Distributional information

- A random variable X has a **Normal** distribution $N(\mu, \sigma^2)$, $\mu \in \mathbb{R}$, $\sigma > 0$, if its probability density function is

$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \qquad -\infty < x < \infty.$$

  $E[X] = \mu$, $Var[X] = \sigma^2$

- A random variable X has a **Gamma** distribution $\text{Gamma}(\alpha, \beta)$, $\alpha > 0$, $\beta > 0$, if its probability density function is

$$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\beta x), \qquad x > 0.$$

  $E[X] = \frac{\alpha}{\beta}$, $Var[X] = \frac{\alpha}{\beta^2}$

- A random variable X has an **Inverse Gamma distribution** $\text{InvGamma}(\alpha, \beta)$, $\alpha > 0$, $\beta > 0$, if its probability density function is

$$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{-(\alpha+1)} \exp\left(-\frac{\beta}{x}\right), \qquad x > 0.$$

  $E[X] = \frac{\beta}{\alpha-1}$ for $\alpha > 1$, $Var[X] = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$ for $\alpha > 2$

- A random variable X has an **Exponential** distribution $\text{Exp}(\lambda)$, $\lambda > 0$, if its probability density function is

$$\lambda \exp(-\lambda x), \qquad x > 0.$$

  $E[X] = \frac{1}{\lambda}$, $Var[X] = \frac{1}{\lambda^2}$

- A random variable X has a **Beta** distribution $\text{Beta}(\alpha, \beta)$, $\alpha > 0$, $\beta > 0$, if its probability density function is

$$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \qquad x \in (0,1).$$

  $E[X] = \frac{\alpha}{\alpha+\beta}$, $Var[X] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

- A random variable X has a **Binomial** distribution $\text{Bin}(n, p)$, $n \in \mathbb{N}_+$, $p \in [0, 1]$, if its probability mass function is

$$\binom{n}{x} p^x (1-p)^{n-x}, \qquad x = 0, 1, 2, \ldots$$

  $E[X] = np$, $Var[X] = np(1-p)$

- A random variable X has a **Geometric** distribution $\mathrm{Geom}(\theta)$, $\theta \in (0, 1]$, if its probability mass function is

$$\theta(1 - \theta)^{x-1}, \qquad x = 1, 2, 3, \ldots$$

$\mathrm{E}[X] = 1/\theta$, $\mathrm{Var}[X] = 1/\theta^2$

- A random variable X has a **Poisson** distribution $\mathrm{Poisson}(\lambda)$, $\lambda > 0$ if its probability mass function is

$$\frac{\exp(-\lambda)\lambda^x}{x!}, \qquad x = 0, 1, 2, \ldots$$

$\mathrm{E}[X] = \lambda$, $\mathrm{Var}[X] = \lambda$

- A random variable X has a **Negative-Binomial** distribution $\mathrm{NegBin}(\alpha, \beta)$, $\alpha > 0$, $\beta > 0$, if its probability mass function is

$$\frac{\Gamma(x + \alpha)}{\Gamma(x + 1)\Gamma(\alpha)} \left(\frac{\beta}{\beta + 1}\right)^\alpha \left(\frac{1}{\beta + 1}\right)^x, \qquad x = 0, 1, 2, \ldots$$

Note: $\Gamma(y) = (y - 1)!$ for integers $y = 1, 2, 3, \ldots$.
$\mathrm{E}[X] = \frac{\alpha}{\beta}$, $\mathrm{Var}[X] = \frac{\alpha(\beta+1)}{\beta^2}$

- A random vector $\boldsymbol{X} = (X_1, \ldots, X_d)$ has a **Multivariate Normal** distribution $\mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $\boldsymbol{\mu} \in \mathbb{R}^d$, $\boldsymbol{\Sigma}$ a symmetric positive definite matrix, if its probability density function is

$$\frac{1}{(2\pi)^{d/2}\sqrt{\det(\boldsymbol{\Sigma})}} \exp\left(-\frac{(\boldsymbol{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\boldsymbol{x} - \boldsymbol{\mu})}{2}\right), \qquad \boldsymbol{x} \in \mathbb{R}^d.$$

$\mathrm{E}[\boldsymbol{X}] = \boldsymbol{\mu}$, $\mathrm{Var}[\boldsymbol{X}] = \boldsymbol{\Sigma}$, $\mathrm{Prec}[\boldsymbol{X}] = \boldsymbol{\Sigma}^{-1}$

- A random vector $(X_1, \ldots, X_K)$ has a **Multinomial** distribution $\mathrm{Multinom}(n, p_1, \ldots, p_K)$, $n \in \mathbb{N}_+$, $p_i \in [0, 1]$, $\sum_{i=1}^K p_i = 1$, if its probability mass function is

$$\frac{n!}{x_1! \ldots x_K!} p_1^{x_1} p_2^{x_2} \ldots p_K^{x_K}, \qquad x_i = 0, 1, 2, \ldots, n, \qquad \sum_{k=1}^K x_k = n$$

$\mathrm{E}[X_i] = np_i$, $\mathrm{Var}[X_i] = np_i(1 - p_i)$, $\mathrm{Cov}[X_i, X_j] = -np_ip_j$ for $i \neq j$

- A random vector $(X_1, \ldots, X_K)$ has a **Dirichlet** distribution $\mathrm{Dirichlet}(\alpha_1, \alpha_2, \ldots, \alpha_K)$, $\alpha_i > 0$, if its probability density function is

$$\frac{1}{B(\boldsymbol{\alpha})} \prod_{i=1}^K x_i^{\alpha_i - 1}, \qquad x_i \in (0, 1) \text{ with } \sum_{i=1}^K x_i = 1$$

where $B(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^K \Gamma(\alpha_i)}{\Gamma(\alpha_0)}$ with $\alpha_0 = \sum_{i=1}^K \alpha_i$.
$\mathrm{E}[X_i] = \frac{\alpha_i}{\alpha_0}$, $\mathrm{Var}[X_i] = \frac{\alpha_i(\alpha_0 - \alpha_i)}{\alpha_0^2(\alpha_0 + 1)}$, $\mathrm{Cov}[X_i, X_j] = \frac{-\alpha_i\alpha_j}{\alpha_0^2(\alpha_0 + 1)}$ for $i \neq j$