Data Mining
Association Analysis: Basic Concepts and Algorithms

Lecture Notes for Chapter 3

Data Mining
by
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Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

{Diaper} → {Beer},
{Milk, Bread} → {Eggs,Coke},
{Beer, Bread} → {Milk},

Implication means co-occurrence, not causality!

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Definition: Association Rule

- **Association Rule**
  - An implication expression of the form X → Y, where X and Y are itemsets
  - Example:
    {Milk, Diaper} → {Beer}
- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold
- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

⇒ Computationally prohibitive!

Mining Association Rules

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

{Milk,Diaper} → {Beer} (s=0.4, c=0.67)
{Milk,Beer} → {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} → {Milk} (s=0.4, c=0.67)
{Beer} → {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} → {Milk,Beer} (s=0.4, c=0.5)
{Milk} → {Diaper,Beer} (s=0.4, c=0.5)

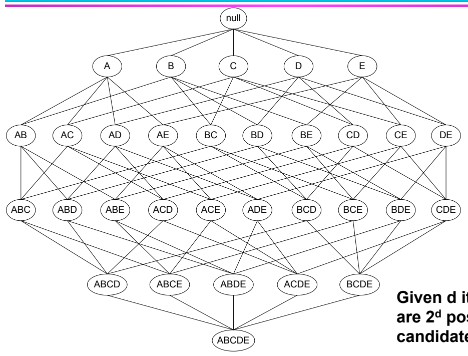
Observations:

- All the above rules are binary partitions of the same itemset: {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Mining Association Rules

- Two-step approach:
  1. Frequent Itemset Generation
     – Generate all itemsets whose support ≥ minsup
  2. Rule Generation
     – Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset
- Frequent itemset generation is still computationally expensive

Frequent Itemset Generation



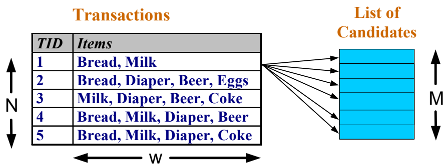
Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- Brute-force approach:
  – Each itemset in the lattice is a candidate frequent itemset
  – Count the support of each candidate by scanning the database

Transactions

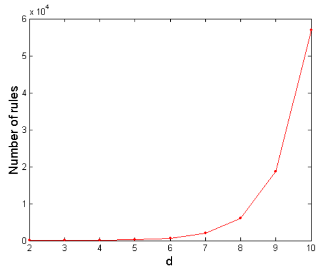
TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke



  – Match each transaction against every candidate
  – Complexity ~ O(NMw) => Expensive since $M = 2^d$!!!

Computational Complexity

- Given d unique items:
  – Total number of itemsets = 2^d
  – Total number of possible association rules:



$$R = \sum_{k=1}^{d-1} \left[\binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j} \right] = 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  – Complete search: $M=2^d$
  – Use pruning techniques to reduce M
- Reduce the number of transactions (N)
  – Reduce size of N as the size of itemset increases
  – Used by DHP and vertical-based mining algorithms
- Reduce the number of comparisons (NM)
  – Use efficient data structures to store the candidates or transactions
  – No need to match every candidate against every transaction

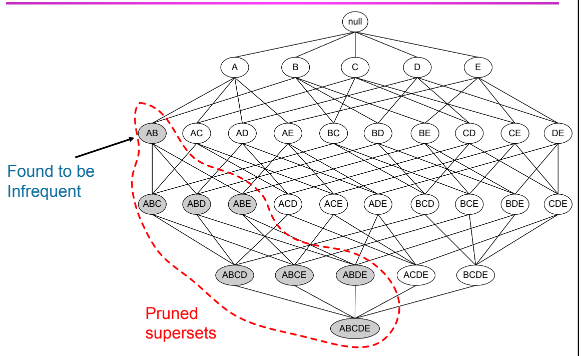
Reducing Number of Candidates

- Apriori principle:
  – If an itemset is frequent, then all of its subsets must also be frequent
- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  – Support of an itemset never exceeds the support of its subsets
  – This is known as the anti-monotone property of support

Illustrating Apriori Principle



Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)

Minimum Support = 3

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	3

If every subset is considered,
${}^6C_1 + {}^6C_2 + {}^6C_3 = 41$
With support-based pruning,
6 + 6 + 1 = 13

Apriori Algorithm

- Method:
  - Let k=1
  - Generate frequent itemsets of length 1
  - Repeat until no new frequent itemsets are identified
    - Generate length (k+1) candidate itemsets from length k frequent itemsets
    - Prune candidate itemsets containing subsets of length k that are infrequent
    - Count the support of each candidate by scanning the DB
    - Eliminate candidates that are infrequent, leaving only those that are frequent

Apriori Algorithm: An Example

minsup = 2

TID	Items
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3

Scan DB

C_1

{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

L_1

{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

C_2

{I1,I2}
{I1,I3}
{I1,I4}
{I1,I5}
{I2,I3}
{I2,I4}
{I2,I5}
{I3,I4}
{I3,I5}
{I4,I5}

C_2

{I1,I2}	4
{I1,I3}	4
{I1,I4}	1
{I1,I5}	2
{I2,I3}	4
{I2,I4}	2
{I2,I5}	2
{I3,I4}	0
{I3,I5}	1
{I4,I5}	0

L_2

{I1,I2}	4
{I1,I3}	4
{I1,I5}	2
{I2,I3}	4
{I2,I4}	2
{I2,I5}	2

Apriori Algorithm: An Example

L_2

{I1,I2}	4
{I1,I3}	4
{I1,I5}	2
{I2,I3}	4
{I2,I4}	2
{I2,I5}	2

$L_2 \bowtie L_2$

{I1,I2,I3}
{I1,I2,I5}
{I1,I3,I5}
{I2,I3,I4}
{I2,I3,I5}
{I2,I4,I5}

Prune

C_3

{I1,I2,I3}
{I1,I2,I5}

C_3

{I1,I2,I3}	2
{I1,I2,I5}	2

L_3

{I1,I2,I3}	2
{I1,I2,I5}	2

$L_3 \bowtie L_3$

{I1,I2,I3,I5}

Reducing Number of Comparisons

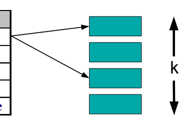
- Candidate counting:
  - Scan the database of transactions to determine the support of each candidate itemset
  - To reduce the number of comparisons, store the candidates in a hash structure
    - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets

Transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

N

Hash Structure



Buckets

Generate Hash Tree

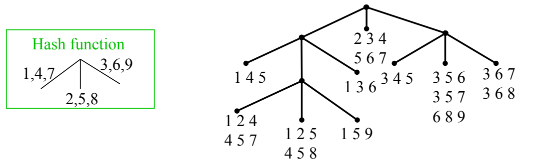
Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

