# Supplementary File of “A Self-guided Reference Vector Strategy for Many-objective Optimization”

## 1. Supplementary Tables and Figures

TABLE A.I
FEATURES OF THE MAF AND MDTLZ TEST PROBLEMS

| Test Problems | $m$ | $n$ | Parameter | Features |
|---|---|---|---|---|
| MaF1 | 3,4,5,7,10 | $m$+$k$-1 | $k$=10 | Linear, Inverted |
| MaF2 | 3,4,5,7,10 | $m$+$k$-1 | $k$=10 | Concave |
| MaF3 | 3,4,5,7,10 | $m$+$k$-1 | $k$=10 | Convex, Multi-modal |
| MaF4 | 3,4,5,7,10 | $m$+$k$-1 | $k$=10 | Concave, Multi-modal, Inverted, badly-scaled |
| MaF5 | 3,4,5,7,10 | $m$+$k$-1 | $k$=10 | Convex, Biased, Badly-scaled |
| MaF6 | 3,4,5,7,10 | $m$+$k$-1 | $k$=10 | Concave, Degenerate |
| MaF7 | 3,4,5,7,10 | $m$+$k$-1 | $k$=20 | Mixed, Disconnected, Multi-modal |
| MaF8 | 3,4,5,7,10 | 2 | —— | Linear, Degenerate |
| MaF9 | 3,4,5,7,10 | 2 | —— | Linear, Degenerate |
| MaF10 | 3,4,5,7,10 | $k$+$l$ | $k=2\times(m\text{-}1)$ , $l$=20 | Mixed, Biased |
| MaF11 | 3,4,5,7,10 | $k$+$l$ | $k=2\times(m\text{-}1)$ , $l$=20 | Concave, Deceptive |
| MaF12 | 3,4,5,7,10 | $k$+$l$ | $k=2\times(m\text{-}1)$ , $l$=20 | Concave, Multi-modal, Biased Deceptive |
| MaF13 | 3,4,5,7,10 | 5 | —— | Concave, Unimodal, Non-separable, Degenerate |
| mDTLZ1 | 3,4,5,7,10 | 2$m$+$k$-1 | $k$=5 | Linear, Inverted, Hardly-dominated boundaries |
| mDTLZ2 | 3,4,5,7,10 | 2$m$+$k$-1 | $k$=5 | Concave, Inverted, Hardly-dominated boundaries |
| mDTLZ3 | 3,4,5,7,10 | 2$m$+$k$-1 | $k$=5 | Concave, Inverted, Hardly-dominated boundaries |
| mDTLZ4 | 3,4,5,7,10 | 2$m$+$k$-1 | $k$=5 | Concave, Inverted, Hardly-dominated boundaries |

TABLE A. II
PARAMETERS SETTINGS OF ALL THE COMPARED ALGORITHMS

| Algorithms | Parameters settings |
|---|---|
| MOEA/D-AWA | $p_c=1.0$ , $p_m=1/n$ , $\eta_c=20$ , $\eta_m=20$ , $nus=0.05N$, $re=0.8$ |
| NSGA-III | $p_c=1.0$ , $p_m=1/n$ , $\eta_c=30$ , $\eta_m=20$ |
| EFR-RR | $p_c=1.0$ , $p_m=1/n$ , $\eta_c=30$ , $\eta_m=20$ , $K=2$ |
| $\theta$-DEA | $p_c=1.0$ , $p_m=1/n$ , $\eta_c=30$ , $\eta_m=20$ , $\theta=5.0$ |
| A-NSGA-III | $p_c=1.0$ , $p_m=1/n$ , $\eta_c=30$ , $\eta_m=20$ , |
| RVEA* | $p_c=1.0$ , $p_m=1/n$ , $\eta_c=30$ , $\eta_m=20$ , $\alpha=2$ , $f_r=0.1$ |
| AR-MOEA | $p_c=1.0$ , $p_m=1/n$ , $\eta_c=20$ , $\eta_m=20$ , |
| VaEA | $p_c=1.0$ , $p_m=1/n$ , $\eta_c=30$ , $\eta_m=20$ , $\sigma=\pi/2(N+1)$ |
| MaOEA/C | $p_c=1.0$ , $p_m=1/n$ , $\eta_c=30$ , $\eta_m=20$ , $\varepsilon=0.8$ |

TABLE A. III
SETTINGS OF THE POPULATION SIZE AND EVALUATIONS

| Objectives ($m$) | Divisions ($H$) | Population Size | $G_{max}$ | Evaluations |
|---|---|---|---|---|
| 3 | 16 | 153 | 600 | 91800 |
| 4 | 8 | 165 | 700 | 115500 |
| 5 | 6 | 210 | 800 | 168000 |
| 7 | 4,2 | 238 | 900 | 214200 |
| 10 | 3,2 | 275 | 1000 | 275000 |

