Phys209: Mathematical Methods in Physics I

Homework 8

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Policies

- Please adhere to the *academic integrity* rules: see my explanations here for further details!
- For the overall grading scheme or any other course-related details, see the syllabus.
- Non-graded question(s) are for your own practice!
- Unless stated otherwise, you are expected to show your derivation of the results.
- The homework is due December 01$^{\text{th}}$ 2023, 23:59 TSI.

(1) Problem One *(6 points)*

Let's assume you are measuring the heat conductance $c(t)$ of a crystal in the lab and you observe it obeys the following time dependence

$$\left(\frac{d^3}{dt^3} + ab^2\right) c(t) = p(t)$$

where $p(t)$ denotes the pressure applied to the crystal as a function of time. Your college on the other hand obtains the following relation based on their measurement:

$$c''(t) = \frac{1}{a} - \frac{1}{a}p(t) - \frac{b^2}{a}c'(t)$$

Taking $a > 0$ and $b > 0$ as time-independent constants, we are going to find out the conductance $c(t)$ as a function of time in this question.

(1.1) (1.2pt)

Combine the given equations to write down an equation for $c(t)$ free of $p(t)$.

(1.2) (1.2pt)

The particular solutions of *differential equations with constant coefficients* usually take the same functional form as the non-homogeneous part of the differential equation. Use this information to **guess** a particular solution and **check** that it indeed satisfies the differential equation.

(1.3) (1.2pt)

Assume that at least one of the solutions should be exponentially suppressed with time, i.e. $e^{-\alpha t}$. Find the value of α which reduces the

order of the given differential equation, and derive the second-order differential equation after the reduction of order.

(1.4) (1.2pt)

Solve the second order differential equation.

(1.5) (1.2pt)

Combine the three homogeneous solutions derived above and the particular solution to write down the most general solution to this differential equation.