Evolutionary Multitasking for Large-Scale Multiobjective Optimization
(Supplementary Document)

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***Abstract*—This is the supplementary document of the paper titled "Evolutionary Multitasking for Large-Scale Multiobjective Optimization", which is submitted to IEEE Transactions on Evolutionary Computation. In this supplementary document, the computational complexity and actual runtimes of the newly proposed multitasking evolutionary transfer optimization algorithm based on discriminative reconstruction neural networks, termed DRNEA, are first analyzed in detail. Then, the parameter sensitivity analysis of DRNEA is experimental studied. Moreover, we provide all the detailed definition of the three different multitasking LMOP suites in our experimental studies, including CPLX, LSMOP, and our proposed RTMF. Finally, the detailed IGD results and some related figures on all tested LMOP suite in our experimental studies are also provided in this file, including the IGD results obtained by DRNEA and its four variants in the ablation study, the IGD results obtained by DRNEA, its four ETO competitors, and its five LMOEA competitors.**

Algorithm 3 General Framework of DRNEA

Input: a set of M LMOPs: {LMOP$_1$, …, LMOP$_M$}, $FE_{\max}$
Output: a set of M populations: P = {P$_1$, P$_2$,…, P$_M$}
1: **for** i = 1 to M **do**
2: initialize P$_i$ with N random solutions for LMOP$_i$
3: initialize a learning model DRN$_i$ for LMOP$_i$
4: **end for**
5: initialize FE = 0 and $\lambda = N/10$
6: **while** $FE \le FE_{\max}$ **do**
7: NS$_l \leftarrow$ the nondominated solutions set of P$_l$, l = 1 to M
8: for i = 1 to M do
9: DRN$_i$ = *Training_DRN*(NS$_1$, …, NS$_M$, i)
10: Q$_i$ = *Transfer_Reproduction* (P, DRN$_i$, λ , FE)
11: U$_i$ = P$_i$ + Q$_i$, set Q$_i = \varnothing$ and P$_i = \varnothing$
12: P$_i$ = *EnvironmentalSelection* (U$_i$)
13: **end for**
14: **end while**
15: **return** {**P$_1$**, **P$_2$**,…, **P$_M$**}

1. Computational Complexity Analysis of DRNEA

From **Algorithm 3** for the general framework of DRNEA, we can find that if only one LMOP is considered, the computational complexity at each generation of DRNEA is mainly determined by the three procedures in lines 9-12, i.e., the training of DRN in line 9, the reproduction based on the trained DRN in line 10, and the environmental selection in line 12. Specifically, training the DRN including two pipelines (i.e., train the encoder and decoder on the nondominated solutions for the target LMOP, and train the encoder and classifier on all the nondominated solutions), which requires a worst time complexity $O(MmNn_{\max}K\prod_{i=1:L} l_i)$. Here, m, N, $n_{\max}$, and K represent the number of objectives for the target LMOP, the population size for each LMOP, the maximal number of variables among these M LMOPs, and the number of neurons in the code layer of DRN, respectively. Besides, the L in $\prod_{i=1:L} l_i$ indicates the number of hidden layers in the encoder (or decoder), and correspondingly l_i represents the number of neurons in the i-th hidden layer. Thus, in order to make the complexity of DRNEA comparable to the general LMOEAs, we set L = 1 and $l_1 = 2K$, which makes the worst time complexity of training the DRN become $O(MmNn_{\max}K^2)$. In addition, the worst time complexity of reproducing the offspring population is $O(\lambda mn_{\max}K^2 + Nmn_{\max})$. At last, the worst time complexity of running environmental selection, like in NSGA-II, is $O(mN^2)$. Thus, since $\lambda < N$, the overall worst time complexity of DRNEA in one generation for the target LMOP is $O(MmNn_{\max}K^2 + mN^2)$, which is comparable to that of NSGA-II, i.e., $O(mNn + mN^2)$, if the values of M and K are significantly small. Therefore, we set K = 10 in this paper is reasonable when considering the time complexity of DRNEA, and we only consider the cases of solving no more than six LMOPs simultaneously in our experimental studies.

To further evaluate the actual runtime of DRNEA and its nine competitor algorithms, their average running times (in minutes: min) from 20 runs are plotted in Fig. S12 for the 3-task LSMOP problems with 2 objectives. Obviously, in Fig. S12, MOMFEA, LMOCSO, WOF, and LSMOF showed the similar running time with the fastest speed on all nine LSMOPs, which take less time than that of MFEA-AKT (may due to the overhead for its adaptive selection of variation operator from six different candidates), followed by MOMFEA-II that needs to learn the pairwise similarity between different two tasks based on the distribution of the populations. The next one that took a little longer was EMT-ET, which needs to determine the positive transfer solutions and to learn the mapping matrix between different tasks (Here, the map-

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ping matrix from task1 to task 2 is different from task 2 to task 1). But the knowledge transfer in EMT-ET is only performed every 50 generations, which will save some computational cost when compared with our proposed DRNEA that may perform transfer learning at each generation. Although multiple DRN modes were online trained at each generation in DRNEA, it was still faster than LMEA and MOEA/PSL. In LMEA to independently solve each task, it needs to analyze the contribution of each variable on optimizing each objective based on the used clustering methods in the large-scale search space, which will significantly slow down the running time. Considering MOEA/PSL, it also needs to train an adaptively changed autoencoder model online to learn the Pareto subspace at each generation, and the epoch of training is set to 10, while the epoch in DRNEA is set to 1.

Moreover, DRNEA, MOMFEA, MOMFEA-II, and MFEA-AKT are implemented using Java codes, while other algorithms are implemented using Matlab codes. Since the source codes of the comparison algorithms are implemented in different programming languages (i.e., Java or Matlab), the comparisons of execution times are only a reference without exact fairness. For example, MOMFEA-II and LMOCSO are implemented with different programming languages, but they show the similar running times. In most cases, the program with Matlab for the same algorithm runs slower than the program based on Java, but Matlab has higher computational efficiency when performing matrix operations. All the solvers were run on a personal computer having an Intel (R) Core (TM) i7-8700 CPU, 3.70GHz (processor), and 32 GB (RAM).

2. Parameter Sensitivity Analysis of DRNEA

As introduced in Section III, the proposed DRNEA contains two important parameters, i.e., K and λ . Here, K is a parameter involved in the learning model DRN to determine the number of neurons in the representation or code layer. Also, K is the dimensionality of the learned Pareto optimal subspace. Thus, the value of parameter K not only affects the architecture and learning ability of the DRN model, but also determines the dimensionality of the learnt subspace for search. In addition, the parameter λ is the number of times to implement solution transfer for each target LMOP at every generation. Here, we conduct the sensitivity analysis of K and λ in affecting the performance of DRNEA. At first, we use DRNEA by setting different values of K, i.e., K = {2, 5, 10, 20, 40, 60, 80, 100, 200}, to simultaneously solve a set of three LSMOP1 problems with m = 3 and n = 512, 1024, and 2048, correspondingly. As shown in Fig. S1, DRNEA is least effective in solving this set of three LSMOP1 problems when the parameter K is set to 2 and 200. On the one hand, when K = 2, the structure of the DRN model becomes relatively simple, and the learning ability of this model is insufficient, which leads to low efficiency for solution transfer. In addition, the 2-dimension of the learned subspace is too low, which may lose too much information of the original search space, resulting in low search efficiency. On the other hand, when K = 200, the architecture of the DRN model will become complex, and the training of

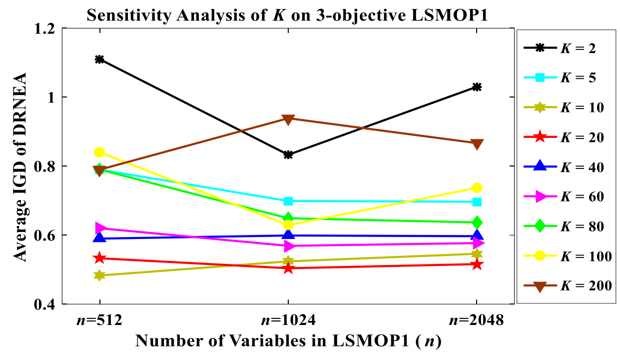

Fig. S1 Illustration of the performance of DRNEA with different values of K on LSMOP1.

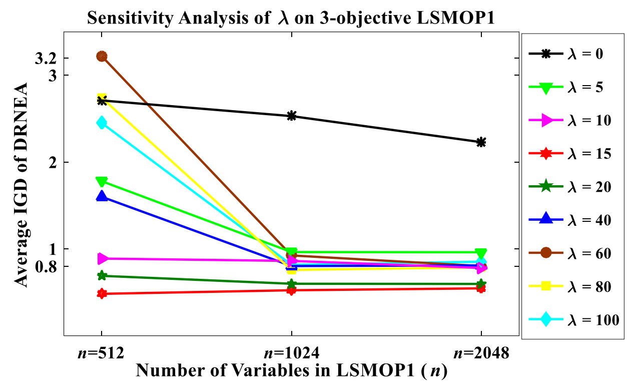

Fig. S2 Illustration of the performance of DRNEA with different values of λ on LSMOP1.

this model may require a large amount of data, which may lead to low accuracy of the obtained DRN model. In addition, the dimensionality of the learned subspace reaches 200, which is also a large-scale search space, and the search strategy adopted (i.e., SBX) still cannot efficiently handle such a large-scale subspace. Therefore, considering both the performance and computational complexity of DRNEA, the value of K should not be set too small (K < 5) or too large (K > 100). From the results shown in Fig. S1, DRNEA obtains the best IGD values when K =10 or K = 20, but considering the time complexity of DRNEA as discussed in the above Section, it is reasonable to set K = 10.

Similarly, we also use DRNEA by setting different values of λ , i.e., λ = {0, 5, 10, 15, 20, 40, 60, 80, 100}, to solve this set of three LSMOP1 problems, as shown in Fig. S2. Specifically, DRNEA performs worst when λ = 0, i.e., there is no knowledge transfer, which validates the efficacy of transfer optimization in solving large-scale multiobjective optimization problems. Besides, DRNEA performs similarly on this set of LSMOP1 problems when λ is in the range of 10 to 20, while its performance deteriorates on 512-dimensional LSMOP1 when λ is set as other cases in this study. In this paper, λ = N/10 and N = 150 are set for 3-objective LMOPs, which are reasonable from the results of this study. In particular, as the successful transfer of solution is determined by the probability value obtained by the DRN's classifier, a large value of λ does not necessarily mean that more solutions will be successfully transferred, but the probability of negative transfer may increase.

3. Experimental Studies on CPLX Test Suite

(1) *Formulation of CPLX*

For the formulation of CPLX, each task is a multiobjective optimization problem with m objectives, defined as follows:

$$\mathbf{F(x)} = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x}))$$

where solution vector $\mathbf{x}$ has n variables, i.e., $\mathbf{x} = (x_1, x_2, \ldots, x_n)$. At first, the n variables are linearly divided into two variable groups, i.e., a group with m-1 position-related variables: $\mathbf{x^p} = (x_1^p, x_2^p, \ldots, x_{m-1}^p)$ and another group with n-m+1 distance-related variables: $\mathbf{x^d} = (x_1^d, x_2^d, \ldots, x_{n-m+1}^d)$. After that, the distance related variables are further uniformly divided into m exclusive variable groups: $\mathbf{x^{d,1}}$ to $\mathbf{x^{d,m}}$. In this way, we can get the formulation model to design the m objective functions in $\mathbf{F(x)}$, as follows:

$$\mathbf{F(x)}: \min \begin{cases} f_1(\mathbf{x}) = h_1(\mathbf{x^p}) + g(\mathbf{x^{d,1}}) \\ f_2(\mathbf{x}) = h_2(\mathbf{x^p}) + g(\mathbf{x^{d,2}}) \\ \ldots \\ f_m(\mathbf{x}) = h_m(\mathbf{x^p}) + g(\mathbf{x^{d,m}}) \end{cases}$$

where functions h_1 to h_m together define the shape of the Pareto front (PF), known as the shape functions, and functions g_1 to g_m define the fitness landscape, known as the landscape function. Four different shape functions are considered in CPLX, which are defined as follows:

1) Convex PF in the case of $m = 2$,

$$\mathbf{H_1(x^p)}: \begin{cases} h_1(\mathbf{x^p}) = x_1 \\ h_2(\mathbf{x^p}) = 1 - \sqrt{x_1} \end{cases}, \ x_1 \in [0,1]$$

2) Concave PF in the case of $m = 2$,

$$\mathbf{H}_2\mathbf{(x^p)}: \begin{cases} h_1(\mathbf{x^p}) = x_1 \\ h_2(\mathbf{x^p}) = 1 - (x_1)^2 \end{cases}, \ x_1 \in [0,1]$$

3) Convex PF in the case of $m \geq 3$,

$$\mathbf{H_3(x^p)}: \begin{cases} h_1(\mathrm{x}^p) = [\cos(0.5\pi x_1)\ldots\cos(0.5\pi x_{m-2})\cos(0.5\pi x_{m-1})]^4 \\ h_2(\mathrm{x}^p) = [\cos(0.5\pi x_1)\ldots\cos(0.5\pi x_{m-2})\sin(0.5\pi x_{m-1})]^4 \\ \ldots \\ h_{m-1}(\mathrm{x}^p) = [\cos(0.5\pi x_1)\sin(0.5\pi x_2)]^4 \\ h_m(\mathrm{x}^p) = [\sin(0.5\pi x_1)]^2 \end{cases}$$

4) Concave PF in the case of $m \geq 3$,

$$\mathbf{H}_4\mathbf{(x^p)}: \begin{cases} h_1(\mathbf{x^p}) = \cos(0.5\pi x_1)\ldots\cos(0.5\pi x_{m-2})\cos(0.5\pi x_{m-1}) \\ h_2(\mathbf{x^p}) = \cos(0.5\pi x_1)\ldots\cos(0.5\pi x_{m-2})\sin(0.5\pi x_{m-1}) \\ \ldots \\ h_{m-1}(\mathbf{x^p}) = \cos(0.5\pi x_1)\sin(0.5\pi x_2) \\ h_m(\mathbf{x^p}) = \sin(0.5\pi x_1) \end{cases}$$

where $x_i \in [0,1]$, and i =1, 2,…, m-1.