TABLE A. IV

THREE COMPARISONS OF RESULTS OF DIFFERENT MAOEAS ON MAF1-MAF13 AND MDTLZ1-MDTLZ4 USING HV

| Problem | $m$ | NSGA-III | NSGA-III/S | $\theta$-DEA | $\theta$-DEA/S | EFR-RR | EFR-RR/S |
|---|---|---|---|---|---|---|---|
| MaF1 | 3 | 2.188E-01(7.43E-04) | **2.218E-01(1.58E-03)+** | 2.054E-01(4.72E-04) | **2.216E-01(9.29E-04)+** | 2.075E-01(1.44E-03) | **2.141E-01(1.55E-03)+** |
| | 4 | 4.683E-02(1.36E-03) | **5.804E-02(4.12E-04)+** | 3.709E-02(4.11E-04) | **5.871E-02(3.78E-04)+** | 3.561E-02(1.51E-03) | **5.264E-02(8.86E-04)+** |
| | 5 | 7.559E-03(5.10E-04) | **1.211E-02(1.78E-04)+** | 5.539E-03(3.82E-04) | **1.243E-02(1.25E-04)+** | 3.535E-03(4.91E-04) | **1.007E-02(2.43E-04)+** |
| | 7 | 2.359E-04(9.71E-06) | **3.179E-04(6.06E-06)+** | 1.852E-04(1.26E-05) | **3.286E-04(5.37E-06)+** | 1.616E-04(1.47E-05) | **1.974E-04(8.28E-06)+** |
| | 10 | 4.264E-07(1.45E-08) | **6.815E-07(1.88E-08)+** | 3.293E-07(5.67E-08) | **7.362E-07(2.15E-08)+** | 1.800E-07(5.13E-08) | **2.994E-07(2.43E-08)+** |
| MaF2 | 3 | 2.450E-01(8.60E-04) | 2.439E-01(1.96E-03)– | 2.467E-01(6.66E-04) | 2.380E-01(2.41E-03)– | 2.437E-01(1.20E-03) | 2.425E-01(1.88E-03)~ |
| | 4 | 2.470E-01(1.52E-03) | 2.319E-01(3.13E-03)– | 2.429E-01(1.26E-03) | 2.356E-01(3.54E-03)– | 2.440E-01(2.76E-03) | **2.469E-01(2.36E-03)~** |
| | 5 | 2.448E-01(2.26E-03) | 2.122E-01(7.34E-03)– | 2.325E-01(3.42E-03) | 2.180E-01(4.78E-03)– | 2.282E-01(3.14E-03) | **2.411E-01(4.27E-03)+** |
| | 7 | 2.223E-01(3.41E-03) | 1.786E-01(6.97E-03)– | 2.179E-01(4.92E-03) | 1.886E-01(7.84E-03)– | 2.200E-01(4.77E-03) | **2.269E-01(5.16E-03)+** |
| | 10 | 2.123E-01(3.96E-03) | 1.565E-01(1.15E-02)– | 1.941E-01(7.62E-03) | 1.759E-01(1.27E-02)– | 1.953E-01(4.18E-03) | **2.171E-01(5.41E-03)+** |
| MaF3 | 3 | 9.612E-01(1.81E-03) | 9.520E-01(1.41E-02)– | 9.591E-01(1.22E-03) | 9.113E-01(5.30E-02)– | 7.628E-01(2.74E-01) | **9.277E-01(2.78E-02)+** |
| | 4 | 9.901E-01(9.77E-03) | 9.897E-01(2.43E-02)~ | 9.890E-01(1.30E-03) | 9.866E-01(8.68E-03)~ | 4.717E-01(4.52E-01) | **5.335E-01(4.64E-01)+** |
| | 5 | 9.940E-01(1.03E-02) | 9.914E-01(4.10E-02)~ | 9.927E-01(1.12E-03) | 9.764E-01(3.28E-02)– | 3.569E-01(4.41E-01) | **4.863E-01(4.90E-01)+** |
| | 7 | 9.969E-01(1.06E-02) | **9.978E-01(1.68E-02)~** | 9.917E-01(1.89E-03) | **9.921E-01(5.32E-03)~** | 0.000E+00(0.00E+00) | **1.114E-01(2.95E-01)+** |
| | 10 | 9.798E-01(1.07E-01) | **9.983E-01(4.51E-03)+** | 9.895E-01(4.60E-03) | 9.276E-01(1.18E-01)– | 2.676E-01(2.19E-01) | 0.000E+00(0.00E+00)– |
| MaF4 | 3 | 5.362E-01(3.92E-03) | **5.393E-01(3.53E-03)+** | 5.350E-01(2.02E-03) | **5.386E-01(4.03E-03)+** | 4.820E-01(1.01E-01) | **5.152E-01(6.82E-03)+** |
| | 4 | 2.472E-01(6.52E-03) | **2.844E-01(3.08E-03)+** | 2.537E-01(6.85E-03) | **2.857E-01(2.60E-03)+** | 1.137E-01(8.54E-02) | **2.355E-01(1.15E-02)+** |
| | 5 | 8.459E-02(4.79E-03) | **1.307E-01(2.08E-03)+** | 7.217E-02(8.80E-03) | **1.311E-01(2.64E-03)+** | 3.259E-02(1.23E-02) | **8.433E-02(7.16E-03)+** |
| | 7 | 8.417E-03(8.62E-04) | **1.785E-02(5.44E-04)+** | 6.051E-03(7.27E-04) | **1.767E-02(5.34E-04)+** | 4.730E-03(1.03E-03) | **5.543E-03(1.25E-03)+** |
| | 10 | 2.368E-04(1.87E-05) | **4.469E-04(3.41E-05)+** | 2.311E-04(2.29E-05) | **4.267E-04(2.57E-05)+** | 2.578E-04(4.05E-05) | **5.089E-05(1.14E-05)+** |
| MaF5 | 3 | 5.707E-01(2.32E-04) | 5.497E-01(8.67E-02)– | 5.708E-01(1.27E-04) | 5.638E-01(1.68E-03)– | 5.619E-01(1.15E-03) | 5.223E-01(7.09E-02)– |
| | 4 | 7.151E-01(1.76E-04) | 6.982E-01(2.31E-03)– | 7.153E-01(1.38E-05) | 6.992E-01(3.35E-03)– | 7.002E-01(1.93E-03) | 6.792E-01(2.96E-03)– |
| | 5 | 8.123E-01(1.51E-04) | 7.879E-01(3.99E-03)– | 8.127E-01(2.27E-05) | 7.887E-01(3.26E-03)– | 7.974E-01(1.85E-03) | 7.708E-01(5.45E-03)– |
| | 7 | 9.126E-01(1.77E-04) | 8.756E-01(4.15E-03)– | 9.130E-01(2.76E-05) | 8.803E-01(2.89E-03)– | 9.059E-01(1.40E-03) | 8.745E-01(5.88E-03)– |
| | 10 | 9.697E-01(6.30E-05) | 9.347E-01(3.96E-03)– | 9.698E-01(2.63E-05) | 9.381E-01(3.54E-03)– | 9.614E-01(9.84E-04) | 9.449E-01(3.96E-03)– |
| MaF6 | 3 | 1.966E-01(6.77E-04) | **2.008E-01(5.08E-05)+** | 1.881E-01(1.72E-03) | **2.004E-01(2.92E-04)+** | 4.893E-02(2.66E-02) | **1.967E-01(1.94E-03)+** |
| | 4 | 1.463E-01(1.36E-03) | **1.518E-01(3.54E-05)+** | 1.357E-01(3.19E-03) | **1.518E-01(3.36E-05)+** | 0.000E+00(0.00E+00) | **1.476E-01(3.53E-03)+** |
| | 5 | 1.244E-01(2.05E-03) | **1.301E-01(1.85E-05)+** | 1.158E-01(1.80E-03) | **1.301E-01(2.04E-05)+** | 0.000E+00(0.00E+00) | **1.254E-01(1.14E-02)+** |
| | 7 | 1.072E-01(2.02E-03) | **1.112E-01(7.80E-06)+** | 1.027E-01(2.13E-03) | **1.174E-01(2.03E-02)+** | 9.450E-03(1.76E-02) | **6.272E-02(5.58E-02)+** |
| | 10 | 3.469E-02(3.53E-02) | **4.313E-02(3.41E-02)+** | 5.896E-02(2.40E-02) | 4.553E-02(1.13E-02)– | 3.610E-03(1.32E-02) | 0.000E+00(0.00E+00)– |
| MaF7 | 3 | 3.225E-01(1.06E-03) | **3.225E-01(1.08E-02)~** | 3.175E-01(8.34E-04) | 3.152E-01(1.23E-02)~ | 2.967E-01(5.00E-03) | **3.002E-01(1.01E-02)+** |
| | 4 | 3.162E-01(2.79E-03) | **3.176E-01(7.76E-03)+** | 3.152E-01(8.25E-03) | 3.126E-01(1.20E-02)~ | 2.826E-01(1.31E-02) | **2.916E-01(9.30E-03)+** |
| | 5 | 3.066E-01(2.18E-03) | **3.099E-01(2.68E-03)~** | 2.780E-01(7.09E-03) | **2.995E-01(3.47E-03)+** | 2.096E-01(2.82E-02) | **2.731E-01(6.24E-03)+** |
| | 7 | 2.684E-01(2.35E-03) | 2.454E-01(3.46E-03)– | 2.371E-01(1.67E-02) | **2.501E-01(1.02E-02)+** | 1.529E-01(1.54E-02) | **1.948E-01(1.15E-02)+** |
| | 10 | 2.367E-01(3.01E-03) | 1.825E-01(8.50E-03)– | 2.305E-01(1.63E-02) | 1.442E-01(1.57E-02)– | 1.390E-01(1.52E-02) | **1.408E-01(1.21E-02)+** |
| MaF8 | 3 | 2.757E-01(1.48E-03) | **2.806E-01(2.95E-03)+** | 2.626E-01(6.89E-03) | **2.800E-01(3.34E-03)+** | 2.550E-01(2.15E-02) | 2.373E-01(2.76E-02)– |
| | 4 | 1.598E-01(2.39E-03) | **1.796E-01(9.58E-04)+** | 1.266E-01(4.99E-03) | **1.798E-01(9.84E-04)+** | 1.319E-01(5.31E-03) | **1.589E-01(1.11E-02)+** |
| | 5 | 1.108E-01(1.95E-03) | **1.283E-01(3.54E-04)+** | 8.232E-02(3.85E-03) | **1.283E-01(2.58E-04)+** | 7.917E-02(6.68E-03) | **1.174E-01(6.66E-03)+** |
| | 7 | 4.273E-02(7.98E-04) | **4.958E-02(6.66E-05)+** | 2.826E-02(2.82E-03) | **4.955E-02(6.07E-05)+** | 2.790E-02(3.16E-03) | **4.479E-02(2.05E-03)+** |
| | 10 | 9.189E-03(2.61E-04) | **1.115E-02(2.38E-05)+** | 5.192E-03(9.50E-04) | **1.114E-02(1.87E-05)+** | 6.917E-03(6.29E-04) | **1.006E-02(3.58E-04)+** |
| MaF9 | 3 | 8.414E-01(1.86E-03) | 8.299E-01(3.36E-03)– | 8.459E-01(1.25E-03) | 8.357E-01(4.28E-03)– | 8.288E-01(3.46E-03) | 8.047E-01(6.39E-02)– |
| | 4 | 2.584E-01(1.19E-02) | **2.797E-01(5.18E-03)+** | 2.761E-01(2.00E-02) | **2.884E-01(1.19E-02)+** | 2.551E-01(2.65E-02) | 2.440E-01(2.10E-02)– |
