# Unparticle: A Review

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(Dated: December 26, 2024)

**ABSTRACT**

We give here a brief review of the proposed hidden sector of an high energy theory which has scale invariant sector at a low energy, which has been dubbed *unparticles*, with some phenomenology and discuss cosmological consequences of this new sector.

## I. INTRODUCTION

Standard Model (SM) as we know of does not have scale invariance as it is evident that scale invariance requires a non definite mass. But there could be a theory in high energy with an unseen sector in infrared (IR) which is exactly scale invariant and weakly interacting.

In 2007, Georgi [1] first came forth with this idea and called the scale invariant stuff *unparticles* to emphasise on the unusual nature of it. In the following, we briefly summarise his reasoning.

Consider an high energy theory with the fields of SM fields together with a Banks-Zaks($BZ$) field which has a nontrivial fixed IR point. These two sets of fields couple with the generic form

$$\frac{1}{M_U^k} O_{sm} O_{BZ} \tag{1}$$

where $M_U^k$ is the mass of the exchanged particle. Below an energy scale $\Lambda_U$ in the effective theory we get the unparticle operators

$$\frac{C_U \Lambda^{d_{BZ}-d_U}}{M_U^k} O_{sm} O_U \tag{2}$$

with $C_U$ being the coefficient function and $d$'s are the scale dimensions of the respective fields. We may justify the use of these two terms by the following

**:** The interactions in (1) does not effect the IR scale invariance

**:** $M_U$ can be large enough to explain the lack of experimental observations.

Now we can move on to producing unparticles. Consider an unparticle state, $|P\rangle$, with momentum $P^\mu$ and the vacuum matrix element

$$\langle 0|O_{\mathcal{U}}(x)O_{\mathcal{U}}^{\dagger}(0)|0\rangle = \int e^{-ipx}\, |\langle 0|O_{\mathcal{U}}(0)|P\rangle\,|^2 \rho\left(P^2\right) \frac{d^4P}{(2\pi)^4}$$

Now we use the IR scale invariance condition that the matrix element scales with $2d_U$ which demands that

$$|\langle 0|O_{\mathcal{U}}(0)|P\rangle\,|^2 \rho\left(P^2\right) = A_{d_{\mathcal{U}}} \Theta(P^0)\Theta(P^2)(P^2)^{d_{\mathcal{U}}-2}$$

Georgi then makes the comparison with the Lorentz invariant phase space of n massless particles

$$(2\pi)^4 \delta^4(P - \sum_{j=1}^{n} p_j) \quad \prod_{j=1}^{n} \delta\left(p_j^2\right) \theta\left(p_j^0\right) \tfrac{d^4 p_j}{(2\pi)^3} \tag{3}$$

$$= A_n \theta\left(P^0\right) \theta\left(P^2\right) \left(P^2\right)^{n-2} \tag{4}$$

Taking the limit $n \to 1$ from above reduces this equation to a 1-unparticle state. This leads to the conclusion

> **Unparticles with scale dimension $d_U$ behave like a collection of $d_U$ massless particles.**

which has the interesting result of having "fractional" particles since $d_U$ can be non-integral.

## II. PHENOMENOLOGY

After Georgi's first publication, there have been numerous phenomenological treatments of unparticles [2]. Georgi himself had followed up [3] with an interesting paper that shows the peculiar nature of unparticles. In this section we briefly summarise its content.

### A. $e^+e^- \to \mu^-\mu^+$ Reaction

Georgi starts with the interaction term

$$\frac{C_{V\mathcal{U}} \Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k} \bar{e}\gamma_\mu e O_{\mathcal{U}}^\mu + \frac{C_{A\mathcal{U}} \Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k} \bar{e}\gamma_\mu \gamma_5 e O_{\mathcal{U}}^\mu \tag{5}$$

with the assumption that the unparticle operator is transverse and hermitian, i.e. $\partial_\mu O_U^\mu = 0$.

Then following the recipe from the previous part, we get the unparticle propagator

$$
\begin{aligned}
&\int e^{iPx}\langle 0|T\left(O^{\mu}_{\mathcal{U}}(x)O^{\nu}_{\mathcal{U}}(0)\right)|0\rangle d^4x \\
&= i\frac{A_{d_{\mathcal{U}}}}{2\pi}\int_0^{\infty}\left(M^2\right)^{d_{\mathcal{U}}-2}\frac{-g^{\mu\nu}+P^{\mu}P^{\nu}/P^2}{P^2-M^2+i\epsilon}dM^2 \\
&= i\frac{A_{d_{\mathcal{U}}}}{2}\frac{-g^{\mu\nu}+P^{\mu}P^{\nu}/P^2}{\sin\left(d_{\mathcal{U}}\pi\right)}\left(-P^2-i\epsilon\right)^{d_{\mathcal{U}}-2}
\end{aligned}
\tag{6}
$$

