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Midterm Examination

Phys209: Mathematical Methods in Physics I

2023/11/30

Please carefully read below before proceeding!

I acknowledge by taking this examination that I am aware of all academic honesty conducts that govern this course and how they also apply for this examination. I therefore accept that I will not engage in any form of academic dishonesty including but not limited to cheating or plagiarism. I waive any right to a future claim as to have not been informed in these matters because I have read the syllabus along with the academic integrity information presented therein.

I also understand and agree with the following conditions:

(1) any of my work *outside the designated areas* in the "fill-in the blank questions" will not be graded;
(2) I take *full responsibility* for any ambiguity in my selection of the correct option in "multiple choice questions";
(3) any of my work *outside the answer boxes* in the "classical questions" will not be graded;
(4) any page which does not contain *both my name and student id* will not be graded.

Signature: ____________________

This exam has a total of 9 questions, some of which are for bonus points. You can obtain a maximum grade of 20+2 from this examination.

Question	Points	Score
1	3	
2	1½	
3	1½	
4	1	
5	2	

Question	Points	Score
6	4	
7	1	
8	6	
9	0	
Total:	20	

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1 Fill-in the blanks

Question: 1 . . . . *(3 points)*

The differential equations are ubiquitous in Physics: we encounter them in the classical mechanics, statistical computations, quantum phenomena, and many other seemingly different branches. Even though differential equations can be vastly different in principle, the majority of them we encounter in Physics include derivatives only with respect to one parameter: such differential equations are called ___***ordinary***___ *differential equations*! In contrast, ___***partial***___ *differential equations* involve derivatives with respect to more than one variables!

Many of the engineering applications of differential equations make use of something called *principle of superposition*! This principle states that if two functions $f_1(x)$ and $f_2(x)$ are solutions to a given differential equation, then their superposition (e.g. $c_1 f_1(x) + c_2 f_2(x)$ for arbitrary coefficients c_i) is also a solution to the same differential equation. This is not applicable to all differential equations; indeed, the differential equation should be ___**linear**___ in the unknown function for this to be true: if it is, for instance, *quartic* or *quintic*, this principle does not hold!

Although most of the differential equations are not generically solvable, those with constant coefficients can always be solved! Indeed, the most general solution is given as a sum of two pieces: ___***homogeneous***___ *solution* and the *particular solution*. The first one is computed by finding the roots of the ___***characteristic***___ *equation*, whereas the latter is given as the convolution of the nonhomogeneous piece in the differential equation with *the* ___***impulse***___ *response*.

2 Choose the correct option

Question: 2 . . . . *(1½ points)*

The domain of a (higher) order function can be thought of the *type* of the input of that (higher) order function, whereas the codomain is the type of the output of that function. For instance, *the derivative operator* is a higher order function which takes functions to functions, hence we can say that both the domain and codomain of the derivative operator are of function type!

Let us denote the (co)domain of an operator $\mathcal{O}$ as (co)dom($\mathcal{O}$): please answer the questions below accordingly.

(a) **(½ point)** Let $f :: \mathbb{C} \to \mathbb{R}$. Choose the correct option.

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- ■ $\text{dom}(f) = \mathbb{C}$
- ☐ $\text{dom}(f) = \mathbb{R}$
- ☐ $\text{dom}(f) = \mathbb{C} \to \mathbb{R}$
- ☐ $\text{codom}(f) = \mathbb{C} \to \mathbb{R}$
- ☐ None

(b) (½ **point**) Let $g :: \mathbb{C} \to (\mathbb{R} \to \mathbb{Z})$. Choose the correct option.

- ☐ $\text{dom}(\text{codom}(g)) = \mathbb{Z}$
- ■ $\text{dom}(\text{codom}(g)) = \mathbb{R}$
- ☐ $\text{dom}(\text{codom}(g)) = \mathbb{C}$
- ☐ $\text{dom}(\text{codom}(g)) = \mathbb{R} \to \mathbb{Z}$
- ☐ $\text{dom}(\text{codom}(g)) = \mathbb{C} \to (\mathbb{R} \to \mathbb{Z})$

(c) (½ **point**) If we are given $\text{dom}(a) = \text{codom}(b)$, which option belove is incorrect? ‘