| | 5 | 2.184E-01(1.19E-02) | **2.535E-03(1.22E-02)**+ | 2.061E-01(2.00E-02) | **2.616E-01(9.29E-03)+** | 9.851E-02(2.65E-02) | 8.568E-02(3.75E-03)– |
| | 7 | 4.092E-02(7.03E-04) | **4.998E-02(1.10E-03)+** | 4.810E-02(7.68E-04) | 4.852E-02(6.71E-04)~ | 3.353E-02(9.70E-04) | 9.615E-03(4.32E-04)– |
| | 10 | 5.184E-03(1.19E-04) | **6.998E-03(1.10E-04)+** | 4.061E-03(2.00E-03) | **5.652E-03(6.71E-04)+** | 2.851E-03(2.65E-04) | **3.615E-03(4.32E-04)+** |
| MaF10 | 3 | 5.926E-01(3.56E-02) | **6.643E-01(5.67E-02)+** | 6.505E-01(4.59E-02) | **6.870E-01(5.19E-02)+** | 5.112E-01(2.12E-02) | 3.688E-01(3.17E-02)– |
| | 4 | 4.307E-01(4.13E-02) | **5.250E-01(3.40E-02)+** | 5.689E-01(4.46E-02) | **5.818E-01(3.27E-02)+** | 4.599E-01(2.32E-02) | 3.033E-01(3.41E-02)– |
| | 5 | 4.293E-01(4.08E-02) | **5.360E-01(3.58E-02)+** | 6.369E-01(2.79E-02) | 6.302E-01(2.27E-02)~ | 4.444E-01(5.27E-02) | 3.224E-01(2.71E-02)– |
| | 7 | 5.391E-01(4.22E-02) | **7.067E-01(3.25E-02)+** | 7.478E-01(1.88E-02) | 7.423E-01(2.20E-02)~ | 6.093E-01(5.36E-02) | 4.749E-01(4.74E-02)– |
| | 10 | 7.628E-01(3.67E-02) | **8.384E-01(1.15E-02)+** | 8.725E-01(8.53E-03) | 8.503E-01(9.02E-03)– | 8.507E-01(2.75E-02) | 7.508E-01(3.67E-02)– |
| MaF11 | 3 | 8.964E-01(6.08E-02) | **9.028E-01(4.64E-02)~** | 8.991E-01(6.13E-02) | 8.715E-01(6.44E-02)– | 9.025E-01(4.60E-02) | 8.994E-01(5.48E-02)~ |
| | 4 | 9.300E-01(7.52E-02) | 9.270E-01(7.09E-02)~ | 9.327E-01(7.59E-02) | 9.144E-01(6.76E-02)– | 9.782E-01(2.55E-03) | 9.363E-01(6.06E-02)– |
| | 5 | 9.717E-01(4.42E-02) | 9.347E-01(6.59E-02)– | 9.845E-01(2.36E-03) | 9.823E-01(2.89E-02)~ | 9.913E-01(1.51E-03) | 9.663E-01(4.32E-02)– |
| | 7 | 9.782E-01(3.24E-02) | 9.775E-01(3.13E-02)~ | 9.334E-01(7.71E-02) | **9.568E-01(4.23E-02)+** | 9.801E-01(4.51E-02) | 9.658E-01(5.34E-02)– |
| | 10 | 9.641E-01(6.12E-02) | **9.708E-01(4.40E-02)+** | 9.004E-01(8.72E-02) | **9.659E-01(4.45E-02)+** | 9.843E-01(3.60E-03) | 9.834E-01(3.16E-02)~ |
| MaF12 | 3 | 5.059E-01(2.56E-02) | **5.116E-01(1.33E-02)+** | 5.081E-01(2.47E-02) | **5.194E-01(2.47E-02)+** | 4.707E-01(9.59E-03) | **4.865E-01(1.80E-02)+** |
| | 4 | 5.885E-01(1.01E-02) | **6.001E-01(2.09E-02)+** | 6.044E-01(2.45E-02) | 6.009E-01(2.03E-02)~ | 5.801E-01(1.80E-03) | **5.794E-01(2.04E-02)~** |
| | 5 | 6.607E-01(1.42E-02) | 6.577E-01(1.71E-02)– | 6.674E-01(1.86E-02) | 6.639E-01(1.63E-02)~ | 6.471E-01(3.43E-03) | **6.565E-01(2.42E-02)+** |
| | 7 | 7.236E-01(1.94E-02) | 6.882E-01(2.11E-02)– | 7.417E-01(2.68E-02) | 6.945E-01(1.86E-02)– | 7.360E-01(2.78E-02) | 7.195E-01(2.88E-02)– |
| | 10 | 7.504E-01(2.28E-02) | 6.869E-01(2.57E-02)– | 7.787E-01(2.71E-02) | 6.813E-01(2.88E-02)– | 7.518E-01(2.79E-02) | **7.503E-01(3.60E-02)~** |
| MaF13 | 3 | 5.033E-01(1.64E-02) | 4.992E-01(3.15E-02)~ | 5.142E-01(1.48E-02) | **5.392E-01(8.53E-03)+** | 3.762E-01(4.07E-02) | 3.745E-01(1.71E-02)~ |
| | 4 | 2.907E-01(1.58E-02) | **3.483E-01(5.56E-03)+** | 2.773E-01(1.31E-02) | **3.486E-01(7.92E-03)+** | 2.170E-02(3.04E-02) | **1.482E-01(5.75E-03)+** |
| | 5 | 1.895E-01(3.28E-02) | **2.262E-01(2.36E-02)+** | 1.257E-01(4.66E-02) | **2.341E-01(2.10E-02)+** | 5.621E-03(9.85E-03) | **8.991E-02(3.36E-02)+** |
| | 7 | 8.824E-02(4.75E-02) | **1.571E-01(3.17E-02)+** | 2.630E-02(1.90E-02) | **1.578E-01(3.20E-02)+** | 1.187E-02(2.18E-02) | **2.447E-02(1.07E-02)+** |
| | 10 | 5.423E-02(4.44E-02) | **1.072E-01(1.90E-02)+** | 6.635E-03(1.44E-02) | **1.094E-01(2.87E-02)+** | 2.297E-05(5.47E-05) | **1.618E-02(2.67E-02)+** |
| mDTLZ1 | 3 | 7.384E-02(4.59E-02) | **1.345E-01(7.40E-02)+** | 1.148E-01(4.96E-02) | 8.138E-02(8.48E-02)– | 1.640E-02(1.78E-02) | **2.113E-01(5.23E-03)+** |
| | 4 | 1.571E-02(6.56E-03) | **1.927E-02(1.31E-02)+** | 2.176E-02(5.34E-03) | **2.185E-02(1.75E-02)~** | 1.179E-03(1.43E-03) | **3.552E-02(4.80E-03)+** |
| | 5 | 1.172E-03(4.95E-04) | **1.827E-03(1.41E-03)+** | 1.973E-03(4.57E-04) | **2.793E-03(8.55E-04)+** | 3.189E-05(9.18E-05) | **1.691E-03(6.97E-04)+** |
| | 7 | 2.782E-06(1.93E-06) | 6.363E-07(9.38E-07)– | 3.957E-06(1.98E-06) | 1.847E-06(2.12E-06)– | 8.749E-08(4.79E-07) | **3.868E-07(1.20E-06)+** |
| | 10 | 8.941E-10(6.37E-10) | 7.660E-10(2.12E-09)– | 2.323E-09(2.03E-09) | 1.831E-10(4.90E-10)– | 0.000E+00(0.00E+00) | **1.235E-10(4.58E-10)+** |
| mDTLZ2 | 3 | 5.300E-01(3.34E-03) | **5.383E-01(3.32E-03)~** | 5.354E-01(3.07E-03) | **5.395E-01(2.70E-03)+** | 5.088E-01(5.04E-03) | **5.249E-01(4.09E-03)+** |
| | 4 | 2.164E-01(1.03E-02) | **2.703E-01(5.87E-03)+** | 2.560E-01(7.62E-03) | **2.780E-01(3.98E-03)+** | 1.649E-01(1.54E-02) | **2.290E-01(7.99E-03)+** |
| | 5 | 5.062E-02(7.11E-03) | **1.111E-01(5.36E-03)+** | 7.911E-02(1.21E-02) | **1.154E-01(2.79E-03)+** | 2.144E-02(6.71E-03) | **6.990E-02(6.70E-03)+** |
| | 7 | 4.366E-03(7.68E-04) | **1.216E-02(8.65E-04)+** | 7.871E-03(1.31E-03) | **1.136E-02(5.89E-04)+** | 1.692E-03(8.17E-04) | **3.522E-03(7.39E-04)+** |
| | 10 | 4.737E-05(2.62E-05) | **2.884E-04(2.22E-05)+** | 2.045E-04(4.28E-05) | **2.504E-04(1.62E-05)+** | 1.072E-04(4.00E-05) | 2.960E-05(1.27E-05)– |
| mDTLZ3 | 3 | 3.667E-01(1.42E-01) | **4.829E-01(1.01E-01)+** | 3.854E-01(1.34E-01) | **4.696E-01(1.59E-01)+** | 9.320E-02(8.56E-02) | **5.033E-01(8.90E-02)+** |
| | 4 | 1.405E-01(4.87E-02) | **1.824E-01(7.93E-02)+** | 1.907E-01(2.82E-02) | **2.599E-01(1.43E-02)+** | 2.119E-02(2.42E-02) | **2.186E-01(1.45E-02)+** |
| | 5 | 3.069E-02(1.35E-02) | **8.926E-02(1.47E-02)+** | 5.882E-02(1.22E-02) | **9.514E-02(8.33E-03)+** | 1.315E-03(3.14E-03) | **4.009E-02(9.43E-03)+** |
| | 7 | 9.070E-04(6.38E-04) | **2.813E-03(9.14E-04)+** | 1.844E-03(8.71E-04) | **2.436E-03(7.11E-04)+** | 1.202E-05(4.60E-05) | **2.220E-04(3.61E-04)+** |
| | 10 | 1.828E-05(1.11E-05) | **1.835E-05(1.44E-05)~** | 2.830E-05(1.20E-05) | 7.688E-06(6.32E-06)– | 0.000E+00(0.00E+00) | **9.258E-12(5.07E-11)+** |
| mDTLZ4 | 3 | 5.298E-01(2.89E-03) | **5.412E-01(1.61E-03)+** | 5.364E-01(1.87E-03) | **5.437E-01(1.48E-03)+** | 5.118E-01(5.85E-03) | **5.230E-01(3.79E-03)+** |
| | 4 | 2.105E-01(1.00E-02) | **2.681E-01(9.68E-03)+** | 2.527E-01(1.11E-02) | **2.813E-01(2.85E-03)+** | 1.656E-01(1.93E-02) | **2.214E-01(8.73E-03)+** |
| | 5 | 3.752E-02(7.45E-03) | **1.045E-01(5.52E-03)+** | 7.149E-02(1.27E-02) | **1.138E-01(2.91E-03)+** | 1.815E-02(7.25E-03) | **6.190E-02(8.90E-03)+** |
| | 7 | 1.672E-03(6.03E-04) | **8.893E-03(8.69E-04)+** | 6.525E-03(1.24E-03) | **9.930E-03(6.81E-04)+** | 1.511E-03(1.34E-03) | **2.301E-03(6.70E-04)+** |
| | 10 | 7.921E-05(2.96E-05) | **1.774E-04(1.59E-05)+** | 2.135E-04(4.43E-05) | **2.962E-04(1.52E-05)+** | 2.152E-04(1.12E-04) | 1.876E-05(1.12E-05)– |
| *best/all* | | 27/85 | 57/85 | 34/85 | 51/85 | 25/85 | 59/85 |
| +/–/~ | | —— | 54/19/12 | —— | 49/25/11 | —— | **56/12/7** |

