2D Conformal Field Theories

Onur Ayberk Çakmak

Middle East Technical University

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Outline

What are Conformal Field Theories?

- Quantum field theories invariant under conformal transformations.
- Transformations preserving the angle but not necessarily distances:

$$g'_{\mu\nu}(x') \to \Omega^2(x) g_{\mu\nu}(x)$$

Importance of 2D CFTs

- In 2D, conformal group becomes infinite dimensional.
- Suitable for analyzing systems critical state. (E.g. $2D$ Ising model at K_c)
- Exact correlation functions without perturbative expansion.

$$\langle \phi_1(z)\phi_2(w)\rangle = \frac{C}{(z-w)^{h_1+h_2}}, \qquad \text{etc.}$$

- Wide range of application in physics.

Symmetries

- Infinite-dimensional Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$$
$$[\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$$

- Anomalous central extension of Witt algebra with "central charge" c.
- L_0, $L_{\pm 1}$ constitute the $\mathfrak{sl}(2,\mathbb{C})$ subalgebra and generate global conformal transformations:
  - L_{-1}: translation $(z \mapsto z + b)$
  - L_0: rotation and dilatation $(z \mapsto az)$
  - L_1: special conformal transformation $\left(z \mapsto \frac{z}{cz+1}\right)$

$$z \mapsto \frac{az+b}{cz+d}$$

Hilbert Space

- **Primary fields:**

$$\phi'(\lambda z, \bar{\lambda}\bar{z}) = \lambda^{-h}\bar{\lambda}^{-\bar{h}}\phi(z, \bar{z})$$

  h, $\bar{h}$: conformal weights.

- **Primary states:** Created by acting on the vacuum state with primary field evaluated at $z = \bar{z} = 0$:

$$\left|h, \bar{h}\right\rangle \equiv \phi(0,0)\left|0\right\rangle, \qquad L_0\left|h, \bar{h}\right\rangle = h\left|h, \bar{h}\right\rangle$$

- **Descendant fields:** Created by acting L_{-n} on primary states.

$$\left|h, \bar{h}\right\rangle \to L_{-1}\left|h, \bar{h}\right\rangle; L_{-1}^2\left|h, \bar{h}\right\rangle\ L_{-2}\left|h, \bar{h}\right\rangle; \ldots$$

Bootstrap Treatment

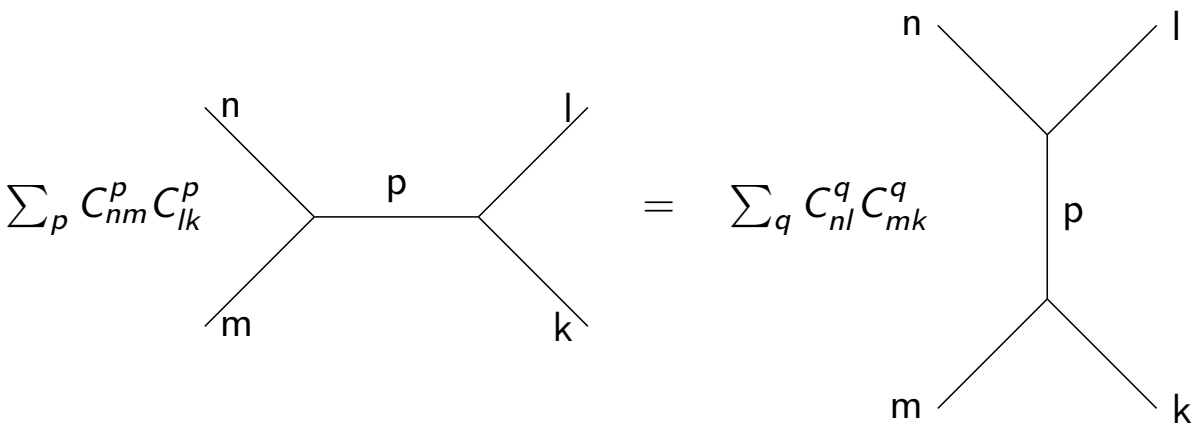
- **Operator Product Expansion (OPE):** Relates local operators:

$$\mathcal{O}_i(z_1)\mathcal{O}_j(z_2) = \sum_k C_{ij}^k |z_1 - z_2|^{h_k - h_i - h_j} \mathcal{O}_k(y)$$

- **4-Point function:**

$$G_{34}^{21}(x) = \lim_{z_1, \bar{z}_1 \to \infty} z_1^{2h_1} \bar{z}_1^{2\bar{h}_1} \langle \phi_1(z_1)\phi_2(1)\phi_3(x)\phi_4(0)\rangle, \quad x = \frac{z_{12}z_{34}}{z_{13}z_{24}}$$

- **Crossing symmetry:** Symmetry arising from equivalence of s and t channels of Feynman diagram of 4-point function.



(Francesco, Mathieu, and Senechal 2012)

Summary and Discussion

- 2D CFTs possess rich mathematical structure.
- The algebra of the symmetry group is the Virasoro algebra.
- Wide area of applications in physics.
- Using bootstrap methods, the related quantities of a CFT can be found.
- Current literature and future directions include AdS3/CFT2 coorespondance (Kraus 2006), worldsheet string theories, entanglement entropy (Sheikh-Jabbari and Yavartanoo 2016), $T\overline{T}$ and $J\overline{T}$ deformed gravity (Guica 2019).

References I

- Belavin, A.A., A.M. Polyakov, and A.B. Zamolodchikov (1984). "Infinite conformal symmetry in two-dimensional quantum field theory". In: *Nuclear Physics B* 241.2, pp. 333–380. DOI: https://doi.org/10.1016/0550-3213(84)90052-X.
- Francesco, P., P. Mathieu, and D. Senechal (2012). *Conformal Field Theory*. Graduate Texts in Contemporary Physics. Springer New York. ISBN: 9781461222569.
- Guica, Monica (2019). "On correlation functions in $J\overline{T}$-deformed CFTs". In: *Journal of Physics A: Mathematical and Theoretical* 52.18, p. 184003. DOI: 10.1088/1751-8121/ab0ef3.
- Kraus, Per (2006). *Lectures on black holes and the AdS_3/ CFT_2 correspondence*. arXiv: hep-th/0609074 [hep-th]. URL: https://arxiv.org/abs/hep-th/0609074.
- Nawata, Satoshi, Runkai Tao, and Daisuke Yokoyama (2024). *Fudan lectures on 2d conformal field theory*. arXiv: 2208.05180 [hep-th].

References II

Sheikh-Jabbari, M. M. and H. Yavartanoo (2016). "Excitation entanglement entropy in two dimensional conformal field theories". In: *Physical Review D* 94.12. ISSN: 2470-0029. DOI: 10.1103/physrevd.94.126006.

Thank You!