In addition, ten different landscape functions are included in CPLX, which are defined as follows:

$$g_1(\mathbf{x^{d,i}}) = \frac{2}{|\mathbf{x^{d,i}}|}\sum_{j=1}^{|\mathbf{x^{d,i}}|}\left[x_j^{d,i} - (x_1^p)^{0.5\left(1.0+\frac{3(No(x_j^{d,i})-2)}{n-2}\right)}\right]^2$$

where i = 1, …, m and $No(x_j^{d,i})$ represents the original ordinal of variable $x_j^{d,i}$ in $\mathbf{x}$ hereafter.

$$g_2(\mathbf{x^{d,i}}) = \frac{2}{|\mathbf{x^{d,i}}|}\sum_{j=1}^{|\mathbf{x^{d,i}}|}\left[x_j^{d,j} - \sin\left(6\pi x_1^p + \frac{No(x_j^{d,j})\pi}{n}\right)\right]^2$$

$$g_3(\mathbf{x^{d,i}}) = \frac{2}{|\mathbf{x^{d,i}}|}\sum_{j=1}^{|\mathbf{x^{d,i}}|}\left[\begin{array}{l} 1+4\left[x_j^{d,i} - (x_1^p)^{0.5\left(1.0+\frac{3(No(x_j^{d,i})-2)}{n-2}\right)}\right]^2 - \\ \cos\left[8\pi\left(x_j^{d,i} - (x_1^p)^{0.5\left(1.0+\frac{3(No(x_j^{d,i})-2)}{n-2}\right)}\right)\right]\end{array}\right]$$

$$g_4(\mathbf{x^{d,i}}) = \frac{2}{|\mathbf{x^{d,i}}|}\sum_{j=1}^{|\mathbf{x^{d,i}}|}\left[x_j^{d,i} - 0.8x_1^p\cos\left(6\pi x_1^p + \frac{No(x_j^{d,i})\pi}{n}\right)\right]^2$$

$$g_5(\mathbf{x^{d,i}}) = \frac{2}{|\mathbf{x^{d,i}}|}\sum_{j=1}^{|\mathbf{x^{d,i}}|}\left[x_j^{d,i} - 0.8x_1^p\sin\left(6\pi x_1^p + \frac{No(x_j^{d,i})\pi}{n}\right)\right]^2$$

$$g_6(\mathbf{x^{d,i}}) = \frac{2}{|\mathbf{x^{d,i}}|}\sum_{j=1}^{|\mathbf{x^{d,i}}|}\left[x_j^{d,i} - 0.8x_1^p\cos\left(3\pi x_1^p + \frac{No(x_j^{d,i})\pi}{3n}\right)\right]^2$$

$$g_7(\mathbf{x^{d,i}}) = \frac{2}{|\mathbf{x^{d,i}}|}\sum_{j=1}^{|\mathbf{x^{d,i}}|}\left[x_j^{d,i} - 2x_2^p\sin\left(2\pi x_1^p + \frac{No(x_j^{d,i})\pi}{n}\right)\right]^2$$

$$g_8(\mathbf{x^{d,i}}) = \frac{2}{|\mathbf{x^{d,i}}|}\sum_{j=1}^{|\mathbf{x^{d,i}}|}\left[\begin{array}{l}\left(0.3(x_1^p)^2\cos\left(24\pi x_1^p + \frac{4No(x_j^{d,i})\pi}{n}\right) + 0.6x_1^p\right) \\ \left(\cos\left(6\pi x_1^p + \frac{index(x_i^d)\pi}{n}\right)\right)(-1) + x_j^{d,i}\end{array}\right]^2$$

$$g_9(\mathbf{x^{d,i}}) = \frac{2}{|\mathbf{x^{d,i}}|}\sum_{j=1}^{|\mathbf{x^{d,i}}|}\left[\begin{array}{l}\left(0.3(x_1^p)^2\cos\left(24\pi x_1^p + \frac{4No(x_j^{d,i})\pi}{n}\right) + 0.6x_1^p\right) \\ \left(\sin\left(6\pi x_1^p + \frac{index(x_i^d)\pi}{n}\right)\right)(-1) + x_j^{d,i}\end{array}\right]^2$$

$$g_{10}(\mathbf{x^{d,i}}) = \frac{2}{|\mathbf{x^{d,i}}|}\left[\begin{array}{l} 4\sum_{j=1}^{|\mathbf{x^{d,i}}|}\left[\mathrm{x}_j^{d,i} - (x_1^p)^{0.5\left(1.0+\frac{3(No(\mathrm{x}_j^{d,i})-2)}{n-2}\right)}\right]^2 + 2 - \\ 2\prod_{j=1}^{|\mathbf{x^{d,i}}|}\cos\frac{20\pi\left(\mathrm{x}_j^{d,i} - (x_1^p)^{0.5\left(1.0+\frac{3(No(x_j^{d,i})-2)}{n-2}\right)}\right)}{\sqrt{No(x_i^d)}}\end{array}\right]$$

Given above, the specific composition of each task in each CPLX problem is summarized below.

$$\text{CPLX1:}\begin{cases}\text{Task 1}[m=2, n=128, \mathbf{H_1(x^p)}, g_1(\mathbf{x^{d,i}})], x^d \in [0,1] \\ \text{Task 2}[m=2, n=256, \mathbf{H_1(x^p)}, g_2(\mathbf{x^{d,i}})], x^d \in [-1,1]\end{cases}$$

$$\text{CPLX2:}\begin{cases}\text{Task 1}[m=2, n=128, \mathbf{H_1(x^p)}, g_1(\mathbf{x^{d,i}})], x^d \in [0,1] \\ \text{Task 2}[m=2, n=128, \mathbf{H}_1\mathbf{(x^p)}, g_3(\mathbf{x^{d,i}})], x^d \in [0,1]\end{cases}$$

$$\text{CPLX3:}\begin{cases}\text{Task 1}[m=2, n=256, \mathbf{H_1(x^p)}, g_2(\mathbf{x^{d,i}})], x^d \in [-1,1] \\ \text{Task 2}\left[m=2, n=256, \mathbf{H}_1\mathbf{(x^p)}, \begin{cases} g_6(\mathbf{x^{d,1}}) \\ g_5(\mathbf{x^{d,2}})\end{cases}\right], x^d \in [-1,1]\end{cases}$$

$$\text{CPLX4:}\begin{cases}\text{Task 1}[m=2, n=256, \mathbf{H_1(x^p)}, g_2(\mathbf{x^{d,i}})], x^d \in [-1,1] \\ \text{Task 2}[m=2, n=256, \mathbf{H}_2\mathbf{(x^p)}, g_2(\mathbf{x^{d,i}})], x^d \in [-1,1]\end{cases}$$

$$\text{CPLX5:}\begin{cases}\text{Task 1}\left[m=2,n=256,\mathbf{H_1(x^p)},\begin{cases}g_4(\mathbf{x^{d,1}})\\ g_5(\mathbf{x^{d,2}})\end{cases}\right],x^d\in[-1,1]\\ \text{Task 2}\left[m=3,n=128,\mathbf{H}_4\mathbf{(x^p)},g_7(\mathbf{x^{d,i}})\right],x^d\in[-2,2]\end{cases}$$

$$\text{CPLX6:}\begin{cases}\text{Task 1}\left[m=2,n=512,\mathbf{H_1(x^p)},\begin{cases}g_4(\mathbf{x^{d,1}})\\ g_5(\mathbf{x^{d,2}})\end{cases}\right],x^d\in[-1,1]\\ \text{Task 2}\left[m=3,n=512,\mathbf{H}_4\mathbf{(x^p)},g_2(\mathbf{x^{d,i}})\right],x^d\in[-1,1]\end{cases}$$

$$\text{CPLX7:}\begin{cases}\text{Task 1}\left[m=2,n=1024,\mathbf{H_1(x^p)},\begin{cases}g_6(\mathbf{x^{d,1}})\\ g_5(\mathbf{x^{d,2}})\end{cases}\right],x^d\in[-1,1]\\ \text{Task 2}\left[m=3,n=1024,\mathbf{H}_3\mathbf{(x^p)},\begin{cases}g_8(\mathbf{x^{d,1}})\\ g_9(\mathbf{x^{d,2}})\\ g_8(\mathbf{x^{d,3}})\end{cases}\right],x^d\in[-1,1]\end{cases}$$

$$\text{CPLX8:}\begin{cases}\text{Task 1}\left[m=2,n=512,\mathbf{H_1(x^p)},\begin{cases}g_8(\mathbf{x^{d,1}})\\ g_9(\mathbf{x^{d,2}})\end{cases}\right],x^d\in[-1,1]\\ \text{Task 2}\left[m=3,n=256,\mathbf{H}_3\mathbf{(x^p)},g_3(\mathbf{x^{d,i}})\right],x^d\in[0,1]\end{cases}$$

$$\text{CPLX9:}\begin{cases}\text{Task 1}\left[m=3,n=512,\mathbf{H}_4\mathbf{(x^p)},\begin{cases}g_8(\mathbf{x^{d,1}})\\ g_9(\mathbf{x^{d,2}})\\ g_8(\mathbf{x^{d,3}})\end{cases}\right],x^d\in[-1,1]\\ \text{Task 2}\left[m=2,n=1024,\mathbf{H}_2\mathbf{(x^p)},g_2(\mathbf{x^{d,i}})\right],x^d\in[0,1]\end{cases}$$

$$\text{CPLX10:}\begin{cases}\text{Task 1}\left[m=2,n=128,\mathbf{H_1(x^p)},g_3(\mathbf{x^{d,i}})\right],x^d\in[0,1]\\ \text{Task 2}\left[m=2,n=128,\mathbf{H_1(x^p)},g_{10}(\mathbf{x^{d,i}})\right],x^d\in[0,1]\end{cases}$$

(2) *Comparison Results on CPLX*

As discussed in Section IV of this paper, the performance of DRNEA is validated to be effective for solving three LSMOPs (customized from the same LSMOP problem with different settings) at the same time but poor for solving six different LSMOPs simultaneously. Here, to further verify our ideas, the performance of DRNEA is used to solve two different LMOP problems concurrently. Specifically, DRNEA is compared with six competitors (NSGA-II, MOMFEA, MOMFEA-II, MFEA-AKT, EMT-ET, and MOEA-PSL) on CPLX problems. CPLX is a newly proposed multitasking optimization test suite, where each CPLX problem is a 2-task problem, and each task is a 2- or 3-objective MOP. The variable dimension (n) of the original problems in CPLX was less than 100. Therefore, in order to make this test suite meet the requirement of having large-scale search spaces, we increased the variable dimension of all the CPLX problems without changing other parameters. Detailed settings of (m, n) can be found in Table S7, which also recodes the detailed IGD values of DRNEA and its six competitors in solving these CPLX problems. From these IGD results in Table S7, DRNEA shows the best performance in 13 out of 20 cases on these ten CPLX problems, which further validate the superior performance of DRNEA in solving these 2-task CPLX problems with large-scale search space.

Also, by Wilcoxon rank-sum test, the IGD-based statistical comparison results of DRNEA and its four LMOEA competitors are provided in the last row of Table S7. Compared to NSGA-II, MOMFEA, MOMFEA-II, MFEA-AKT, EMT-ET, and MOEA/PSL, DRNEA performs worse/better/similarly in 1/17/2, 1/19/0, 1/18/1, 4/15/1, 3/16/1, and 2/14/4 out of 20 test CPLX cases, which validate the superior performance of DRNEA with knowledge transfer in solving these LMOPs. Specifically, for each 2-task CPLX problem, NSGA-II handles each task independently without knowledge transfer, while MOMFEA, MOMFEA-II, and MFEA-AKT solve two tasks simultaneously based on the framework of NSGA-II and carries out implicitly knowledge transfer. Besides, EMT-ET runs the strategy of explicit knowledge transfer to solve the two tasks in each CPLX, simultaneously. In MOEA/PSL, without knowledge transfer, a Pareto optimal subspace is also learned via an autoencoder for effectively handling the large-scale search space. From the results shown in Table S7, it is reasonable to conclude that DRNEA with a powerful learning model, i.e., DRN, can significantly improve the performance of LMOEAs in solving these CPLX problems with large-scale search space.

4. Basic Definition of LSMOP

In LSMOP, the n variables are linearly divided into two variable groups at first, i.e., a group with m-1 position-related variables: $\mathbf{x^p} = (x_1^p, x_2^p, ..., x_{m-1}^p)$ and another group with n-m+1 distance-related variables: $\mathbf{x^d} = (x_1^d, x_2^d, ..., x_{n-m+1}^d)$. After that, the distance-related variables are further nonuniformly divided into m variable groups: $\mathbf{x^{p,1}}$ to $\mathbf{x^{p,m}}$. Thereafter, each distance-based second level groups are further uniformly divided into K = 5 components, i.e., $(\mathbf{x}^{\mathbf{p},1,1}, \mathbf{x}^{\mathbf{p},1,2}, ..., \mathbf{x}^{\mathbf{p},1,5}), ...,$ $(\mathbf{x}^{\mathbf{p},m,1}, \mathbf{x}^{\mathbf{p},m,2}, ..., \mathbf{x}^{\mathbf{p},m,5})$. Then, the i-th objective is defined on $(\mathbf{x}^{\mathbf{p},i,1}, \mathbf{x}^{\mathbf{p},i,2}, ..., \mathbf{x}^{\mathbf{p},i,5})$ with the combination of five base single-objective functions respectively defined on these five components with the following formulation:

$$\mathbf{F(x)}:\min\begin{cases}f_1(\mathbf{x}) = h_1(\mathbf{x^p})(1+\frac{1}{5}\sum_{j=1}^{5} b_{1,j}(\mathbf{x^{d,1,j}}))\\ f_2(\mathbf{x}) = h_2(\mathbf{x^p})(1+\frac{1}{5}\sum_{j=1}^{5} b_{2,j}(\mathbf{x^{d,2,j}}))\\ \quad\dots\\ f_m(\mathbf{x}) = h_m(\mathbf{x^p})(1+\frac{1}{5}\sum_{j=1}^{5} b_{m,j}(\mathbf{x^{d,m,j}}))\end{cases}$$

Therefore, for the same LSMOP problem with different numbers of variables, the variable grouping results may be different, as a result, different but related optimization tasks can be obtained by setting different values of n.

5. Definition of RMTF Test Suite.

RMTF: a set of real-world multitasking function (to simulate practical applications), where each task in every RMTF problem is a large-scale 2-objective optimization problem. Here, the real-world applications in neural network optimization and portfolio optimization are considered to customize the RMTF benchmarks, which are elaborated in the following.

Neural network optimization problem for classification [5]: In general, the objective of optimizing a neural network for classification is to find the optimal weights that can minimize the error function on a prepared training dataset. However, if the optimization of neural networks only considers the objective of minimizing the error function, it may often over-fit the training data, which means that the network has a

Table A1
Summary of the datasets used in training a deep neural network for bi-classification tasks for customizing RMTF1-RMTF6

Dataset Index	Name	No. Features (N_F)	No. Samples (N_S)	Training Proportion
D1	Climate [1]	18	540	80%
D2	Raisin-Grains [2]	7	901	80%
D3	Rice-Osmancik-Cammeo [3]	7	3811	50%
D4	Statlog-Australian [1]	14	690	80%
D5	Statlog-German [1]	23	1000	80%
D6	WaveForm01 [1]	40	3346	50%
D7	WaveForm02 [1]	40	3347	50%
D8	WaveForm12 [1]	40	3308	50%
D9	Speaker-Accent-US-UK [1]	12	210	80%
D10	Speaker-Accent-FR-ES [1]	12	59	80%
D11	Speaker-Accent-GE-IT [1]	12	60	80%
D12	Dry-Bean-Barbunya-Sira [4]	16	3958	50%
D13	Dry-Bean-Cali-Horoz [4]	16	3558	50%
D14	Dry-Bean-Seker-Bombay [4]	16	2549	50%

very good approximation accuracy on the training data, but a very poor one on unseen data. Thus, many methods have been developed to improve the generalization performance of neural networks. A very popular technique to improve the generalization performance is known as regularization, which usually adds a penalty term to the error function. From the multi-objective optimization point of view, adding a regularization term in the error function is equivalent to combining two objectives using a weighted aggregation formulation [5]. In RMTF, the formulation of neural network optimization is defined as follows:

$$\mathbf{F(x)}:\min\begin{cases} f_1(\mathbf{x})=E(\mathbf{x}), \text{ indicates the classification error} \\ f_2(\mathbf{x})=R(\mathbf{x}), \text{ indicates the regularization term} \end{cases}$$

For the error functions, three common cost functions in training neural networks, like the mean squared error (MSE), the mean absolute error (MAE), and the root mean squared error (RMSE), are considered to customize RMTF problems, which are respectively defined as follows:

$$MSE(\mathbf{x})=\frac{1}{N}\sum_{i=1}^{N}\left[y_i^t-y_i\right]^2$$

where N is the number of training samples selected from the dataset D, y_i^t is the one dimensional target output or category label of the i-th sample, considering that the neural network has only one output for the case of bi-classification problem hereafter.

$$MAE(\mathbf{x})=\frac{1}{N}\sum_{i=1}^{N}\left|y_i^t-y_i\right|$$

$$RMSE(\mathbf{x})=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[y_i^t-y_i\right]^2}$$

where y^i is the network output for the i-th sample, which is further determined by the adopted activation function and the structure of the neural network. In this paper, five different activation functions are used to define the RMTF problems, which are listed as follows:

A1) Logistic function: sigm(z)

$$\mathrm{sigm}(z)=\frac{1}{1+e^{-z}}$$

A2) Hyperbolic tangent function: tanh(z)

$$\tanh(z)=\frac{e^z-e^{-z}}{e^z+e^{-z}}$$

A3) Gaussian function: gaus(z)

$$\mathrm{gaus}(z)=e^{(-z^2)}$$

A4) Soft sign function: ssig(z)

$$\mathrm{ssig}(z)=\frac{z}{1+|z|}$$

A5) Decaying sine unit function [6]: dsiu(z)

$$\mathrm{dsiu}(z)=\begin{cases} \frac{z}{2}, & \text{if } z=\pi \parallel z=-\pi \\ \frac{\pi}{2}\left(\frac{\sin(z-\pi)}{z-\pi}-\frac{\sin(z+\pi)}{z+\pi}\right), & \text{otherwise} \end{cases}$$

Besides, the neural network with two hidden layers are created, and we use two parameters (h_1 and h_2) to control the number of neurons in these two hidden layers, respectively.