TABLE A.V

COMPARISON OF RESULTS OF MAOEA/SRV AND SIX COMPETITIVE MAOEAS ON MAF1-MAF13 USING HV

| Problem | *m* | MOEA/D-AWA | A-NSGA-III | RVEA* | AR-MOEA | VaEA | MaOEA/C | MaOEA/SRV |
|---|---|---|---|---|---|---|---|---|
| MaF1 | 3 | 2.261E-01(3.82E-04)~ | **2.305E-01(1.17E-04)+** | 2.269E-01(5.14E-04)~ | 2.270E-01(4.70E-04)~ | 2.257E-01(6.95E-04)~ | 2.263E-01(5.78E-04)~ | 2.262E-01(6.70E-04) |
| | 4 | 5.322E-02(3.90E-04)– | 5.308E-02(9.86E-04)– | 4.975E-02(2.26E-03)– | 5.767E-02(6.09E-04)– | 5.776E-02(5.38E-04)– | **6.088E-02(2.96E-04)~** | 6.067E-02(2.66E-04) |
| | 5 | 9.899E-03(1.55E-04)– | 7.264E-03(5.69E-04)– | 7.631E-03(5.88E-04)– | 1.181E-02(4.51E-04)– | 1.111E-02(2.49E-04)– | 1.284E-02(1.50E-04)~ | **1.303E-02(9.92E-05)** |
| | 7 | 1.958E-04(3.57E-05)– | 1.426E-04(1.89E-05)– | 8.215E-03(7.44E-04)– | 1.163E-02(2.93E-04)– | 2.080E-04(1.02E-05)– | 3.193E-04(6.07E-06)– | **3.399E-04(5.64E-06)** |
| | 10 | 1.403E-07(6.10E-08)– | 4.341E-07(5.19E-08)– | 1.018E-07(2.25E-08)– | 1.428E-07(3.78E-07)– | 3.664E-07(2.36E-08)– | 5.557E-07(3.49E-08)– | **6.455E-07(2.36E-08)** |
| MaF2 | 3 | 2.502E-01(1.27E-04)~ | 2.432E-01(9.66E-04)– | 2.463E-01(7.64E-04)– | 2.452E-01(8.70E-04)– | 2.476E-01(1.10E-03)– | **2.512E-01(2.82E-04)+** | 2.509E-01(4.24E-04) |
| | 4 | 2.571E-01(8.18E-04)– | 2.348E-01(1.80E-03)– | 2.380E-01(1.43E-03)– | 2.375E-01(1.02E-03)– | 2.483E-01(2.49E-03)– | **2.601E-01(8.86E-04)+** | 2.587E-01(1.22E-03) |
| | 5 | 2.479E-01(1.64E-03)– | 1.859E-01(2.47E-03)– | 1.907E-01(1.05E-03)– | 1.893E-01(6.04E-04)– | 2.367E-01(3.52E-03)– | **2.607E-01(1.48E-03)+** | 2.542E-01(2.15E-03) |
| | 7 | 2.247E-01(1.83E-03)– | 2.261E-01(3.63E-03)– | 1.907E-01(1.47E-03)– | 1.892E-01(1.32E-03)– | 2.075E-01(5.16E-03)– | **2.472E-01(2.11E-03)+** | 2.346E-01(5.24E-03) |
| | 10 | 2.005E-01(3.52E-03)– | 2.259E-01(5.09E-03)~ | 1.810E-01(1.16E-03)– | 2.062E-01(3.03E-03)– | 2.022E-01(6.24E-03)– | **2.439E-01(2.27E-03)+** | 2.220E-01(2.02E-03) |
| MaF3 | 3 | **9.637E-01(1.12E-03)+** | 9.603E-01(1.77E-03)+ | 9.604E-01(1.38E-03)+ | 9.611E-01(1.36E-03)+ | 9.592E-01(1.51E-03)+ | 9.061E-01(8.47E-03)– | 9.131E-01(4.23E-02) |
| | 4 | 9.943E-01(9.04E-04)+ | 9.932E-01(2.73E-03)+ | 9.943E-01(4.23E-04)+ | **9.945E-01(3.37E-04)+** | 9.934E-01(2.03E-03)+ | 9.576E-01(2.12E-02)– | 9.748E-01(5.40E-03) |
| | 5 | 9.992E-01(6.27E-05)+ | 9.983E-01(9.72E-04)+ | **9.993E-01(9.96E-05)+** | 9.993E-01(4.29E-05)+ | 9.978E-01(9.87E-04)+ | 9.798E-01(1.91E-02)~ | 9.774E-01(2.85E-02) |
| | 7 | **1.000E+0(1.34E-05)+** | 9.794E-01(2.89E-02)– | 9.993E-01(5.44E-05)+ | 9.993E-01(4.07E-05)+ | 9.997E-01(1.58E-04)+ | 9.875E-01(8.56E-03)~ | 9.879E-01(5.57E-03) |
| | 10 | 1.000E+0(1.02E-05)+ | 4.468E-01(4.87E-01)– | **1.000E+0(1.55E-06)+** | 1.000E+00(8.16E-07)+ | 9.995E-01(1.29E-03)+ | 9.906E-01(7.97E-03)+ | 7.260E-01(1.80E-01) |
| MaF4 | 3 | 5.365E-01(1.66E-03)– | 5.333E-01(7.45E-03)– | 5.400E-01(3.29E-03)~ | 5.373E-01(2.62E-03)– | 5.404E-01(2.27E-03)~ | 5.240E-01(1.52E-02)– | **5.405E-01(2.08E-03)** |
| | 4 | 1.948E-01(1.12E-02)– | 2.316E-01(2.67E-02)– | 2.838E-01(3.11E-03)~ | 2.554E-01(3.34E-03)– | 2.794E-01(5.10E-03)– | 2.623E-01(1.57E-02)– | **2.860E-01(2.55E-03)** |
| | 5 | 2.721E-02(5.95E-03)– | 7.727E-02(9.00E-03)– | 1.106E-01(1.12E-02)– | 8.934E-02(3.21E-03)– | 1.196E-01(2.22E-03)– | 1.183E-01(8.08E-03)– | **1.315E-01(1.82E-03)** |
| | 7 | 1.869E-03(1.19E-03)– | 7.124E-03(1.11E-03)– | 1.135E-01(7.06E-03)– | 8.901E-02(4.02E-03)– | 1.009E-02(6.28E-04)– | 1.631E-02(1.11E-03)– | **1.824E-02(2.78E-04)** |
| | 10 | 2.249E-05(2.49E-05)– | 2.657E-04(1.69E-05)– | 9.089E-06(4.51E-06)– | 3.123E-06(1.20E-06)– | 1.437E-04(1.32E-05)– | **4.765E-04(4.19E-05)+** | 4.658E-04(2.11E-05) |
| MaF5 | 3 | 5.338E-01(8.50E-02)– | 5.541E-01(3.96E-02)– | 5.297E-01(8.07E-02)– | 5.470E-01(7.05E-02)– | 5.660E-01(1.11E-03)~ | 5.659E-01(9.43E-04)~ | **5.663E-01(6.66E-04)** |
| | 4 | 6.790E-01(6.26E-02)– | 6.885E-01(4.82E-02)– | 6.832E-01(3.19E-02)– | 6.824E-01(8.27E-02)– | **7.084E-01(1.61E-03)~** | 7.043E-01(2.38E-03)~ | 7.036E-01(1.81E-03) |
| | 5 | 7.521E-01(4.59E-02)– | 8.083E-01(2.13E-02)+ | 7.615E-01(3.46E-02)– | **8.121E-01(3.18E-04)+** | 7.920E-01(2.17E-03)~ | 7.995E-01(2.43E-03)~ | 7.986E-01(1.82E-03) |
| | 7 | 8.594E-01(3.50E-02)– | **9.108E-01(7.71E-04)+** | 7.697E-01(2.26E-02)– | 8.124E-01(6.46E-04)– | 8.876E-01(2.92E-03)– | 8.999E-01(2.18E-03)~ | 8.993E-01(1.62E-03) |
| | 10 | 9.075E-01(2.40E-02)– | 9.638E-01(6.50E-04)~ | 9.197E-01(9.87E-03)– | 9.639E-01(2.68E-03)~ | 9.430E-01(4.12E-03)– | 9.580E-01(3.21E-03)– | **9.640E-01(9.94E-04)** |
| MaF6 | 3 | 1.996E-01(7.12E-05)– | 1.984E-01(3.15E-04)– | 2.002E-01(1.33E-04)– | **2.008E-01(3.33E-05)~** | 2.007E-01(1.29E-04)~ | 2.005E-01(2.51E-04)– | 2.008E-01(9.85E-05) |
| | 4 | 1.501E-01(1.89E-04)– | 1.471E-01(1.08E-03)– | 1.476E-01(1.22E-03)– | 1.517E-01(1.95E-05)~ | 1.517E-01(1.15E-04)~ | 1.517E-01(1.65E-04)~ | **1.518E-01(4.21E-05)** |
| | 5 | 1.264E-01(2.77E-03)– | 1.256E-01(1.67E-03)– | 1.241E-01(6.92E-04)– | 1.300E-01(2.24E-04)– | 1.300E-01(4.09E-05)– | 1.300E-01(9.79E-05)– | **1.301E-01(1.92E-05)** |
| | 7 | 1.104E-01(5.84E-05)– | 1.061E-01(4.64E-03)– | 1.250E-01(1.31E-03)+ | **1.301E-01(3.00E-04)+** | 1.111E-01(2.53E-05)~ | 1.110E-01(1.54E-04)~ | 1.111E-01(1.12E-05) |
| | 10 | 1.005E-01(3.29E-05)– | 7.756E-02(1.91E-02)– | 9.798E-02(8.91E-04)– | 1.002E-01(1.17E-03)– | 7.374E-02(2.42E-02)– | 1.007E-01(6.68E-04)– | **1.008E-01(5.63E-04)** |
| MaF7 | 3 | 3.160E-01(1.41E-02)– | 2.786E-01(8.14E-04)– | 2.729E-01(2.62E-02)– | 2.770E-01(1.18E-02)– | 3.226E-01(1.08E-02)+ | **3.238E-01(1.18E-03)+** | 3.221E-01(1.04E-02) |
| | 4 | 2.983E-01(6.12E-03)– | 2.637E-01(4.12E-03)– | 2.844E-01(2.13E-03)– | 2.683E-01(1.34E-02)– | 3.201E-01(6.08E-03)– | 3.213E-01(5.48E-03)~ | **3.249E-01(6.46E-03)** |
| | 5 | 2.600E-01(6.09E-03)– | 2.508E-01(4.82E-03)– | 2.735E-01(1.57E-03)– | 2.550E-01(3.07E-03)– | 3.045E-01(2.58E-03)– | 3.054E-01(2.18E-03)– | **3.076E-01(2.65E-03)** |
| | 7 | 2.066E-01(2.37E-02)– | 2.239E-01(3.76E-03)– | 2.635E-01(2.39E-03)+ | 2.542E-01(1.85E-03)– | 2.533E-01(3.79E-03)~ | **2.652E-01(3.01E-03)+** | 2.573E-01(1.13E-02) |
| | 10 | 1.988E-01(4.43E-03)– | 1.835E-01(7.96E-03)– | 1.530E-01(1.70E-02)– | 1.665E-01(5.05E-03)– | 1.933E-01(8.84E-03)– | 2.005E-01(9.70E-03)~ | **2.007E-01(1.41E-02)** |
| MaF8 | 3 | **2.863E-01(7.29E-04)+** | 2.750E-01(1.53E-03)– | 2.696E-01(5.01E-02)– | 2.809E-01(1.07E-03)~ | 2.839E-01(7.87E-04)+ | 2.769E-01(2.20E-03)– | 2.802E-01(3.11E-03) |
| | 4 | 1.789E-01(3.28E-03)~ | 1.685E-01(5.96E-03)– | 1.630E-01(6.10E-03)– | 1.790E-01(1.14E-03)+ | **1.793E-01(7.77E-04)+** | 1.710E-01(1.73E-03)– | 1.785E-01(1.30E-03) |
| | 5 | 1.270E-01(1.28E-03)~ | 1.134E-01(1.77E-03)– | 9.585E-02(9.19E-03)– | 1.242E-01(1.30E-04)– | **1.279E-01(2.16E-04)~** | 1.198E-01(8.08E-04)– | 1.279E-01(5.48E-04) |
| | 7 | 4.300E-02(1.08E-03)– | 4.361E-02(1.07E-03)– | 9.700E-02(5.95E-03)+ | **1.238E-01(5.22E-04)+** | 4.936E-02(6.41E-05)~ | 4.322E-02(4.92E-04)– | 4.958E-02(9.10E-05) |
| | 10 | 7.812E-03(7.20E-04)– | 9.288E-03(3.07E-04)– | 3.464E-03(9.69E-04)– | 1.083E-02(1.04E-04)– | 1.110E-02(2.46E-05)~ | 8.816E-03(2.02E-04)– | **1.118E-02(2.69E-05)** |
| MaF9 | 3 | 7.157E-01(9.15E-03)– | 8.413E-01(2.60E-03)+ | 8.419E-01(3.23E-03)+ | **8.453E-01(9.34E-04)+** | 8.399E-01(1.84E-03)+ | 8.317E-01(3.44E-03)~ | 8.329E-01(4.71E-03) |
| | 4 | 1.337E-01(3.00E-02)– | 2.720E-01(1.24E-02)– | 2.683E-01(6.78E-02)– | 2.897E-01(3.65E-03)– | 2.746E-01(4.77E-03)– | 2.914E-01(2.83E-03)~ | **2.928E-01(1.09E-02)** |
| | 5 | 9.607E-02(2.62E-02)– | 2.337E-01(4.81E-02)– | 2.641E-01(1.61E-02)– | **3.186E-01(1.50E-03)+** | 8.205E-02(4.87E-02)– | 2.298E-01(8.08E-03)– | 3.026E-01(1.11E-03) |
| | 7 | 9.318E-02(6.21E-02)– | 7.309E-02(1.42E-02)– | 2.652E-01(1.35E-02)– | **3.184E-01(2.02E-03)+** | 9.318E-02(1.33E-02)– | 2.134E-01(3.24E-03)– | 2.720E-01(8.37E-03) |
| | 10 | 8.795E-03(1.04E-03)– | 8.408E-03(2.29E-03)– | 6.505E-03(1.69E-03)– | 1.623E-02(2.10E-04)– | 1.472E-02(1.31E-05)– | **1.686E-02(9.68E-05)~** | 1.685E-02(4.62E-05) |
| MaF10 | 3 | 6.379E-01(5.90E-03)– | 6.055E-01(1.48E-02)– | 6.152E-01(1.82E-02)– | 6.465E-01(1.39E-02)– | 6.337E-01(4.95E-02)– | 6.496E-01(4.27E-02)– | **7.221E-01(6.18E-02)** |
| | 4 | 6.878E-01(3.36E-03)– | 5.761E-01(1.87E-02)– | 5.821E-01(3.02E-02)– | 6.342E-01(1.54E-02)– | 4.356E-01(3.30E-02)– | 6.432E-01(1.68E-02)– | **7.057E-01(3.21E-02)** |
| | 5 | 7.254E-01(3.37E-02)– | 6.077E-01(2.41E-02)– | 6.072E-01(4.17E-02)– | 6.806E-01(6.57E-03)– | 4.153E-01(3.24E-02)– | 6.900E-01(1.39E-02)– | **7.563E-01(1.29E-02)** |
| | 7 | 7.999E-01(5.57E-03)– | 6.893E-01(1.92E-02)– | 6.771E-01(3.46E-02)– | 7.312E-01(1.46E-02)– | 5.326E-01(3.79E-02)– | 7.388E-01(1.21E-02)– | **8.020E-01(8.37E-03)** |
| | 10 | 8.136E-01(9.18E-03)– | 6.833E-01(3.70E-02)– | 6.881E-01(3.58E-02)– | 7.606E-01(3.44E-02)– | 8.109E-01(2.98E-02)– | 8.206E-01(9.68E-03)– | **8.406E-01(9.85E-03)** |
| MaF11 | 3 | 9.014E-01(2.63E-02)~ | 8.896E-01(1.81E-02)– | 8.965E-01(1.10E-02)– | 9.047E-01(1.34E-02)~ | 8.995E-01(5.20E-02)~ | **9.171E-01(3.50E-02)+** | 9.081E-01(6.66E-02) |
| | 4 | 9.240E-01(4.72E-02)– | 9.336E-01(2.13E-02)– | 9.317E-01(2.94E-02)– | 9.443E-01(9.77E-03)+ | 9.397E-01(6.24E-02)~ | **9.481E-01(5.59E-03)+** | 9.398E-01(5.38E-02) |
| | 5 | 9.398E-01(3.43E-02)– | 9.560E-01(2.19E-03)– | 9.565E-01(4.50E-03)– | 9.634E-01(7.96E-03)~ | **9.711E-01(3.17E-03)+** | 9.519E-01(7.31E-02)– | 9.624E-01(4.69E-03) |
| | 7 | 9.647E-01(1.16E-02)– | 9.691E-01(1.90E-02)+ | 9.469E-01(4.08E-02)– | 9.693E-01(1.21E-02)+ | 9.695E-01(4.42E-02)+ | **9.701E-01(5.35E-02)+** | 9.630E-01(3.12E-02) |
| | 10 | 9.736E-01(2.91E-02)~ | 9.748E-01(1.18E-02)~ | 9.582E-01(7.86E-02)– | 9.793E-01(2.47E-02)+ | 9.807E-01(3.25E-02)+ | **9.914E-01(1.07E-03)+** | 9.746E-01(5.29E-02) |
| MaF12 | 3 | 5.069E-01(2.63E-02)– | 4.733E-01(2.19E-02)– | 5.070E-01(2.79E-03)– | 4.893E-01(2.07E-02)– | 4.967E-01(2.33E-02)– | 5.137E-01(2.17E-02)– | **5.213E-01(1.67E-02)** |
| | 4 | 6.174E-01(2.48E-02)– | 5.982E-01(2.46E-02)– | 6.062E-01(9.57E-03)– | 6.118E-01(1.11E-02)– | 5.906E-01(1.97E-02)– | 6.002E-01(2.43E-02)– | **6.238E-01(2.68E-02)** |
| | 5 | 6.956E-01(3.85E-02)– | 7.026E-01(1.20E-02)– | 6.692E-01(9.15E-03)– | 6.802E-01(7.90E-03)– | 6.465E-01(1.72E-02)– | 6.684E-01(2.48E-02)– | **7.089E-01(1.60E-02)** |
| | 7 | 7.664E-01(2.80E-02)– | 7.677E-01(1.03E-02)– | 6.928E-01(1.08E-02)– | 7.251E-01(8.88E-03)– | 6.945E-01(1.37E-02)– | 7.321E-01(2.93E-02)– | **7.744E-01(2.24E-02)** |
| | 10 | 7.858E-01(1.95E-02)– | 8.006E-01(1.47E-02)~ | 7.096E-01(2.27E-02)– | 7.532E-01(1.45E-02)– | 7.150E-01(2.42E-02)– | 7.598E-01(3.60E-02)– | **8.028E-01(2.96E-02)** |
| MaF13 | 3 | 5.101E-01(1.69E-02)– | 5.136E-01(1.29E-02)– | 5.294E-01(8.48E-03)– | 5.287E-01(7.38E-03)– | 5.307E-01(1.03E-02)– | 5.391E-01(9.07E-03)~ | **5.475E-01(9.86E-03)** |
| | 4 | 3.277E-01(1.16E-02)– | 3.149E-01(1.53E-02)– | 1.374E-01(7.50E-02)– | 3.529E-01(5.98E-03)– | 3.436E-01(8.70E-03)– | 3.262E-01(1.07E-02)– | **3.646E-01(7.17E-03)** |
| | 5 | 2.360E-01(1.10E-02)– | 2.351E-01(9.75E-03)– | 1.043E-01(6.84E-02)– | **2.749E-01(4.96E-03)+** | 2.694E-01(4.22E-03)+ | 2.371E-01(1.01E-03)– | 2.482E-01(2.13E-03) |
| | 7 | 1.468E-01(7.76E-03)– | 1.613E-01(9.88E-03)– | 1.130E-01(6.24E-02)– | **2.771E-01(3.01E-03)+** | 1.818E-01(5.88E-03)– | 2.432E-01(1.45E-03)– | 2.588E-01(2.55E-03) |
| | 10 | 1.057E-01(7.07E-03)– | 1.139E-01(9.35E-03)– | 6.428E-02(3.14E-02)– | **1.372E-01(2.73E-03)+** | 1.252E-01(4.80E-03)– | 1.309E-01(9.82E-03)~ | 1.346E-01(2.02E-03) |
| *best/all* | | 3/65 | 2/65 | 2/65 | 11/65 | 4/65 | 14/65 | 29/65 |
| +/–/~ | | 6/54/5 | 8/53/4 | 9/53/3 | 18/40/7 | 13/39/13 | 13/34/18 | – – |

