Phys209: Mathematical Methods in Physics I

Homework 9

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Policies

- Please adhere to the *academic integrity* rules: see my explanations here for further details!
- For the overall grading scheme or any other course-related details, see the syllabus.
- Non-graded question(s) are for your own practice!
- Unless stated otherwise, you are expected to show your derivation of the results.
- The homework is due December 8th 2023, 23:59 TSI.

(1) Taylor Series *(6 points)*

In this homework, we will go over series expansion solutions to differential equations. As you may remember, we have explicitly solved in class the differential equation $f''(x)-xf(x)=0$ by *Taylor expanding* the solution around 0, i.e. by taking $f(x)=\sum\limits_{k=0}^{\infty} a_k x^k$. We also discussed in class that Taylor expansions around different points are also possible!

In this homework, we are going to solve the differential equation $f''(x)-(x+1)f(x)=0$ by taking $f(x)=\sum\limits_{k=0}^{\infty} a_k (x+1)^k$

(1.1) (1pt)

Insert the expansion of $f(x)$ into the differential equation; change the dummy variables of summations as you see fit; and then bring the differential equation to the form

$$\sum_k (\cdots)\,(x+1)^k = 0$$

(1.2) (3pt)

From the *orthogonality* of $(x+1)^k$ for different k values, obtain the recursion relations for the unknown coefficients a_k. (Your recursion relations might be something like $a_{3n+1}=(\dots)a_1$ and so on.)

(1.3) (1pt)

Insert the results for a_k into $f(x)=\sum\limits_{k=0}^{\infty} a_k (x+1)^k$ and write down it in the form

$$f(x)=c_1 f_1(x)+c_2 f_2(x)$$

where c_i are the undetermined a_i , and *importantly*, $f_{1,2}(x)$ are a_k independent!

(1.4) (1pt)

Compute the Wronskian determinant of $f_1(x)$ and $f_2(x)$. Are they linearly independent?

(2) Bonus question

(not graded)

Previous question can be solved with the Mathematica code

```
DSolve[f''[x] - (x + 1) f[x] == 0, f[x], x]
```

which gives the full result $f(x)$. We can extract the individual results $f_{1,2}(x)$ and compute their Wronskian determinant as follows:

```
With[{
        solution = DSolve[f''[x] - (x + 1) f[x] == 0, f[x], x][[1, 1,
            2]]
},
With[{
        solution1 = solution /. {C[2] -> 0, C[1] -> 1},
        solution2 = solution /. {C[1] -> 0, C[2] -> 1}
},
Wronskian[{solution1, solution2}, x]
]
]
```