Regarding the regularization of the neural network, it will cause the network to have smaller weights and biases, which will force its response to be smoother and further to control the effective complexity of the neural network. In RMTF, the widely used two types of regularization are considered:

L1 regularization consists of the sum of the absolute values of all the parameters in the neural network, which is defined as

$$R_1(\mathbf{x})=\frac{1}{n}\sum_{i=1}^{n}|x_i|$$

L2 regularization consists of the squared sum of all the parameters in the neural network, which is defined as:

$$R_2(\mathbf{x})=\frac{1}{n}\sum_{i=1}^{n}(x_i)^2$$

In addition, the input of the neural network is determined by the prepared training dataset. The classification error depends on the function represented by the neural network, and it is measured on the dataset. Here, the 14 datasets used in RMTF for bi-classification are summarized in Table A1.

Given above, we propose six multitasking real-world LMOPs, termed RMTF1 to RMTF6, to simulate the neural network optimization on different bi-classification tasks (considering from different aspects, like the dataset, the error function, the activation function, the regularization function, and the structure of the network), as follows:

$$\text{RMTF1:}\begin{cases}\text{Task1}\left[\text{D1},E_{RMSE}(\mathbf{x}),\text{gaus}(z),R_1(\mathbf{x}),h_1=10,h_2=5,n=251\right]\\ \text{Task2}\left[\text{D1},E_{RMSE}(\mathbf{x}),\text{sigm}(z),R_1(\mathbf{x}),h_1=10,h_2=5,n=251\right]\end{cases}$$

$$\text{RMTF2:}\begin{cases}\text{Task1}\left[\text{D2},E_{RMSE}(\mathbf{x}),\tanh(z),R_1(\mathbf{x}),h_1=50,h_2=20,n=1441\right]\\ \text{Task2}\left[\text{D3},E_{MAE}(\mathbf{x}),\text{dsiu}(z),R_2(\mathbf{x}),h_1=50,h_2=20,n=1441\right]\end{cases}$$

$$\text{RMTF3:}\begin{cases}\text{Task1}\left[\text{D4},E_{MSE}(\mathbf{x}),\tanh(z),R_1(\mathbf{x}),h_1=10,h_2=5,n=211\right]\\ \text{Task2}\left[\text{D5},E_{MSE}(\mathbf{x}),\tanh(z),R_1(\mathbf{x}),h_1=10,h_2=5,n=311\right]\end{cases}$$

$$\text{RMTF4:}\begin{cases}\text{Task1}\left[\text{D6},E_{MSE}(\mathbf{x}),\text{ssig}(z),R_1(\mathbf{x}),h_1=20,h_2=10,n=1041\right]\\ \text{Task2}\left[\text{D7},E_{MAE}(\mathbf{x}),\text{ssig}(z),R_1(\mathbf{x}),h_1=20,h_2=10,n=1041\right]\\ \text{Task3}\left[\text{D8},E_{RMSE}(\mathbf{x}),\text{ssig}(z),R_1(\mathbf{x}),h_1=20,h_2=10,n=1041\right]\end{cases}$$

$$\text{RMTF5:}\begin{cases}\text{Task1}\left[\text{D9},E_{MSE}(\mathbf{x}),\text{dsiu}(z),R_1(\mathbf{x}),h_1=10,h_2=5,n=191\right]\\ \text{Task2}\left[\text{D10},E_{MAE}(\mathbf{x}),\text{dsiu}(z),R_1(\mathbf{x}),h_1=10,h_2=5,n=191\right]\\ \text{Task3}\left[\text{D11},E_{RMSE}(\mathbf{x}),\text{dsiu}(z),R_1(\mathbf{x}),h_1=10,h_2=5,n=191\right]\end{cases}$$

$$\text{RMTF6:}\begin{cases}\text{Task1}\left[\text{D12},E_{MSE}(\mathbf{x}),\text{sigm}(z),R_2(\mathbf{x}),h_1=10,h_2=5,n=231\right]\\ \text{Task2}\left[\text{D13},E_{MSE}(\mathbf{x}),\tanh(z),R_2(\mathbf{x}),h_1=12,h_2=6,n=289\right]\\ \text{Task3}\left[\text{D14},E_{MSE}(\mathbf{x}),\text{dsiu}(z),R_2(\mathbf{x}),h_1=14,h_2=7,n=351\right]\end{cases}$$

Specifically, the meaning of variables in RMTF1-6 is the weights of the deep neural network. Thus, the number of variables n can be computed as follows:

$$n=\underbrace{(N_F+1)\times h_1}_{G_1}+\underbrace{(h_1+1)\times h_2}_{G_2}+\underbrace{(h_2+1)}_{G_3}$$

Thus, the variables of each task can be divided into three exclusive groups. Theoretically, we can specify a different activation function for each neuron to nonlinearly rectify its output. In our customized RMTF1-6, we used the same activation function for the neurons associated with the variables in the same group. Since it is a bi-classification task, we always use sigm(z) as the activation function for the output neuron, i.e., variables in G_3, while other neurons use the corresponding activation function as stipulated in the above definitions.

Portfolio optimization [7]: This problem aims to find the portfolio of instruments having the largest expected return and the lowest risk, which is defined as follows:

$$\mathbf{F(x)}:\min\begin{cases}f_1(\mathbf{x})=\sum_{i=1}^{n}\sum_{j=1}^{n}\mathbf{x}_i\sigma_{i,j}\mathbf{x}_j\\ f_2(\mathbf{x})=1-\sum_{i=1}^{n}\mathbf{x}_i r_i\end{cases}$$

where $\mathbf{x}$ is the decision vector referring to the ratio of each instrument in the portfolio, $\sigma_{i,j}$ indicates the covariance between the i-th and j-th instruments, and r_i represents the expected return of the i-th instrument. Thus, $f_1(\mathbf{x})$ can be regarded as the total risk of the portfolio, while $f_2(\mathbf{x})$ represents the negative value of the expected return.

Similar to [8], we define a 2-task portfolio optimization problem, termed RMTF7, where each task is a 2-objective optimization problem with the above formulation defined on a

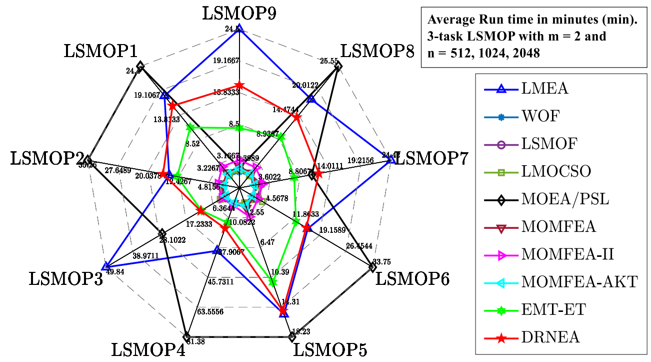

Fig. S12 Illustration the average runtimes of DRNEA and its nine competitors in solving the 3-task LSMOP1-9 problems with m = 2 and n = 512, 1024, 2048

prepared dataset. Specifically, the two datasets, named PoData1 and PoData2, are the minutely closing price of EUR/CHF taken from MT4. Thus, the RMTF7 is defined as follows:

$$\text{RMTF7:}\begin{cases}\text{Task1 }\left[\text{PoData1},n=1000\right]\\ \text{Task2 }\left[\text{PoData2},n=5000\right]\end{cases}$$

In particular, the meaning of variables in RMTF7 is portfolio of the instruments. The source codes of RMTF implemented by both Java and Matlab can be downloadable in the website: https://github.com/songbai-liu, which also provides all the datasets used in defining RMTF benchmarks.

6. Further Studies on DRNEA

The overall performance of DRNEA has been verified in the above experimental studies. Nevertheless, it is still worth studying how the contribution of the transferred solutions in DRNEA to each task of the same multitask LMOP varies with the evolutionary process. Thus, we record the number of surviving solutions transferred from other tasks at each of hundred generations in the current population. Here, the 3-task and 3-objective LSMOP1, LSMOP3, LSMOP5, and LSMOP7 are considered in this study, as shown in Fig. S12 provided in the supplementary document. From Fig. S12, we can learn two observations about DRNEA in solving these 3-task LSMOPs: i) transfer via DRN may be more contributed to solving higher-dimensional tasks (n = 1024, 2048 here) because their numbers of transferred solutions surviving in populations is larger than that in solving lower-dimensional tasks (n = 512), except for LSMOP7; and ii) the contribution of transfer via DRN to solving these four LSMOPs gradually decreases with evolution, and even has no contribution at the middle and late stages of evolution because none of the transferred solutions survive in their current evolutionary population. The reasons behind this may be multifaceted: i) the number of transfer attempts per generation is fixed (i.e., λ for each task), and the success of each attempt is determined by the learned similarities (i.e., $rand(0,1) < p_i$); ii) in the four test LSMOPs here, it is evident that the problem with lower dimension is easier to be solved so that the population with better performance can be obtained faster, and its solution has a higher probability of survival when it is transferred to help the optimization of related but more challenging problems; iii) the surviving-transferred solutions can help the optimization of the target task, but the help-efficiency will inevitably decrease with the evolution, as the population for target task is constantly evolving to be optimal. Therefore, properly evaluating the specific benefit (as shown in Fig. 1) brought by the transfer optimization manner in solving the multitask problems is an interesting topic worthy of further study in our future work.

Moreover, as pointed out in [24], setting different termination conditions in performance comparison may lead to different experimental conclusions. In our experimental studies, the termination condition is set as $FE_{\max} = 10^5$ and $FE_{\max} = 1.5 \times 10^5$ for cases of m = 2 and m = 3, respectively. From the results shown in Fig. S12, we can find that the advantages of ETO manner in DRNEA may be weakened with the increasing of $FE_{\max}$, as there is no benefit brought by transfer learning in the performance of solving 3-task LSMOP problems at the middle to late stages of evolution based on the settings above. It may be reasonable as the main impetus of using ETO manner to solve multiple different problems (or tasks) simultaneously, compared to solving them independently, is that we expect to improve both the final performance and the efficiency of optimization. Of course, when assessing the effectiveness of a new ETO algorithm, the setting of termination conditions is indeed a point worth our attention, especially in the cases of solving LMOPs, while a relative fair setting is that considering the evaluation of each ETO algorithm as an anytime solver under multiple termination conditions.

At last, due to page limitations, the parameter sensitivity analysis of the times of transfer in each generation for every target LMOP (i.e., λ in DRNEA) is provided in the supplementary document of this paper. Moreover, the complexity analysis of DRNEA is also presented in the supplementary document. Fig. S12 of the supplementary document depicts the average running times of DRNEA and its nine competitors in solving 3-task LSMOP1-LSMOP9 problems in the case of m = 2, and the results show that the average actual running time of DRNEA to solve these problems is indeed much longer than that of its most competitors (e.g., LMOCSO, WOF, LSMOF, MOMFEA, and MFEA-AKT), but it is acceptable when compared with LMEA, MOEA/PSL, and EMT-ET.

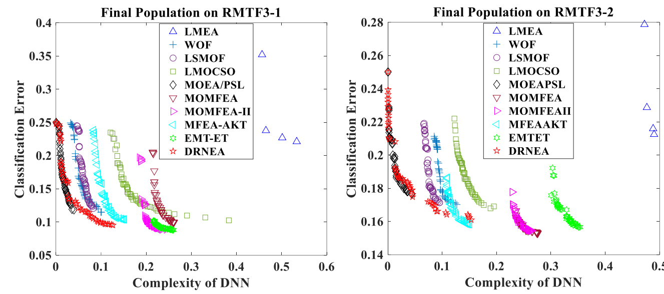

Fig. 8 Final nondominated solution sets obtained by DRNEA and its nine competitors on 2-task RMTF3 problem for training DNN.

References

[1] UCI Machine Learning Repository in http://archive.ics.uci.edu/ml/index.php.

[2] I. Cinar, M. Koklu and S. Tasdemir, "Classification of Raisin Grains Using Machine Vision and Artificial Intelligence Methods", Gazi Journal of Engineering Sciences, vol. 6, no. 3, pp. 200-209, December, 2020. Dataset: https://www.muratkoklu.com/datasets/.

[3] I. Cinar and M. Koklu, "Classification of Rice Varieties Using Artificial Intelligence Methods." International Journal of Intelligent Systems and Applications in Engineering, 7(3), 188-194, 2019.

[4] M. Koklu and I. A. Oakan, "Multiclass Classification of Dry Beans Using Computer Vision and Machine Learning Techniques." Computers and Electronics in Agriculture, 174, 105507, 2020. DOI: https://doi.org/10.1016/j.compag.2020.105507.

[5] Y. Jin and B. Sendhoff, "Pareto-based multiobjective machine learning: An overview and case studies," IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews), vol. 38, no. 3, pp. 397–415, 2008.

[6] M. M. Noel, S. Bharadwaj, V. Muthiah-Nakarajan, P. Dutta, G. B. Amali, "Biologically Inspired Oscillating Activation Functions Can Bridge the Performance Gap between Biological and Artificial Neurons," 2021. arXiv:2111.04020.

[7] A. Ponsich, A. L. Jaimes, and C. A. C. Coello, "A survey on multiobjective evolutionary algorithms for the solution of the portfolio optimization problem and other finance and economics applications," IEEE Transactions on Evolutionary Computation, vol. 17, no. 3, pp. 321–344, 2013.

[8] Y. Tian, C. Lu, X. Zhang, K. C. Tan, and Y. Jin, "Solving Large-Scale Multiobjective Optimization Problems With Sparse Optimal Solutions via Unsupervised Neural Networks," IEEE Transactions on Cybernetics, vol. 51, no. 6, pp. 3115-3128, 2021.