TABLE A.VI
COMPARISON OF RESULTS OF MAOEA/SRV AND SIX COMPETITIVE MAOEAS ON MAF1-MAF13 USING IGD

| Problem | *m* | MOEA/D-AWA | A-NSGA-III | RVEA* | AR-MOEA | VaEA | MaOEA/C | MaOEA/SRV |
|---|---|---|---|---|---|---|---|---|
| MaF1 | 3 | 0.0385(5.25E-04)– | 0.0337(1.12E-04)~ | 0.0349(5.00E-04)~ | 0.0335(4.59E-04)~ | 0.0335(3.60E-04)~ | 0.0333(4.38E-04)~ | **0.0332(4.07E-04)** |
| | 4 | 0.0973(1.09E-03)– | 0.0929(1.71E-03)– | 0.0944(6.46E-03)– | 0.0823(1.53E-03)– | 0.0759(6.92E-04)~ | 0.0727(9.15E-04)~ | **0.0722(6.98E-04)** |
| | 5 | 0.1360(1.68E-03)– | 0.1577(9.02E-03)– | 0.1493(7.09E-03)– | 0.1132(1.54E-03)– | 0.1097(1.51E-03)~ | 0.1044(1.65E-03)~ | **0.1035(2.18E-04)** |
| | 7 | 0.2098(2.65E-02)– | 0.2108(1.04E-02)– | 0.2378(9.41E-02)– | 0.2295(7.64E-02)– | 0.1691(2.12E-03)– | 0.1583(1.99E-03)~ | **0.1560(1.48E-03)** |
| | 10 | 0.3486(6.49E-02)– | 0.2567(6.03E-03)– | 0.3163(1.69E-02)– | 0.2243(1.53E-03)– | 0.2226(1.91E-03)– | 0.2158(3.15E-03)~ | **0.2033(4.28E-03)** |
| MaF2 | 3 | 0.0244(1.18E-04)~ | 0.0253(5.37E-04)– | 0.0248(1.47E-04)~ | 0.0252(8.12E-04)– | 0.0246(2.16E-04)~ | **0.0242(3.82E-04)~** | 0.0249(1.98E-04) |
| | 4 | 0.0644(5.73E-04)~ | 0.0654(1.66E-03)– | 0.0669(7.01E-04)– | 0.0625(9.67E-04)+ | 0.0624(1.33E-03)+ | **0.0618(9.09E-04)+** | 0.0637(1.68E-03) |
| | 5 | 0.1035(1.59E-03)– | 0.1057(1.83E-03)– | **0.0848(4.20E-04)+** | 0.0967(8.59E-04)+ | 0.0979(2.16E-03)+ | 0.0958(1.84E-03)+ | 0.0991(1.08E-03) |
| | 7 | 0.1381(2.61E-03)+ | 0.1534(4.71E-03)– | 0.1621(7.18E-03)– | 0.1643(8.11E-03)– | **0.1280(2.31E-03)+** | 0.1605(6.57E-03)– | 0.1490(8.56E-03) |
| | 10 | **0.1740(3.95E-03)+** | 0.2390(5.45E-02)– | 0.2201(8.21E-03)– | 0.1841(4.13E-03)+ | 0.1743(4.62E-03)+ | 0.2619(2.07E-02)– | 0.1941(6.35E-03) |
| MaF3 | 3 | **0.0319(1.48E-03)+** | 0.0418(3.57E-03)+ | 0.0431(2.83E-03)+ | 0.0376(1.23E-03)+ | 0.0446(2.34E-03)+ | 0.1769(2.32E-02)– | 0.0805(5.04E-02) |
| | 4 | **0.0570(2.16E-03)+** | 0.0625(7.80E-03)+ | 0.0666(3.08E-03)+ | 0.0605(1.36E-03)+ | 0.0738(9.98E-03)+ | 0.1580(2.16E-02)– | 0.0995(4.77E-02) |
| | 5 | 0.0715(1.05E-03)+ | 0.0833(2.42E-02)+ | 0.0722(2.01E-03)+ | **0.0710(9.91E-04)+** | 0.1150(2.41E-02)+ | 0.1469(1.91E-02)– | 0.1246(8.20E-02) |
| | 7 | 0.0896(1.65E-03)+ | 0.1839(5.54E-02)– | **0.0848(1.45E-03)+** | 0.1711(3.67E-02)– | 0.1210(7.42E-03)– | 0.1454(1.26E-02)– | 0.1114(1.05E-02) |
| | 10 | 0.0995(5.32E-03)– | 34.3319(6.28E+1)– | 0.0766(1.64E-03)– | 0.0816(1.04E-03)– | 0.1154(7.52E-03)– | 0.1481(1.04E-02)– | **0.0643(3.11E-03)** |
| MaF4 | 3 | 0.2393(6.46E-03)– | 0.2623(1.09E-02)– | 0.2685(2.66E-02)– | 0.2719(6.99E-03)– | 0.2581(9.67E-03)– | 0.4406(1.16E-01)– | **0.2199(2.24E-03)** |
| | 4 | 1.0390(2.86E-02)– | 1.2100(3.50E-01)– | 0.8798(4.71E-02)– | 0.8870(1.21E-02)– | 0.8462(2.53E-02)– | 1.3251(2.22E-01)– | **0.8108(1.01E-02)** |
| | 5 | 2.5827(1.93E-01)– | 2.6059(5.14E-01)– | 2.6440(1.59E-01)– | 2.6506(4.90E-02)– | 2.9119(5.08E-02)– | 2.7573(2.76E-01)– | **2.2584(1.78E-01)** |
| | 7 | 9.8505(2.46E-01)– | 11.2630(7.16E-01)– | 10.4631(3.37E-01)– | 11.2630(7.16E-01– | 10.4007(4.30E-01)– | 12.0181(1.15E+0)– | **9.4495(4.63E-01)** |
| | 10 | 70.1767(6.13E+0)– | 95.1792(6.98E+0)– | 79.1648(2.20E+0)– | 91.3284(7.26E+0)– | 52.2148(1.72E+0)– | 96.7162(8.56E+0)– | **40.2131(1.38E+0)** |
| MaF5 | 3 | 0.3501(4.88E-01)– | 0.2791(2.86E-01)– | 0.4981(6.78E-01)– | 0.3597(5.00E-01)– | 0.2002(2.62E-03)~ | 0.2110(3.83E-03)~ | **0.2002(3.11E-03)** |
| | 4 | 1.2197(9.40E-01)– | 1.1055(8.00E-01)– | 0.8819(2.19E-01)– | 1.1352(9.19E-01)– | **0.7580(7.51E-03)+** | 0.7804(1.49E-02)~ | 0.7887(1.74E-02) |
| | 5 | 3.1656(1.77E+0)– | 2.1044(2.99E-01)– | 2.5427(1.44E+0)– | 2.0456(2.78E-03)– | **1.7372(2.33E-02)+** | 1.8372(4.32E-02)~ | 1.8058(2.73E-02) |
| | 7 | 9.5998(3.57E+0)– | 8.0390(1.21E-01)– | 9.0350(1.53E-01)– | 8.122(1.45E-01)– | **7.1667(1.49E-01)+** | 7.5133(1.45E-01)+ | 7.6001(1.14E-01) |
| | 10 | 73.7198(2.64E+1)– | 65.6854(4.32E+0)+ | 92.1055(8.04E+0)– | 110.0818(7.74E+0)– | **47.8608(1.33E+0)+** | 62.4180(9.98E+0)+ | 70.5402(1.07E+0) |
| MaF6 | 3 | 0.0048(9.03E-05)– | 0.0068(9.78E-04)– | 0.0041(1.89E-04)– | 0.0038(3.36E-05)~ | 0.0037(4.03E-04)~ | 0.0040(7.53E-04)– | **0.0035(2.17E-04)** |
| | 4 | 0.0059(2.32E-04)– | 0.0145(2.64E-03)– | 0.0139(4.61E-03)– | **0.0037(3.16E-05)+** | 0.0043(6.89E-04)~ | 0.0046(1.27E-03)~ | 0.0042(4.35E-04) |
| | 5 | 0.0078(4.21E-03)– | 0.0151(4.91E-03)– | 0.0188(6.92E-03)– | **0.0030(8.17E-06)+** | 0.0040(5.25E-04)~ | 0.0039(1.33E-03)~ | 0.0039(2.38E-04) |
| | 7 | 0.0042(2.02E-04)– | 0.0293(7.43E-02)– | 0.0261(5.32E-02)– | 0.0031(7.43E-02)~ | 0.0049(8.87E-04)– | **0.0022(8.59E-04)+** | 0.0029(4.21E-04) |
| | 10 | **0.0035(4.80E-04)+** | 0.2923(6.79E-02)– | 0.0345(6.25E-03)– | 0.0059(6.84E-03)+ | 0.2503(1.30E-01)– | 0.0068(1.08E-02)~ | 0.0065(2.35E-02) |
| MaF7 | 3 | 0.0802(9.88E-03)– | 0.0550(2.08E-03)– | 0.1396(2.41E-01)– | 0.0816(9.15E-02)– | 0.0617(5.37E-02)– | **0.0469(1.79E-03)~** | 0.0473(8.90E-04) |
| | 4 | 0.2424(5.90E-02)– | 0.1609(1.04E-02)– | **0.1316(3.56E-03)+** | 0.1980(1.48E-01)– | 0.1545(3.82E-02)– | 0.1531(3.80E-02)– | 0.1317(4.26E-03) |
| | 5 | 0.4484(9.90E-02)– | 0.2848(1.14E-02)– | **0.2186(5.41E-03)+** | 0.2583(3.82E-03)+ | 0.2733(5.38E-03)~ | 0.2921(1.60E-02)– | 0.2668(3.02E-02) |
| | 7 | 0.6563(4.30E-02)– | 0.5065(1.74E-02)~ | 0.5769(1.06E-02)– | **0.5059(1.32E-02)~** | 0.5166(1.36E-02)– | 0.7303(6.44E-02)– | 0.5069(1.19E-01) |
| | 10 | 1.1769(1.55E-02)– | 0.9648(6.89E-02)– | 0.8526(8.70E-02)~ | 1.5354(1.49E-01)– | 0.9593(3.82E-02)– | 1.8873(2.97E-01)– | **0.8320(6.34E-02)** |
| MaF8 | 3 | 0.0614(1.66E-03)– | 0.0725(4.27E-03)– | 0.0974(1.76E-01)– | 0.0581(1.94E-03)– | 0.0576(1.96E-03)~ | 0.0698(4.39E-03)– | **0.0567(4.59E-03)** |
| | 4 | 0.1648(8.76E-03)– | 0.1392(1.95E-02)– | 0.1546(2.05E-02)– | 0.0839(2.82E-03)– | 0.0776(3.02E-03)~ | 0.1058(5.11E-03)– | **0.0772(2.52E-03)** |
| | 5 | 0.1452(7.28E-03)– | 0.1381(6.82E-03)– | 0.2812(9.53E-02)– | 0.0886(1.49E-03)– | 0.0841(2.20E-03)~ | 0.1310(6.12E-03)– | **0.0837(3.06E-03)** |
| | 7 | 0.2584(5.16E-02)– | 0.1859(1.21E-02)– | 0.8549(8.16E-02)– | 0.1154(1.01E-02)– | **0.1027(2.43E-03)~** | 0.1906(1.20E-02)– | 0.1037(2.49E-03) |
| | 10 | 0.4777(1.41E-01)– | 0.3110(4.80E-02)– | 0.9841(2.55E-01)– | 0.1211(3.60E-03)– | **0.1155(2.09E-03)~** | 0.2570(1.52E-02)– | 0.1169(1.44E-03) |
| MaF9 | 3 | 0.0497(2.31E-03)~ | 0.0503(3.42E-03)~ | 0.0505(5.81E-03)– | **0.0472(7.99E-04)+** | 0.0529(1.11E-03)~ | 0.0581(3.41E-03)– | 0.0496(1.19E-03) |
| | 4 | 0.1674(9.85E-03)– | 0.1701(2.35E-02)– | 0.2051(1.88E-01)– | 0.1225(8.54E-03)+ | **0.1128(1.45E-02)+** | 0.1178(4.88E-03)+ | 0.1295(1.97E-02) |
| | 5 | 0.1659(8.47E-03)+ | 0.2941(1.42E-01)+ | 0.1891(4.45E-02)+ | **0.0867(2.45E-03)+** | 0.1032(2.22E-03)+ | 0.1333(1.02E-02)+ | 0.9820(2.49E-01) |
| | 7 | 0.2417(5.93E-02)+ | 0.3588(1.25E-01)+ | 0.4728(3.54E-01)+ | 0.1587(1.45E-02)+ | **0.1424(2.94E-03)+** | 0.1753(7.07E-03)+ | 0.8540(2.82E-01) |
| | 10 | 1.2213(1.45E-01)– | 0.5938(1.89E-01)~ | 0.7717(1.79E-01)– | **0.1642(5.90E-03)+** | 0.2046(6.17E-03)+ | 0.2714(1.49E-02)+ | 0.5249(1.15E-01) |
| MaF10 | 3 | 0.1634(1.42E-02)– | 0.3263(2.66E-02)– | 0.3401(4.55E-02)– | 0.2268(2.12E-02)– | 0.6570(6.64E-02)– | 0.9625(1.53E-01)– | **0.1443(2.22E-01)** |
| | 4 | 0.3288(1.95E-02)– | 0.6354(3.89E-02)– | 0.7647(5.63E-02)– | 0.4163(2.26E-02)– | 1.2193(9.23E-02)– | 1.2247(5.86E-02)– | **0.3136(1.23E-01)** |
| | 5 | 0.5968(9.87E-02)~ | 0.8047(4.64E-02)– | 1.0569(1.08E-01)– | 0.5961(3.03E-02)~ | 1.4096(8.69E-02)– | 1.2258(2.09E-02)– | **0.5918(9.45E-02)** |
| | 7 | **0.8874(5.67E-02)+** | 0.9946(4.31E-02)+ | 1.6192(1.21E-01)– | 0.9386(4.31E-02)+ | 1.3829(7.80E-02)– | 1.3277(3.04E-02)– | 1.1964(3.92E-02) |
| | 10 | 1.4402(1.32E-01)– | 1.3958(8.38E-02)– | 1.6721(9.48E-02)– | 1.1979(9.20E-02)~ | 1.4720(5.00E-02)– | 1.4841(3.34E-02)– | **1.1357(7.43E-02)** |
| MaF11 | 3 | 0.1600(1.16E-02)+ | 0.1798(3.93E-02)+ | 0.2579(3.19E-02)+ | **0.1524(3.01E-03)+** | 0.2435(5.38E-02)+ | 0.2436(6.41E-02)+ | 0.3109(1.13E-01) |
| | 4 | 0.4378(5.95E-02)+ | 0.5617(1.54E-01)+ | 0.7409(1.42E-01)– | **0.3731(2.26E-02)+** | 0.5503(8.07E-02)+ | 0.5961(1.19E-01)~ | 0.5951(5.72E-02) |
| | 5 | 0.5101(8.43E-02)+ | 0.9540(4.75E-01)+ | 1.4971(4.74E-01)+ | 0.4758(2.29E-02)+ | 1.0317(2.37E-01)+ | **0.3913(3.05E-01)+** | 1.5887(5.86E-01) |
| | 7 | 2.5210(2.48E-01)+ | 4.1588(1.63E+00)+ | 6.9393(2.77E+0)– | 3.1323(1.66E+00)+ | 3.1297(1.22E+00)+ | **1.8964(1.80E-01)+** | 4.8276(1.58E+00) |
| | 10 | **2.0449(3.23E-01)+** | 6.3017(1.68E+00)– | 12.9325(7.24E-01)– | 2.3455(2.06E-01)+ | 4.9428(7.64E-01)+ | 3.6910(2.77E-01)+ | 5.001(1.44E+00) |
| MaF12 | 3 | 0.1999(1.85E-02)– | 0.2091(1.96E-02)– | 0.1874(1.88E-03)– | 0.1878(1.80E-02)– | 0.2039(2.50E-02)– | 0.1911(2.17E-02)– | **0.1789(2.40E-02)** |
| | 4 | 0.6049(2.26E-02)– | 0.5487(7.72E-03)– | 0.5346(5.78E-03)– | 0.5315(2.50E-03)– | 0.5337(1.37E-02)– | 0.5412(1.16E-02)– | **0.5224(1.27E-02)** |
| | 5 | 1.1298(3.49E-02)– | 0.9648(5.64E-03)– | 0.9702(6.85E-03)– | 0.9824(1.57E-03)– | 0.9467(1.15E-02)– | 0.9508(4.67E-03)– | **0.9375(6.99E-03)** |
| | 7 | 2.2099(2.38E-02)– | 2.0974(1.27E-02)– | 2.1492(2.52E-02)– | 2.2146(3.39E-02)– | 2.0530(1.47E-02)~ | 2.1060(1.93E-02)– | **2.0491(2.27E-02)** |
| | 10 | 4.1902(2.87E-02)~ | 4.4184(6.86E-02)– | 4.3549(1.76E-02)– | 4.2997(8.15E-03)– | 4.0897(2.03E-02)~ | 4.2999(7.72E-02)~ | **3.9547(1.01E-02)** |
| MaF13 | 3 | 0.0722(9.23E-03)– | 0.0702(6.79E-03)– | 0.0588(3.88E-03)~ | 0.0610(3.68E-03)– | 0.0661(7.34E-03)– | 0.0664(5.64E-03)– | **0.0588(3.70E-03)** |
| | 4 | 0.1364(1.59E-02)– | 0.1630(2.43E-02)– | 0.3815(1.73E-01)– | 0.0946(6.00E-03)– | 0.0909(6.66E-03)– | 0.1116(9.84E-03)– | **0.0821(6.23E-03)** |
| | 5 | 0.1889(2.47E-02)– | 0.1866(2.15E-02)– | 0.4238(1.13E-01)– | 0.0949(7.49E-03)– | **0.0839(3.63E-03)+** | 0.1246(7.83E-03)– | 0.1155(1.83E-02) |
| | 7 | 0.2149(3.08E-02)– | 0.2172(2.95E-02)– | 0.6646(6.73E-02)– | 0.1172(2.43E-02)~ | **0.0897(3.53E-03)+** | 0.1789(1.61E-02)– | 0.1176(1.15E-02) |
| | 10 | 0.1929(1.70E-02)– | 0.2395(1.84E-02)– | 0.4750(1.89E-01)– | 0.1102(5.97E-03)~ | **0.0914(3.90E-03)+** | 0.2368(1.88E-02)– | 0.1004(4.01E-03) |
| *best/all* | | 6/65 | 0/65 | 4/65 | 9/65 | 12/65 | 6/65 | 28/65 |
| +/–/~ | | 15/44/6 | 11/50/4 | 12/48/5 | 22/35/8 | 22/29/14 | 13/37/15 | – – |