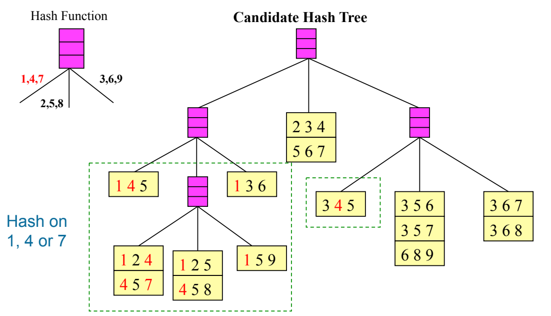
You need:

• Hash function

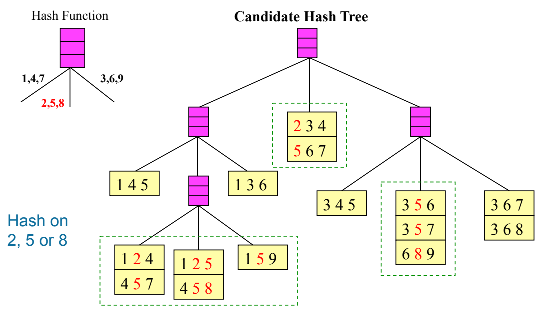
• Max leaf size: max number of itemsets stored in a leaf node (if number of candidate itemsets exceeds max leaf size, split the node)



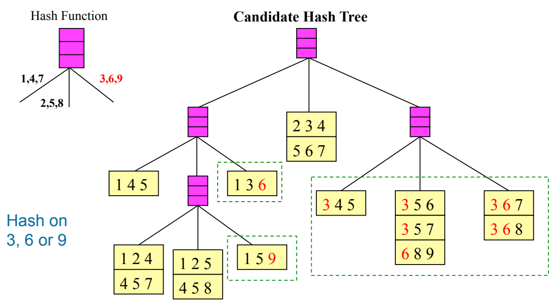
Association Rule Discovery: Hash tree



Association Rule Discovery: Hash tree

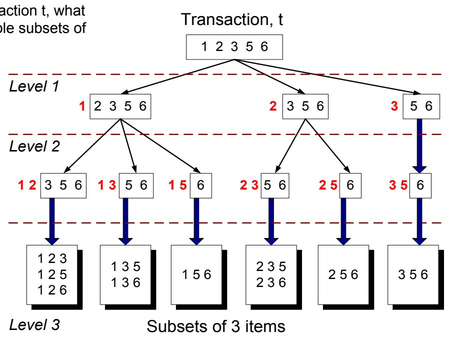


Association Rule Discovery: Hash tree

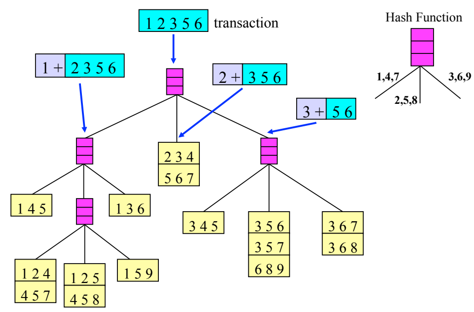


Subset Operation

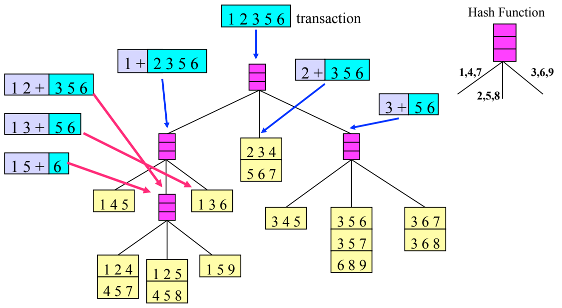
Given a transaction t, what are the possible subsets of size 3?



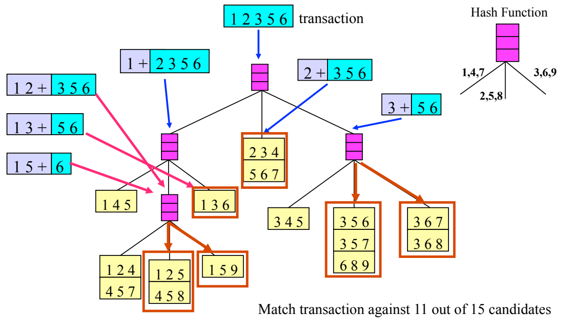
Subset Operation Using Hash Tree



Subset Operation Using Hash Tree



Subset Operation Using Hash Tree



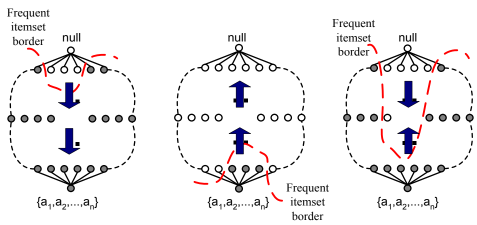
Match transaction against 11 out of 15 candidates

Factors Affecting Complexity

- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - more space is needed to store support count of each item
  - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
  - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
  - transaction width increases with denser data sets
  - This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

Alternative Methods for Frequent Itemset Generation

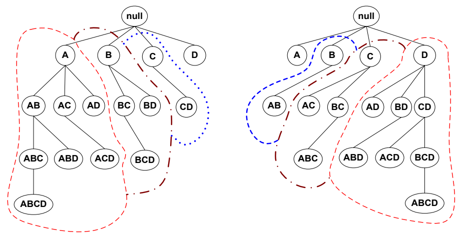
- Traversal of Itemset Lattice
  - General-to-specific vs Specific-to-general



(a) General-to-specific (b) Specific-to-general (c) Bidirectional

Alternative Methods for Frequent Itemset Generation

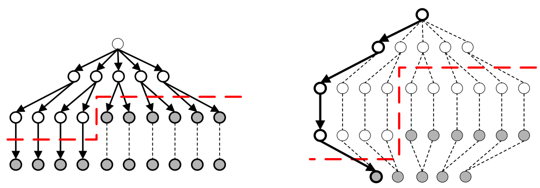
- Traversal of Itemset Lattice
  - Equivalent Classes



(a) Prefix tree (b) Suffix tree

Alternative Methods for Frequent Itemset Generation

- Traversal of Itemset Lattice
  - Breadth-first vs Depth-first



(a) Breadth first (b) Depth first

Alternative Methods for Frequent Itemset Generation

- Representation of Database
  - horizontal vs vertical data layout

Horizontal Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

FP-growth Algorithm

- Use a compressed representation of the database using an FP-tree
- Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets

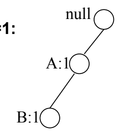
FP-tree Construction

- Scan DB once, find frequent 1-itemset
- Sort frequent items in frequency descending order, F-list
- Scan DB again, construct FP-tree

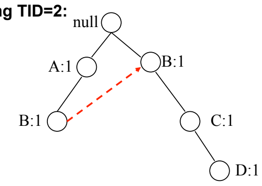
FP-tree construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{A}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

After reading TID=1:



After reading TID=2:



F-list=<A:8, B:8, C:6, D:5, E:3>

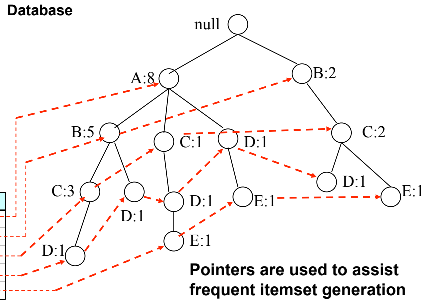
FP-Tree Construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{A}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

Transaction Database

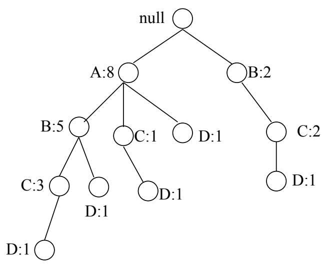
Header table

Item	Pointer
A	
B	
C	
D	
E	



Pointers are used to assist frequent itemset generation

FP-growth



Conditional Pattern base for D:
P = {(A:1,B:1,C:1), (A:1,B:1), (A:1,C:1), (A:1), (B:1,C:1)}

Recursively apply FP-growth on P

Frequent Itemsets found (with sup > 1):
AD, BD, CD, ACD, BCD

FP-growth

Database *DB*

TID	Items
T100	*f, a, c, d, g, i, m, p*
T200	*a, b, c, f, l, m, o*
T300	*b, f, h, j, o, w*
T400	*b, c, k, s, p*
T500	*a, f, c, e, l, p, m, n*

Scan *DB*

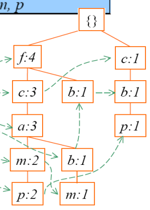
F-list <*(f:4), (c:4), (a:3), (b:3), (m:3), (p:3)*>

Scan *DB*

TID	(Ordered) frequent items
T100	*f, c, a, m, p*
T200	*f, c, a, b, m*
T300	*f, b*
T400	*c, b, p*
T500	*f, c, a, m, p*

Header Table

item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



FP-growth

TID	*Items bought*	*(ordered) frequent items*
100	*{f, a, c, d, g, i, m, p}*	*{f, c, a, m, p}*
200	*{a, b, c, f, l, m, o}*	*{f, c, a, b, m}*
300	*{b, f, h, j, o, w}*	*{f, b}*
400	*{b, c, k, s, p}*	*{c, b, p}*
500	*{a, f, c, e, l, p, m, n}*	*{f, c, a, m, p}*

F-list=f-c-a-b-m-p

Frequent Patterns containing *p*: *p*:3, *cp*:3

Frequent Patterns containing *m* but no *p*: *m*:3, *am*:3, *cm*:3, *fm*:3, *cam*:3, *fam*:3, *fcm*:3, *fcam*:3

Frequent Patterns containing *b* but no *p nor m*: *b*:3

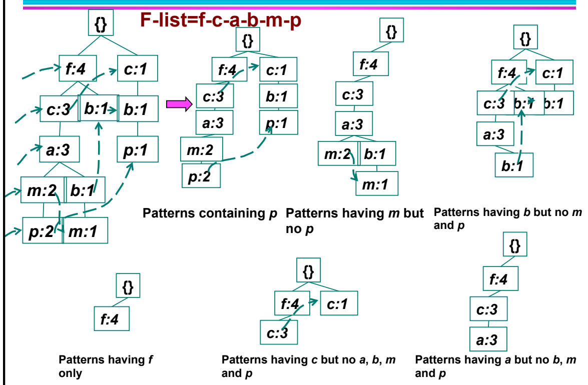
Frequent Patterns containing *a* but no *p nor m,b*: *a*:3, *fa*:3, *ca*:3, *fca*:3

Frequent Patterns containing *c* but no *p nor m,b,a*: *c*:3, *fc*:3

Frequent Patterns containing *f* but no *p nor m,b,a,c*: *f*:4

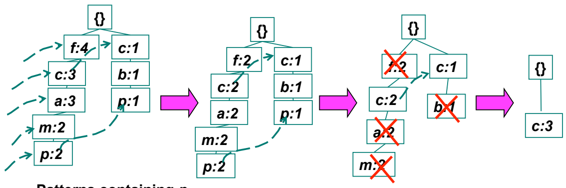
FP-growth

F-list=f-c-a-b-m-p



FP-growth

- Frequent patterns containing p are {p:3, cp:3}



Patterns containing *p*

minsup=3

ECLAT

- For each item, store a list of transaction ids (tids)

Horizontal Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

TID-list

ECLAT

- Determine support of any k-itemset by intersecting tid-lists of two of its (k-1) subsets.

A
1
4
5
6
7
8
9

$\wedge$

B
1
2
5
7
8
10

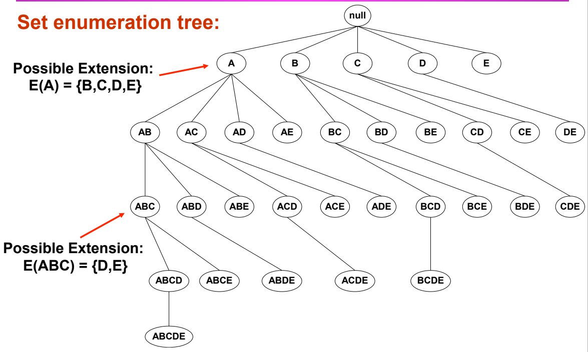
$\rightarrow$

AB
1
5
7
8

- 3 traversal approaches:
  - top-down, bottom-up and hybrid
- Advantage: very fast support counting
- Disadvantage: intermediate tid-lists may become too large for memory

Tree Projection

Set enumeration tree:



Tree Projection

- Items are listed in lexicographic order
- Each node P stores the following information:
  - Itemset for node P
  - List of possible lexicographic extensions of P: E(P)
  - Pointer to projected database of its ancestor node
  - Bitvector containing information about which transactions in the projected database contain the itemset

Projected Database

Original Database:

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

Projected Database for node A:

TID	Items
1	{B}
2	{}
3	{C,D,E}
4	{D,E}
5	{B,C}
6	{B,C,D}
7	{}
8	{B,C}
9	{B,D}
10	{}

For each transaction T, projected transaction at node A is T ∩ E(A)

Compact Representation of Frequent Itemsets

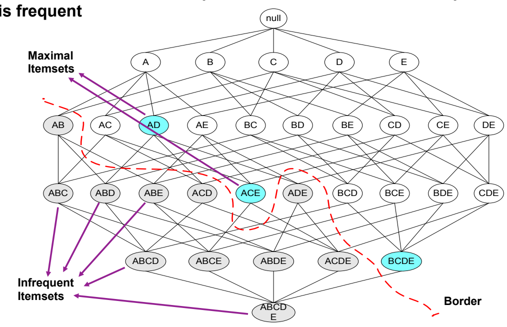
- Some itemsets are redundant because they have identical support as their supersets

TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1

- Number of frequent itemsets $= 3 \times \sum_{k=1}^{10} \binom{10}{k}$
- Need a compact representation