TABLE S2

IGD RESULTS OF THREE DIFFERENT DRNEAS ON LSMOP1-LSMOP9 PROBLEMS WITH 2 OBJECTIVES AND 512 TO 2048 VARIABLES

Problems Set	*n*	DRNEAV1	DRNEAV2	DRNEAV3	DRNEAV4	DRNEA
Set of LSMOP1 (m=2)	512	2.311E+0(3.31E-1)-	3.260E-1(1.32E-2)~	6.812E-1(4.41E-1)-	2.233E+0(5.12E-1)-	**2.887E-1(3.62E-1)**
	1024	5.040E+0(2.09E+0)-	8.417E-1(3.32E-1)-	5.409E-1(2.62E-1)-	3.029E+0(6.09E-1)-	**3.212E-1(1.74E-1)**
	2048	6.099E+0(1.68E+0)-	1.314E+0(3.11E-1)-	6.145E-1(1.45E-1)-	3.583E+0(2.80E-1)-	**4.413E-1(1.39E-1)**
Set of LSMOP2 (m=2)	512	5.471E-2(1.94E-3)-	3.876E-2(1.56E-3)~	4.183E-2(1.59E-3)-	5.888E-2(1.26E-3)-	**3.679E-2(1.41E-3)**
	1024	7.232E-2(1.68E-2)-	2.568E-2(1.45E-3)-	1.277E-2(3.40E-3)~	3.103E-2(4.45E-3)-	**1.125E-2(2.19E-3)**
	2048	7.147E-2(1.89E-2)-	1.771E-2(2.30E-3)-	8.710E-3(8.06E-4)~	2.111E-2(2.33E-3)-	**8.011E-3(3.96E-4)**
Set of LSMOP3 (m=2)	512	1.594E+1(1.19E+0)-	4.755E+0(5.28E+0)-	8.745E+0(9.28E+0)-	6.354E+0(5.07E+2)-	**1.563E+0(5.46E-3)**
	1024	5.568E+3(3.47E+3)-	7.901E+1(1.04E+2)-	7.851E+0(1.24E+2)-	9.241E+0(1.10E+3)-	**1.553E+0(3.16E-3)**
	2048	9.762E+3(3.02E+3)-	4.619E+2(6.00E+2)-	1.579E+0(1.94E-2)~	9.284E+0(1.29E+3)-	**1.525E+0(3.15E-2)**
Set of LSMOP4 (m=2)	512	9.382E-2(2.97E-3)-	5.391E-2(5.66E-3)~	6.267E-2(1.90E-2)-	1.070E-1(7.69E-3)-	**5.037E-2(1.42E-2)**
	1024	9.038E-2(1.39E-2)-	4.726E-2(8.21E-3)-	4.867E-2(1.22E-2)-	6.443E-2(2.29E-3)-	**4.041E-2(4.73E-3)**
	2048	7.337E-2(2.03E-2)-	**2.564E-2(3.10E-3)~**	3.132E-2(2.31E-3)~	4.021E-2(1.21E-3)-	2.711E-2(1.99E-3)
Set of LSMOP5 (m=2)	512	6.364E+0(7.59E-1)-	2.665E+0(1.17E+0)-	1.695E+0(9.71E-1)-	1.030E+0(3.49E+0)-	**3.077E-1(2.07E-1)**
	1024	1.204E+1(1.80E+0)-	2.861E+0(1.09E+0)-	1.304E+0(5.88E-1)-	1.179E+0(2.94E+0)-	**5.781E-1(3.70E-1)**
	2048	2.039E+1(6.30E+0)-	3.839E+0(9.50E-1)-	1.522E+0(7.06E-1)-	1.797E+0(2.39E+0)-	**7.006E-1(3.41E-1)**
Set of LSMOP6 (m=2)	512	1.761E+0(1.83E+0)-	7.308E-1(2.87E-2)-	7.390E-1(2.92E-2)-	1.748E+0(4.14E+1)-	**7.075E-1(1.90E-2)**
	1024	5.155E+2(2.54E+2)-	1.393E+1(2.82E+1)-	1.877E+1(3.79E+1)-	2.706E+2(6.25E+2)-	**7.306E-1(3.81E-1)**
	2048	1.308E+3(4.89E+2)-	2.105E+1(4.18E+1)-	7.463E-1(1.26E-4)-	2.817E+1(2.91E+1)-	**7.463E-1(1.16E-5)**
Set of LSMOP7 (m=2)	512	4.106E+3(2.09E+3)-	4.030E+2(4.43E+2)-	1.036E+3(1.04E+3)-	6.050E+0(1.57E+0)-	**1.464E+0(4.16E-3)**
	1024	1.441E+4(4.11E+3)-	2.437E+3(2.15E+3)-	5.067E+2(7.07E+2)-	6.519E+0(1.51E+0)-	**1.489E+0(3.33E-3)**
	2048	2.278E+4(5.55E+3)-	2.222E+3(2.00E+3)-	1.588E+0(1.10E-2)~	9.046E+0(2.32E+0)-	**1.543E+0(4.82E-3)**
Set of LSMOP8 (m=2)	512	2.667E+0(4.80E-1)-	2.967E+0(1.02E+0)-	2.669E+0(4.72E-1)-	1.141E+0(3.42E+0)-	**1.650E-1(6.79E-2)**
	1024	5.792E+0(7.11E-1)-	3.352E+0(7.40E-1)-	1.659E+0(5.51E-1)-	1.237E+0(3.11E+0)-	**1.638E-1(7.14E-2)**
	2048	8.397E+0(4.89E-1)-	3.234E+0(8.45E-1)-	1.658E+0(1.90E+0)-	1.206E+0(3.89E+0)-	**4.136E-1(1.11E-1)**
Set of LSMOP9 (m=2)	512	8.485E-1(1.46E-2)-	**4.038E-1(1.06E-1)+**	4.715E-1(1.92E-1)~	2.375E+0(8.64E-1)-	4.304E-1(2.43E-2)
	1024	1.027E+0(8.02E-2)-	4.258E-1(8.55E-2)-	4.496E-1(2.73E-2)-	2.682E+0(2.26E+0)-	**1.965E-1(1.02E-2)**
	2048	1.907E+0(1.20E-1)-	5.336E-1(3.03E-1)-	6.823E-1(5.41E-1)-	3.850E+0(1.44E+0)-	**4.626E-1(2.51E-2)**
Best/All		0/27	2/27	0/27	0/27	25/27
+/−/~		0/27/0	1/22/4	0/21/6	0/27/0	----------

TABLE S3

IGD RESULTS OF THREE DIFFERENT DRNEAS ON LSMOP1-LSMOP9 PROBLEMS WITH 2 OBJECTIVES AND 512 TO 2048 VARIABLES

Problems Set	*n*	DRNEAV1	DRNEAV2	DRNEAV3	DRNEAV4	DRNEA
Set of LSMOP1 (m=3)	512	4.450E+0(8.41E-1)-	9.615E-1(1.46E-1)-	7.516E-1(3.23E-1)-	1.480E+0(1.49E+0)-	**4.697E-1(1.73E-2)**
	1024	3.636E+0(3.80E-1)-	1.019E+0(2.02E-1)-	8.163E-1(3.11E-1)-	2.030E+0(1.70E+0)-	**5.157E-1(1.10E-2)**
	2048	6.234E+0(3.07E-1)-	1.038E+0(2.26E-1)-	7.642E-1(7.96E-2)-	2.822E+0(1.54E+0)-	**5.192E-1(1.08E-1)**
Set of LSMOP2 (m=3)	512	6.490E-2(2.35E-3)-	5.513E-2(1.08E-3)~	**5.255E-2(1.11E-3)~**	6.475E-2(2.34E-3)-	5.342E-2(7.81E-4)
	1024	1.146E-1(2.25E-2)-	4.842E-2(4.44E-3)~	5.075E-2(7.45E-3)~	5.342E-2(1.46E-3)-	**4.559E-2(5.95E-3)**
	2048	1.252E-1(2.87E-2)-	4.976E-2(5.41E-3)~	4.963E-2(7.98E-3)~	4.694E-2(1.56E-3)~	**4.013E-2(8.14E-3)**
Set of LSMOP3 (m=3)	512	1.977E+1(6.64E+0)-	1.404E+0(4.40E-1)-	8.108E-1(6.69E-2)-	1.228E+1(1.00E+0)-	**7.423E-1(5.78E-3)**
	1024	1.523E+1(4.58E+0)-	2.124E+0(2.16E+0)-	8.618E-1(1.59E-4)-	2.277E+1(3.74E+0)-	**7.894E-1(2.46E-2)**
	2048	5.382E+1(5.78E+1)-	2.177E+0(1.20E+0)-	8.489E-1(2.71E-2)~	1.464E+1(9.95E+1)-	**8.391E-1(8.60E-3)**
Set of LSMOP4 (m=3)	512	1.898E-1(3.05E-3)-	1.551E-1(3.14E-3)~	1.993E-1(4.81E-3)-	1.965E-1(5.21E-3)-	**1.537E-1(4.54E-3)**
	1024	2.366E-1(4.48E-2)-	1.006E-1(1.20E-2)~	1.093E-1(3.55E-3)~	1.237E-1(2.61E-3)-	**9.081E-2(3.24E-3)**
	2048	1.875E-1(3.81E-2)-	8.234E-2(3.23E-3)-	7.242E-2(1.89E-3)~	8.167E-2(2.95E-3)-	**7.040E-2(2.97E-3)**
Set of LSMOP5 (m=3)	512	5.857E+0(8.09E-1)-	9.875E-1(1.84E-1)-	7.560E-1(1.94E-1)-	2.138E+0(6.51E+0)-	**4.207E-1(1.76E-1)**
	1024	9.988E+0(1.03E+0)-	1.020E+0(1.76E-1)-	6.725E-1(1.33E-1)-	2.140E+0(8.25E+0)-	**4.987E-1(1.00E-1)**
	2048	1.642E+1(5.30E+0)-	8.967E-1(1.72E-1)-	6.259E-1(5.57E-1)-	2.224E+0(5.67E+0)-	**5.321E-1(4.89E-1)**
Set of LSMOP6 (m=3)	512	1.955E+3(6.61E+2)-	1.734E+2(3.66E+2)-	1.544E+0(3.42E-1)-	7.963E+1(3.88E+2)-	**1.420E+0(6.22E-2)**
	1024	1.791E+4(1.73E+4)-	5.146E+2(6.97E+2)-	1.489E+0(2.38E-2)~	8.823E+1(3.98E+2)-	**1.467E+0(1.10E-1)**
	2048	2.495E+4(1.19E+4)-	1.831E+3(2.15E+3)-	1.585E+0(1.42E-1)-	1.233E+1(1.32E+2)-	**1.495E+0(6.81E-3)**
Set of LSMOP7 (m=3)	512	1.025E+1(1.66E+1)-	1.011E+0(2.85E-2)-	9.695E-1(1.62E-2)-	5.905E+1(7.35E+1)-	**6.965E-1(3.15E-2)**
	1024	4.665E+3(2.73E+3)-	7.220E+1(2.88E+1)-	8.403E-1(1.56E-2)-	6.710E+1(6.44E+1)-	**7.305E-1(9.34E-3)**
	2048	8.453E+2(1.22E+3)-	1.078E+1(6.06E+1)-	7.881E-1(3.07E-3)-	7.608E+1(8.07E+1)-	**7.194E-1(3.44E-3)**
Set of LSMOP8 (m=3)	512	7.580E-1(8.25E-3)-	4.475E-1(5.71E-2)-	4.015E-1(1.25E-1)-	5.203E-1(2.84E-2)-	**2.086E-1(7.90E-2)**
	1024	7.770E-1(2.32E-1)-	4.480E-1(9.57E-2)-	3.521E-1(4.39E-2)~	5.590E-1(5.89E-2)-	**3.023E-1(5.56E-2)**
	2048	7.742E-1(5.54E-2)-	5.419E-1(1.46E-2)-	3.344E-1(2.45E-2)-	5.488E-1(7.78E-2)-	**2.714E-1(1.91E-2)**
Set of LSMOP9 (m=3)	512	1.107E+0(8.02E-2)-	1.062E+0(5.86E-1)-	8.326E-1(2.04E-1)-	1.807E+0(4.93E+0)-	**7.166E-1(6.48E-2)**
	1024	2.627E+0(1.39E-1)-	1.018E+0(1.13E+0)-	9.055E-1(3.90E-1)-	1.659E+0(6.68E+0)-	**6.801E-1(3.06E-2)**
	2048	5.476E+0(7.13E-1)-	1.109E+0(5.69E-1)-	9.243E-1(1.05E-1)-	1.983E+0(6.62E+0)-	**7.514E+0(1.09E-2)**
Best/All		0/27	0/27	1/27	0/27	26/27
+/−/~		0/27/0	0/22/5	0/19/8	0/26/1	----------

TABLE S4
IGD RESULTS OF DRNEA AND ITS FIVE LMOEA COMPETITORS ON LSMOP1-9 PROBLEMS

Problems Set	*n*	LMEA	WOF-NSGA-II	LSMOF	LMOCSO	MOEA/PSL	DRNEA
Set of	512	1.097E+1(4.06E-1)-	**2.474E-1(1.79E-2)+**	6.104E-1(3.06E-2)-	6.477E-1(5.08E-2)-	1.014E+0(5.15E-1)-	2.887E-1(3.62E-1)
LSMOP1	1024	1.114E+1(2.69E-1)-	**2.801E-1(1.46E-2)~**	6.349E-1(1.07E-2)-	1.441E+0(8.17E-2)-	1.183E+0(4.02E-1)-	3.212E-1(1.74E-1)
(*m*=2)	2048	1.137E+1(2.08E-1)-	**3.298E-1(1.86E-2)+**	6.394E-1(2.21E-2)-	1.993E+0(1.37E-1)-	1.192E+0(4.45E-1)-	4.413E-1(1.39E-1)
Set of	512	7.232E-2(5.09E-4)-	3.153E-2(9.19E-4)~	**2.383E-2(6.76E-4)+**	4.595E-2(9.28E-4)-	2.867E-2(1.16E-3)+	3.679E-2(1.41E-3)
LSMOP2	1024	3.980E-2(2.81E-4)-	1.929E-2(1.38E-3)~	1.854E-2(2.67E-4)~	2.547E-2(3.44E-4)-	1.689E-2(4.85E-4)~	**1.125E-2(2.19E-3)**
(*m*=2)	2048	2.247E-2(2.60E-4)-	1.295E-2(1.11E-3)-	1.307E-2(3.87E-4)-	1.427E-2(3.40E-4)-	1.034E-2(3.05E-4)-	**8.011E-3(3.96E-4)**
Set of	512	1.073E+3(1.13E+3)-	**1.043E+0(1.58E-1)+**	1.566E+0(6.50E-4)~	1.342E+1(4.22E+0)-	1.115E+1(1.36E+1)-	1.563E+0(5.46E-3)
LSMOP3	1024	1.359E+3(1.06E+3)-	**1.450E+0(1.38E-1)+**	1.574E+0(7.11E-4)~	1.553E+3(3.81E+3)-	7.357E+0(1.82E+1)-	1.553E+0(3.16E-3)
(*m*=2)	2048	2.176E+3(1.65E+3)-	1.569E+0(1.47E-2)~	1.577E+0(2.83E-4)~	9.074E+0(2.82E+0)-	2.246E+1(3.36E+1)-	**1.525E+0(3.15E-2)**
Set of	512	1.336E-1(5.24E-4)-	9.526E-2(3.84E-3)-	5.079E-2(9.63E-4)~	8.895E-2(7.31E-4)-	8.384E-2(1.73E-3)-	**5.037E-2(1.42E-2)**
LSMOP4	1024	7.706E-2(1.61E-4)-	5.795E-2(1.05E-3)-	**3.267E-2(1.20E-3)+**	5.261E-2(4.69E-4)-	4.777E-2(3.54E-4)~	4.041E-2(4.73E-3)
(*m*=2)	2048	4.422E-2(3.76E-4)-	3.234E-2(6.45E-4)-	**2.325E-2(8.50E-4)~**	2.960E-2(2.14E-4)-	2.718E-2(4.13E-4)~	2.711E-2(1.99E-3)
Set of	512	2.330E+1(5.51E-1)-	5.667E-1(7.56E-2)-	7.584E-1(0.00E+0)-	1.227E+0(1.78E-1)-	2.466E+0(1.43E+0)-	**3.077E-1(2.07E-1)**
LSMOP5	1024	2.364E+1(4.34E-1)-	7.515E-1(1.32E-3)-	7.584E-1(0.00E+0)-	3.433E+0(4.06E-1)-	2.550E+0(1.83E+0)-	**5.781E-1(3.70E-1)**
(*m*=2)	2048	2.417E+1(2.26E-1)-	7.533E-1(2.14E-3)-	7.584E-1(0.00E+0)-	4.932E+0(4.05E-1)-	1.935E+0(1.63E+0)-	**7.006E-1(3.41E-1)**
Set of	512	2.075E+3(1.81E+3)-	6.874E-1(1.63E-1)~	**5.146E-1(9.04E-4)+**	8.006E-1(1.06E-2)-	8.862E-1(3.19E-1)-	7.075E-1(1.90E-2)
LSMOP6	1024	1.504E+3(1.54E+3)-	6.839E-1(1.68E-1)+	**5.076E-1(4.76E-5)+**	7.847E-1(2.94E-3)~	7.944E-1(3.98E-1)~	7.306E-1(3.81E-1)
(*m*=2)	2048	4.270E+3(3.84E+3)-	7.599E-1(4.12E-3)-	**5.046E-1(2.76E-5)+**	7.706E-1(7.17E-4)~	5.843E-1(1.99E-1)+	7.463E-1(1.16E-5)
Set of	512	8.125E+4(3.87E+3)-	1.464E+0(8.04E-4)~	1.465E+0(1.46E-3)~	4.613E+2(5.44E+1)-	8.761E+2(1.69E+3)-	**1.464E+0(4.16E-3)**
LSMOP7	1024	8.637E+4(2.62E+3)-	**1.473E+0(3.03E-4)~**	1.473E+0(5.43E-4)~	9.343E+2(1.25E+2)-	1.735E+3(3.71E+3)-	1.489E+0(3.33E-3)
(*m*=2)	2048	9.046E+4(1.69E+3)-	1.478E+0(1.70E-4)~	**1.476E+0(5.01E-4)~**	4.502E+3(1.10E+3)-	1.587E+3(2.33E+3)-	1.543E+0(4.82E-3)
Set of	512	1.967E+1(4.58E-1)-	2.448E-1(4.70E-2)-	7.584E-1(0.00E+0)-	1.117E+0(1.54E-1)-	1.067E+0(3.95E-1)-	**1.650E-1(6.79E-2)**
LSMOP8	1024	2.017E+1(3.70E-1)-	6.280E-1(4.42E-2)-	7.584E-1(0.00E+0)-	2.939E+0(5.56E-1)-	1.283E+0(8.42E-1)-	**1.638E-1(7.14E-2)**
(*m*=2)	2048	2.050E+1(2.20E-1)-	7.359E-1(2.17E-2)-	7.584E-1(0.00E+0)-	3.966E+0(5.14E-1)-	1.372E+0(8.65E-1)-	**4.136E-1(1.11E-1)**
Set of	512	3.256E+1(1.30E+0)-	**2.666E-1(2.43E-1)+**	6.392E-1(1.05E-3)-	4.961E-1(3.35E-2)~	6.305E-1(3.02E-2)-	4.304E-1(2.43E-2)
LSMOP9	1024	3.446E+1(1.04E+0)-	2.437E-1(2.67E-1)-	6.366E-1(1.46E-3)-	7.580E-1(1.85E-1)-	6.401E-1(3.51E-4)-	**1.965E-1(1.02E-2)**
(*m*=2)	2048	3.606E+1(9.71E-1)-	**2.168E-1(2.42E-1)+**	6.346E-1(1.43E-3)-	3.040E+0(1.14E+0)-	6.430E-1(8.37E-3)-	4.626E-1(2.51E-2)
Best/All		0/27	8/27	7/27	0/27	0/27	12/27
+/-/=		0/27/0	7/12/8	5/13/9	0/24/3	2/21/4	------