- ☐ $a :: \mathbb{R} \to \mathbb{R}$, $b :: \mathbb{C} \to \mathbb{R}$
- ☐ $a :: \mathbb{C} \to (\mathbb{R} \to \mathbb{R} \to \mathbb{R})$, $b :: \mathbb{C} \to \mathbb{C}$
- ■ $a :: \mathbb{C} \to \mathbb{C}$, $b :: \mathbb{C} \to (\mathbb{R} \to \mathbb{R} \to \mathbb{R})$
- ☐ $a :: (\mathbb{R} \to \mathbb{Z}) \to (\mathbb{Z} \to \mathbb{R})$, $b :: (\mathbb{Z} \to \mathbb{R}) \to (\mathbb{R} \to \mathbb{Z})$
- ☐ $a :: (\mathbb{Z} \to \mathbb{R}) \to (\mathbb{R} \to \mathbb{Z})$, $b :: (\mathbb{R} \to \mathbb{Z}) \to (\mathbb{Z} \to \mathbb{R})$

Question: 3 . . . . *(1½ points)*

The kernel of an operator $\mathcal{O}$ is denoted as $\ker(\mathcal{O})$ and is *the set of all functions* f_i which satisfy $\mathcal{O} \cdot f_i = x \to 0$.

(a) (½ **point**) What is $\ker\left(x^2 \dfrac{\mathrm{d}}{\mathrm{d}x} + 1\right)$?

- ☐ $\{x \to 0, x \to \exp\left(x^{-2}\right)\}$
- ☐ $\{0, \exp\left(x^{-2}\right)\}$
- ■ $\{x \to 0, x \to \exp\left(x^{-1}\right)\}$
- ☐ $\{0, \exp\left(x^{-1}\right)\}$
- ☐ None

(b) (½ **point**) Let $\cup$, $\cap$, and $\times$ denote the set operations union, intersection, and Cartesian product respectively. Then which of the following statement is always true?

- ☐ $\ker(\mathcal{D}_1 \cdot \mathcal{D}_2) \subset \ker(\mathcal{D}_1) \cup \ker(\mathcal{D}_2)$
- ☐ $\ker(\mathcal{D}_1 \cdot \mathcal{D}_2) \subset \ker(\mathcal{D}_1) \cap \ker(\mathcal{D}_2)$
- ☐ $\ker(\mathcal{D}_1 \cdot \mathcal{D}_2) \subset \ker(\mathcal{D}_1) \times \ker(\mathcal{D}_2)$
- ☐ $\ker(\mathcal{D}_1) \subset \ker(\mathcal{D}_1 \cdot \mathcal{D}_2)$
- ■ $\ker(\mathcal{D}_2) \subset \ker(\mathcal{D}_1 \cdot \mathcal{D}_2)$

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The *commutator* of two operators A and B is denoted as $[A, B]$, and is defined as

$$[A, B] = f \to A \cdot B \cdot f - B \cdot A \cdot f \tag{1}$$

for any *function* $f = x \to f(x)$.

(c) **(½ point)** Which of the following is a true statement?

- ☐ $\left[\frac{\mathrm{d}}{\mathrm{d}x}, x\frac{\mathrm{d}}{\mathrm{d}x}\right] \cdot \cos = \cos$
- ☐ $\left[\frac{\mathrm{d}}{\mathrm{d}x}, x\frac{\mathrm{d}}{\mathrm{d}x}\right] \cdot \cos = x \to \sin(x)$
- ☐ $\left[\frac{\mathrm{d}}{\mathrm{d}x}, x\frac{\mathrm{d}}{\mathrm{d}x}\right] \cdot \cos = -sin(x)$
- ■ $\left[\frac{\mathrm{d}}{\mathrm{d}x}, x\frac{\mathrm{d}}{\mathrm{d}x}\right] \cdot \cos = x \to -sin(x)$
- ☐ $\left[\frac{\mathrm{d}}{\mathrm{d}x}, x\frac{\mathrm{d}}{\mathrm{d}x}\right] \cdot \cos(x) = sin(x)$

Question: 4 .... *(1 points)*

The order of a differential equation is given by the highest number of derivatives acting on the unknown function *when the differential equation is brought to its simplest form.* In general, it is hard to determine the simplest form without actually solving the equation; for this question, ***we will assume*** that the application of Leibniz rule (i.e. $(fg)' = f'g + fg'$) will not reduce the order further; for instance, please take $f''(x)f'(x)$ as an order-2 term even though there exist the relation $f''(x)f'(x) = -f'''(x)f(x) + \frac{\mathrm{d}(f''(x)f(x))}{\mathrm{d}x}$ with a third-derivative term in it.

