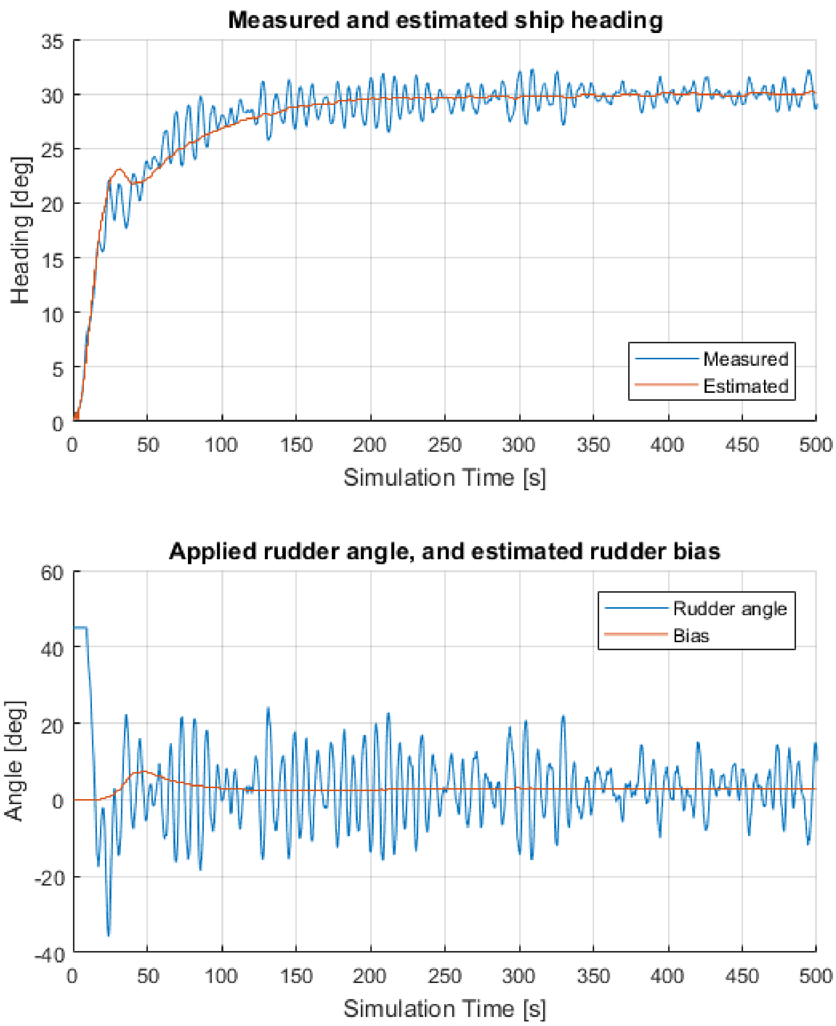


Figure 21: Plot of ship with PD-regulator and Kalman Filter estimation of heading and rudder input, with bias cancellation from the Kalman Filter.

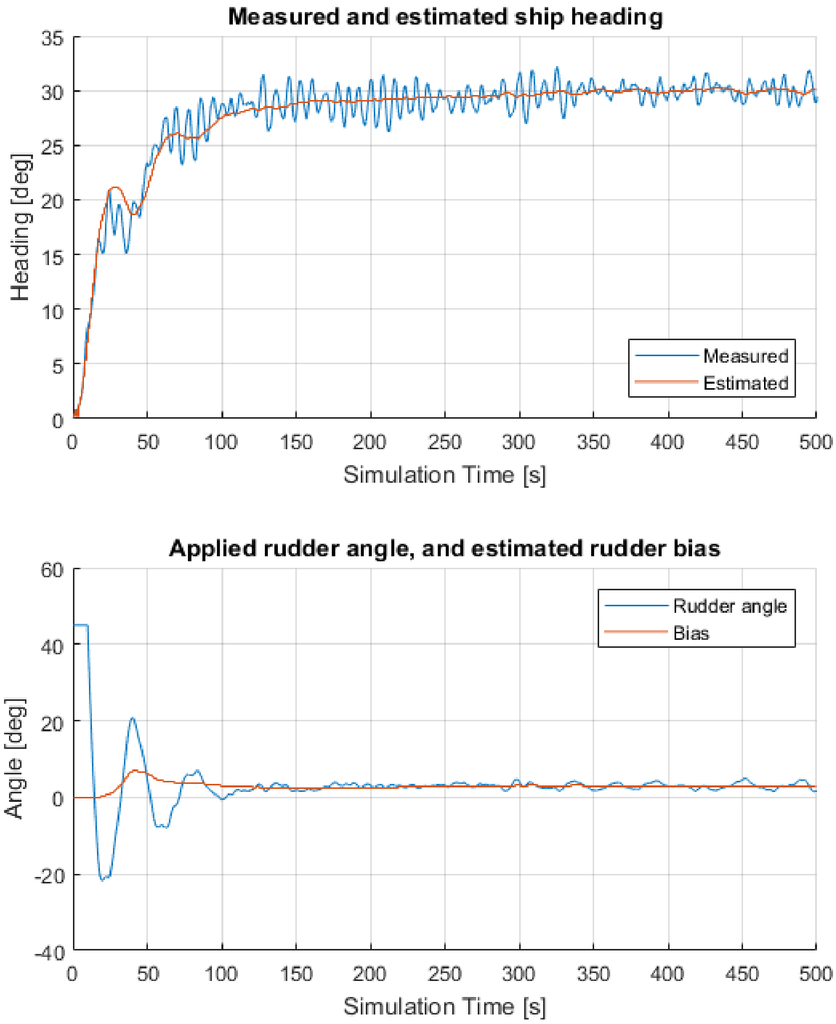


Figure 22: Plot of ship with PD-regulator and Kalman Filter estimation of heading and rudder input.