TABLE S5
IGD RESULTS OF DRNEA AND ITS FIVE LMOEA COMPETITORS ON LSMOP1-9 PROBLEMS

Problems Set	*n*	LMEA	WOF-NSGA-II	LSMOF	LMOCSO	MOEAPSL	DRNEA
Set of	512	8.496E+0(4.84E-1)-	**4.454E-1(5.13E-2)+**	4.786E-1(5.59E-3)~	1.281E+0(1.05E-1)-	1.030E+0(2.76E-1)-	4.697E-1(1.73E-2)
LSMOP1	1024	9.885E+0(3.52E-1)-	**4.502E-1(5.61E-2)+**	5.139E-1(1.03E-2)~	1.467E+0(9.31E-2)-	1.102E+0(3.20E-1)-	5.157E-1(1.10E-2)
(*m*=3)	2048	1.102E+1(4.96E-1)-	**5.106E-1(8.13E-2)~**	5.349E-1(1.20E-2)~	1.526E+0(6.53E-2)-	1.256E+0(5.30E-1)-	5.192E-1(1.08E-1)
Set of	512	7.020E-2(1.43E-3)-	6.376E-2(1.46E-3)-	7.904E-2(2.58E-3)-	**5.026E-2(5.78E-4)+**	5.451E-2(1.37E-3)~	5.342E-2(7.81E-4)
LSMOP2	1024	5.621E-2(2.72E-3)-	4.739E-2(1.33E-3)~	5.984E-2(2.39E-3)-	**3.996E-2(2.50E-4)+**	4.614E-2(2.25E-3)~	4.559E-2(5.95E-3)
(*m*=3)	2048	5.029E-2(3.29E-3)-	4.151E-2(2.83E-3)~	5.355E-2(3.51E-3)-	**3.517E-2(9.35E-5)+**	4.011E-2(2.40E-3)~	4.013E-2(8.14E-3)
Set of	512	2.033E+2(1.42E+2)-	8.619E-1(6.77E-6)-	8.577E-1(5.48E-3)-	1.019E+1(1.52E+0)-	1.321E+0(8.74E-1)-	**7.423E-1(5.78E-3)**
LSMOP3	1024	2.388E+2(1.48E+2)-	8.618E-1(2.63E-5)-	8.618E-1(4.69E-5)-	1.430E+1(2.32E+0)-	4.752E+0(4.35E+0)-	**7.894E-1(2.46E-2)**
(*m*=3)	2048	2.806E+2(3.64E+2)-	9.026E-1(1.29E-1)-	8.618E-1(6.39E-5)~	1.421E+1(3.19E+0)-	2.650E+0(3.50E+0)-	**8.391E-1(8.60E-3)**
Set of	512	1.960E-1(1.52E-3)-	1.974E-1(4.29E-3)-	2.030E-1(3.91E-3)-	**1.479E-1(1.86E-3)~**	1.505E-1(4.04E-3)~	1.537E-1(4.54E-3)
LSMOP4	1024	1.202E-1(1.56E-3)-	1.149E-1(3.38E-3)-	1.318E-1(2.59E-3)-	9.114E-2(4.17E-4)~	1.008E-1(2.49E-3)~	**9.081E-2(3.24E-3)**
(*m*=3)	2048	7.822E-2(1.62E-3)-	7.178E-2(2.38E-3)~	8.591E-2(2.72E-3)-	**5.833E-2(2.55E-4)+**	7.055E-2(2.80E-3)~	7.040E-2(2.97E-3)
Set of	512	1.545E+1(7.48E-1)-	4.864E-1(2.97E-1)-	7.482E-1(6.91E-2)-	2.615E+0(1.39E-1)-	9.036E-1(1.48E-1)-	**4.207E-1(1.76E-1)**
LSMOP5	1024	1.822E+1(4.43E-1)-	**4.900E-1(2.02E-1)~**	7.904E-1(8.35E-2)-	2.933E+0(8.76E-2)-	1.076E+0(4.18E-1)-	4.987E-1(1.00E-1)
(*m*=3)	2048	1.972E+1(5.11E-1)-	6.494E-1(2.12E-1)-	7.878E-1(5.63E-2)-	3.141E+0(9.73E-2)-	9.375E-1(1.43E-1)-	**5.321E-1(4.89E-1)**
Set of	512	2.718E+4(4.05E+3)-	1.728E+0(1.73E-3)-	**7.733E-1(2.17E-2)+**	8.521E+1(7.35E+1)-	1.422E+2(1.88E+2)-	1.420E+0(6.22E-2)
LSMOP6	1024	3.362E+4(2.92E+3)-	1.763E+0(2.02E-3)-	**7.938E-1(5.47E-2)+**	4.249E+2(1.34E+2)-	9.928E+1(3.09E+2)-	1.467E+0(1.10E-1)
(*m*=3)	2048	3.468E+4(2.77E+3)-	1.715E+0(2.03E-1)-	**7.864E-1(2.62E-2)+**	4.746E+2(1.18E+2)-	1.107E+2(3.45E+2)-	1.495E+0(6.81E-3)
Set of	512	1.068E+3(9.34E+2)-	7.539E-1(8.75E-5)-	7.876E-1(1.61E-2)-	7.850E-1(6.91E-2)-	9.013E-1(4.98E-2)-	**6.965E-1(3.15E-2)**
LSMOP7	1024	9.535E+2(7.30E+2)-	7.547E-1(3.92E-4)~	7.404E-1(1.29E-2)~	7.348E-1(3.83E-2)~	9.029E-1(3.16E-1)-	**7.305E-1(9.34E-3)**
(*m*=3)	2048	2.405E+3(3.13E+3)-	7.580E-1(3.50E-4)-	7.228E-1(1.37E-2)~	7.320E-1(8.50E-2)~	7.842E-1(3.77E-2)-	**7.194E-1(3.44E-3)**
Set of	512	5.282E-1(7.27E-2)-	2.130E-1(2.07E-2)~	3.328E-1(3.30E-2)-	4.034E-1(4.96E-3)-	2.878E-1(3.98E-2)-	**2.086E-1(7.90E-2)**
LSMOP8	1024	5.199E-1(7.45E-2)-	3.225E-1(2.27E-1)~	3.134E-1(2.42E-2)~	3.944E-1(3.67E-3)-	3.517E-1(2.47E-1)-	**3.023E-1(5.56E-2)**
(*m*=3)	2048	5.726E-1(1.84E-1)-	3.188E-1(2.30E-1)-	3.366E-1(5.67E-2)-	3.859E-1(3.29E-3)-	2.781E-1(6.10E-2)~	**2.714E-1(1.91E-2)**
Set of	512	3.028E+1(7.12E-1)-	7.216E-1(6.53E-3)~	8.461E-1(7.13E-2)-	**6.505E-1(9.57E-3)**+	7.165E-1(4.88E-2)~	7.166E-1(6.48E-2)
LSMOP9	1024	3.743E+1(1.44E+0)-	7.249E-1(1.03E-2)-	8.119E-1(8.79E-2)-	1.628E+1(1.08E+1)-	8.010E-1(5.43E-2)-	**6.801E-1(3.06E-2)**
(*m*=3)	2048	4.217E+1(1.05E+0)-	7.503E-1(3.11E-2)~	7.778E-1(8.79E-2)~	2.813E+1(7.86E+0)-	**7.368E-1(3.34E-2)+**	7.514E+0(1.09E-2)
Best/All		0/27	4/27	3/27	6/27	1/27	13/27
+/-/=		0/27/0	2/15/10	3/16/8	5/18/4	1/18/8	------

TABLE S6
IGD RESULTS OF DRNEA AND ITS FOUR ETO COMPETITORS ON LSMOP1-9 PROBLEMS

Problems	n	MOMFEA	MOMFEA-II	MFEA-AKT	EMTET	DRNEA
LSMOP1 (m=2)	512	1.82E+0(1.82E-1)-	1.52E+0(1.16E-1)-	1.21E+0(7.31E-2)-	5.913E-1(9.72E-2)-	**2.887E-1(3.62E-1)**
	1024	2.70E+0(1.83E-1)-	2.29E+0(2.19E-1)-	1.25E+0(9.06E-2)-	6.373E-1(6.30E-2)-	**3.212E-1(1.74E-1)**
	2048	4.21E+0(2.16E-1)-	3.72E+0(2.82E-1)-	1.37E+0(8.58E-2)-	7.047E-1(1.26E-3)-	**4.413E-1(1.39E-1)**
LSMOP2 (m=2)	512	5.52E-2(1.41E-3)-	5.35E-2(1.16E-3)-	3.72E-2(1.52E-3)~	**1.291E-2(1.28E-4)+**	3.679E-2(1.41E-3)
	1024	3.51E-2(3.61E-4)-	3.46E-2(4.02E-4)-	2.03E-2(6.70E-4)-	**9.304E-3(1.40E-4)+**	1.125E-2(2.19E-3)
	2048	2.07E-2(3.17E-4)-	2.05E-2(1.82E-4)-	1.26E-2(3.53E-4)-	7.894E-2(4.12E-2)~	**8.011E-3(3.96E-4)**
LSMOP3 (m=2)	512	1.24E+1(1.35E+0)-	1.12E+1(1.40E+0)-	2.44E+1(1.15E+0)-	1.568E+0(2.66E-3)-	**1.563E+0(5.46E+0)**
	1024	1.64E+1(6.15E-1)-	1.55E+1(1.04E+0)-	2.68E+1(1.12E+0)-	1.575E+0(1.71E-3)-	**1.553E+0(3.16E-3)**
	2048	2.12E+1(8.44E-1)-	2.08E+1(8.73E-1)-	2.96E+1(9.27E-1)-	1.582E+0(1.14E-3)~	**1.525E+0(3.15E-3)**
LSMOP4 (m=2)	512	9.99E-2(1.90E-3)-	9.76E-2(2.77E-3)-	9.13E-2(2.28E-3)-	8.708E-2(1.29E-3)-	**5.037E-2(1.42E-2)**
	1024	6.13E-2(8.48E-4)-	6.15E-2(9.20E-4)-	5.33E-2(1.04E-3)-	5.639E-2(9.48E-4)-	**4.041E-2(4.73E-3)**
	2048	3.77E-2(4.50E-4)-	3.79E-2(2.93E-4)-	3.10E-2(5.90E-4)~	1.567E-1(8.56E-2)-	**2.711E-2(1.99E-3)**
LSMOP5 (m=2)	512	7.78E+0(4.95E-1)-	6.64E+0(4.51E-1)-	2.86E+0(3.03E-1)-	7.584E-1(7.89E-7)-	**3.077E-1(2.07E-1)**
	1024	1.09E+1(6.20E-1)-	9.98E+0(4.82E-1)-	3.05E+0(3.09E-1)-	7.584E-1(2.30E-7)-	**5.781E-1(3.70E-1)**
	2048	1.48E+1(5.73E-1)-	1.30E+1(2.95E-1)-	3.20E+0(2.83E-1)-	7.584E-1(1.85E-7)-	**7.006E-1(3.41E-1)**
LSMOP6 (m=2)	512	6.90E-1(1.28E-1)~	**6.68E-1(1.27E-1)+**	7.51E-1(1.81E-3)-	1.127E+0(3.59E-1)-	**7.075E-1(1.90E-2)**
	1024	7.72E-1(6.16E-3)~	7.32E-1(7.08E-2)~	7.57E-1(7.80E-4)~	1.301E+0(3.04E-1)-	**7.306E-1(3.81E-1)**
	2048	**6.90E-1(1.29E-1)+**	7.31E-1(1.10E-1)~	7.58E-1(4.74E-4)~	1.392E+0(2.31E-1)-	**7.463E-1(1.16E-5)**
LSMOP7 (m=2)	512	1.69E+4(4.01E+3)-	8.71E+3(4.80E+3)-	3.26E+2(6.58E+1)-	**1.457E+0(1.94E-3)~**	1.464E+0(4.16E-3)
	1024	2.55E+4(5.75E+3)-	2.43E+4(3.36E+3)-	6.94E+2(7.61E+1)-	**1.471E+0(1.14E-3)~**	1.489E+0(3.33E-3)
	2048	3.80E+4(3.04E+3)-	3.67E+4(4.15E+3)-	1.09E+3(1.68E+2)-	**1.478E+0(6.53E-4)+**	1.543E+0(4.82E-3)
LSMOP8 (m=2)	512	6.05E+0(4.63E-1)-	5.03E+0(5.23E-1)-	1.64E+0(8.37E-2)-	7.584E-1(7.40E-7)-	**1.650E-1(6.79E-2)**
	1024	8.45E+0(4.03E-1)-	7.26E+0(4.05E-1)-	1.89E+0(1.44E-1)-	7.584E-1(7.64E-7)-	**1.638E-1(7.14E-2)**
	2048	1.12E+1(1.97E-1)-	9.88E+0(3.48E-1)-	2.10E+0(1.21E-1)-	7.584E-1(2.89E-7)-	**4.136E-1(1.11E-1)**
LSMOP9 (m=2)	512	9.32E-1(2.38E-2)-	8.52E-1(1.24E-2)-	9.83E-1(1.30E-1)-	6.402E-1(5.77E-5)-	**4.304E-1(2.43E-2)**
	1024	1.27E+0(1.22E-1)-	1.00E+0(8.98E-2)-	1.19E+0(1.59E-1)-	6.389E-1(3.24E-4)-	**1.965E-1(1.02E-2)**
	2048	2.58E+0(2.42E-1)-	2.00E+0(1.06E-1)-	1.47E+0(2.50E-1)-	6.383E-1(6.44E-4)-	**4.626E-1(2.51E-2)**
Best/All		1/27	1/27	0/27	0/27	25/27
+/−/=		1/24/2	1/24/2	0/23/4	3/20/4	-------

