Lecture 11: Practical use of MPC: Output feedback, target calculation and offset-free control

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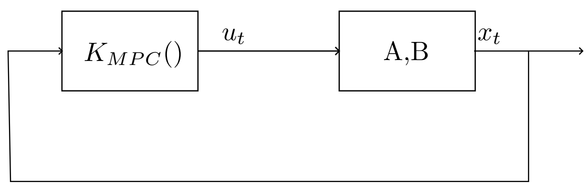
1 MPC controller - state feedback

MPC controller algorithm:

1. For t = 0, 1, 2, ..., ∞ do
2. Determine x_t
3. Solve open-loop opt. prob. with initial state $x_0 = x_t$
4. $\rightarrow u_0, u_1, u_2, \dots, u_{N-1}$
5. Set $u_t = u_0$
6. end



Observation: MPC controller is non-linear state feedback. We determine x_t with measurements, or state estimate.

Reference tracking (regulation) Say that we have a plant that we want to design an MPC for. A typical control objective is that we want controlled variables $\gamma_t \rightarrow \gamma_{ref}$ where $\gamma_t = Hx_t$. Note: Controlled variables $\gamma_t = Hx_t$ may be different from measured variables $y_t = Cx_t$.

Steady state:

$$\begin{aligned} x_s &= Ax_s + Bu_s \rightarrow x_s = (I - A)^{-1}Bu_s \\ \gamma_s &= Hx_s = H(I - A)^{-1}Bu_s \end{aligned}$$

Example:

$$\begin{aligned} A &= \begin{bmatrix} 0.8 & 0.4 \\ -0.1 & 0.9 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 0.5 \\ 0 & 2 \end{bmatrix}, \ H = \begin{bmatrix} 1 & -1 \end{bmatrix} \\ \gamma_s &= \begin{bmatrix} 3.33 & 8.33 \end{bmatrix} u_s \end{aligned}$$

Observe: (i) Input constraints limit possible γ_s. $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \leq u \leq \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow 0 \leq \gamma_s \leq 11.66$. With input constraints, we cannot control our system to wherever we want. (ii) Several u_s give same γ_s (in this case). If $u_s = \begin{bmatrix} 0 \\ 0.24 \end{bmatrix} \rightarrow \gamma_s = 2.0$. Also, $u_s = \begin{bmatrix} 0.17 \\ 0.17 \end{bmatrix} \rightarrow \gamma_s = 2.0$

The solution to this problem is target calculation. It can be formulated as a QP.

$$\begin{aligned}
\min \quad & \frac{1}{2} u_s^\top R_s u_s \\
\text{s.t.} \quad & x_s = Axs + Bu_s \\
& Hx_s = \gamma_{ref} \\
& x^{low} \leq x_s \leq x^{high} \\
& u^{low} \leq u_s \leq u^{high}
\end{aligned}$$

2 Offset-free control

Also known as integral control. This is a sort of robust control where you remove the effect of unknown disturbances. This i sthe rolw of 'I' in PID control, which we don't have in MPC so far. An unmodelled disturbance d_t will give an offset in the controlled variable γ_t. Model with disturbance

$$\begin{aligned}
x_{t+1} &= Ax_t + Bu_t + A_d d_t \\
y_t &= Cx_t + C_d d_t
\end{aligned}$$

The disturbance model:

$$d_{t+1} = d_t$$

Idea: Augment the model in MPC to get

$$\begin{aligned}
\begin{bmatrix} x_{t+1} \\ d_{t+1} \end{bmatrix} &= \begin{bmatrix} A & A_d \\ 0 & I \end{bmatrix} \begin{bmatrix} x_t \\ d_t \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_t \\
y_t &= \begin{bmatrix} c & c_d \end{bmatrix} \begin{bmatrix} x_t \\ d_t \end{bmatrix}
\end{aligned}$$

Offset-free MPC:

- Use state estimator with the above model to estimate $\hat{x}_t and \hat{d}_t$
- Use the model above as model in MPC.

Note:

- Requires (A_{a_1}, C_a) to be observable. A practical requirement to solve the problem of having to measure too many disturbances: $\dim(\hat{d}_t) \leq \dim(y_t)$
- Typical industrial practice: $A_d = 0,\ C_d = I$ ('bias update'). OFten works well, especially when the process is slow. But it does not always work. The advantage of this method is that you do not need a state estimator.
- Target calculation must be modified to depend on $\hat{d}$.