TABLE A. VII
COMPARISON OF RESULTS OF MAOEA/SRV AND TWO MAOEA/SRV VARIANTS ON MAF1-MAF13 USING HV

| Problem | *m* | MSRV-I | MSRV-II | MaOEA/SRV |
|---|---|---|---|---|
| MaF1 | 3 | 2.219E-01(7.91E-04)– | 2.049E-01(1.09E-02)– | **2.262E-01(6.70E-04)** |
| | 4 | 5.555E-02(3.18E-04)– | 4.293E-02(1.04E-02)– | **6.067E-02(2.66E-04)** |
| | 5 | 1.205E-02(7.94E-05)– | 7.327E-03(2.93E-03)– | **1.303E-02(9.92E-05)** |
| | 7 | 3.281E-04(4.86E-06)– | 1.901E-05(3.17E-05)– | **3.399E-04(5.64E-06)** |
| | 10 | 6.444E-07(2.13E-08)~ | 2.264E-08(4.14E-08)– | **6.455E-07(2.36E-08)** |
| MaF2 | 3 | 2.407E-01(3.00E-04)– | 1.934E-01(1.54E-02)– | **2.509E-01(4.24E-04)** |
| | 4 | 2.484E-01(1.00E-03)– | 1.993E-01(1.15E-02)– | **2.587E-01(1.22E-03)** |
| | 5 | 2.500E-01(1.54E-03)~ | 2.055E-01(9.18E-03)– | **2.542E-01(2.15E-03)** |
| | 7 | 2.339E-01(3.50E-03)~ | 6.389E-02(8.56E-03)– | **2.346E-01(5.24E-03)** |
| | 10 | 2.201E-01(3.22E-03)~ | 5.987E-02(5.57E-03)– | **2.220E-01(2.02E-03)** |
| MaF3 | 3 | 9.110E-01(4.68E-02)– | 9.118E-01(2.00E-02)~ | **9.131E-01(4.23E-02)** |
| | 4 | 9.716E-01(7.86E-03)– | 9.090E-01(6.86E-02)– | **9.748E-01(5.40E-03)** |
| | 5 | **9.799E-01(1.34E-02)+** | 9.203E-01(5.60E-02)– | 9.774E-01(2.85E-02) |
| | 7 | 9.878E-01(4.83E-03)~ | 8.333E-01(1.04E-01)– | **9.879E-01(5.57E-03)** |
| | 10 | **7.314E-01(1.98E-01)+** | 4.769E-01(2.18E-01)– | 7.260E-01(1.80E-01) |
| MaF4 | 3 | 5.335E-01(3.04E-03)– | 4.280E-01(1.82E-02)– | **5.405E-01(2.08E-03)** |
| | 4 | 2.758E-01(2.58E-03)– | 2.046E-01(1.01E-02)– | **2.860E-01(2.55E-03)** |
| | 5 | 1.215E-01(1.67E-03)– | 8.491E-02(3.50E-03)– | **1.315E-01(1.82E-03)** |
| | 7 | 1.732E-02(3.61E-04)– | 1.077E-02(4.22E-04)– | **1.824E-02(2.78E-04)** |
| | 10 | 4.619E-04(1.77E-05)– | 2.671E-04(4.94E-05)– | **4.658E-04(2.11E-05)** |
| MaF5 | 3 | 5.557E-01(1.11E-03)– | 5.314E-01(3.88E-02)– | **5.663E-01(6.66E-04)** |
| | 4 | 6.927E-01(1.76E-03)– | 6.669E-01(5.28E-03)– | **7.036E-01(1.81E-03)** |
| | 5 | 7.875E-01(1.85E-03)– | 7.624E-01(7.93E-03)– | **7.986E-01(1.82E-03)** |
| | 7 | 8.890E-01(1.65E-03)– | 8.721E-01(6.11E-03)– | **8.993E-01(1.62E-03)** |
| | 10 | 9.610E-01(1.06E-03)~ | 9.446E-01(3.22E-03)– | **9.640E-01(9.94E-04)** |
| MaF6 | 3 | 2.002E-01(6.34E-05)– | 1.892E-01(8.26E-03)– | **2.008E-01(9.85E-05)** |
| | 4 | 1.508E-01(4.34E-05)– | 1.410E-01(8.93E-03)– | **1.518E-01(4.21E-05)** |
| | 5 | 1.300E-01(1.96E-05)~ | 1.204E-01(7.34E-03)– | **1.301E-01(1.92E-05)** |
| | 7 | 1.110E-01(1.17E-05)~ | 1.075E-01(2.67E-03)– | **1.111E-01(1.12E-05)** |
| | 10 | 5.083E-02(1.13E-02)– | 3.184E-03(1.74E-02)– | **1.008E-01(5.63E-04)** |
| MaF7 | 3 | 3.115E-01(1.03E-02)– | 2.674E-01(2.05E-02)– | **3.221E-01(1.04E-02)** |
| | 4 | 3.128E-01(8.96E-03)– | 2.657E-01(3.16E-02)– | **3.249E-01(6.46E-03)** |
| | 5 | 3.007E-01(3.18E-03)– | 2.963E-01(2.01E-02)– | **3.076E-01(2.65E-03)** |
| | 7 | **2.622E-01(6.11E-03)+** | 1.644E-01(2.54E-02)– | 2.573E-01(1.13E-02) |
| | 10 | 1.989E-01(1.38E-02)– | 1.256E-01(1.18E-02)– | **2.007E-01(1.41E-02)** |
| MaF8 | 3 | 2.727E-01(3.37E-03)– | 2.290E-01(2.75E-02)– | **2.802E-01(3.11E-03)** |
| | 4 | 1.733E-01(1.33E-03)– | 1.370E-01(2.22E-02)– | **1.785E-01(1.30E-03)** |
| | 5 | 1.270E-01(3.24E-04)~ | 1.010E-01(6.42E-03)– | **1.279E-01(5.48E-04)** |
| | 7 | 4.955E-02(8.37E-05)~ | 3.863E-02(1.75E-03)– | **4.958E-02(9.10E-05)** |
| | 10 | **1.119E-02(2.16E-05)~** | 8.436E-03(4.32E-04)– | 1.118E-02(2.69E-05) |
| MaF9 | 3 | 8.134E-01(3.97E-03)– | 6.251E-01(4.38E-02)– | **8.329E-01(4.71E-03)** |
| | 4 | 2.404E-01(1.31E-02)– | 8.909E-02(3.63E-02)– | **2.928E-01(1.09E-02)** |
| | 5 | 8.198E-02(1.48E-02)– | 4.746E-02(1.66E-02)– | **3.026E-01(1.11E-03)** |
| | 7 | 7.111E-02(1.52E-02)– | 7.075E-03(1.63E-03)– | **2.720E-01(8.37E-03)** |
| | 10 | 1.434E-02(4.56E-05)– | 2.337E-04(2.38E-05)– | **1.685E-02(4.62E-05)** |
| MaF10 | 3 | **7.312E-01(6.09E-02)+** | 5.654E-01(2.56E-02)– | 7.221E-01(6.18E-02) |
| | 4 | **7.058E-01(3.55E-02)~** | 5.597E-01(3.20E-02)– | 7.057E-01(3.21E-02) |
| | 5 | 7.524E-01(1.22E-02)– | 5.929E-01(2.85E-02)– | **7.563E-01(1.29E-02)** |
| | 7 | 7.944E-01(8.36E-03)– | 2.789E-01(3.18E-02)– | **8.020E-01(8.37E-03)** |
| | 10 | 8.382E-01(8.48E-03)– | 2.053E-01(2.43E-02)– | **8.406E-01(9.85E-03)** |
| MaF11 | 3 | 8.858E-01(5.90E-02)– | 8.266E-01(4.50E-02)– | **9.081E-01(6.66E-02)** |
| | 4 | 9.273E-01(5.40E-02)– | 7.943E-01(6.71E-02)– | **9.398E-01(5.38E-02)** |
| | 5 | 9.492E-01(4.18E-02)– | 8.588E-01(4.98E-02)– | **9.624E-01(4.69E-03)** |
| | 7 | 9.533E-01(3.06E-02)– | 7.360E-01(9.30E-02)– | **9.630E-01(3.12E-02)** |
| | 10 | 9.719E-01(2.02E-03)~ | 6.953E-01(5.63E-02)– | **9.746E-01(5.29E-02)** |
| MaF12 | 3 | 5.039E-01(2.67E-02)– | 4.902E-01(5.08E-02)– | **5.213E-01(1.67E-02)** |
| | 4 | 6.188E-01(2.76E-02)– | 5.846E-01(4.13E-02)– | **6.238E-01(2.68E-02)** |
| | 5 | 6.897E-01(2.52E-02)– | 6.791E-01(2.00E-02)– | **7.089E-01(1.60E-02)** |
| | 7 | 7.690E-01(1.48E-02)– | 7.398E-01(3.29E-02)– | **7.744E-01(2.24E-02)** |
| | 10 | 7.969E-01(3.41E-02)– | 7.378E-01(2.90E-02)– | **8.028E-01(2.96E-02)** |
| MaF13 | 3 | 5.285E-01(9.10E-03)– | 4.400E-01(3.52E-02)– | **5.475E-01(9.86E-03)** |
| | 4 | 3.433E-01(9.09E-03)– | 2.712E-01(4.45E-02)– | **3.646E-01(7.17E-03)** |
| | 5 | 2.231E-01(2.13E-02)– | 1.732E-01(4.42E-02)– | **2.482E-01(2.13E-03)** |
| | 7 | 1.541E-01(4.23E-02)– | 1.050E-01(4.60E-02)– | **2.588E-01(2.55E-03)** |
| | 10 | 1.126E-01(2.16E-02)– | 7.348E-02(3.13E-02)– | **1.346E-01(2.02E-03)** |
| *best*/*all* | | 6/65 | 0/65 | 59/65 |
| +/–/~ | | 4/48/13 | 0/64/1 | – – |

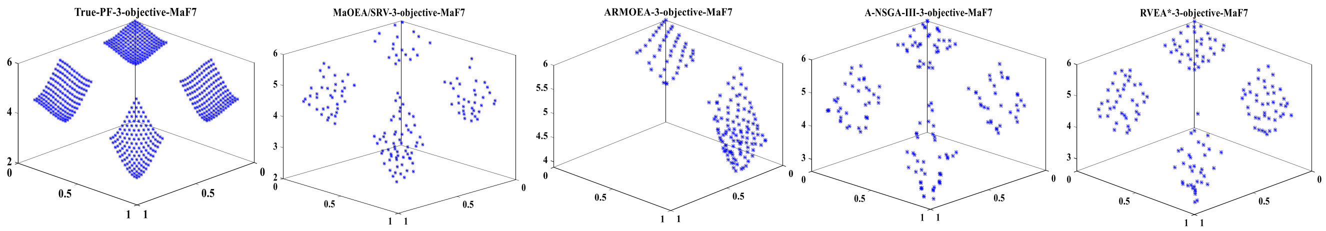


Fig. A. 1 Final solution sets achieved by four MaOEAs and the true PF on the 3-objective MaF7 problem.