(a) **(½ point)** Which of the following differential equations has the highest order?

- ☐ $\left[\frac{\mathrm{d}}{\mathrm{d}x}\Big(f''(x) + f'(x) + f(x)\Big) + \Big(f''(x) + f'(x) + f(x)\Big)\right]^2 = 0$
- ☐ $\left[\frac{\mathrm{d}}{\mathrm{d}x}\Big(f''(x) + f'(x) + f(x)\Big)\right]^2 + \Big(f''(x) + f'(x) + f(x)\Big) = 0$
- ■ $\frac{\mathrm{d}^2}{\mathrm{d}x^2}\Big(f''(x) + f'(x) + f(x)\Big) + \Big(f''(x) + f'(x) + f(x)\Big) = 0$
- ☐ $\frac{\mathrm{d}}{\mathrm{d}x}\Big(f''(x) + f'(x) + f(x)\Big)^2 + \Big(f''(x) + f'(x) + f(x)\Big) = 0$
- ☐ $\frac{\mathrm{d}}{\mathrm{d}x}\Big(f''(x) + f'(x) + f(x)\Big) + \Big(f''(x) + f'(x) + f(x)\Big)^2 = 0$

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(b) **(½ point)** Which of the following *formal expressions of* differential equations has the highest order?

Hint: For any function of the form $g\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)$, *consider the Taylor expansion of* g *as if its argument is an ordinary variable. Such a formal manipulation is permissible.*

- ☐ $\left[\exp\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)\right] f(x) = 0$
- ☐ $\left[\frac{1}{1+\frac{\mathrm{d}}{\mathrm{d}x}}\right] f(x) = 0$
- ☐ $\left[\cos\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)\right] f(x) = 0$
- ☐ $\left[\sum_{n=0}^{\infty} \frac{\mathrm{d}^n}{\mathrm{d}x^n}\right] f(x) = 0$
- ■ **All of them have the infinite order.**

3 Show your derivation

Question: 5 .... *(2 points)*

(a) **(1 point)** Show that $\ker(\mathcal{D}_1 \cdot \mathcal{D}_2) = \ker(\mathcal{D}_1) \cup \ker(\mathcal{D}_2)$ if $\mathcal{D}_1 \neq \mathcal{D}_2$ and if $[\mathcal{D}_1, \mathcal{D}_2] \cdot f = x \to 0$ for any f. Assume $\mathcal{D}_i \cdot (x \to 0) = x \to 0$.

Hint: The set $\ker(\mathcal{D})$ *should have* $n+1$ *elements in it if the operator* $\mathcal{D}$ *is of order* n*; for instance,* $\ker\left(\frac{\mathrm{d}}{\mathrm{d}x} - 1\right) = \{x \to 0, x \to e^x\}$, $\ker\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - 1\right) = \{x \to 0, x \to e^x, x \to e^{-x}\}$, *and so on.*

Solution: By definition, $f \in \ker(\mathcal{D}_1 \cdot \mathcal{D}_2)$ if $\mathcal{D}_1 \cdot \mathcal{D}_2 \cdot f = x \to 0$. But as $\mathcal{D}_1 \cdot (x \to 0) = x \to 0$, $\mathcal{D}_2 \cdot f = x \to 0$ is sufficient for $\mathcal{D}_1 \cdot \mathcal{D}_2 \cdot f = x \to 0$, hence

$$\ker(\mathcal{D}_2) \subset \ker(\mathcal{D}_1 \cdot \mathcal{D}_2) \tag{2}$$

But as $\mathcal{D}_1$ and $\mathcal{D}_2$ commute, we can reorder $\mathcal{D}_1 \cdot \mathcal{D}_2$ as $\mathcal{D}_2 \cdot \mathcal{D}_1$ hence we also have

$$\ker(\mathcal{D}_1) \subset \ker(\mathcal{D}_1 \cdot \mathcal{D}_2) \tag{3}$$

which leads to the statement

$$(\ker(\mathcal{D}_1) \cup \ker(\mathcal{D}_2)) \subset \ker(\mathcal{D}_1 \cdot \mathcal{D}_2) \tag{4}$$

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We are given that $\mathcal{D}_1 \neq \mathcal{D}_2$, hence their kernel should be distinct except the trivial solution $x \to 0$, therefore their union has $n_1 + n_2 + 1$ elements where n_i is order of $\mathcal{D}_i$ ($x \to 0$ is a common element). But we also expect $n_1 + n_2 + 1$ elements for the right hand side, so these two sets have to be equal to each other:

$$\ker(\mathcal{D}_1) \cup \ker(\mathcal{D}_2) = \ker(\mathcal{D}_1 \cdot \mathcal{D}_2) \tag{5}$$

(b) **(½ point)** What happens when $\mathcal{D}_1 = \mathcal{D}_2$? Comment on the relation between the sets $\ker(\mathcal{D}^2)$ and $\ker(\mathcal{D})$.