Maximal Frequent Itemsets

An itemset is maximal frequent if none of its immediate supersets is frequent



Closed Itemsets

- An itemset is closed if none of its immediate supersets has the same support as the itemset

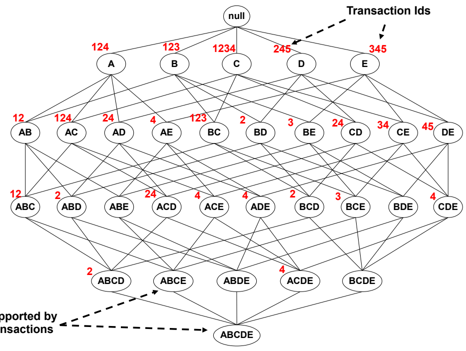
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

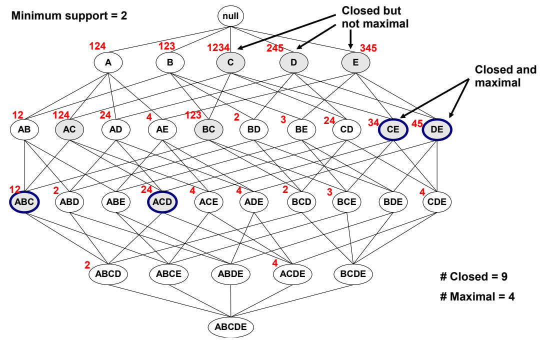
Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

Maximal vs Closed Itemsets

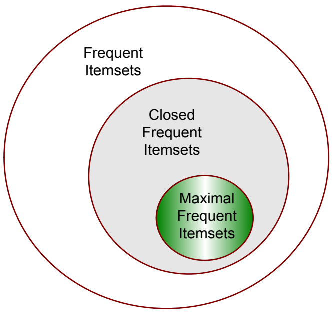
TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Maximal vs Closed Frequent Itemsets

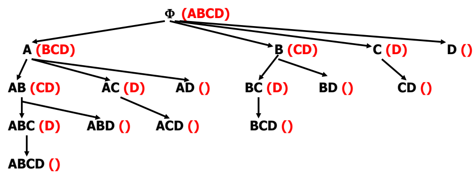


Maximal vs Closed Itemsets



MaxMiner Algorithm

- Each node n in the set enumeration tree consists of two itemsets:
  - the head, h(n)
  - the tail, t(n)
  - E.g. h(n)={A}, t(n)={B,C,D}

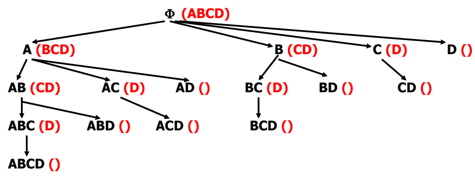


MaxMiner Algorithm

- Algorithm
  - Initially, generate one node N with h(N) = Φ and t(N) = {A,B,C,D}.
  - Consider expanding N, (local pruning)
    - If for some i ∈ t(N), h(N) ∪ {i} is NOT frequent, remove i from t(N) before expanding N.
    - If h(N) ∪ t(N) is frequent, do not expand N.
  - Apply global pruning techniques…

MaxMiner Algorithm

- Global Pruning Technique
  - When a max pattern is identified (e.g. ABCD), prune all nodes (B, C and D) where h(N)∪t(N) is a sub-set of ABCD.



MaxMiner Algorithm: Example

Φ (ABCDE~~F~~)

A (BCDE) B (CDE) C (DE) D (E) E ()

Items	Frequency
A	2
B	2
C	3
D	3
E	2
~~F~~	~~1~~
~~ABCDE~~	~~1~~

Tid	Items
10	A,B,C,D,E
20	B,C,D,E,
30	A,C,D,F

minsup=2

Max patterns:

MaxMiner Algorithm: Example

Φ (ABCDE~~F~~)

A (~~B~~CDE~~F~~) B (CDE) C (DE) D (E) E ()

Node A

Items	Frequency
~~AB~~	~~1~~
AC	2
AD	2
~~AE~~	~~1~~
ACD	2

Tid	Items
10	A,B,C,D,E
20	B,C,D,E,
30	A,C,D,F

minsup=2

Max patterns:

ACD

MaxMiner Algorithm: Example

Φ (ABCDE~~F~~)

A (~~B~~CDE~~F~~) B (CDE) ~~C (DE) D (E) E ()~~

Node B

Items	Frequency
BC	2
BD	2
BE	2
BCDE	2

Tid	Items
10	A,B,C,D,E
20	B,C,D,E,
30	A,C,D,F

minsup=2

Max patterns:

ACD
BCDE

CLOSET Algorithm

- Creating F_list: <c:4,e:4,f:4,a:3,d:2>
- Divide search space
  - Patterns containing d
  - Patterns containing a but not d
  - Patterns containing f but not a, d
  - …
- Find frequent closed pattern recursively
- Among the transactions having d, cfa is frequent closed → cfad is a frequent closed pattern

Min_sup=2

TID	Items
10	a, c, d, e, f
20	a, b, e
30	c, e, f
40	a, c, d, f
50	c, e, f

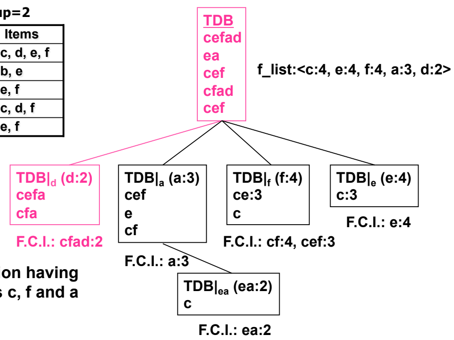
DB|*d*

TID	Items
10	c, e, f, a
40	c, f, a

CLOSET Algorithm

Min_sup=2

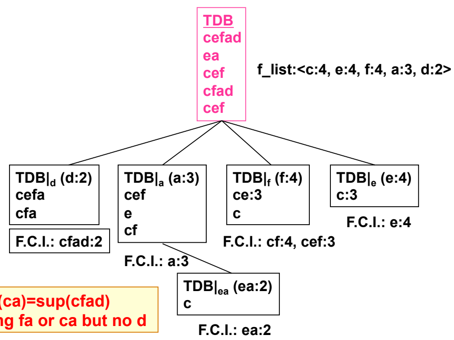
TID	Items
10	a, c, d, e, f
20	a, b, e
30	c, e, f
40	a, c, d, f
50	c, e, f



Local frequent items: c, f, a

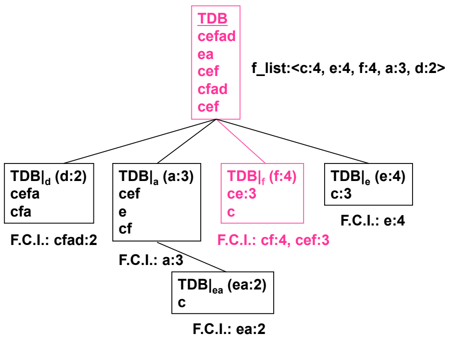
Every transaction having d also contains c, f and a