TABLE S7
IGD RESULTS OF DRNEA AND ITS THREE ETO COMPETITORS ON LSMOP1-9 PROBLEMS

Problems	n	MOMFEA	MOMFEA-II	MFEA-AKT	DRNEA
LSMOP1 (m=3)	512	4.783E+0(5.75E-1)-	5.063E+0(6.77E-1)-	1.558E+0(1.53E-1)-	**4.697E-1(1.73E-2)**
	1024	6.673E+0(3.53E-1)-	6.842E+0(4.37E-1)-	1.561E+0(1.16E-1)-	**5.157E-1(1.10E-2)**
	2048	8.340E+0(6.84E-1)-	8.405E+0(5.78E-1)-	1.888E+0(1.53E-1)-	**5.192E-1(1.08E-1)**
LSMOP2 (m=3)	512	6.353E-2(1.19E-3)-	6.258E-2(1.84E-3)-	**4.787E-2(1.24E-3)+**	5.342E-2(7.81E-4)
	1024	5.138E-2(1.50E-3)-	5.204E-2(1.57E-3)-	**4.341E-2(1.34E-3)~**	4.559E-2(5.95E-3)
	2048	4.710E-2(1.27E-3)-	4.686E-2(1.46E-3)-	4.273E-2(1.79E-3)-	**4.013E-2(8.14E-3)**
LSMOP3 (m=3)	512	2.092E+1(5.58E+0)-	1.879E+1(6.01E+0)-	1.011E+1(6.07E-1)-	**7.423E-1(5.78E-3)**
	1024	2.192E+1(5.21E+0)-	2.216E+1(4.94E+0)-	1.057E+1(7.72E-1)-	**7.894E-1(2.46E-2)**
	2048	2.457E+1(5.24E+0)-	2.709E+1(4.37E+0)-	1.088E+1(1.29E+0)-	**8.391E-1(8.60E-3)**
LSMOP4 (m=3)	512	2.013E-1(3.09E-3)-	1.989E-1(2.60E-3)-	1.601E-1(4.77E-3)-	**1.537E-1(4.54E-3)**
	1024	1.230E-1(1.45E-3)-	1.219E-1(1.96E-3)-	9.798E-2(2.97E-3)-	**9.081E-2(3.24E-3)**
	2048	7.139E-2(1.73E-3)~	7.110E-2(1.20E-3)~	**6.579E-2(2.01E-3)+**	7.040E-2(2.97E-3)
LSMOP5 (m=3)	512	8.119E+0(6.90E-1)-	6.694E+0(1.07E+0)-	3.992E+0(6.32E-1)-	**4.207E-1(1.76E-1)**
	1024	1.298E+1(1.03E+0)-	1.330E+1(6.37E-1)-	4.319E+0(7.73E-1)-	**4.987E-1(1.00E-1)**
	2048	2.175E+1(1.12E+0)-	2.045E+1(1.24E+0)-	7.214E+0(1.30E+0)-	**5.321E-1(4.89E-1)**
LSMOP6 (m=3)	512	3.548E+3(9.69E+2)-	2.291E+3(5.75E+2)-	7.047E+2(2.87E+2)-	**1.420E+0(6.22E-2)**
	1024	1.377E+4(3.76E+3)-	8.974E+3(1.70E+3)-	1.531E+3(5.38E+2)-	**1.467E+0(1.10E-1)**
	2048	2.884E+4(4.11E+3)-	2.362E+4(3.29E+3)-	3.151E+3(9.65E+2)-	**1.495E+0(6.81E-3)**
LSMOP7 (m=3)	512	1.793E+0(1.46E+0)-	1.183E+0(3.40E-1)-	9.733E-1(2.26E-2)-	**6.965E-1(3.15E-2)**
	1024	1.670E+0(1.85E+0)-	1.083E+0(5.01E-1)-	8.501E-1(5.46E-3)-	**7.305E-1(9.34E-3)**
	2048	1.482E+0(1.66E+0)-	8.314E-1(2.83E-1)-	8.049E-1(5.39E-3)-	**7.194E-1(3.44E-3)**
LSMOP8 (m=3)	512	7.647E-1(2.50E-3)-	7.464E-1(4.21E-2)-	7.108E-1(6.66E-2)-	**2.086E-1(7.90E-2)**
	1024	7.546E-1(1.58E-2)-	7.283E-1(4.53E-2)-	7.348E-1(4.13E-2)-	**3.023E-1(5.56E-2)**
	2048	7.510E-1(1.62E-2)-	7.578E-1(2.13E-4)-	7.305E-1(3.16E-2)-	**2.714E-1(1.91E-2)**
LSMOP9 (m=3)	512	2.118E+0(1.77E-1)-	1.551E+0(1.07E-1)-	4.572E+0(5.37E-1)-	**7.166E-1(6.48E-2)**
	1024	4.344E+0(3.05E-1)-	3.108E+0(1.94E-1)-	5.944E+0(3.92E-1)-	**6.801E-1(3.06E-2)**
	2048	7.242E+0(6.76E-1)-	5.871E+0(2.87E-1)-	6.637E+0(6.13E-1)-	**7.514E+0(1.09E-2)**
Best/All		0/27	0/27	3/27	24/27
+/−/=		0/26/1	0/26/1	2/24/1	-------

TABLE S8
IGD RESULTS OF DRNEA AND ITS SIX COMPETITORS ON CPLX PROBLEMS

Problems	(*m*, *n*)	NSGA-II	MOMFEA	MOMFEA-II	MFEA-AKT	EMT-ET	MOEA/PSL	DRNEA
CPLX1	2, 128	5.753E-2(6.92E-3)-	5.385E-2(8.58E-3)-	5.155E-2(9.46E-3)-	5.760E-2(7.36E-3)-	5.988E-2(8.45E-3)-	5.821E-2(4.90E-3)-	**1.034E-2(7.20E-4)**
	2, 256	1.283E-1(2.42E-2)-	1.208E-1(2.32E-2)-	1.170E-1(7.88E-3)-	1.196E-1(2.93E-2)-	1.303E-1(4.08E-2)-	3.186E-1(2.05E-1)-	**9.833E-2(1.24E-2)**
CPLX2	2, 128	4.950E-2(6.63E-3)-	5.369E-2(7.47E-3)-	4.999E-2(4.55E-3)-	6.397E-2(8.60E-3)-	5.132E-2(8.32E-3)-	4.829E-2(6.39E-3)-	**1.004E-2(5.96E-4)**
	2, 128	3.644E-1(3.07E-2)-	4.240E-1(4.71E-2)-	3.976E-1(3.53E-2)-	3.532E-1(1.65E-2)-	4.572E-1(9.91E-2)-	7.121E-1(2.77E-1)-	**2.712E-1(1.83E-2)**
CPLX3	2, 256	1.217E-1(1.99E-2)-	1.317E-1(2.75E-2)-	1.218E-1(2.59E-2)-	1.392E-1(4.74E-2)-	1.218E-1(2.72E-2)-	3.673E-1(2.78E-1)-	**9.625E-2(9.32E-3)**
	2, 256	3.163E-1(7.19E-2)-	1.442E-1(1.71E-2)-	1.330E-1(1.82E-2)-	1.518E-1(9.85E-3)-	3.068E-1(6.63E-2)-	2.921E-1(4.33E-2)-	**8.313E-2(5.96E-3)**
CPLX4	2, 256	1.221E-1(1.58E-2)-	1.297E-1(3.07E-2)-	1.253E-1(3.07E-2)-	1.077E-1(1.08E-3)-	1.153E-1(1.64E-2)-	2.581E-1(1.13E-1)-	**7.749E-2(9.35E-3)**
	2, 256	1.984E-1(7.31E-2)-	1.382E-1(3.29E-2)-	1.320E-1(3.07E-2)-	1.168E-1(1.43E-3)-	1.293E-1(2.31E-2)-	3.476E-1(2.60E-1)-	**8.749E-2(1.27E-2)**
CPLX5	2, 256	1.862E-1(4.08E-2)-	1.535E-1(1.30E-2)-	1.542E-1(2.08E-2)-	1.580E-1(2.59E-2)-	1.797E-1(2.81E-2)-	1.594E-1(6.15E-2)-	**1.086E-1(1.90E-2)**
	3, 128	4.785E-1(4.12E-2)-	2.101E-1(2.38E-2)+	**2.030E-1(8.32E-3)**+	2.220E-1(2.84E-2)+	4.135E-1(1.03E-1)-	4.250E-1(2.13E-1)-	2.383E-1(8.59E-3)
CPLX6	2, 512	2.195E-1(2.96E-2)-	2.223E-1(3.25E-2)-	2.033E-1(2.09E-2)-	1.533E-1(1.75E-2)-	2.146E-1(1.52E-2)-	1.757E-1(4.01E-2)-	**1.302E-1(5.17E-3)**
	3, 512	5.502E-1(8.19E-2)-	5.891E-1(1.82E-1)-	5.063E-1(1.52E-1)-	9.087E-1(2.29E-1)-	5.058E-1(1.58E-1)-	8.169E-1(1.50E-1)-	**3.227E-1(6.76E-2)**
CPLX7	2, 1024	2.735E-1(2.42E-2)-	2.626E-1(1.44E-2)-	2.124E-1(1.16E-2)=	**2.087E-1(4.29E-3)**=	2.144E-1(1.92E-2)=	2.113E-1(3.59E-2)=	2.112E-1(2.13E-2)
	3, 1024	3.551E-1(2.53E-2)-	1.703E-1(1.03E-2)-	1.755E-1(9.86E-3)-	**1.091E-1(1.39E-2)**+	1.552E-1(1.78E-2)-	1.294E-1(5.94E-2)=	1.316E-1(7.54E-3)
CPLX8	2, 512	1.574E-1(1.15E-2)-	1.947E-1(8.89E-3)-	1.803E-1(9.46E-3)-	1.549E-1(7.18E-3)-	1.600E-1(1.07E-2)-	1.281E-1(2.77E-2)=	**1.269E-1(5.45E-3)**
	3, 256	7.593E-1(5.55E-2)=	8.393E-1(6.05E-2)-	8.401E-1(6.86E-2)-	6.754E-1(5.57E-2)+	**3.284E-1(6.42E-2)**+	6.137E-1(3.94E-1)+	7.530E-1(6.28E-2)
CPLX9	3, 512	3.129E-1(2.02E-2)=	4.732E-1(4.05E-2)-	3.837E-1(2.64E-2)-	**2.617E-1(1.24E-2)**+	3.090E-1(2.37E-2)+	2.715E-1(6.42E-3)+	3.295E-1(1.56E-2)
	2, 1024	3.393E-1(1.56E-1)+	4.668E-1(6.29E-2)-	4.356E-1(1.09E-1)-	4.223E-1(7.79E-2)-	**1.940E-1(3.86E-2)**+	4.497E-1(2.53E-1)-	3.877E-1(1.15E-1)
CPLX10	2, 128	3.655E-1(4.10E-2)-	2.853E-1(2.03E-2)-	2.843E-1(2.22E-2)-	2.709E-1(2.64E-2)-	4.397E-1(1.28E-1)-	**2.000E-1(1.44E-2)**=	2.017E-1(2.42E-2)
	2, 128	2.443E-1(9.85E-3)-	2.469E-1(2.87E-2)-	2.335E-1(2.04E-2)-	2.418E-1(2.98E-2)-	2.438E-1(1.59E-2)-	1.117E-1(6.29E-2)-	**6.039E-2(6.85E-3)-**
Best/All		0/20	0/20	1/20	3/20	2/20	1/20	13/20
+/−/=		1/17/2	1/19/0	1/18/1	4/15/1	3/16/1	2/14/4	-------

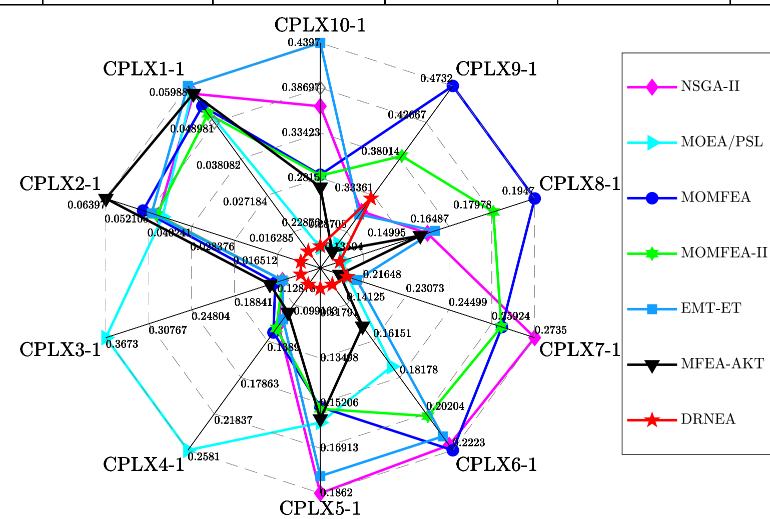

Fig. S4 Illustration of the IGD results obtained by DRNEA and its six competitors on the task 1 of ten CPLX problems.

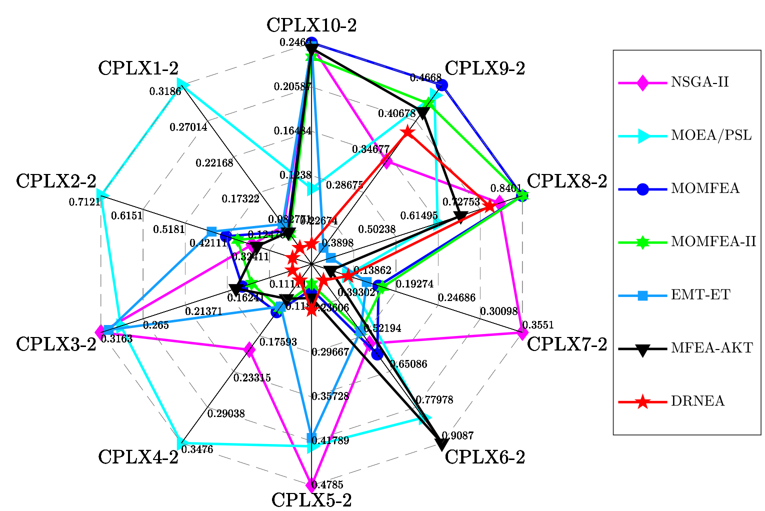

Fig. S5 Illustration of the IGD results obtained by DRNEA and its six competitors on the task 2 of ten CPLX problems.