Solution: If $\mathcal{D}_1 = \mathcal{D}_2$, we are stuck at the step

$$\ker(\mathcal{D}) \subset \ker(\mathcal{D}^2) \tag{6}$$

This means that there needs to be further solutions to $\mathcal{D}^2 \cdot f = x \to 0$ then those in $\mathcal{D} \cdot f = x \to 0$: this is precisely the *repeated roots* case we have seen in the class.

(c) **(½ point)** What happens when $[\mathcal{D}_1, \mathcal{D}_2] \neq x \to 0$ for any f? Comment on the relation between the sets $\ker(\mathcal{D}_1 \cdot \mathcal{D}_2)$ and $\ker(\mathcal{D}_2)$.

Solution: If $\mathcal{D}_1$ and $\mathcal{D}_2$ do not commute, then we can write down

$$\ker(\mathcal{D}_2) \subset \ker(\mathcal{D}_1 \cdot \mathcal{D}_2) \tag{7}$$

but we cannot necessarily write it down for $\mathcal{D}_1$, hence

$$\ker(\mathcal{D}_1) \not\subset \ker(\mathcal{D}_1 \cdot \mathcal{D}_2) \tag{8}$$

Question: 6 .... *(4 points)*

Consider the differential equation

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \alpha(x)\frac{\mathrm{d}}{\mathrm{d}x} + \beta(x)\right) f(x) = 0 \tag{9}$$

and define the function g such that

$$g :: \mathbb{C} \to \mathbb{C} \tag{10a}$$
$$g = f(x) \to x \tag{10b}$$

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What is the differential equation that the function g satisfies?
Hint: Following chain rules would be helpful!

$$\frac{\mathrm{d}}{\mathrm{d}x} = \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^{-1}\frac{\mathrm{d}}{\mathrm{d}y} \tag{11a}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} = -\left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^{-3}\frac{\mathrm{d}^2 x}{\mathrm{d}y^2}\frac{\mathrm{d}}{\mathrm{d}y} + \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^{-2}\frac{\mathrm{d}^2}{\mathrm{d}y^2} \tag{11b}$$

Solution: Let us call $f(x) = y$ for simplicity and observe the chain rule:

$$\frac{\mathrm{d}}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}}{\mathrm{d}y} = \frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}\frac{\mathrm{d}}{\mathrm{d}y} = \frac{1}{g'(y)}\frac{\mathrm{d}}{\mathrm{d}y} \tag{12}$$

which leads to

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} = \frac{1}{g'(y)}\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{1}{g'(y)}\frac{\mathrm{d}}{\mathrm{d}y}\right) = -\frac{g''(y)}{(g'(y))^3}\frac{\mathrm{d}}{\mathrm{d}y} + \frac{1}{(g'(y))^2}\frac{\mathrm{d}^2}{\mathrm{d}y^2} \tag{13}$$

Therefore

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \alpha(x)\frac{\mathrm{d}}{\mathrm{d}x} + \beta(x)\right)f(x) = 0 \tag{14}$$

becomes

$$\left(-\frac{g''(y)}{(g'(y))^3}\frac{\mathrm{d}}{\mathrm{d}y} + \frac{1}{(g'(y))^2}\frac{\mathrm{d}^2}{\mathrm{d}y^2} + \alpha(g(y))\frac{1}{g'(y)}\frac{\mathrm{d}}{\mathrm{d}y} + \beta(g(y))\right)y = 0 \tag{15}$$

which is

$$-\frac{g''(y)}{(g'(y))^3} + \alpha(g(y))\frac{1}{g'(y)} + \beta(g(y))y = 0 \tag{16}$$

Question: 7 .... *(1 points)*

Let's assume that you are given the *characteristic equation*

$$xa^3 + ra + b = 0 \tag{17}$$

for the variable a and the constants $x, r,$ and b. What is the homogeneous differential equation for the function h for which this is the characteristic equation?