CLOSET Algorithm

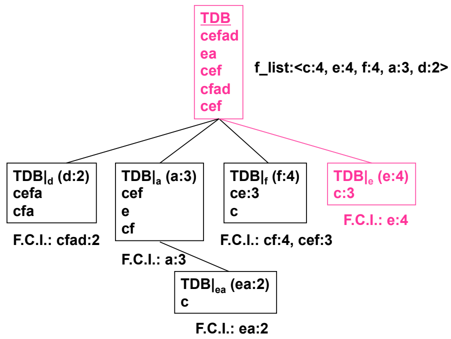


sup(fa)=sup(ca)=sup(cfad)
No FCI having fa or ca but no d

CLOSET Algorithm



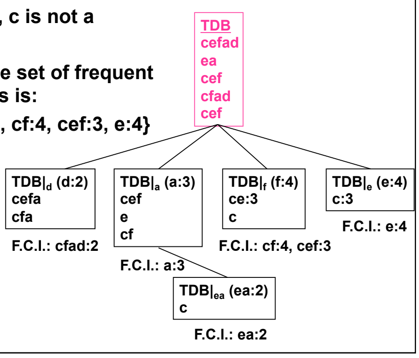
CLOSET Algorithm



CLOSET Algorithm

- **sup(c)=sup(cf), c is not a closed itemset**
- **In summary, the set of frequent closed itemsets is:**

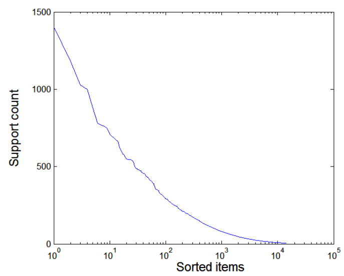
{acdf:2, a:3, ae:2, cf:4, cef:3, e:4}



Effect of Support Distribution

- Many real data sets have skewed support distribution

Support distribution of a retail data set



Effect of Support Distribution

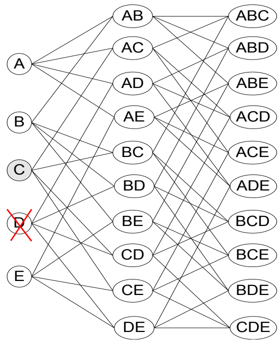
- How to set the appropriate *minsup* threshold?
  - If *minsup* is set too high, we could miss itemsets involving interesting rare items (e.g., expensive products)
  - If *minsup* is set too low, it is computationally expensive and the number of itemsets is very large
- Using a single minimum support threshold may not be effective

Multiple Minimum Support

- How to apply multiple minimum supports?
  - MS(i): minimum support for item i
  - e.g.: MS(Milk)=5%, MS(Coke) = 3%, MS(Broccoli)=0.1%, MS(Salmon)=0.5%
  - MS({Milk, Broccoli}) = min (MS(Milk), MS(Broccoli)) = 0.1%
  - Challenge: Support is no longer anti-monotone
    - Suppose: Support(Milk, Coke) = 1.5% and Support(Milk, Coke, Broccoli) = 0.5%
    - {Milk,Coke} is infrequent but {Milk,Coke,Broccoli} is frequent

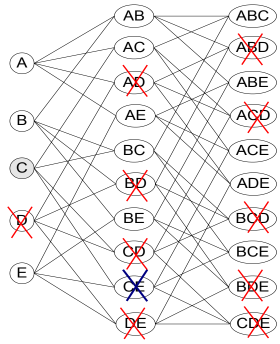
Multiple Minimum Support

Item	MS(I)	Sup(I)
A	0.10%	0.25%
B	0.20%	0.26%
C	0.30%	0.29%
D	0.50%	0.05%
E	3%	4.20%



Multiple Minimum Support

Item	MS(I)	Sup(I)
A	0.10%	0.25%
B	0.20%	0.26%
C	0.30%	0.29%
D	0.50%	0.05%
E	3%	4.20%



Multiple Minimum Support (Liu 1999)

- Order the items according to their minimum support (in ascending order)
  - e.g.: MS(Milk)=5%, MS(Coke) = 3%, MS(Broccoli)=0.1%, MS(Salmon)=0.5%
  - Ordering: Broccoli, Salmon, Coke, Milk
- Need to modify Apriori such that:
  - L_1 : set of frequent items
  - F_1 : set of items whose support is ≥ MS(1) where MS(1) is $\min_i$(MS(i))
  - C_2 : candidate itemsets of size 2 is generated from F_1 instead of L_1

Multiple Minimum Support (Liu 1999)

- Modifications to Apriori:
  - In traditional Apriori,
    - A candidate (k+1)-itemset is generated by merging two frequent itemsets of size k
    - The candidate is pruned if it contains any infrequent subsets of size k
  - Pruning step has to be modified:
    - Prune only if subset contains the first item
    - e.g.: Candidate={Broccoli, Coke, Milk} (ordered according to minimum support)
    - {Broccoli, Coke} and {Broccoli, Milk} are frequent but {Coke, Milk} is infrequent
      - Candidate is not pruned because {Coke,Milk} does not contain the first item, i.e., Broccoli.

Rule Generation

- Given a frequent itemset L, find all non-empty subsets f ⊂ L such that f → L – f satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

ABC →D,	ABD →C,	ACD →B,	BCD →A,
A →BCD,	B →ACD,	C →ABD,	D →ABC
AB →CD,	AC → BD,	AD → BC,	BC →AD,
BD →AC,	CD →AB,		

- If |L| = k, then there are $2^k – 2$ candidate association rules (ignoring L → ∅ and ∅ → L)

Rule Generation

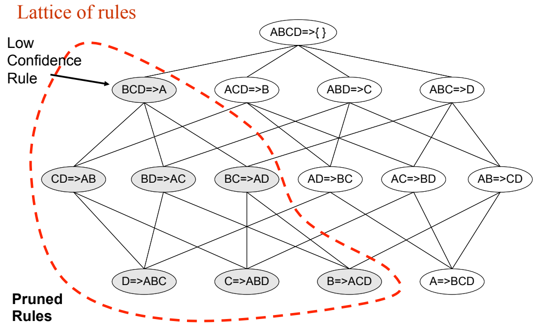
- How to efficiently generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property

    c(ABC →D) can be larger or smaller than c(AB →D)
  - But confidence of rules generated from the same itemset has an anti-monotone property
  - e.g., L = {A,B,C,D}:

    $$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

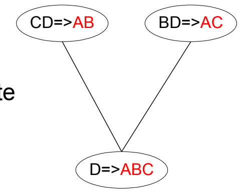
    - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation for Apriori Algorithm



