ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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Spinning Conformal Correlations

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Outline

Operator Product Expansion (OPE) associativity is used only in 4-point CFT correlation functions of scalars.

Aim

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Motivation: Analogy between CFT correlation functions in Mellin representations and scattering amplitudes

Points in d-dimensional space -> null vectors in d+2 dimensional space

$$P^A = \lambda(1, x^2, x^a)$$

Properties:

- Null vector on the lightcone $P^2 = 0$
- Homogeneity
- Symmetric and Traceless
- Transverse: $(P \cdot F)_{A_2 \ldots A_l} \equiv P^A F_{AA_2 \ldots Al}$

Conformal groups -> Lorentz groups

$$\langle \Psi_1(P_1)\Psi_2(P_2)\Psi_3(P_3)\rangle = F(P_1, P_2, P_3)$$

1. $SO(d+1, 1)$ symmetry: F depends only on $(P_i - P_j)^2 = P_i^2 + P_j^2 - 2P_i\cdot$
2. On the null cone: $P_i^2 = 0$, hence $F(P_1, P_2, P_3) = F(P_{12}, P_{23}, P_{31})$,

$$P_{ij} = -2P_i \cdot P_j.$$

$$\langle \Psi_1(\lambda P_1)\Psi_2(P_2)\Psi_3(P_3)\rangle = \lambda^{-\Delta_1}\langle \Psi_1(P_1)\Psi_2(P_2)\Psi_3(P_3)\rangle,$$

$$F(\lambda P_{12}, P_{23}, P_{31}) = \lambda^{-\Delta_1} F(P_{12}, P_{23}, P_{31}),$$

$$F(\lambda P_{12}, \lambda^{-1} P_{23}, P_{31}) = \lambda^{\Delta_3-\Delta_1} F(P_{12}, P_{23}, P_{31}).$$

$$F(P_{12}, P_{23}, P_{31}) = N P_{12}^{a} P_{23}^{b} P_{31}^{c},$$

$$\langle \Psi_1(P_1)\Psi_2(P_2)\Psi_3(P_3)\rangle = \frac{N}{(P_{12})^{\frac{\Delta_1+\Delta_2-\Delta_3}{2}}(P_{23})^{\frac{\Delta_2+\Delta_3-\Delta_1}{2}}(P_{31})^{\frac{\Delta_3+\Delta_1-\Delta_2}{2}}}.$$

Projection to Physical Space: $f(x) = F(P_x), \quad P_{ij} \to x_{ij}^2.$

Tensors are converted to polynomials

$f_{a_1 \dots a_l}$ symmetric <-> $f(z) \equiv f_{a_1 \dots a_l} z^{a_1} \dots z^{a_l}$

$F_{A_1 \dots A_l}(P)$ symmetric <-> $F(P, Z) \equiv F_{A_1 \dots A_l} Z^{A_1} \dots z^{A_l}$

$$f_{a_1 \ldots a_l}(x) = \frac{\partial P^{A_1}}{\partial x^{a_1}} \ldots \frac{\partial P^{A_l}}{\partial x^{a_l}} F_{A_1 \ldots A_l}(P_x)$$

References

Thank You

for your attention.

Do you have any question?