TABLE S9
HV RESULTS OF DRNEA AND ITS FIVE LMOEA COMPETITORS ON RMTF1-7 PROBLEMS

Problems Set	*n*	LMEA	WOF-SMPSO	LSMOF	LMOCSO	MOEA/PSL	DRNEA
RMTF1	251	4.024E-01(6.55E-3)-	7.131E-01(9.30E-3)-	7.032E-01(8.60E-3)-	6.739E-01(9.05E-3)-	7.460E-01(1.10E-3)=	**7.499E-01(3.60E-3)**
(*m*=2)	251	4.052E-01(6.64E-3)-	7.124E-01(1.42E-2)-	7.156E-01(7.62E-3)-	6.660E-01(4.01E-3)-	7.457E-01(1.11E-3)=	**7.510E-01(1.40E-3)**
RMTF2	1441	3.639E-01(1.00E-2)-	6.660E-01(1.72E-2)=	6.527E-01(3.87E-3)-	6.065E-01(8.71E-3)-	**7.025E-01(4.62E-3)=**	6.992E-01(2.52E-3)
(*m*=2)	1441	3.716E-01(1.34E-2)-	8.322E-01(3.83E-2)=	8.200E-01(2.17E-2)=	7.358E-01(1.32E-2)-	**8.779E-01(1.87E-2)+**	8.348E-01(3.02E-2)
RMTF3	211	4.466E-01(1.25E-2)-	8.615E-01(1.39E-2)-	8.583E-01(1.13E-2)-	7.964E-01(1.07E-2)-	8.860E-01(1.18E-2)-	**9.074E-01(2.17E-3)**
(*m*=2)	311	4.386E-01(6.47E-3)-	7.953E-01(1.61E-2)-	7.881E-01(6.06E-3)-	7.438E-01(1.00E-2)-	8.370E-01(6.54E-3)=	**8.450E-01(4.42E-3)**
RMTF4	1041	4.485E-01(9.81E-3)-	8.586E-01(3.99E-2)-	8.013E-01(1.20E-2)-	7.703E-01(1.16E-2)-	8.772E-01(2.08E-2)=	**8.894E-01(1.67E-2)**
(*m*=2)	1041	3.520E-01(1.24E-2)-	**7.721E-01(2.16E-2)+**	6.896E-01(2.16E-2)-	6.282E-01(1.73E-2)-	7.149E-01(3.27E-2)-	7.233E-01(2.23E-2)
	1041	3.438E-01(1.23E-2)-	7.102E-01(2.78E-2)=	6.647E-01(1.38E-2)-	6.055E-01(1.26E-2)-	7.132E-01(1.50E-2)=	**7.163E-01(3.36E-2)**
RMTF5	191	4.519E-01(8.64E-3)-	8.737E-01(2.79E-2)-	8.680E-01(1.78E-2)-	8.138E-01(1.07E-2)-	8.773E-01(1.74E-2)-	**9.216E-01(1.69E-2)**
(*m*=2)	191	3.769E-01(2.32E-2)-	8.631E-01(2.61E-2)-	7.885E-01(5.39E-2)-	7.769E-01(2.47E-2)-	8.251E-01(2.92E-2)-	**9.013E-01(2.39E-2)**
	191	3.206E-01(1.08E-2)-	6.703E-01(3.88E-2)-	6.159E-01(1.31E-2)-	6.367E-01(1.68E-2)-	6.717E-01(1.69E-2)-	**7.125E-01(1.20E-2)**
RMTF6	231	3.880E-01(9.48E-3)-	8.332E-01(2.25E-2)-	7.794E-01(2.00E-2)-	7.373E-01(1.37E-2)-	8.234E-01(1.47E-2)-	**8.598E-01(1.14E-2)**
(*m*=2)	289	4.383E-01(1.38E-2)-	9.120E-01(2.31E-2)-	9.229E-01(8.17E-3)-	8.210E-01(1.78E-2)-	9.504E-01(1.01E-2)=	**9.583E-01(2.55E-3)**
	351	4.557E-01(5.68E-3)-	9.186E-01(2.04E-2)-	8.929E-01(1.86E-2)-	8.300E-01(1.06E-2)-	9.701E-01(2.27E-2)=	**9.791E-01(6.27E-3)**
RMTF7	1000	9.014E-02(3.79E-5)-	9.198E-02(2.26E-4)-	9.150E-02(1.84E-4)-	9.177E-02(1.75E-4)-	1.058E-01(1.36E-2)-	**1.221E-01(1.42E-3)**
(*m*=2)	5000	9.001E-02(5.75E-5)-	9.114E-02(3.21E-5)-	9.109E-02(2.23E-5)-	9.101E-02(2.21E-5)-	1.016E-01(1.35E-2)-	**1.183E-01(2.83E-3)**
Best/All		0/17	1/17	0/17	0/17	0/27	12/27
+/-/=		0/17/0	1/13/3	0/16/1	0/17/0	1/8/8	------

TABLE S10
HV RESULTS OF DRNEA AND ITS FOUR MULTITASKING ETO COMPETITORS ON RMTF1-7 PROBLEMS

Problems Set	*n*	MOMFEA	MOMFEA-II	MFEA-AKT	EMTET	DRNEA
RMTF1	251	6.195E-01(9.48E-3)-	6.265E-01(7.58E-3)-	6.853E-01(1.32E-2)-	5.931E-01(1.55E-2)-	**7.499E-01(3.60E-3)**
(*m*=2)	251	6.193E-01(8.42E-3)-	6.268E-01(8.05E-3)-	6.809E-01(1.26E-2)-	5.967E-01(8.23E-3)-	**7.510E-01(1.40E-3)**
RMTF2	1441	4.836E-01(6.14E-3)-	4.872E-01(7.52E-3)-	6.253E-01(6.53E-3)-	4.641E-01(7.58E-3)-	**6.992E-01(2.52E-3)**
(*m*=2)	1441	5.597E-01(7.48E-3)-	5.643E-01(8.35E-3)-	7.790E-01(1.16E-2)-	5.311E-01(9.19E-3)-	**8.348E-01(3.02E-2)**
RMTF3	211	7.468E-01(1.55E-2)-	7.631E-01(7.36E-3)-	8.292E-01(7.09E-3)-	7.363E-01(1.08E-2)-	**9.074E-01(2.17E-3)**
(*m*=2)	311	6.760E-01(1.12E-2)-	6.838E-01(8.18E-3)-	7.706E-01(7.68E-3)-	6.511E-01(1.26E-2)-	**8.450E-01(4.42E-3)**
RMTF4	1041	6.843E-01(7.16E-3)-	7.136E-01(7.86E-3)-	8.155E-01(1.70E-2)-	6.315E-01(5.60E-3)-	**8.894E-01(1.67E-2)**
(*m*=2)	1041	6.131E-01(1.58E-2)-	6.613E-01(7.37E-3)-	6.951E-01(1.96E-2)-	5.934E-01(1.23E-2)-	**7.233E-01(2.23E-2)**
	1041	5.777E-01(7.96E-3)-	6.076E-01(8.47E-3)-	6.767E-01(1.62E-2)-	5.438E-01(1.09E-2)-	**7.163E-01(3.36E-2)**
RMTF5	191	8.149E-01(1.15E-2)-	8.353E-01(9.68E-3)-	8.577E-01(3.34E-3)-	7.692E-01(1.75E-2)-	**9.216E-01(1.69E-2)**
(*m*=2)	191	7.886E-01(1.55E-2)-	8.355E-01(1.19E-2)-	8.005E-01(2.29E-2)-	7.781E-01(2.27E-2)-	**9.013E-01(2.39E-2)**
	191	6.270E-01(1.05E-2)-	6.513E-01(1.72E-2)-	6.523E-01(1.22E-2)-	6.076E-01(1.19E-2)-	**7.125E-01(1.20E-2)**
RMTF6	231	6.806E-01(1.07E-2)-	7.053E-01(1.37E-2)-	7.710E-01(1.27E-2)-	6.470E-01(1.26E-2)-	**8.598E-01(1.14E-2)**
(*m*=2)	289	7.345E-01(7.58E-3)-	7.499E-01(1.39E-2)-	8.685E-01(9.33E-3)-	6.768E-01(1.60E-2)-	**9.583E-01(2.55E-3)**
	351	7.188E-01(7.81E-3)-	7.320E-01(1.25E-2)-	8.828E-01(1.10E-2)-	6.804E-01(9.56E-3)-	**9.791E-01(6.27E-3)**
RMTF7	1000	9.186E-02(4.03E-5)-	9.186E-02(1.90E-5)-	9.199E-02(5.68E-5)-	9.173E-02(5.52E-5)-	**1.221E-01(1.42E-3)**
(*m*=2)	5000	9.115E-02(8.63E-6)-	9.115E-02(1.48E-5)-	9.116E-02(1.89E-5)-	9.112E-02(1.89E-5)-	**1.183E-01(2.83E-3)**
Best/All		0/17	0/17	0/17	0/17	17/17
+/-/=		0/17/0	0/17/0	0/17/0	0/17/0	------

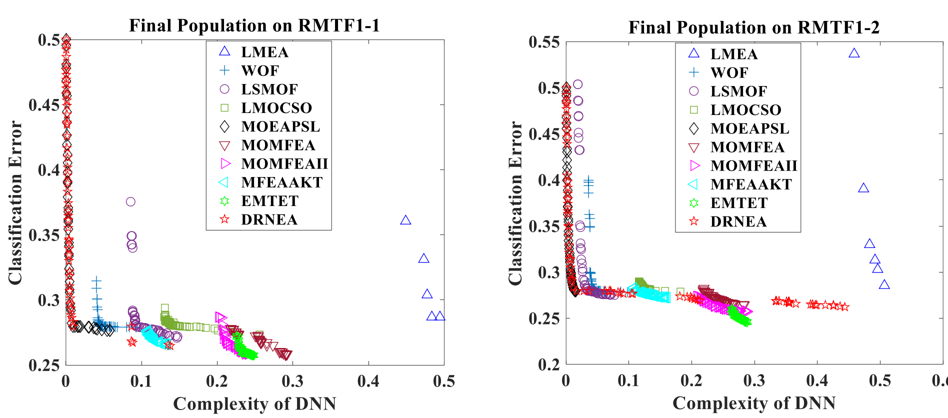

Fig. S6 Final nondominated solution sets obtained by DRNEA and its nine competitors on 2-task RMTF1 problem for training DNN.

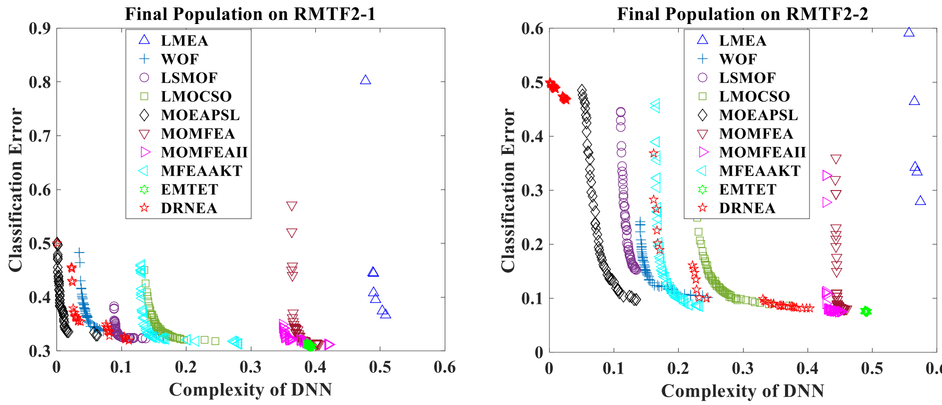

Fig. S7 Final nondominated solution sets obtained by DRNEA and its nine competitors on 2-task RMTF2 problem for training DNN.

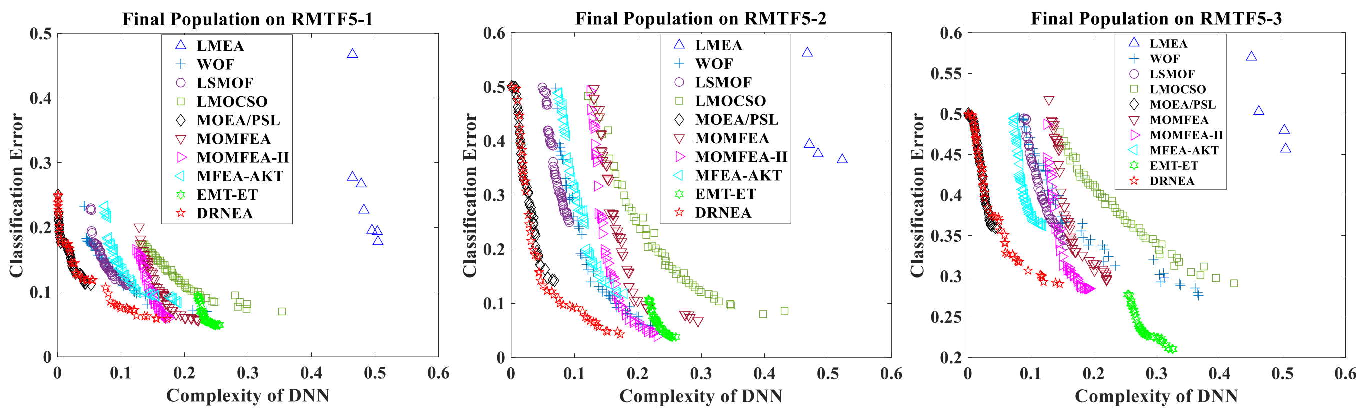

Fig. S8 Final nondominated solution sets obtained by DRNEA and its nine competitors on 3-task RMTF5 problem for training DNN

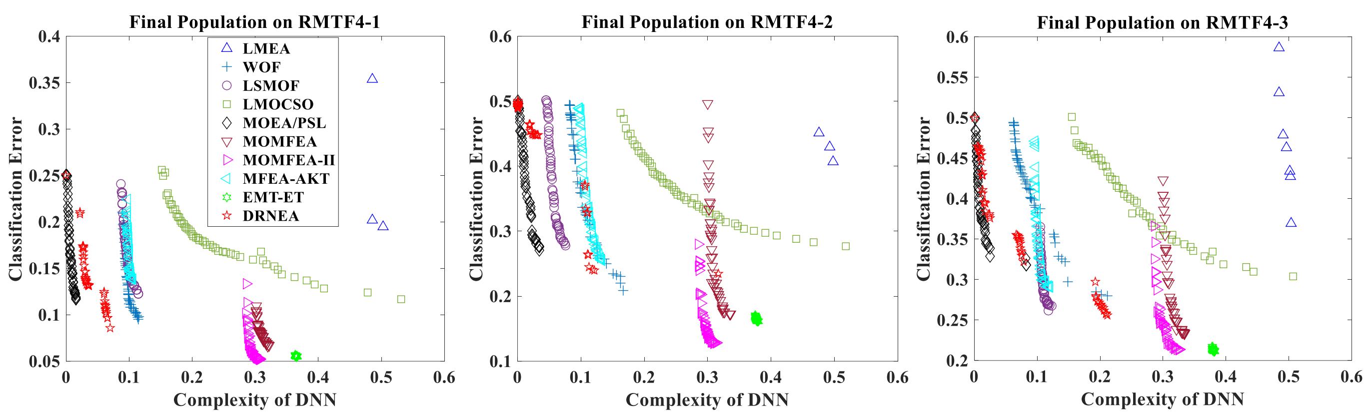

Fig. S9 Final nondominated solution sets obtained by DRNEA and its nine competitors on 3-task RMTF4 problem for training DNN.

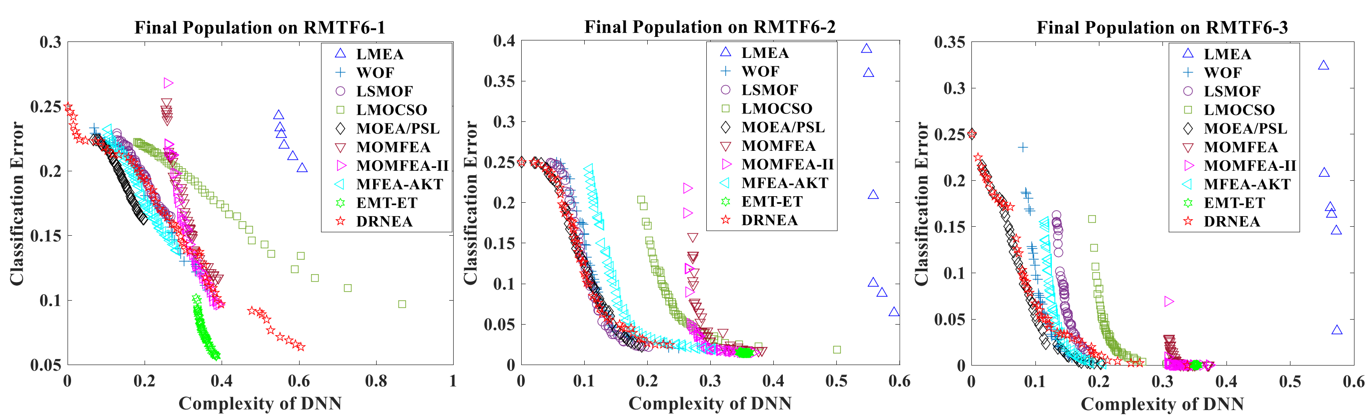

Fig. S10 Final nondominated solution sets obtained by DRNEA and its nine competitors on 3-task RMTF6 problem for training DNN.

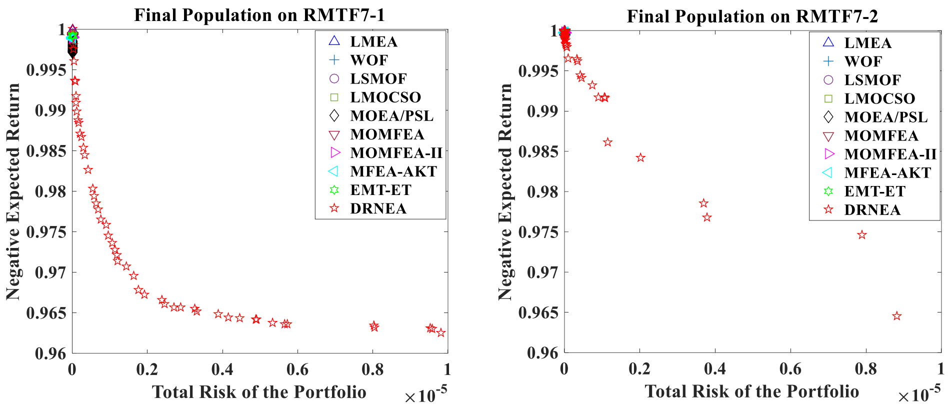

Fig. S11 Final nondominated solution sets obtained by DRNEA and its nine competitors on 2-task RMTF7 problem for portfolio optimization.