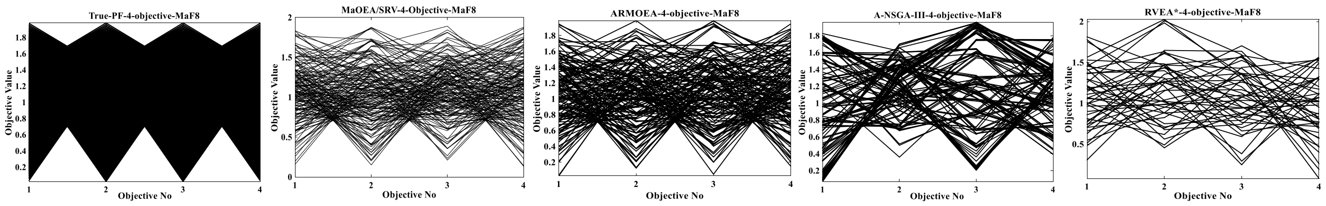


Fig. A. 2 Final solution sets achieved by four MaOEAs and the true PF on the 4-objective MaF8 problem

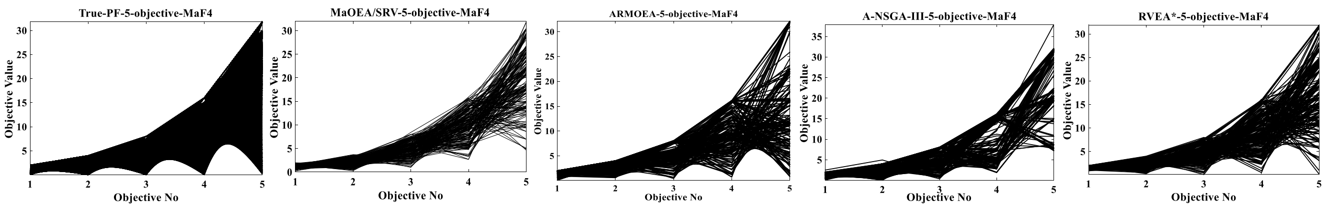


Fig. A. 3 Final solution sets achieved by four MaOEAs and the true PF on the 5-objective MaF4 problem

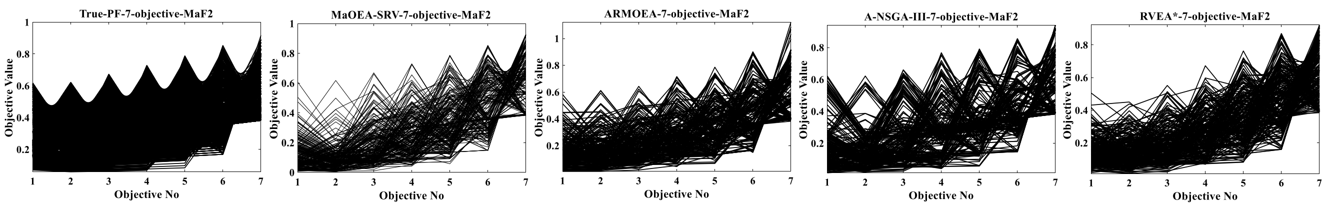


Fig. A. 4 Final solution sets achieved by four MaOEAs and the true PF on the 7-objective MaF2 problem

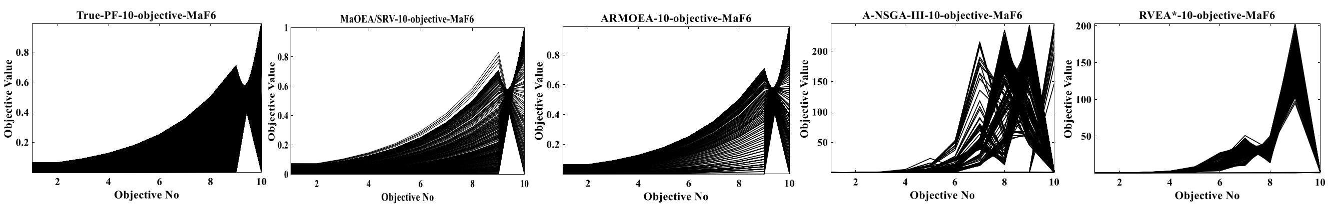


Fig. A. 5 Final solution sets achieved by four MaOEAs and the true PF on the 10-objective MaF6 problem

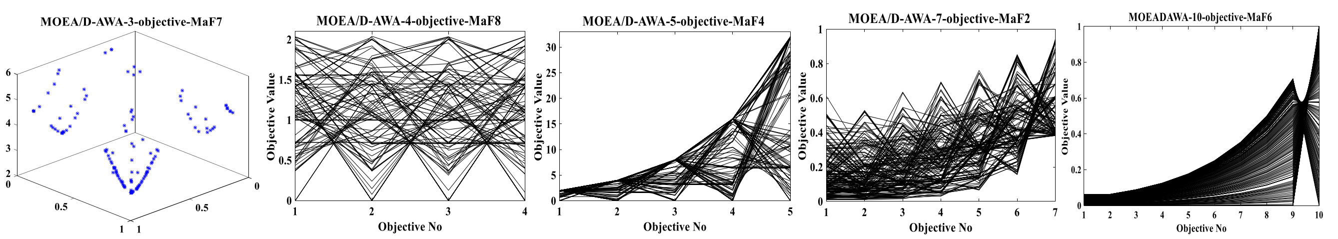


Fig. A. 6 Final solution sets achieved by MOEA/D-AWA on the above considered problems

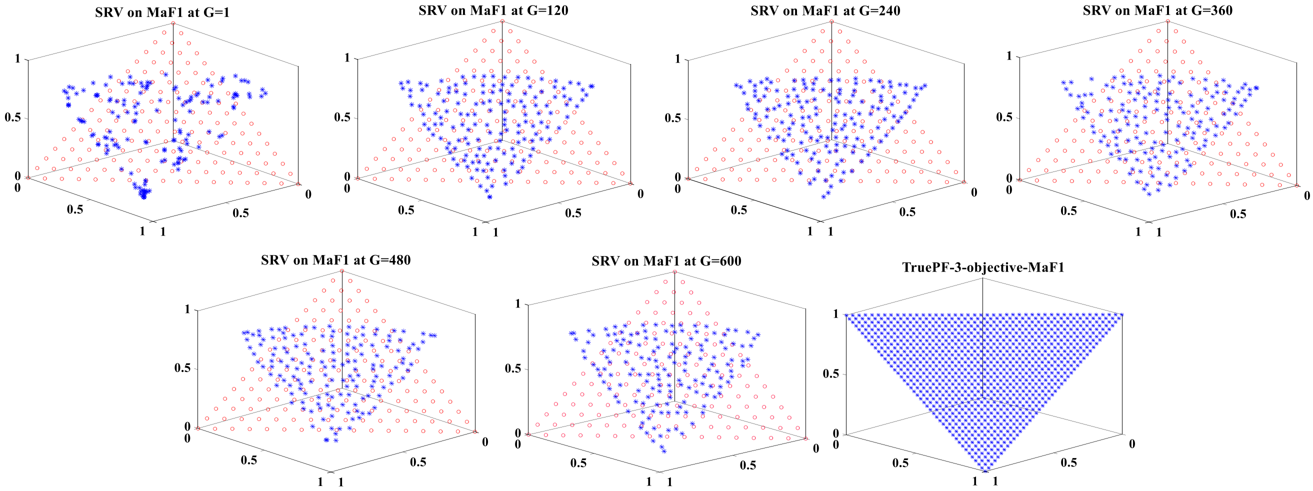


Fig. A7 The dynamically changed RVs (blue points) extracted by SRV on the 3-objective MaF1 problem at different generations, which are different from the preset evenly distributed RVs (red points) and can properly track the shapes of the true PF.

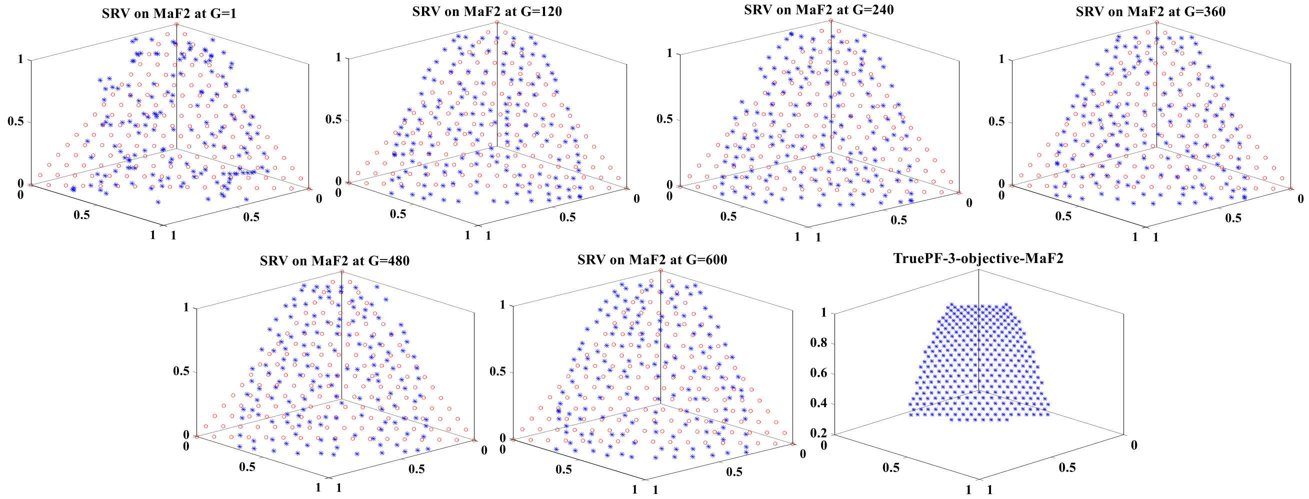


Fig. A8 The dynamically changed RVs (blue points) extracted by SRV on the 3-objective MaF2 problem at different generations, which are different from the preset evenly distributed RVs (red points) and can properly track the shapes of the true PF.

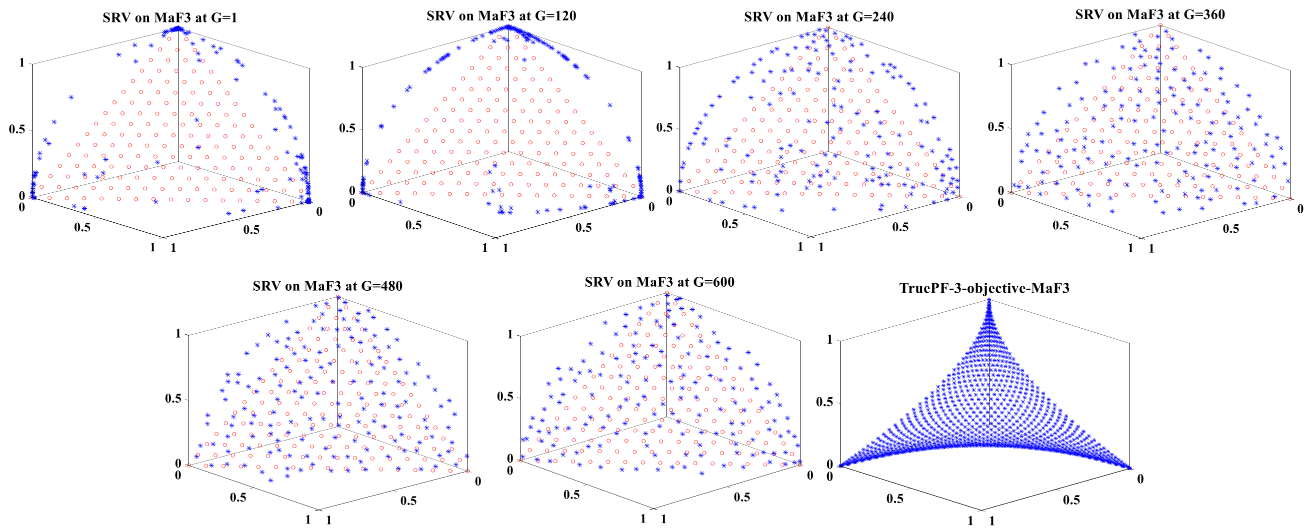


Fig. A9 The dynamically changed RVs (blue points) extracted by SRV on the 3-objective MaF3 problem at different generations, which are different from the preset evenly distributed RVs (red points) and can properly track the shapes of the true PF.

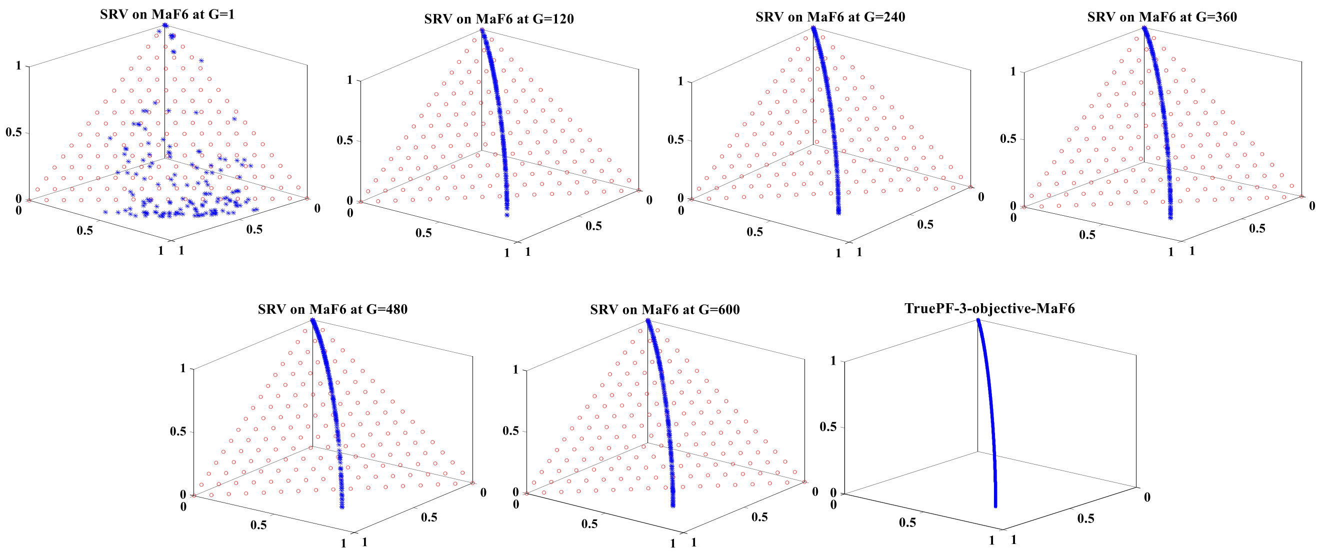


Fig. A10 The dynamically changed RVs (blue points) extracted by SRV on the 3-objective MaF6 problem at different generations, which are different from the preset evenly distributed RVs (red points) and can properly track the shapes of the true PF.

