1. *Find the metric that minimizes any of the errors, by calculus.*

A. (MSE) – $\sum(\textbf{actual} - \textbf{predicted})^2$

Set y_i as the actual data

Predicted data base on the simplest Econometrics Model, which is $\widehat{y_i} = \beta_0$

So our goal is:

$$\min \sum(\text{actual} - \text{predicted})^2$$

which also is:

$$\min \sum_{i=1}^{n}(y_i - \widehat{y_i})^2$$
$$\min \sum_{i=1}^{n}(y_i - \beta_0)^2$$

For solve this function, we need make first order derivative of the function to be zero, so:

$$\frac{d}{d\beta_0}\left(\sum_{i=1}^{n}(y_i - \beta_0)^2\right) = \sum_{i=1}^{n} -2(y_i - \beta_0) = 0$$
$$\sum_{i=1}^{n}(y_i - \beta_0) = 0$$
$$n\beta_0 = \sum_{i=1}^{n} y_i$$
$$\beta_0 = \frac{\sum_{i=1}^{n} y_i}{n}$$

Therefore, we can see that the β_0 is actually the average of the y.

According to our data, we have

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
165	160	175	180	155	150	110	195	160	185

So, the β_0 will be:

$$\beta_0 = \frac{\sum_{i=1}^{10} y_i}{10} = \frac{165+160+175+180+155+150+110+195+160+185}{10} = 163.50$$

Therefore, our simplest Econometrics Model $\widehat{y_i} = 163.50$

B. MSPE – $\sum\left(\frac{\boldsymbol{actual-predicted}}{\boldsymbol{actual}}\right)^{2}$

Set y_i as the actual data

Predicted data base on the simplest Econometrics Model, which is $\widehat{y_i} = \beta_0$

So our goal is:

$$\min \sum\left(\frac{\text{actual}-\text{predicted}}{\text{actual}}\right)^{2}$$

which also is:

$$\min \sum_{i=1}^{n}\left(\frac{y_i-\widehat{y_i}}{y_i}\right)^{2}$$

$$\min \sum_{i=1}^{n}\left(\frac{y_i-\beta_0}{y_i}\right)^{2}$$

For solve this function, we need make first order derivative of the function to be zero, so:

$$\frac{d}{d\beta_0}\left(\sum_{i=1}^{n}\left(\frac{y_i-\beta_0}{y_i}\right)^{2}\right) = \sum_{i=1}^{n} -\frac{2(y_i-\beta_0)}{{y_i}^2} = 0$$

$$-2\sum_{i=1}^{n}\frac{y_i-\beta_0}{{y_i}^2} = -2\sum_{i=1}^{n}\left(\frac{1}{y_i}-\frac{\beta_0}{{y_i}^2}\right) = 0$$

$$-2\left(\sum_{i=1}^{n}\frac{1}{y_i}-\sum_{i=1}^{n}\frac{\beta_0}{{y_i}^2}\right) = 0$$

$$\sum_{i=1}^{n}\frac{1}{y_i}-\sum_{i=1}^{n}\frac{\beta_0}{{y_i}^2} = 0$$

$$\beta_0\sum_{i=1}^{n}\frac{1}{{y_i}^2} = \sum_{i=1}^{n}\frac{1}{y_i}$$

$$\beta_0 = \frac{\sum_{i=1}^{n}\frac{1}{y_i}}{\sum_{i=1}^{n}\frac{1}{{y_i}^2}}$$

According to our data, we have

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
165	160	175	180	155	150	110	195	160	185

So, the β_0 will be:

$$\beta_0 = \frac{\sum_{i=1}^{n}\frac{1}{y_i}}{\sum_{i=1}^{n}\frac{1}{{y_i}^2}} = \frac{0.0625725}{0.0004026} = 155.42191$$

Therefore, our simplest Econometrics Model $\widehat{y_i} = 155.42$

C. MAE - $\sum |actual - predicted|$

Set y_i as the actual data
Predicted data base on the simplest Econometrics Model, which is $\hat{y_i} = \beta_0$
So our goal is:

$$\min \sum |\text{actual} - \text{predicted}|$$

which also is:

$$\min \sum_{i=1}^{n} |y_i - \hat{y_i}|$$
$$\min \sum_{i=1}^{n} |y_i - \beta_0|$$

Since this function include the absolute symbol, we have to move it for to do first order derivative. So,

$$\min \sum_{i=1}^{n} \sqrt{(y_i - \beta_0)^2}$$

Then we make the first order derivative of this function to be zero:

$$\frac{d}{d\beta_0}\left(\sum_{i=1}^{n} \sqrt{(y_i - \beta_0)^2}\right) = \sum_{i=1}^{n}\left(-\frac{y_i - \beta_0}{\sqrt{(y_i - \beta_0)^2}}\right) = 0$$