TABLE S11

IGD RESULTS OF DRNEA AND TWO WOF VARIANTS ON 2-OBJECTIVE LSMOP1-9 PROBLEMS

Problems Set	n	WOF-NSGA-II	WOF-SMPSO	DRNEA
Set of	512	**2.474E-1(1.79E-2)**	4.133E-1(3.94E-2)	2.887E-1(3.62E-1)
LSMOP1	1024	**2.801E-1(1.46E-2)**	4.023E-1(7.70E-2)	3.212E-1(1.74E-1)
(m=2)	2048	**3.298E-1(1.86E-2)**	5.089E-1(9.73E-2)	4.413E-1(1.39E-1)
Set of	512	3.153E-2(9.19E-4)	**2.629E-2(3.01E-3)**	3.679E-2(1.41E-3)
LSMOP2	1024	1.929E-2(1.38E-3)	1.869E-2(4.44E-4)	**1.125E-2(2.19E-3)**
(m=2)	2048	1.295E-2(1.11E-3)	**7.345E-3(2.73E-3)**	8.011E-3(3.96E-4)
Set of	512	**1.043E+0(1.58E-1)**	1.502E+0(7.21E-2)	1.563E+0(5.46E-3)
LSMOP3	1024	**1.450E+0(1.38E-1)**	1.573E+0(1.97E-2)	1.553E+0(3.16E-3)
(m=2)	2048	1.569E+0(1.47E-2)	1.576E+0(5.74E-3)	**1.525E+0(3.15E-2)**
Set of	512	9.526E-2(3.84E-3)	6.294E-2(4.59E-3)	**5.037E-2(1.42E-2)**
LSMOP4	1024	5.795E-2(1.05E-3)	4.150E-2(3.00E-3)	**4.041E-2(4.73E-3)**
(m=2)	2048	3.234E-2(6.45E-4)	**1.410E-2(6.25E-3)**	2.711E-2(1.99E-3)
Set of	512	5.667E-1(7.56E-2)	5.454E-1(2.08E-1)	**3.077E-1(2.07E-1)**
LSMOP5	1024	7.515E-1(1.32E-3)	6.427E-1(1.05E-1)	**5.781E-1(3.70E-1)**
(m=2)	2048	7.533E-1(2.14E-3)	7.417E-1(8.61E-4)	**7.006E-1(3.41E-1)**
Set of	512	6.874E-1(1.63E-1)	**5.711E-1(1.31E-1)**	7.075E-1(1.90E-2)
LSMOP6	1024	6.839E-1(1.68E-1)	**6.487E-1(7.21E-2)**	7.306E-1(3.81E-1)
(m=2)	2048	7.599E-1(4.12E-3)	**2.171E-1(1.59E-1)**	7.463E-1(1.16E-5)
Set of	512	1.464E+0(8.04E-4)	1.505E+0(1.68E-3)	**1.464E+0(4.16E-3)**
LSMOP7	1024	**1.473E+0(3.03E-4)**	1.514E+0(1.85E-3)	1.489E+0(3.33E-3)
(m=2)	2048	**1.478E+0(1.70E-4)**	1.506E+0(2.74E-2)	1.543E+0(4.82E-3)
Set of	512	2.448E-1(4.70E-2)	6.601E-1(1.36E-1)	**1.650E-1(6.79E-2)**
LSMOP8	1024	6.280E-1(4.42E-2)	6.791E-1(1.02E-1)	**1.638E-1(7.14E-2)**
(m=2)	2048	7.359E-1(2.17E-2)	7.421E-1(3.06E-6)	**4.136E-1(1.11E-1)**
Set of	512	**2.666E-1(2.43E-1)**	5.097E-1(3.32E-4)	4.304E-1(2.43E-2)
LSMOP9	1024	2.437E-1(2.67E-1)	5.088E-1(8.50E-4)	**1.965E-1(1.02E-2)**
(m=2)	2048	**2.168E-1(2.42E-1)**	5.083E-1(7.48E-4)	4.626E-1(2.51E-2)
Best/All		9/27	6/27	12/27

TABLE S1

PARAMETERS SETTINGS OF ALL THE COMPARED ALGORITHMS

Algorithm	Parameter's settings
MOEA/PSL	$p_c=1.0$, $p_m=1/n$, $\eta_c=20$, $\eta_m=20$, $epochs$ = 10, η =0.1
LSMOF	CR = 1.0, F = 0.5, $p_m=1/n$, $\eta_m=15$, NCA = 20, NIA = 6
LMEA	$p_c=1.0$, $p_m=1/n$, $\eta_c=20$, $\eta_m=20$, $nSel$ = 5, $nPer$ = 5, $nCor$ = 6
WOF-SMPSO	$\omega\in[0.1,0.5]$, $c_1,c_2\in[1.5,2.5]$, t_1 = 1000, t_2 = 500, q = m+1, $\delta=0.5$, γ = 4, PT = 3, GS = 3
WOF-NSGA-II	$p_c=1.0$, $p_m=1/n$, $\eta_c=20$, $\eta_m=20$, t_1 = 1000, t_2 = 500, q = m+1, $\delta=0.5$, γ = 4, PT = 3, GS = 3
LMOCSO	$p_m=1/n$, $\eta_m=20$, $\alpha=2$
MOMFEA	$p_c=1.0$, $p_m=1/n$, $\eta_c=20$, $\eta_m=20$, rmp = 0.9
MOMFEA-II	$p_c=1.0$, $p_m=1/n$, $\eta_c=20$, $\eta_m=20$
MFEA-AKT	$p_c=1.0$, $p_m=1/n$, $\eta_c=20$, $\eta_m=20$, $\lambda=0.25, \omega=0.25, \alpha=0.3$, rmp = 0.9
EMT/ET	$p_c=1.0$, $p_m=1/n$, $\eta_c=20$, $\eta_m=15$, n = 8, p = 0.5, $\lambda\sim N(0,2)$
DRNEA	$p_c=1.0$, $p_m=1/n$, $\eta_c=20$, $\eta_m=20$, $epochs$ = 1, η =0.1, $\lambda=N/10$
NSGA-II	$p_c=1.0$, $p_m=1/n$, $\eta_c=20$, $\eta_m=20$

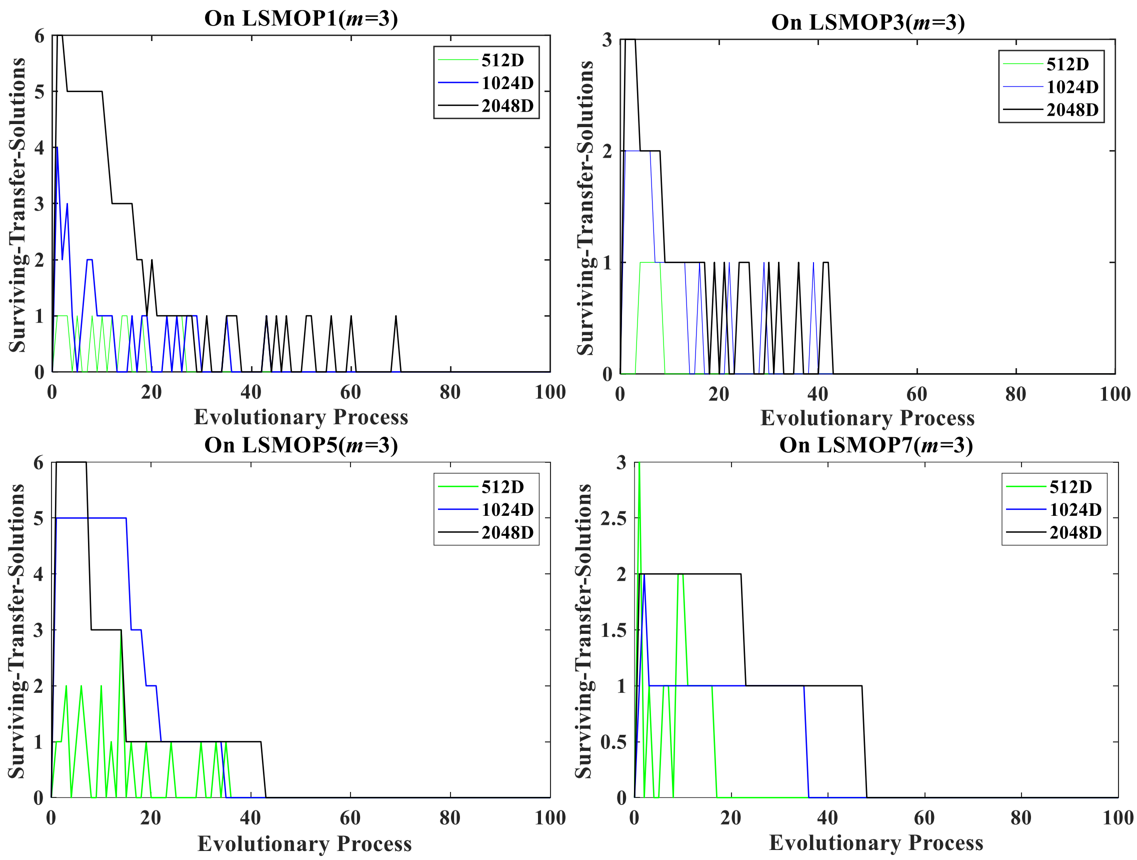

Fig. S12 Illustration of the surviving transferred solutions in the current population at each generation.

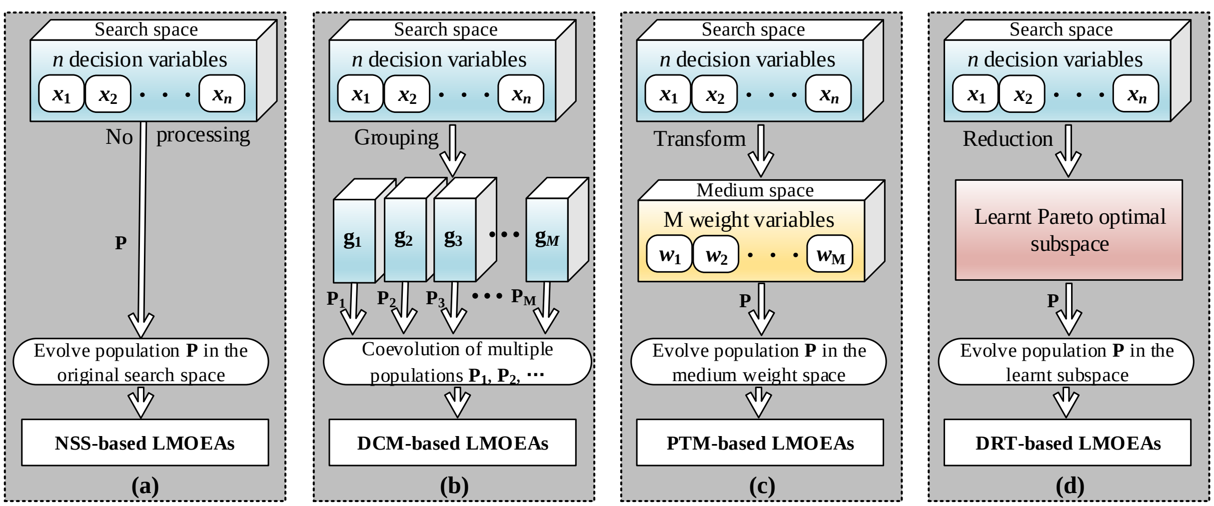

Fig. S13 Simple examples to show the main motivations in the four different types of LMOEA.

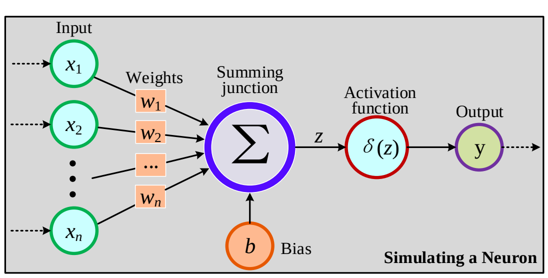

Fig. S14 Simulation of a neuron.

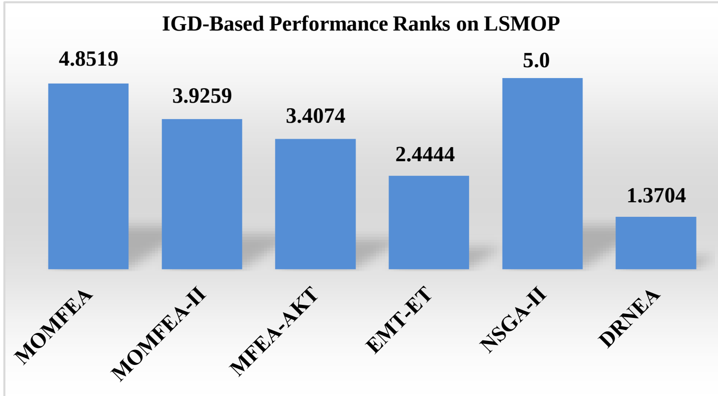

Fig. S15 Illustration of the performance ranks of DRNEA and its four ETO competitors over all LSMOP test problems in the case of $m = 2$.

Here, we rank DRNEA and its four ETO by Friedman's test based on their obtained IGD results on LSMOPs with $m = 2$. As shown in Fig. S15, DRNEA obtains the best performance rank with score 1.3704, which is lower than that obtained by any of its ETO competitors. This confirms the superiority of DRNEA over its ETO competitors in solving these test LSMOPs. Besides, we can find from Fig. S15 that these four ETO competitors perform better than NSGA-II, which validates the effectiveness of transfer optimization in handling LSMOPs. From Fig. S15, we can also conclude that, in this study, EMT-ET with explicit transfer performs better than other three ETO competitors with implicit transfer, because it gets a lower ranking score of 2.4444.

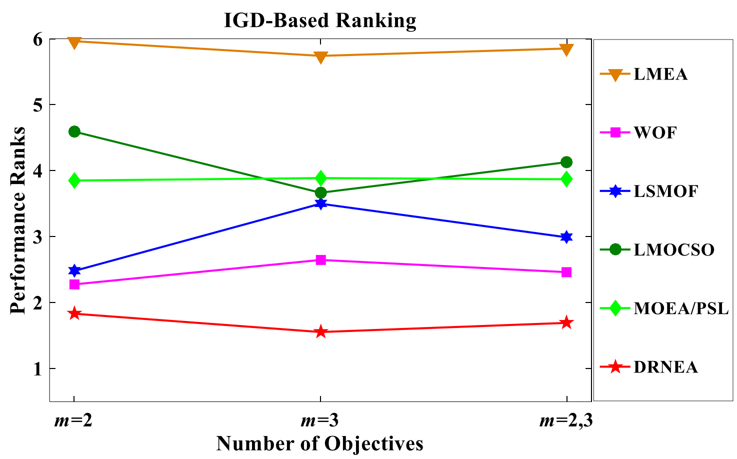

Fig. S16 Illustration of the performance ranks of DRNEA and its five LMOEA competitors over all LSMOP test problems for different cases of *m*.

Here, DRNEA and its five LMOEA competitors are ranked by Friedman's test based on the IGD results with different settings of *m*, and DRNEA obtains the lowest ranks in all cases, as shown in Fig. S16. Thus, the superiority of DRNEA over its five LMOEA competitors is demonstrated on these LSMOP benchmarks. Furthermore, these LMOEAs are specially designed for solving LMOPs, but each of them has its own pitfall in handling the large-scale search space, which are discussed in Section II.B. Therefore, this experimental study shows that with knowledge learned and transferred across related LMOPs, the search and optimization capability of LMOEAs can be improved, i.e., LMOEA with ETO manner is a desirable trick to handle the large-scale search space.