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Solution:

$$\left(x\frac{\mathrm{d}^3}{\mathrm{d}\#^3} + r\frac{\mathrm{d}}{\mathrm{d}\#} + b\right) h(\#) = 0 \text{ for any letter } \# \notin \{x, r, b\} \tag{18}$$

Question: 8 .... *(6 points)*

Let's assume you are measuring the heat conductance $c(t)$ of a crystal in the lab and you observe it obeys the following time dependence

$$\left(\frac{\mathrm{d}^3}{\mathrm{d}t^3} + ab^2\right) c(t) = p(t) \tag{19}$$

where $p(t)$ denotes the pressure applied to the crystal as a function of time. Your college on the other hand obtains the following relation based on their measurement:

$$c''(t) = \frac{1}{a} - \frac{1}{a}p(t) - \frac{b^2}{a}c'(t) \tag{20}$$

Taking $a > 0$ and $b > 0$ as time-independent constants, find out the conductance $c(t)$ as a function of time t.

Hint 1: Your result should not depend on $p(t)$.

Hint 2: Assume that you expect a solution with exponential suppression in time, i.e. $e^{-\alpha t}$.

*Hint 3: Particular solutions are unique: if you can guess the particular solution **and** show that it satisfies the differential equation, I would accept that!*

Solution: Let's first combine the given information to get rid of $p(t)$:

$$\left(\frac{\mathrm{d}^3}{\mathrm{d}t^3} + a\frac{\mathrm{d}^2}{\mathrm{d}t^2} + b^2\frac{\mathrm{d}}{\mathrm{d}t} + ab^2\right) c(t) = 1 \tag{21}$$

In this equation, we immediately see that the particular solution is

$$c_p(t) = \frac{1}{ab^2} \tag{22}$$

Therefore, we actually only need to find the homogeneous solution. Nevertheless, we will also systematically compute particular solution below in case you fail to see this, but if you guess the correct solution, that is also a viable solution as mentioned in hint 3.

We are given the information that one of the solutions should be exponentially suppressed, so we can use reduction of order:

$$c(t) = e^{-\alpha t} f(t) \tag{23}$$

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for which we have

$$c'(t) = -\alpha e^{-\alpha t} f(t) + e^{-\alpha t} f'(t) \tag{24a}$$
$$c''(t) = \alpha^2 e^{-\alpha t} f(t) - 2\alpha e^{-\alpha t} f'(t) + e^{-\alpha t} f''(t) \tag{24b}$$
$$c'(t) = -\alpha^3 e^{-\alpha t} f(t) + 3\alpha^2 e^{-\alpha t} f'(t) - 3\alpha e^{-\alpha t} f''(t) + e^{-\alpha t} f'''(t) \tag{24c}$$

When we insert this into the equation

$$\begin{aligned} 1 &= \left(\frac{\mathrm{d}^3}{\mathrm{d}t^3} + a\frac{\mathrm{d}^2}{\mathrm{d}t^2} + b^2\frac{\mathrm{d}}{\mathrm{d}t} + ab^2\right)\left(e^{-\alpha t} f(t)\right) \\ &= \left(-\alpha^3 + a\alpha^2 + b^2(-\alpha) + ab^2\right) e^{-\alpha t} f(t) \\ &\quad + \left(3\alpha^2 + a(-2\alpha) + b^2\right) e^{-\alpha t} f'(t) \\ &\quad + \left(-3\alpha + a\right) e^{-\alpha t} f''(t) \\ &\quad + e^{-\alpha t} f'''(t) \end{aligned} \tag{25}$$

To apply reduction of order, we need to choose α such that the overall coefficient of $f(t)$ is zero: this requires $\alpha = a$. With this choice, we obtain

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} - 2a\frac{\mathrm{d}}{\mathrm{d}t} + b^2 + a^2\right) g(t) = e^{at} \tag{26}$$

where $g(t) = f'(t)$. The homogeneous solution to this equation is given by the characteristic equation

$$r^2 - 2ar + b^2 + a^2 = 0 \tag{27}$$

which can be rewritten as

$$(r-a)^2 = -b^2 \tag{28}$$

hence $r = a \pm ib$, leading to the homogeneous solution:

$$g_h(t) = c_1 e^{(a+ib)t} + c_2 e^{(a-ib)t} \tag{29}$$

Let us now compute the particular solution. We first find out the impulse response in Laplace domain:

$$I(s) = \frac{1}{s^2 - 2as + a^2 + b^2} \tag{30}$$

which should be rewritable as $\frac{d_1}{s-a-ib} + \frac{d_2}{s-a+ib}$. By equating these, we end up with

$$I(s) = \frac{1}{2ib}\frac{1}{s-a-ib} - \frac{1}{2ib}\frac{1}{s-a+ib} \tag{31}$$