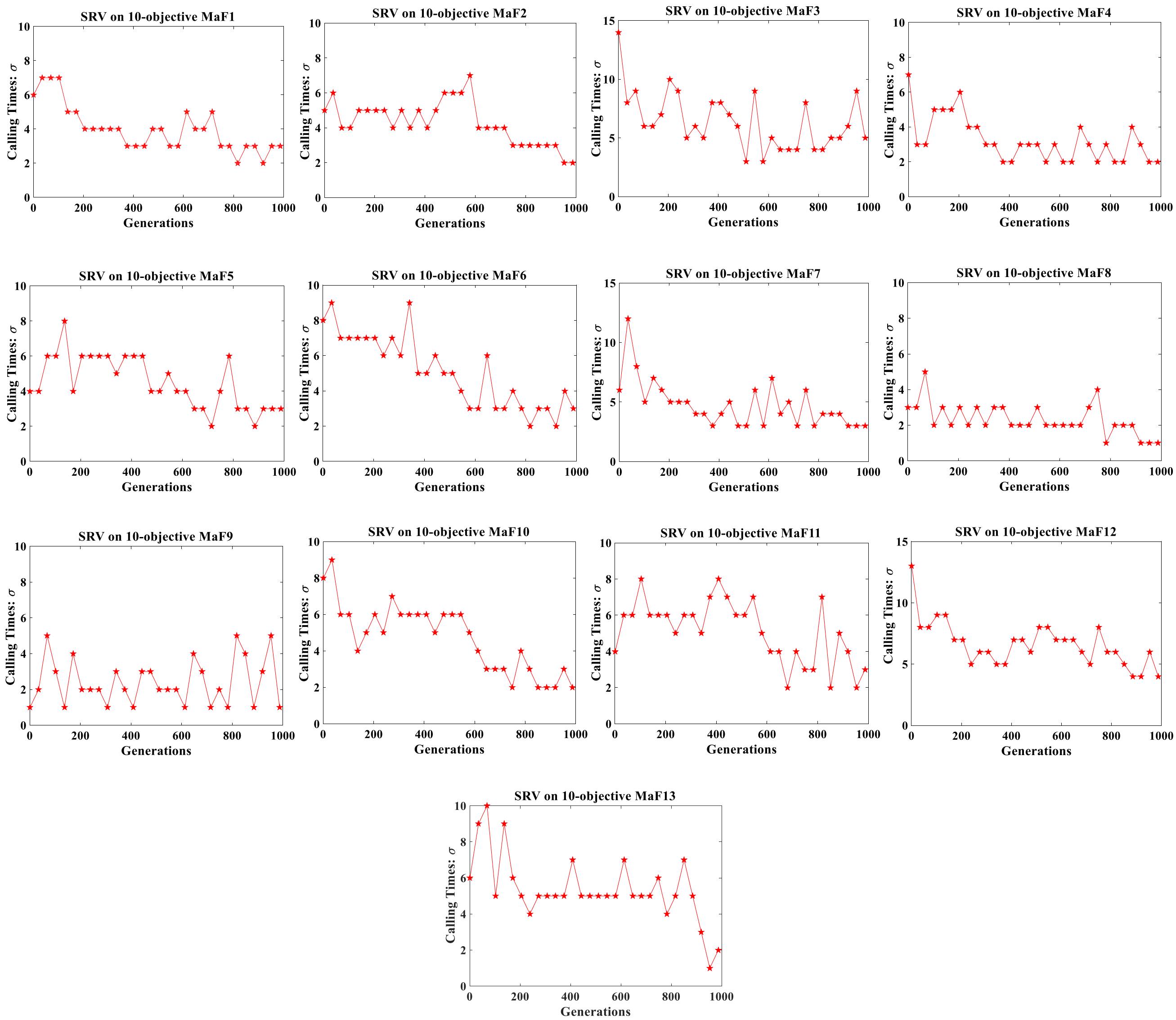


Fig. A11 The times $\sigma$ of calling the self-guided adjustment method for centroids in SRV on solving the MaF1-MaF13 problems with 10 objectives ($m$=10)

**2. Elitism selection strategy (ESS) in some MaOEA/Ds (Please read Section II.A of this paper in advance)**

After running APP, each individual $x \in P_c$ is associated to its closest RV and $P_c$ is divided into $N$ subsets $S_1^R, S_2^R, ..., S_N^R$, where $S_i^R$ includes all the individuals associated to the $i$-th RV $r^i$. For example, each $x \in P_c$ can be associated to its $K$ closest RVs in EFR-RR [1], while each RV in MOEA/D-SAS [2] is associated with its $L$ closest individuals (here $K$ and $L$ are the integers preset by the users). ESS endeavors to select $N$ promising individuals based on the subsets $S_1^R, S_2^R, ..., S_N^R$ from APP. In each $S_i^R$ associated to $r^i$ ($i = 1, 2, ..., N$), two indicators $I_c$ and $I_d$ are designed to respectively reflect the convergence and diversity information for each individual $x$ ($x \in S_i^R$). Then, the ranking and selection of individuals in $S_i^R$ are run based on their $I_c$ and $I_d$ values. Based on the processing principle, ESS can be classified into the following two categories.

- Processing $I_c$ and $I_d$ separately. First, all individuals in each $S_i^R$ are sorted by their $I_c$ values, and then each individual can get an $I_c$ rank (i.e., 1, 2, and so on). Second, all individuals in $S_1^R, S_2^R, ..., S_N^R$ are further classified into multiple subsets (i.e., $S_1^{Ic}, S_2^{Ic}$, and so on) by collecting the individuals with the same $i$-th $I_c$ rank into $S_i^{Ic}$ ($i$ starts from 1 to the maximal $I_c$ rank). Third, starting from $S_1^{Ic}$, then $S_2^{Ic}$ and so on, each subset is chosen to construct a temporary set $S$ until its size exceeds $N$ for the first time, and we assume that the last subset is included into $S$ as $S_l^{Ic}$ (i.e., $S = S_1^{Ic} + S_2^{Ic} + ... + S_l^{Ic}$). Then, all individuals in the first $l$-1 subsets of $S$ are selected to ensure convergence, while some individuals are selected from $S_l^{Ic}$ based on their $I_d$ values to maintain diversity. Assuming that one individual $x \in S_i^R$ is associated to $r^i$ ($i = 1, 2, ..., N$), the $I_c$ and $I_d$ values in MOEA/D-LWS [3] are decided by the localized weighted sum function and $\theta(x^j, r^i)$ in (6), respectively. In RPD-NSGA-II [4], the $I_c$ and $I_d$ values for each individual $x \in S_i^R$ are defined by the RP-dominance (i.e., $d_1(x, r^i)$ in (5)) and the RV density of $r^i$. NSGA-III [5] can be classified into this type as individuals in each subset are first sorted by their non-dominated fronts, which is regarded as $I_c$, while $d_2(x, r^i)$ in (4) is used as $I_d$. In ASEA [6], each subset is sorted by its adaptive sorting-based selection, which defines $I_c$ based on the Euclidean distance from an individual $x$ to the ideal point $z^*$ termed $EdI(x)$, as computed by (7). Then, a certain number of individuals with better $I_c$ ranks in $S_i^R$ are further sorted by $\theta(x, r^i)$ as $I_d$. In MOEA/D-SAS [2], all individuals in $S_i^R$ are sorted by their aggregated function values (e.g., weighted sum and Tchebycheff (TCH) functions [7]) as $I_c$. For the selection in the last subset $S_l^{Ic}$, the angle of two individuals $(x, y)$ is used as $I_d$, as computed by (8). In EFR-RR [1], $d_2(x, r^i)$ in (4) is used as $I_d$ to find the $K$ nearest RVs for each individual $x \in P_c$, which helps to maintain diversity. Then, each individual can obtain the $K$ modified TCH-based ranks based on its $K$ nearest RVs, in which the best rank is assigned as its $I_c$ rank to run the environmental selection.
- Processing $I_c$ and $I_d$ simultaneously. This kind of ESS fuses $I_c$ and $I_d$ into a single fitness function, which is used to sort the individuals in each $S_i^R$ to get multiple subsets ($S_1^{Icd}, S_2^{Icd}$ and so on). Starting from $S_1^{Icd}$, then $S_2^{Icd}$ and so on, each subset is selected to build a temporary set $S$ until its size exceeds $N$ for the first time, and we assume that the last subset is included into $S$ as $S_l^{Icd}$ (i.e., $S = S_1^{Icd} + S_2^{Icd} + ... + S_l^{Icd}$). In general, a random selection is run in the last subset $S_l^{Icd}$ to guarantee diversity. In $\theta$-DEA [8], individuals in $S_i^R$ are sorted by using $\theta$-dominance (i.e., $d_1(x, r^i) + \theta \cdot d_2(x, r^i)$), which uses a parameter $\theta$ to combine $d_1(x, r^i)$ in (5) and $d_2(x, r^i)$ in (4). Similar to $\theta$-DEA, individuals in $S_i^R$ are sorted by an angle-penalized distance (APD) in RVEA [9], which consists of $EdI(x)$ in (7) and $\theta(x, r^i)$ in (6), $i = 1, 2, ..., N$. In SPEA/R [10], each individual $x \in S_i^R$ is assigned a local fitness value that consists of a local strength value and a local density of $x$. Unlike RVEA and $\theta$-DEA that run a random selection in $S_l^{Icd}$, the best $N - |S - S_l^{Icd}|$ individuals with the local fitness in SPEA/R will be selected from $S_l^{Icd}$.

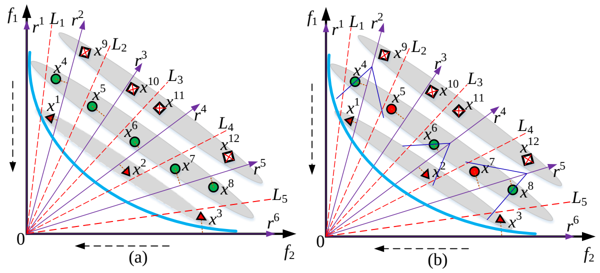

Fig. A12 Examples of the APP and ESS for (a) NSGA-III and (b) $\theta$ -DEA

In Fig. A12, the environmental selection mechanism is illustrated on a 2-objective problem ($m$ = 2) with a population size $N$ = 6 for (a) NSGA-III (processing $I_c$ and $I_d$ separately) and (b) $\theta$-DEA (processing $I_c$ and $I_d$ simultaneously), using six evenly distributed RVs ($r^1, r^2, ..., r^6$). At first, a union population with six parents and six offspring ($U = \{x^1, ..., x^{12}\}$) is divided into three fronts based on the non-domination relationship, i.e., $F_1 = \{x^1, x^2, x^3\}$, $F_2 = \{x^4, ..., x^8\}$ and $F_3 = \{x^9, ..., x^{12}\}$. For both NSGA-III and $\theta$-DEA, their candidate populations ($P_c$) will include $F_1$ and $F_2$, as the sizes of $P_c$ ($|P_c| = |F_1 + F_2| = 8$) have been already larger than 6. Then, their APPs and ESSs are run to select six individuals for the next generation. First, in their APPs, based on the principle that individuals should be associated to their closest RVs using the distance metric $d_2(x, r^i)$ in (4), $P_c$ is divided into six subsets: $S_1^R = \varnothing$, $S_2^R = \{x^1, x^4\}$, $S_3^R = \{x^5\}$, $S_4^R = \{x^2, x^6\}$, $S_5^R = \{x^7, x^8\}$, and $S_6^R = \{x^3\}$, where $L_1, L_2, ..., L_5$ are five boundary lines of these subsets in Fig. A12. Second, the ESS of NSGA-III will select all individuals from the first non-dominated front, i.e., $S_1^{Ic} = F_1 = \{x^1, x^2, x^3\}$, and then choose other two individuals from $S_3^R$ and $S_5^R$ in $S_2^{Ic} = F_2$, as their solutions have not been selected yet, which helps to promote diversity. Thus, $x^5$ in $S_3^R$ and $x^8$ in $S_5^R$ are included based on their $I_d$ values using $d_2(x, r^i)$ in (4). Moreover, one of the remaining individuals in $S_2^{Ic} = F_2$ (i.e., {$x^4$, $x^6$, $x^7$}) is randomly chosen. When running ESS in $\theta$-DEA, $\theta$-dominance is used to sort the individuals in each subset $S_i^R$ ($i = 1, 2, ..., 6$), and then two subsets can be constructed based on their $\theta$-dominance ranks to collect the individuals with the same rank, i.e., $S_1^{Icd} = \{x^1, x^2, x^3, x^5, x^7\}$ and $S_2^{Icd} = \{x^4, x^6, x^8\}$. At last, all individuals in $S_1^{Icd}$ are selected, and only one individual in $S_2^{Icd}$ is randomly chosen.

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