The cross section for this reaction, after rescaling the coefficients in Eq. (5) to the $Z$ mass, reads

$$
\begin{aligned}
|\mathcal{M}|^2 = 2\left(q^2\right)^2 \Big[&\left(\left|\Delta_{VV}\left(q^2\right)\right|^2+\left|\Delta_{AA}\left(q^2\right)\right|^2+\left|\Delta_{VA}\left(q^2\right)\right|^2+\left|\Delta_{AV}\left(q^2\right)\right|^2\right)\left(1+\cos^2\theta\right) \\
&+\left(\mathrm{Re}\left(\Delta^*_{VV}\left(q^2\right)\Delta_{AA}\left(q^2\right)\right)+\mathrm{Re}\left(\Delta^*_{VA}\left(q^2\right)\Delta_{AV}\left(q^2\right)\right)\right)4\cos\theta\Big]
\end{aligned}
\tag{7}
$$

where $q=\sqrt{s}$ is the center of mass energy and $\theta$ is the angle between the outgoing particles in CoM and

$$
\Delta_{xy}(q^2)=\sum_{j=\gamma,Z,\mathcal{U}} d^e_{xj}d^{\mu *}_{yj}\Delta_j(q^2) \tag{8}
$$

where $\Delta_j$'s are the propagators for unparticle, $Z$ boson and photon with $d$'s given by

| $d_{xj}$ | $\gamma$ | $Z$ | $\mathcal{U}$ |
|---|---|---|---|
| $V$ | $e$ | $\frac{e}{\sin\theta\cos\theta}\left(-1/4+\sin^2\theta\right)$ | $\frac{c_{V\mathcal{U}}}{M_Z^{d_{\mathcal{U}}-1}}$ |
| $A$ | 0 | $\frac{e/4}{\sin\theta\cos\theta}$ | $\frac{c_{A\mathcal{U}}}{M_Z^{d_{\mathcal{U}}-1}}$ |

Note that this is for the lepton flavor conserving interactions. Now we can look for the case where $c_{VU}=0$ which means we only get $Z$ interference since vector coupling vanishes and photon and unparticle exchanges do not contribute to the total cross section.

Fig.(1) displays the total cross section difference for various values of $d_{\mathcal{U}}$ between 1 and 2. The most interesting thing about this graph is the dramatic fluctuation with respect to different $d_{\mathcal{U}}$'s. This can be explained if we consider the phase of the unparticle propagator along the physical cut

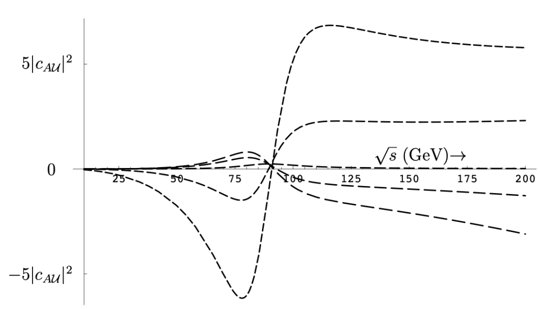
$$
\phi_{d_{\mathcal{U}}}=-(d_{\mathcal{U}}-1)\pi \tag{9}
$$

which is positive for $1<d_{\mathcal{U}}<3/2$ and negative for $3/2<d_{\mathcal{U}}<2$. So considering the $Z$ propagator which is negative below the $Z$ pole and positive above, away from the $Z$ pole we get destructive interference for $1<d_{\mathcal{U}}<3/2$ and vice versa for $3/2<d_{\mathcal{U}}<2$.

The $d_{\mathcal{U}}=3/2$ case is also interesting, which can be seen in Fig. 2. Now the phase simplifies to $\phi=-\pi/2$ so that the particle interferes with only the imaginary part of the $Z$ amplitude. Here the constructive interference peaks around the $Z$ pole and dies down away from the pole. We also see this interference pattern when we consider the vector coupling.

In a recent study [4] the authors investigate the possible effects of the unparticles in this scattering reaction within the MuonE experiment. Their analysis suggests the constraints on the scaling dimension $1<d_{\mathcal{U}}<1.4$

FIG. 1. The fractional change in total cross section versus $\sqrt{s}$ for $d_{\mathcal{U}}=1.1,1.3,1.5,1.7,1.9$. Taken from [3].



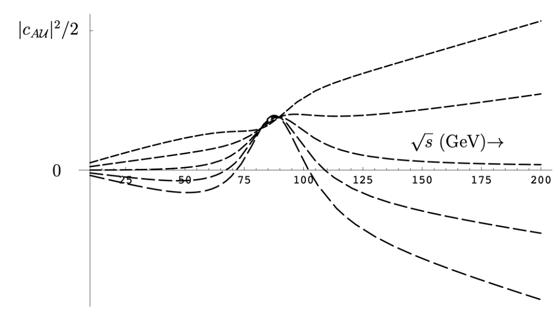




FIG. 2. The fractional change in total cross section versus $\sqrt{s}$ for close values of $d_{\mathcal{U}}=3/2$. Note the different vertical scale with Fig. 1

and the energy scale $1 < \mu < 12$ GeV for which the scale invariance is broken. Although Georgi's treatment in [3] was for a scalar unparticle, their finding is that they contribute a small effect to be detected. However, the vector unparticles can induce a significant disparity at MuonE so that the unparticles can be observed in the future experiments.

