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Moore-Read state and Fractional Quantum Hall Effect-Conformal Field Theory Correspondence

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Outline

Introduction

- What is Fractional Hall Effect (FQHE)?
- FQHE Wave Functions
- Their Conformal Field Theory (CFT) Realization and Construction

What is Quantum Hall Effect

Quantum Hall effect is a breakthrough for condensed matter theory, and its Hamiltonian is

$$\mathbf{H} = \frac{1}{2m}(\vec{\mathbf{p}} + e\vec{A})^2 + \frac{e^2}{r} \tag{1}$$

However, it is impossible to solve this Hamiltonian exactly.

Wave Function in a Symmetric Gauge

When we consider the symmetric gauge $\vec{A} = \frac{B}{2}(-y, x, 0)$, wave functions at LLL takes the form,

$$\Psi_{LLL}(z_i, \bar{z}_i) = f(z_1, .., z_N)e^{-\sum_{i=1}^{N}|z_i|^2/4\ell_B^2} \tag{2}$$

where $f(z_i)$ is any holomorphic function.

Laughlin Wave Function

Laughlin made a bold move and wrote directly the wave function,

$$\Psi_L(z_i) = \prod_{i<j}^{N} (z_i - z_j)^m e^{-\sum_{i=1}^{N} |z_i|^2/4\ell_B^2} \tag{3}$$

for $\nu = \frac{1}{m}$ filling fraction, where m is odd.
This approximation have greater than %99 overlap in the ground state.

Moore/Read Pfaffian State

States of filling factor with even denominator such as $\nu = \frac{5}{2}, \frac{7}{2}$ can be described as,

$$\Psi_{Pf}(z_i) = Pf(\frac{1}{z_i - z_j}) \prod_{i<j}^{N} (z_i - z_j)^m e^{-\sum_{i=1}^{N} |z_i|^2 / 4\ell_B^2} \tag{4}$$

where m is even, and $det(M) = Pf(M)^2$.

Vertex Operator Construction

These states can be realized with CFT language, introduce vertex operator as

$$V_1(z) = :e^{i\sqrt{m}\varphi_1(z)}: \tag{5}$$

and two point correlator of free massless boson field φ_1 as,

$$\langle \varphi_1(z)\varphi_1(w)\rangle = -ln(z-w) \tag{6}$$

Vertex Operator Construction

Now, Laughlin wave function can be written as,

$$\Psi_L(z_i) = \langle 0 | \mathbf{R} \Big\{ V_1(z_1) V_1(z_2) .... V_1(z_N) e^{-i\sqrt{m}\rho_m \int d^2 z' \varphi_1(z')} \Big\} |0\rangle$$

$$\equiv \langle V_1(z_1) V_1(z_2) ... V_1(z_N) \rangle_{\frac{1}{m}} \tag{7}$$

$$= \prod_{i<j}^{N} (z_i - z_j)^m e^{-\sum_{i=1}^{N} |z_i|^2 / 4\ell_B^2}$$

where $\mathbf{R}$ is radial ordering, ρ_m is constant background particle density and $|z_1| \geq |z_2| \geq .... \geq |z_N|$.

Vertex Operator Construction

Pfaffian states can be realized with CFT language, introduce vertex operator as

$$V_1(z) = :\chi_1(z) e^{i\sqrt{m}\varphi_1(z)}: \tag{8}$$

and two point correlator of free massless real Majorana fermions (1+1D) φ_1 as,

$$\langle \chi_1(z)\chi_1(w)\rangle = \frac{1}{z-w} \tag{9}$$

Vertex Operator Construction

Now, Pfaffian wave function can be written as,

$$\Psi_{Pf}(z_i) = \langle 0| \, \mathbf{R}\Big\{ V_1(z_1)V_1(z_2)....V_1(z_N)e^{-i\sqrt{m}\bar{\rho}\int d^2z'\varphi_1(z')} \Big\} \, |0\rangle$$

$$\equiv \langle V_1(z_1)V_1(z_2)...V_1(z_N)\rangle_{\frac{1}{m}} \tag{10}$$

$$= Pf(\frac{1}{z_i - z_j})\prod_{i<j}^{N}(z_i - z_j)^m e^{-\sum_{i=1}^{N}|z_i|^2/4\ell_B^2}$$

where $\mathbf{R}$ is radial ordering, $\bar{\rho}$ is constant background particle density and again $|z_1| \geq |z_2| \geq .... \geq |z_N|$.

Bootstrap and QHE

- $SU(2)_1$ WZW: Bulk and edge physics of FQHE can be studied
- Criticality: Quantum Hall transitions such as Anderson localization and topological insulating phases
- Conformal Blocks: They can solve some calculations of fractionalized excitations such as correlations

Why Topological Order is Important

- High T_c Superconductors
- Quantum Info and Quantum Computing
- Edge and Bulk Physics of Polymers

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Thanks for your attention