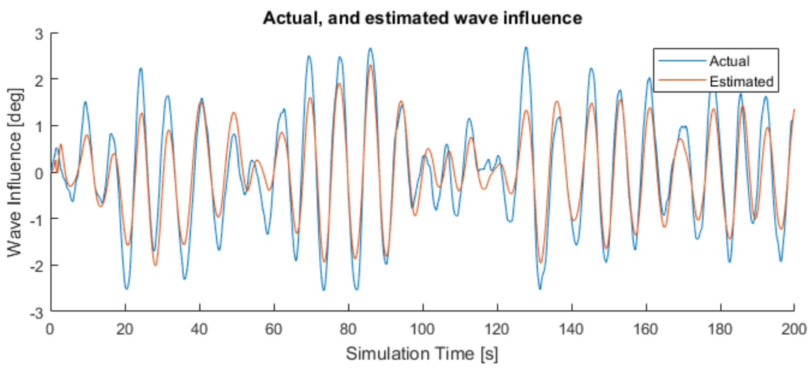
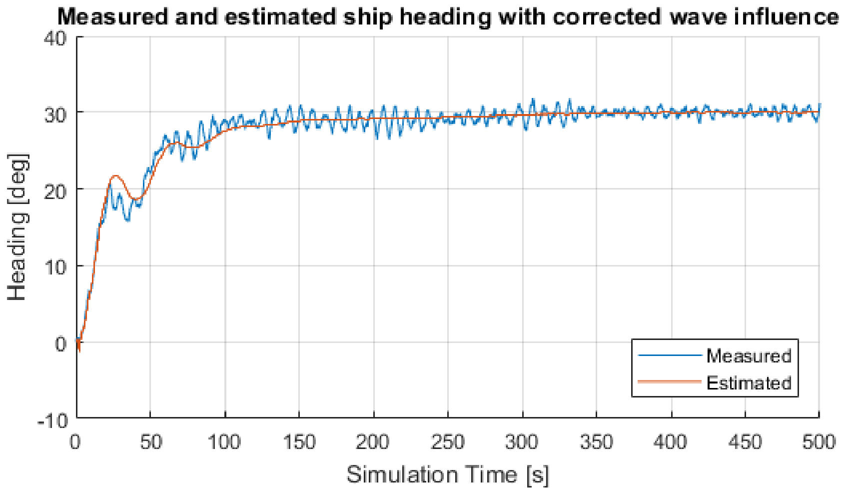


Figure 23: Plot of wave influence and Kalman Filter estimated wave influence with Kalman Filtered states as input to the regulator. With wave and current disturbance.



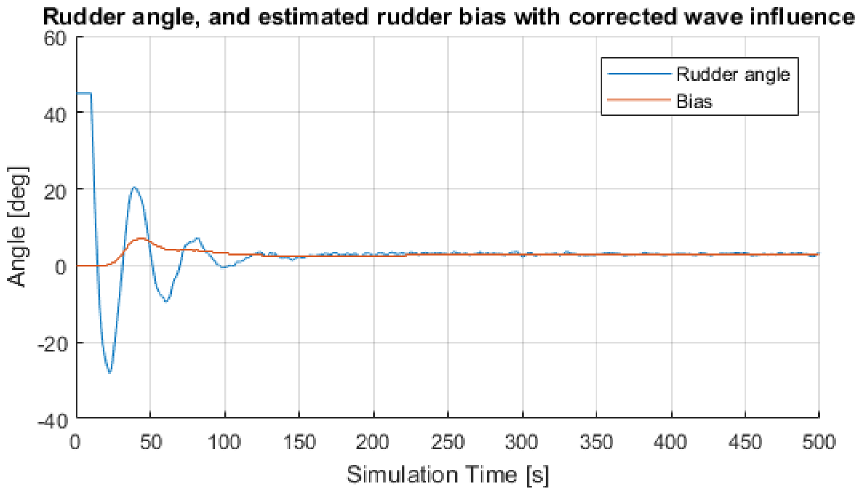


Figure 24: Plot of ship with PD-regulator and Kalman Filter estimation of heading and rudder input, with corrected parameters found in part 2.

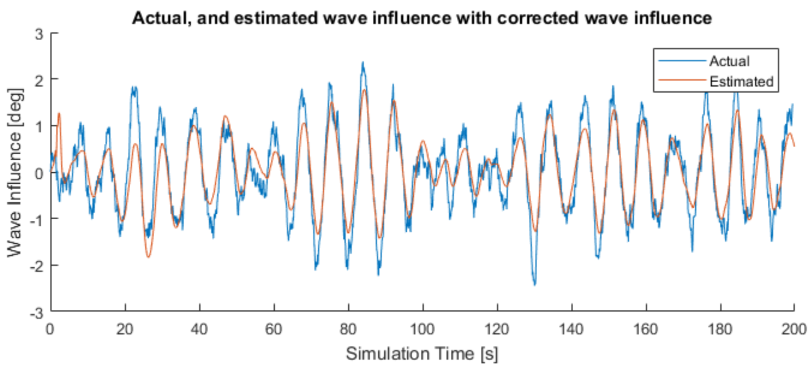


Figure 25: Plot of wave influence and Kalman Filter estimated wave influence based on corrected parameters found in part 2.