For $-\frac{y_i - \beta_0}{\sqrt{(y_i - \beta_0)^2}}$, we just have to possible result, which is

$$\begin{cases} -\frac{y_i - \beta_0}{\sqrt{(y_i - \beta_0)^2}} = -1, \quad when\ y_i > \beta_0 \\ -\frac{y_i - \beta_0}{\sqrt{(y_i - \beta_0)^2}} = 1, \quad when\ y_i < \beta_0 \\ -\frac{y_i - \beta_0}{\sqrt{(y_i - \beta_0)^2}} = \frac{0}{0} = 0, when\ y_i = \beta_{0,}\ (L'Hospital's\ rule) \end{cases}$$

In order to satisfy $\sum_{i=1}^{n}\left(-\frac{y_i - \beta_0}{\sqrt{(y_i - \beta_0)^2}}\right) = 0$, when n is even, we should guarantee $\frac{n}{2}$ of $-\frac{y_i - \beta_0}{\sqrt{(y_i - \beta_0)^2}}$ should be -1, and $\frac{n}{2}$ of $-\frac{y_i - \beta_0}{\sqrt{(y_i - \beta_0)^2}}$ should be 1. Therefore, if we set the data with ascending order, the solution will be the range, which is between $\boldsymbol{\frac{n}{2}th}$ number's value and $\boldsymbol{\frac{n+2}{2}th}$ number's value.

On the other hand, when n is odd, one of them should be 0, and $\frac{n-1}{2}$ of $-\frac{y_i - \beta_0}{\sqrt{(y_i - \beta_0)^2}}$ should be -1, and $\frac{n-1}{2}$ of $-\frac{y_i - \beta_0}{\sqrt{(y_i - \beta_0)^2}}$ should be 1. Base on this concept, we assume the **median** of data is the optimal solution.

According to our data, we have

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
165	160	175	180	155	150	110	195	160	185

In order to obtain the median, we have to rearrange the data.

1	2	3	4	5	6	7	8	9	10
y_7	y_6	y_5	y_9	y_2	y_1	y_3	y_4	y_{10}	y_8
110	150	155	160	160	165	175	180	185	195

Therefore, as the result n is even, the β_0 should be:

$$160 \leq \beta_0 \leq 165$$

D. MAPE - $\sum\left|\frac{(\text{actual}-\text{predicted})}{\text{actual}}\right|$

Set y_i as the actual data
Predicted data base on the simplest Econometrics Model, which is $\widehat{y_i} = \beta_0$
So our goal is:

$$\min \sum\left|\frac{(\text{actual}-\text{predicted})}{\text{actual}}\right|$$

which also is:

$$\min \sum_{i=1}^{n}\left|\frac{(y_i-\widehat{y_i})}{y_i}\right|$$

$$\min \sum_{i=1}^{n}\left|\frac{(y_i-\beta_0)}{y_i}\right|$$

Since this function include the absolute symbol, we have to move it for to do first order derivative. So,

$$\min \sum_{i=1}^{n}\sqrt{\left(\frac{(y_i-\beta_0)}{y_i}\right)^2}$$

Then we make the first order derivative of this function to be zero:

$$\frac{d}{d\beta_0}\left(\sum_{i=1}^{n}\sqrt{\left(\frac{(y_i-\beta_0)}{y_i}\right)^2}\right) = \sum_{i=1}^{n} -\frac{\sqrt{{y_i}^2}(y_i-\beta_0)}{{y_i}^2\sqrt{(y_i-\beta_0)^2}} = 0$$

As the result of y is height, which always positive, $\sqrt{{y_i}^2} = y_i$. So,

$$\sum_{i=1}^{n} -\frac{(y_i-\beta_0)}{y_i\sqrt{(y_i-\beta_0)^2}} = 0$$

$$\begin{cases} -\frac{(y_i-\beta_0)}{y_i\sqrt{(y_i-\beta_0)^2}} = -\frac{1}{y_i}, & when\ y_i > \beta_0 \\ -\frac{(y_i-\beta_0)}{y_i\sqrt{(y_i-\beta_0)^2}} = \frac{1}{y_i}, & when\ y_i < \beta_0 \\ -\frac{(y_i-\beta_0)}{y_i\sqrt{(y_i-\beta_0)^2}} = \frac{0}{0} = 0, when\ y_i = \beta_{0,} & (L'Hospital's\ rule) \end{cases}$$

According to this, we cannot to make sure the derivative will be to zero, because it depends on the value of y_i. For this reason, we cannot get the estimation of MAPE by calculus.