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hence impulse response in position space is

$$i(t) = \begin{cases} \frac{1}{2ib}e^{(a+ib)t} - \frac{1}{2ib}e^{(a-ib)t} & t > 0 \\ 0 & t < 0 \end{cases} \tag{32}$$

The particular solution then reads as the usual convolution:

$$\begin{aligned} g_p(t) &= \int_0^\infty i(t-t')e^{at'}dt' \\ &= \int_0^t \left[\frac{1}{2ib}e^{(a+ib)(t-t')} - \frac{1}{2ib}e^{(a-ib)(t-t')}\right] e^{at'}dt' \\ &= \frac{e^{(a+ib)t}}{2ib}\int_0^t e^{-ibt'}dt' - \frac{e^{(a-ib)t}}{2ib}\int_0^t e^{ibt'}dt' \\ &= \frac{e^{(a+ib)t}}{2ib}\frac{e^{-ibt}}{-ib} - \frac{e^{(a-ib)t}}{2ib}\frac{e^{ibt}}{ib} \\ &= \frac{e^{at}}{b^2} \end{aligned} \tag{33}$$

By combining homogeneous and particular solutions, we can write down full $g(t)$:

$$g(t) = c_1 e^{(a+ib)t} + c_2 e^{(a-ib)t} + \frac{e^{at}}{b^2} \tag{34}$$

By integrating this, we obtain $f(t)$:

$$f(t) = d_1 e^{(a+ib)t} + d_2 e^{(a-ib)t} + \frac{e^{at}}{ab^2} + d_3 \tag{35}$$

for the new unknown variables $d_{1,2}$ and the integration constant d_3. We know have the final solution via $c(t) = e^{-at}f(t)$:

$$f(t) = d_1 e^{ibt} + d_2 e^{-ibt} + d_3 e^{-at} + \frac{1}{ab^2} \tag{36}$$

which can also be rewritten as

$$f(t) = d_1 \cos(bt) + d_2 \sin(bt) + d_3 e^{-at} + \frac{1}{ab^2} \tag{37}$$

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Bonus Question: 9 .... *(2 points)*

(a) **(1 point)** Insert T (True) or F (False) in the spaces left to (i), (ii), and (iii). Remember that the logical "and" operation (denoted by $\wedge$) satisfies the following: $T \wedge T = T$, $T \wedge F = F$, $F \wedge T = F$, $F \wedge F = F$.

<u>F</u> (i): This exam is hard.
<u>T</u> (ii): (iii) is true.
<u>T</u> (iii): (iii) $\neq$ (i) $\wedge$ (ii).

(b) **(½ point)** Let's say you would like to solve the differential equation $g\left(x, \frac{\mathrm{d}}{\mathrm{d}x}\right) f(x) = h(x)$ for $g\left(x, \frac{\mathrm{d}}{\mathrm{d}x}\right) = \cos(x)\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \sin(x)$ and $h(x) = e^x$. Which Mathematica code below would achieve that?

- ■ `DSolve[Cos[x]f''[x]+Sin[x]f[x]==Exp[x],f[x],x]`
- ☐ `DSolve[cos[x]f''[x]+sin[x]f[x]==exp[x],f[x],x]`
- ☐ `DiffSolve[Cos[x]f''[x]+Sin[x]f[x]==Exp[x],f[x],x]`
- ☐ `DiffSolve[cos[x]f''[x]+sin[x]f[x]==exp[x],f[x],x]`
- ☐ None

(c) **(½ point)** Which code below would compute $(\mathcal{L} \cdot f)(s)$ for $f = x \to \cos(x)\sin(x)^2$ where $\mathcal{L}$ denotes the Laplace transformation?

- ☐ `LTransform[Cos[x] Sin[x]^2, x, s]`
- ☐ `LTransform[cos[x] sin[x]^2, x, s]`
- ■ `LaplaceTransform[Cos[x] Sin[x]^2, x, s]`
- ☐ `LaplaceTransform[cos[x] sin[x]^2, x, s]`
- ☐ None

« « « Congratulations, you have made it to the end! » » »