NTNU
Kunnskap for en bedre verden

TMA4135 - Matematikk 4D

Execise #4

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Problem 1

a)

$$\text{Max} z = 5x_1 + 3x_2 + x_3$$

s.t.:

$$x_1 + x_2 + 3x_3 \leq 6$$
$$5x_1 + 3x_2 + 6x_3 \leq 15$$
$$x_1, x_2, x_3 \geq 0$$

Standard form:

$$x_1 + x_2 + 3x_3 + s_1 = 6$$
$$5x_1 + 3x_2 + 6x_3 + s_2 = 15$$

	z	x_1	x_2	x_3	s_1	s_2	RHS	Forholdstest
z	1	-5	-3	-1	0	0	0	
s_1	0	1	1	3	1	0	6	6
s_2	0	5	3	6	0	1	15	3

Ta x_1 inn i basis og s_1 ut.

	z	x_1	x_2	x_3	s_1	s_2	RHS	Forholdstest
z	1	0	0	5	0	1	15	
s_1	0	0	0.4	1.8	1	-0.2	3	0
s_2	0	1	0.6	1.2	0	0.2	3	0

Svar: z = 15, $(x_1, x_2, x_3) = (3, 0, 0)$

b)
Ja, den optimale løsningen sier at bare 1 produkt må produseres.

c)
Marketing kan se på løsninger som ikke er fullstendig optimale, men nær nok. Disse løsningene kan bruke litt mindre x_1 og litt mer av de to andre.

d)
Dual:

$$\pi_1 = 1 \quad (\text{for restriksjon } x_1 + x_2 + 3x_3 \leq 6)$$
$$\pi_2 = 2 \quad (\text{for restriksjon } 5x_1 + 3x_2 + 6x_3 \leq 15)$$

Disse representerer økning i objektfunksjonen for en enhets økning. En økning i den første ressursen vil øke profitten med 1, mens en økning i den andre vil øke profitt med 2.

Problem 2

1. From complementary slackness:

$2x_1 + x_2 + x_3 = 10$ (binding) $\implies v_1 = -1$ (from given solution)

$Cx_1 + x_3 = -1 \implies v_2 = 8 > 0$

$x_2 + 2x_3 = B \implies v_3 = 0$

2. From dual constraints:

$2v_1 - Cv_2 \leq 3 \implies -2 - 8C \leq 3 \implies C = 1$ (to match primal constraint)

$v_1 + v_2 + 2v_3 \geq A \implies -1 + 8 + 0 \geq A \implies A \leq 7$

3. From primal constraints:

$x_2 + 2x_3 = B \implies 11 - 2 = B \implies B = 9$

4. From objective function equality:

$3x_1 - x_2 + Ax_3 = 10v_1 - v_2 + 5v_3$

$-11 + A(-1) = -10 - 8 + 0 \implies A = 3$

5. For $v_3 : E = 0$ (free variable)

6. For $v_1 : v_1 + v_2 + 2v_3 = D \implies -1 + 8 + 0 = D \implies D = 7$

Problem 3

Primal Problem:

$\max z = x_1 + 2x_2 + x_3 + x_4$

subject to:

$2x_1 + x_2 + 5x_3 + x_4 \leq 8$ (corresponds to y_1)

$2x_1 + 2x_2 + 4x_4 = 12$ (corresponds to y_2)

$3x_1 + x_2 + 2x_3 \geq 18$ (corresponds to y_3)

$x_1, x_2, x_4 \geq 0, \quad x_3 \leq 0$

Dual Problem:

$\min w = 8y_1 + 12y_2 + 18y_3$

subject to:

$2y_1 + 2y_2 + 3y_3 \geq 1$

$y_1 + 2y_2 + y_3 \geq 2$

$5y_1 + 2y_3 \leq 1$

$y_1 + 4y_2 \geq 1$

$y_1 \geq 0, \quad y_3 \geq 0, \quad y_2$ unrestricted

Problem 4

Primal Problem:

$\min z = 2x_1 + 2x_2 + x_3 - x_4 + x_5$

subject to:

$x_1 + 2x_2 - x_3 + 2x_4 = 6 \quad (v_1)$

$2x_1 + x_3 + x_5 \geq 4 \quad (v_2)$

$3x_2 - 2x_3 + 3x_4 = 7 \quad (v_3)$

$x_1, x_2, x_3, x_4, x_5 \geq 5$

a) Dual feasibility:

Dual constraints:

$v_1 + 2v_2 \leq 2$

$2v_1 + 3v_3 \leq 2$

$-v_1 + v_2 - 2v_3 \leq 1$

$2v_1 + 3v_3 \leq -1$

$v_2 \leq 1$

Checking feasibility with $v = (1, 0, -1)$:

$1 + 2(0) \leq 2 \quad \text{(Satisfied)}$

$2(1) + 3(-1) \leq 2 \quad \text{(Satisfied)}$

$-1 + 0 - 2(-1) \leq 1 \quad \text{(Satisfied)}$

$2(1) + 3(-1) \nleq -1 \quad \text{(Not satisfied)}$

$0 \leq 1 \quad \text{(Satisfied)}$

The dual solution is not feasible.

b) Corresponding primal solution:

From complementary slackness:

$x_1 + 2x_2 - x_3 + 2x_4 = 6$

$2x_1 + x_3 + x_5 = 4$

$3x_2 - 2x_3 + 3x_4 = 7$

This system is underdetermined. No unique solution.

Any solution must satisfy $x_1, x_2, x_3, x_4, x_5 \geq 5$ to be primal feasible.

c) Optimality:

The dual solution is not feasible, therefore it cannot be optimal.