Higher-Spin Gravity and Symmetry

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What are Higher-Spin Fields?

- Aim is to explore the most general gauge theories that can incorporate any spin
- Another aspect is to study all possible fundamental interactions and their underlying symmetries and mathematical structures.
- A consistent theory gravity of interacting fields with HS fields $s > 2$

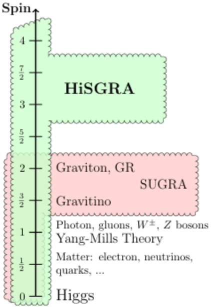

Figure: [BBC+22]

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History

- Fierz and Pauli (1939): formulation of consistent massive higher spin E.O.M
- Sing-Hagen Lagrangian(1974) (need auxiliary dynamical s-2,s-3 fields) [SH74]
- Fronsdal equation(1978): massless case, only spin s and s-2 fields couple [Vuk18]
- Vasiliev(1990): the exact non-linear equations of motion of the theory. Admitting a vacuum AdS solution.

  The shortest route to Vasiliev's equations is 40 pages.

  [DS15]

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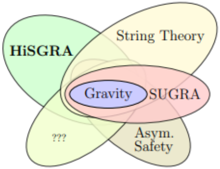
Why Higher-Spin?



- Much larger symmetry than CFT's
- Is in between string theory and supergravity: $[m]^2$ of string excitations $\sim \frac{N}{\alpha'}$

$$lim_{\alpha'} \rightarrow 0 \ || \ massless\ modes\ of\ supergravity$$

$$lim_{\alpha'} \rightarrow \infty \ || \ massless\ excitations \rightarrow HS\ theories?$$

- All HS gauge theories have graviton inherently $\rightarrow$ generalizes GR with infinite tower of spin states. [BBC+22]
- The symmetries due to higher spin may naturally give rise to QG.

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Higher Spins from Free Scalar CFT

$$S = \int d^d x \, \frac{1}{2}(\partial\phi)^2 \tag{1}$$

Admits a larger symmetry, the HS algebra. To see this, consider 1 HS operator for each even spin conserved current.
HS currents:

$$J_{\mu_1\mu_2\ldots\mu_s} = \sum_{k=0}^{k} c_{sk}\partial_{\{\mu_1\ldots.\mu_k}\phi\partial_{\mu_{k+1}\ldots.\mu_s\}}\phi \tag{2}$$

where s = 2,4,6... and parenthesis denote traceless symmetrization. Corresponding to irreps of SO(d) of spin s.(due to ϕ being a singlet it cannot have odd spin).

Higher Spins from Free Scalar CFT

fix c_{sk} by:

$$0 = \partial^{\mu_1} J_{\mu_1 \mu_2 \dots \mu_s} \tag{3}$$

Index-free notation is more useful in local coordinates:

$$J_s(x, \epsilon) = J_{\mu_1 \mu_2 \dots \mu_s} \epsilon^{\mu_1} \dots . \epsilon^{\mu_s}$$

where ϵ is null,
by keeping the track of nullity, one can get rid of polarization:

$$J_{\mu_1 \mu_2 \dots \mu_s} \propto D_{\mu_1} \dots D_{\mu_s} J_s(x, \epsilon) \tag{4}$$

where

$$D_\mu = (\frac{d}{2} - 1 + \epsilon^\nu \frac{\partial}{\partial \epsilon^\nu} - \frac{1}{2} \epsilon^\mu \frac{\partial}{\partial \epsilon^\nu} \frac{\partial}{\partial \epsilon_\nu}) \tag{5}$$

Higher Spins from Free Scalar CFT

The conservation equation is now:

$$\partial^\mu D_\mu J_s(x, \epsilon) = 0 \tag{6}$$

in this language, spin op's are:

$$J_s(x, \epsilon) = \sum_{k=0}^{s} c_{sk} (\epsilon\partial)^k \phi (\epsilon\partial)^{s-k} \phi = \phi f_s(\epsilon \overleftarrow{\partial}, \epsilon \overrightarrow{\partial}) \phi \tag{7}$$

Inserting this into eq. 6 we get the Gegenbauer differential equation for f_s which we already know the solution.

Having spin-s currents, we can generate conserved charges over them. Can also be obtained in the same way as $T^{\mu\nu}$.(conf. killing tensors)

$$\partial_{(\mu_1}\xi_{\mu_2\ldots\mu_s)} = \frac{s-1}{d+2s-4} g_{(\mu_1\mu_2}\xi_{\mu_3\ldots\mu_s)} \tag{8}$$

ordinary conserved current:

$$J_\mu^{\xi_{s-1}} = J_{\mu_1\mu_2\ldots\mu_s}\xi^{\mu_2\ldots\mu_s} \tag{9}$$

and thus:

$$\partial^\mu J_\mu^{\xi_{s-1}} = 0 \tag{10}$$

for s=2 $\rightarrow$ generators of the conformal group.