## III. UNGRAVITY

It is also worth mentioning perhaps the most interesting part about the unparticles: they can couple to the stress-energy tensor which in turns creates a new fundamental force called "ungravity" mediated by the exchange of unparticles between massive particles. Proposed shortly after Georgi's original publication in [5], the authors compute the first order corrections to the Newtonian potential created by unparticles which turns out to be proportional to

$$\left(\frac{R_G}{r}\right)^{2d_{\mathcal{U}}-1} \tag{10}$$

where $R_G$ is a characteristic length in which ungravity becomes significant. More importantly, if the scaling dimension is constrained to be $d_{\mathcal{U}} > 3$ for a spin 2 unparticle, the inverse square law gains a modification of the type

$$\frac{1}{r^{4+2\delta}}, \delta > 0 \tag{11}$$

Noting that the extra dimension modifications have the type $O(1/r^3)$, future short range experiments of gravity can distinguish between these two extensions of gravitational theories. One of the suggested such experiments uses the Galileo Navigation Satellite System [6, 7]

Ungravity, if it exists, can have some cosmological consequences too. As it has been explored in [8–10], ungravity modifications to the Friedmann equations in the late time region gives rise to an effective cosmological constant which could contribute to the dark energy content of the universe. Furthermore, unparticles also play a role in black hole physics [11, 12] and it has been suggested that they can also be a dark matter candidate [13, 14].

## IV. CONCLUSION

The concept that an unseen sector of a high energy theory that is scale invariant and weakly interacting with SM is an intriguing proposition. After the realization that this unparticle stuff behaves like a collection of $d_{\mathcal{U}}$ massless particles, it has gain a significant amount of attention from both public and physicists. Many phenomenology work has been done afterwards and various experimental mechanisms has been suggested to observe them. Here, we have first reiterated the original logical steps done by Georgi and computed the total cross section for a reaction that involves an unparticle exchange. We have finally listed some consequences of unparticles in the cosmological scale which was mainly the contributions to the dark energy and dark matter content of the universe, which involved black hole physics in the latter case.


[1] H. Georgi, Unparticle physics, Physical Review Letters **98**, 10.1103/physrevlett.98.221601 (2007).

[2] K. Cheung, W.-Y. Keung, T.-C. Yuan, P. Ko, and D. Ki Hong, Unparticle phenomenology—a mini review, in *AIP Conference Proceedings* (AIP, 2008) p. 156–161.

[3] H. Georgi, Another odd thing about unparticle physics, Physics Letters B **650**, 275–278 (2007).

[4] D. N. Le, V. D. Le, D. T. Le, and V. C. Le, Unparticle effects at the muone experiment (2023), arXiv:2304.04439 [hep-ph].

[5] H. Goldberg and P. Nath, Scalar modifications to gravity from unparticle effects may be testable, Physical Review Letters **100**, 10.1103/physrevlett.100.031803 (2008).

[6] O. BERTOLAMI and J. PÁRAMOS, Using global positioning systems to test extensions of general relativity, International Journal of Modern Physics D **20**, 1617–1641 (2011).

[7] J. Páramos and O. Bertolami, The galileo satellite constellation and modifications to the inverse-square law for gravitation (2009), arXiv:0910.3413 [gr-qc].

[8] L. R. Diaz-Barron and M. Sabido, On emergent gravity, ungravity and , Physics Letters B **818**, 136365 (2021).

[9] H. Wei, Relaxing the cosmological constraints on unparticle dark component, The European Physical Journal C **62**, 579–586 (2009).

[10] D.-C. Dai, S. Dutta, and D. Stojkovic, Unparticle dark energy, Physical Review D **80**, 10.1103/physrevd.80.063522 (2009).

[11] D.-C. Dai and D. Stojkovic, Hawking radiation of unparticles, Physical Review D **80**, 10.1103/physrevd.80.064042 (2009).

[12] J. R. Mureika, Unparticle-enhanced black holes at the LHC, Phys. Lett. B **660**, 561 (2008), arXiv:0712.1786 [hep-ph].

[13] H. Davoudiasl, Constraining unparticle physics with cosmology and astrophysics, Physical Review Letters **99**, 10.1103/physrevlett.99.141301 (2007).

[14] T. Kikuchi and N. Okada, Unparticle dark matter, Physics Letters B **665**, 186–189 (2008).