Spinning Conformal Correlators

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Based on the work by Miguel S. Costa, João Penedones, David Poland, and Slava Rychkov

Outline

Why Conformal Field Theory?

- **CFT Basics:** Quantum field theories invariant under scaling, rotations, and translations.
- **Special Cases:**
  - In 2D: Well-developed using the Virasoro algebra.
  - In higher dimensions: Exact solutions are rare.
- **Applications:**
  - Statistical mechanics: Critical phenomena.
  - AdS/CFT correspondence: Quantum gravity and holography.
  - Fundamental physics: Strongly interacting systems.

The Problem

- Current progress in CFTs focuses mainly on scalar operators.
- Spinning operators, such as conserved currents and the stress-energy tensor, remain challenging to analyze.
- **Key Questions:**
  - How can we handle symmetric traceless tensors in d-dimensional CFTs systematically?
  - Can we extend bootstrap methods to higher-spin operators?

Embedding Space Formalism

- Conformal symmetry $SO(d+1,1)$ becomes Lorentz symmetry in $(d+2)$-dimensional Minkowski space.
- Physical points correspond to null rays in embedding space.
- **Example: Scalar Correlator**

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\rangle = \frac{\text{const}}{(x_{12}^2)^{\frac{\Delta_1+\Delta_2-\Delta_3}{2}}(x_{23}^2)^{\cdots}}$$

- Embedding space simplifies symmetry-based computations.

- Tensors are replaced with polynomials using polarization vectors Z.
- Symmetric traceless tensors $T_{a_1 \dots a_l}$ are encoded as:

$$T(Z) = T_{a_1 \dots a_l} Z^{a_1} \cdots Z^{a_l}.$$

- Properties like tracelessness and transversality are automatically encoded in $T(Z)$.
- Index-free representations reduce computational complexity.

Two-Point Functions

- Two-point functions are uniquely constrained by symmetry.
- For a spin-l primary operator:

$$\langle O_{A_1\ldots A_l}(P_1) O_{B_1\ldots B_l}(P_2)\rangle \propto \frac{\text{Symmetric and Traceless Terms}}{(P_{12})^{\Delta}}.$$

- Embedding correlators are projected to physical space for final results.
- Simplified computations allow handling of higher-spin cases.

- Three-point functions combine scalars and spinning operators.
- Scalar-Scalar-Spin-l example:

$$\langle O_1(P_1) O_2(P_2) O_3(P_3, Z) \rangle \propto \frac{(Z \cdot P_1)(P_2 \cdot P_3) - (Z \cdot P_2)(P_1 \cdot P_3)}{P_{12}^{\Delta_1 + \Delta_2 - \Delta_3} \cdots}.$$

- **Building Blocks:**

$$V_{i,jk} = \frac{(Z_i \cdot P_j)(P_k \cdot P_i) - (Z_i \cdot P_k)(P_j \cdot P_i)}{P_j \cdot P_k},$$

$$H_{ij} = -2\big[(Z_i \cdot Z_j)(P_i \cdot P_j) - (Z_i \cdot P_j)(Z_j \cdot P_i)\big].$$

Connections to Scattering Amplitudes

- Correlators in d-dimensional CFT resemble scattering amplitudes in $(d+1)$-dimensional spacetime.
- Example: AdS/CFT correspondence maps boundary correlators to bulk scattering processes.
- **Implications:**
  - Tools from scattering amplitudes (e.g., recursion relations) can be applied to CFT.
  - Deepens our understanding of quantum field theory and holography.

Summary

- Developed a formalism for efficiently computing spinning correlators in CFT.
- Simplified computations using embedding space and polarization vectors.
- Established connections with scattering amplitudes and holography.

Future Directions

- Investigate conserved tensor operators (e.g., stress-energy tensors, currents).
- Explore connections to Mellin amplitudes.
- Apply the formalism to holographic theories, including higher-spin gravity.

Thank You!
Do you have any questions?