Rule Generation for Apriori Algorithm

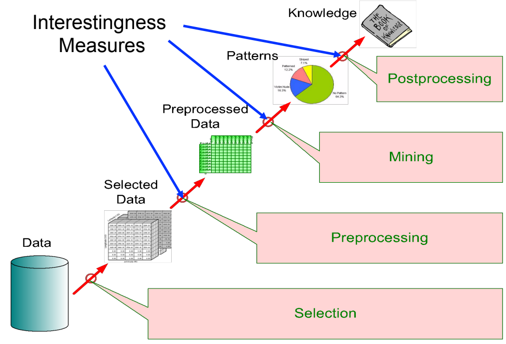
- Candidate rule is generated by merging two rules that share the same prefix in the rule consequent
- join(CD=>AB,BD=>AC) would produce the candidate rule D => ABC
- Prune rule D=>ABC if its subset AD=>BC does not have high confidence



Pattern Evaluation

- Association rule algorithms tend to produce too many rules
  - many of them are uninteresting or redundant
  - Redundant if {A,B,C} → {D} and {A,B} → {D} have same support & confidence
- Interestingness measures can be used to prune/rank the derived patterns
- In the original formulation of association rules, support & confidence are the only measures used

Application of Interestingness Measure



Computing Interestingness Measure

- Given a rule X → Y, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for X → Y

	Y	$\overline{Y}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{X}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	$\lvert T \rvert$

f_{11}: support of X and Y
f_{10}: support of X and $\overline{Y}$
f_{01}: support of $\overline{X}$ and Y
f_{00}: support of $\overline{X}$ and $\overline{Y}$

Used to define various measures

- support, confidence, lift, Gini, J-measure, etc.

Drawback of Confidence

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea → Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

⇒ Although confidence is high, rule is misleading

⇒ P(Coffee|$\overline{\text{Tea}}$) = 0.9375

Statistical Independence

- Population of 1000 students
  - 600 students know how to swim (S)
  - 700 students know how to bike (B)
  - 420 students know how to swim and bike (S,B)

  - P(S∧B) = 420/1000 = 0.42
  - P(S) × P(B) = 0.6 × 0.7 = 0.42

  - P(S∧B) = P(S) × P(B) => Statistical independence
  - P(S∧B) > P(S) × P(B) => Positively correlated
  - P(S∧B) < P(S) × P(B) => Negatively correlated

Statistical-based Measures

- Measures that take into account statistical dependence

$$Lift = \frac{P(Y|X)}{P(Y)}$$

$$Interest = \frac{P(X,Y)}{P(X)P(Y)}$$

$$PS = P(X,Y) - P(X)P(Y)$$

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1-P(X)]P(Y)[1-P(Y)]}}$$

Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea → Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

⇒ Lift = 0.75/0.9= 0.8333 (< 1, therefore is negatively associated)

Drawback of Lift & Interest

	Y	$\overline{Y}$	
X	10	0	10
$\overline{X}$	0	90	90
	10	90	100

$$Lift = \frac{0.1}{(0.1)(0.1)} = 10$$

	Y	$\overline{Y}$	
X	90	0	90
$\overline{X}$	0	10	10
	90	10	100

$$Lift = \frac{0.9}{(0.9)(0.9)} = 1.11$$

Statistical independence:

If P(X,Y)=P(X)P(Y) => Lift = 1

There are lots of measures proposed in the literature

Some measures are good for certain applications, but not for others

What criteria should we use to determine whether a measure is good or bad?

What about Apriori-style support based pruning? How does it affect these measures?

#	Measure	Formula
1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i),-\sum_j P(B_j)\log P(B_j))}$
8	J-Measure (J)	$\max\Big(P(A,B)\log(\frac{P(B\|A)}{P(B)})+P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}),$ $P(A,B)\log(\frac{P(A\|B)}{P(A)})+P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})\Big)$
9	Gini index (G)	$\max\Big(P(A)[P(B\|A)^2+P(\overline{B}\|A)^2]+P(\overline{A})[P(B\|\overline{A})^2+P(\overline{B}\|\overline{A})^2]$ $-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A\|B)^2+P(\overline{A}\|B)^2]+P(\overline{B})[P(A\|\overline{B})^2+P(\overline{A}\|\overline{B})^2]$ $-P(A)^2-P(\overline{A})^2\Big)$
10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B\|A),P(A\|B))$
12	Laplace (L)	$\max\Big(\frac{NP(A,B)+1}{NP(A)+2},\frac{NP(A,B)+1}{NP(B)+2}\Big)$
13	Conviction (V)	$\max\Big(\frac{P(A)P(\overline{B})}{P(A\overline{B})},\frac{P(B)P(\overline{A})}{P(B\overline{A})}\Big)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B)-P(A)P(B)$
17	Certainty factor (F)	$\max\Big(\frac{P(B\|A)-P(B)}{1-P(B)},\frac{P(A\|B)-P(A)}{1-P(A)}\Big)$
18	Added Value (AV)	$\max(P(B\|A)-P(B),P(A\|B)-P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})}\times\frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\|A)-P(B),P(A\|B)-P(A))$

Properties of A Good Measure

- Piatetsky-Shapiro:
  3 properties a good measure M must satisfy:
  - M(A,B) = 0 if A and B are statistically independent
  - M(A,B) increase monotonically with P(A,B) when P(A) and P(B) remain unchanged
  - M(A,B) decreases monotonically with P(A) [or P(B)] when P(A,B) and P(B) [or P(A)] remain unchanged

Comparing Different Measures

10 examples of contingency tables:

Example	f_{11}	f_{10}	f_{01}	f_{00}
E1	8123	83	424	1370
E2	8330	2	622	1046
E3	9481	94	127	298
E4	3954	3080	5	2961
E5	2886	1363	1320	4431
E6	1500	2000	500	6000
E7	4000	2000	1000	3000
E8	4000	2000	2000	2000
E9	1720	7121	5	1154
E10	61	2483	4	7452

Rankings of contingency tables using various measures:

#	ϕ	λ	α	Q	Y	κ	M	J	G	s	c	L	V	I	IS	PS	F	AV	S	ζ	K
E1	1	1	3	3	3	1	2	2	1	3	5	5	4	6	2	2	4	6	1	2	5
E2	2	2	1	1	1	2	1	3	2	2	1	1	1	8	3	5	1	8	2	3	6
E3	3	3	4	4	4	3	3	8	7	1	4	4	6	10	1	8	6	10	3	1	10
E4	4	7	2	2	2	5	4	1	3	6	2	2	2	4	4	1	2	3	4	5	1
E5	5	4	8	8	8	4	7	5	4	7	9	9	9	3	6	3	9	4	5	6	3
E6	6	6	7	7	7	7	6	4	6	9	8	8	7	2	8	6	7	2	7	8	2
E7	7	5	9	9	9	6	8	6	5	4	7	7	8	5	5	4	8	5	6	4	4
E8	8	9	10	10	10	8	10	10	8	4	10	10	10	9	7	7	10	9	8	7	9
E9	9	9	5	5	5	9	9	7	9	8	3	3	3	7	9	9	3	7	9	9	8
E10	10	8	6	6	6	10	5	9	10	10	6	6	5	1	10	10	5	1	10	10	7

Property under Variable Permutation

	B	$\overline{B}$
A	p	q
$\overline{A}$	r	s

$\Longrightarrow$

	A	$\overline{A}$
B	p	r
$\overline{B}$	q	s

Does M(A,B) = M(B,A)?