Higher Spins from Free Scalar CFT

For higher spins, we get further generators that are higher derivative symmetries. Action of charges on fields are:

$$[Q_s, \phi] \sim \xi^{\mu_1 \cdots \mu_{s-1}} \partial_{\mu_1} \ldots \partial_{\mu_s} \phi \tag{11}$$

Remark

There is 1 charge for each conf. Killing tensor. Consequently, the number of HS generators at each spin s is the dimension of the representation at spin s.

Example

By counting the DOF, in d=3 there are:

$$s(2s-1)(2s+1)/3$$

generators.

Higher Spins from Free Scalar CFT

In general, commutator of charges produce charges of greater spins:

$$[Q_4, Q_4] \sim Q_2 + Q_4 + Q_6 \tag{12}$$

Once a spin-4 charge is present, you must also have spin-6.

Important Remark

- One needs an infinite tower of charges to get a closed algebra
- The algebra is infinite-dimensional intrinsically

Already clear from free CFT, but one can ask if there are any other CFT's with higher-spin symmetry.

Maldecena and Zhiboedov(2013)

[MZ13] Given a CFT in dim. bigger than 3 with J_2 and at least one HS current, J_s, one can show that it is actually a free CFT in disguise.

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Free O(N) Vector Model and AdS

$$S = \int d^d x \frac{1}{2} (\partial_\mu \phi^i)^2 \tag{13}$$

$\rightarrow$ global $O(N)$ symmetry. Break down the HS current operators to different representations:

$$J_s^{ij} \rightarrow J_s + J_s^{\{ij\}} + J_s^{[ij]} \tag{14}$$

J_s with even spin are $O(N)$ singlets, $J_s^{\{ij\}}$ are the symmetric traceless and $J_s^{[ij]}$ odd spin are the antisymmetric representation.

Remark

The conformal $T^{\mu\nu}$ is the $O(N)$ singlet and $J_1^{[ij]}$ is the current in the adjoint rep of $O(N)$.

Higher Spin/ Vector Model Duality

We are only interested in correlation functions of $O(N)$ invariant operators. Define single trace operators: $J_0 = \phi^i \phi^i$. with dimension $\Delta = d - 2$. Thus, as a whole set:

$$single \;\; trace: \;\; J_0 + \sum_{even\ s} J_s$$

$$(\Delta, S): \;\; (d-2, 0) + \sum_{even\ s} (d-2+s, s)$$

established by Flato-Fronsdal (1980).

CFT to AdS

Now assume the AdS/CFT duality holds to full extent.

Expect

The singlet sector in CFT is dual to some gravitational theory in AdS:

- Single trace operators in CFT $\leftrightarrow$ Single particle states in AdS
- single trace spectrum should match the gauge fields in bulk

Example

- Generalize this to spin-s: one has massless gauge fields corresponding to conserved spin-s

  $J_s : \partial J_s = 0 \leftrightarrow$ *massless gauge field* $\delta\phi_s \sim \nabla\epsilon_{s-1}$

CFT to AdS [Gio16]

In addition: $\mathrm{J}_0 = \phi^i \phi^i \leftrightarrow \textit{Scalar field}\,\phi$ with $m^2 = \Delta(\Delta - d)/l_{AdS}^2$

Remark

This is the spectrum of the Vasiliev HS theory in AdS_{d+1} in [Vas03]
The interactions can be done by Wick contractions on the CFT side.

Remark

The features of Vasiliev theory admits CFT arguments. So one can conjecture:

$$\textit{Free } O(N) \textit{ Vector Model} \leftrightarrow \textit{Higher Spin Gravity in AdS}$$

singlet sector of the well-known critical 3-d O(N) model with the $(\phi^i\phi^i)^2$ interaction is dual to the minimal bosonic theory in AdS4 containing massless gauge fields of even spin in large N. (Klebanov, Polyakov(2002))[KP02]

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Future Directions [BBC+22]

- HS in low dimensions: 2D HS Gravity is not explored. (Konstantin Alkalaeva, Xavier Bekaertd)[AB20]
- HS symmetry of weakly coupled $\mathcal{N} = 4$ SYM
- Conformal HiSGRA
- Slightly broken HS symmetry fixes 3,4 point correlators
- CMT side [GLSS22]

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