Symmetric measures:

- support, lift, collective strength, cosine, Jaccard, etc

Asymmetric measures:

- confidence, conviction, Laplace, J-measure, etc

Property under Row/Column Scaling

Grade-Gender Example (Mosteller, 1968):

	Male	Female	
High	2	3	5
Low	1	4	5
	3	7	10

	Male	Female	
High	4	30	34
Low	2	40	42
	6	70	76
	2x	10x	

Mosteller:

Underlying association should be independent of the relative number of male and female students in the samples

Property under Inversion Operation

	A	B	C	D	E	F
Transaction 1	1	0	0	1	0	0
▪	0	0	1	1	1	0
▪	0	0	1	1	1	0
▪	0	0	1	1	1	0
▪	0	1	1	0	1	1
▪	0	0	1	1	1	0
	0	0	1	1	1	0
	0	0	1	1	1	0
	0	0	1	1	1	0
Transaction N	1	0	0	1	0	0

(a) (b) (c)

Example: ϕ-Coefficient

- ϕ-coefficient is analogous to correlation coefficient for continuous variables

	Y	$\overline{Y}$	
X	60	10	70
$\overline{X}$	10	20	30
	70	30	100

	Y	$\overline{Y}$	
X	20	10	30
$\overline{X}$	10	60	70
	30	70	100

$$\phi = \frac{0.6 - 0.7 \times 0.7}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}} = 0.5238$$

$$\phi = \frac{0.2 - 0.3 \times 0.3}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}} = 0.5238$$

ϕ Coefficient is the same for both tables

Property under Null Addition

	B	$\overline{B}$
A	p	q
$\overline{A}$	r	s

$\Longrightarrow$

	B	$\overline{B}$
A	p	q
$\overline{A}$	r	s + k

Invariant measures:

- support, cosine, Jaccard, etc

Non-invariant measures:

- correlation, Gini, mutual information, odds ratio, etc

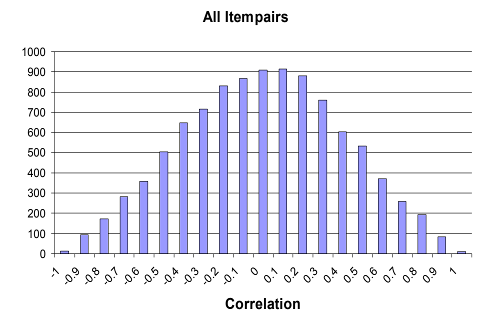
Different Measures have Different Properties

Symbol	Measure	Range	P1	P2	P3	O1	O2	O3	O3'	O4
Φ	Correlation	-1 … 0 … 1	Yes	Yes	Yes	Yes	No	Yes	Yes	No
λ	Lambda	0 … 1	Yes	No	No	Yes	No	No*	Yes	No
α	Odds ratio	0 … 1 … ∞	Yes*	Yes	Yes	Yes	Yes	Yes*	Yes	No
Q	Yule's Q	-1 … 0 … 1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Y	Yule's Y	-1 … 0 … 1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
κ	Cohen's	-1 … 0 … 1	Yes	Yes	Yes	Yes	No	No	Yes	No
M	Mutual Information	0 … 1	Yes	Yes	Yes	Yes	No	No*	Yes	No
J	J-Measure	0 … 1	Yes	No	No	No	No	No	No	No
G	Gini Index	0 … 1	Yes	No	No	No	No	No*	Yes	No
s	Support	0 … 1	No	Yes	No	Yes	No	No	No	No
c	Confidence	0 … 1	No	Yes	No	Yes	No	No	No	Yes
L	Laplace	0 … 1	No	Yes	No	Yes	No	No	No	No
V	Conviction	0.5 … 1 … ∞	No	Yes	No	Yes**	No	No	Yes	No
I	Interest	0 … 1 … ∞	Yes*	Yes	Yes	Yes	No	No	No	No
IS	IS (cosine)	0 .. 1	No	Yes	Yes	Yes	No	No	No	Yes
PS	Piatetsky-Shapiro's	-0.25 … 0 … 0.25	Yes	Yes	Yes	Yes	No	Yes	Yes	No
F	Certainty factor	-1 … 0 … 1	Yes	Yes	Yes	No	No	No	Yes	No
AV	Added value	0.5 … 1 … 1	Yes	Yes	Yes	No	No	No	No	No
S	Collective strength	0 … 1 … ∞	No	Yes	Yes	Yes	No	Yes*	Yes	No
ζ	Jaccard	0 .. 1	No	Yes	Yes	Yes	No	No	No	Yes
K	Klosgen's	$\left(\sqrt{\frac{2}{\sqrt{3}}-1}\right)\left(2-\sqrt{3}-\frac{1}{\sqrt{3}}\right)\ldots 0 \ldots \frac{2}{3\sqrt{3}}$	Yes	Yes	Yes	No	No	No	No	No

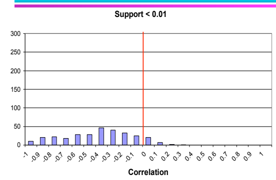
Support-based Pruning

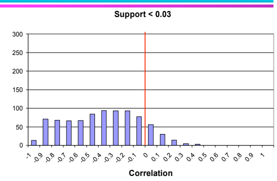
- Most of the association rule mining algorithms use support measure to prune rules and itemsets
- Study effect of support pruning on correlation of itemsets
  - Generate 10000 random contingency tables
  - Compute support and pairwise correlation for each table
  - Apply support-based pruning and examine the tables that are removed

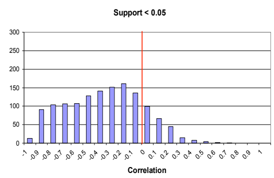
Effect of Support-based Pruning



Effect of Support-based Pruning







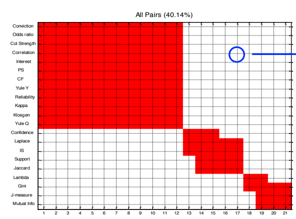
Support-based pruning eliminates mostly negatively correlated itemsets

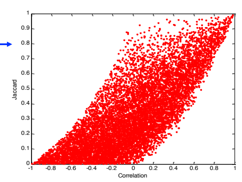
Effect of Support-based Pruning

- Investigate how support-based pruning affects other measures
- Steps:
  - Generate 10000 contingency tables
  - Rank each table according to the different measures
  - Compute the pair-wise correlation between the measures

Effect of Support-based Pruning

- Without Support Pruning (All Pairs)



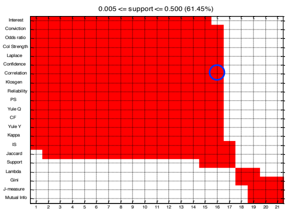


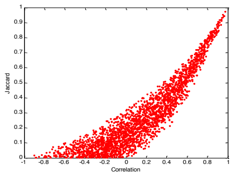
Scatter Plot between Correlation & Jaccard Measure

- Red cells indicate correlation between the pair of measures > 0.85
- 40.14% pairs have correlation > 0.85

Effect of Support-based Pruning

- 0.5% ≤ support ≤ 50%



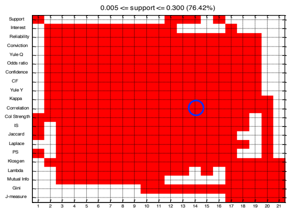


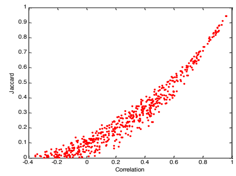
Scatter Plot between Correlation & Jaccard Measure:

- 61.45% pairs have correlation > 0.85

Effect of Support-based Pruning

- 0.5% ≤ support ≤ 30%





Scatter Plot between Correlation & Jaccard Measure

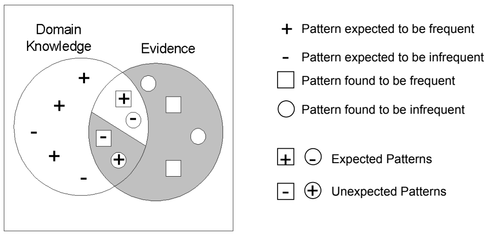
- 76.42% pairs have correlation > 0.85

Subjective Interestingness Measure

- Objective measure:
  - Rank patterns based on statistics computed from data
  - e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).
- Subjective measure:
  - Rank patterns according to user's interpretation
    - A pattern is subjectively interesting if it contradicts the expectation of a user (Silberschatz & Tuzhilin)
    - A pattern is subjectively interesting if it is actionable (Silberschatz & Tuzhilin)

Interestingness via Unexpectedness

- Need to model expectation of users (domain knowledge)



- Need to combine expectation of users with evidence from data (i.e., extracted patterns)

Interestingness via Unexpectedness

- Web Data (Cooley et al 2001)
  - Domain knowledge in the form of site structure
  - Given an itemset F = $\{X_1, X_2, \ldots, X_k\}$ (X_i : Web pages)
    - L: number of links connecting the pages
    - lfactor = L / (k × k-1)
    - cfactor = 1 (if graph is connected), 0 (disconnected graph)
  - Structure evidence = cfactor × lfactor
  - Usage evidence = $\dfrac{P(X_1 \cap X_2 \cap \ldots \cap X_k)}{P(X_1 \cup X_2 \cup \ldots \cup X_k)}$
  - Use Dempster-Shafer theory to combine domain knowledge and evidence from data

Continuous and Categorical Attributes

How to apply association analysis formulation to non-asymmetric binary variables?

Session Id	Country	Session Length (sec)	Number of Web Pages viewed	Gender	Browser Type	Buy
1	USA	982	8	Male	IE	No
2	China	811	10	Female	Netscape	No
3	USA	2125	45	Female	Mozilla	Yes
4	Germany	596	4	Male	IE	Yes
5	Australia	123	9	Male	Mozilla	No
…	…	…	…	…	…	…

Example of Association Rule:

{Number of Pages ∈[5,10) ∧ (Browser=Mozilla)} → {Buy = No}

Handling Categorical Attributes

- Transform categorical attribute into asymmetric binary variables

- Introduce a new “item” for each distinct attribute-value pair
  - Example: replace Browser Type attribute with
    - Browser Type = Internet Explorer
    - Browser Type = Mozilla
    - Browser Type = Mozilla

Handling Categorical Attributes

- Potential Issues
  - What if attribute has many possible values
    - Example: attribute country has more than 200 possible values
    - Many of the attribute values may have very low support
      - Potential solution: Aggregate the low-support attribute values
  - What if distribution of attribute values is highly skewed
    - Example: 95% of the visitors have Buy = No
    - Most of the items will be associated with (Buy=No) item
      - Potential solution: drop the highly frequent items

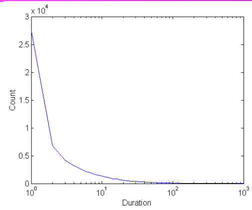
Handling Continuous Attributes

- Different kinds of rules:
  - Age∈[21,35) ∧ Salary∈[70k,120k) → Buy
  - Salary∈[70k,120k) ∧ Buy → Age: μ=28, σ=4

- Different methods:
  - Discretization-based
  - Statistics-based
  - Non-discretization based
    - minApriori

Handling Continuous Attributes

- Use discretization
- Unsupervised:
  - Equal-width binning
  - Equal-depth binning
  - Clustering



- Supervised:

Attribute values, v

Class	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
Anomalous	0	0	20	10	20	0	0	0	0
Normal	150	100	0	0	0	100	100	150	100

$\underbrace{v_1\ v_2}_{bin_1}$ $\underbrace{v_3\ v_4\ v_5}_{bin_2}$ $\underbrace{v_6\ v_7\ v_8\ v_9}_{bin_3}$

Discretization Issues

- Size of the discretized intervals affect support & confidence

  {Refund = No, (Income = $51,250)} → {Cheat = No}

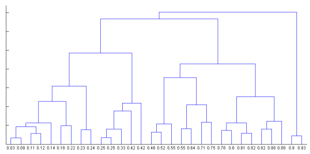
  {Refund = No, (60K ≤ Income ≤ 80K)} → {Cheat = No}

  {Refund = No, (0K ≤ Income ≤ 1B)} → {Cheat = No}

  - If intervals too small
    - may not have enough support
  - If intervals too large
    - may not have enough confidence
- Potential solution: use all possible intervals

Discretization Issues

- Execution time
  - If intervals contain n values, there are on average $O(n^2)$ possible ranges



- Too many rules

  {Refund = No, (Income = $51,250)} → {Cheat = No}

  {Refund = No, (51K ≤ Income ≤ 52K)} → {Cheat = No}

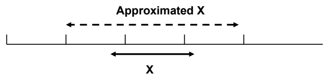
  {Refund = No, (50K ≤ Income ≤ 60K)} → {Cheat = No}

Approach by Srikant & Agrawal

- Preprocess the data
  - Discretize attribute using equi-depth partitioning
    - Use *partial completeness measure* to determine number of partitions
    - Merge adjacent intervals as long as support is less than max-support
- Apply existing association rule mining algorithms
- Determine interesting rules in the output

Approach by Srikant & Agrawal

- Discretization will lose information



  - Use *partial completeness measure* to determine how much information is lost

C: frequent itemsets obtained by considering all ranges of attribute values
P: frequent itemsets obtained by considering all ranges over the partitions

P is *K-complete* w.r.t C if $P \subseteq C$, and $\forall X \in C$, $\exists X' \in P$ such that:
1. X' is a generalization of X and support (X') $\le K \times$ support(X) $(K \ge 1)$
2. $\forall Y \subseteq X$, $\exists Y' \subseteq X'$ such that support (Y') $\le K \times$ support(Y)

Given *K (partial completeness level)*, can determine number of intervals (N)

Interestingness Measure

{Refund = No, (Income = \$51,250)} $\rightarrow$ {Cheat = No}

{Refund = No, (51K $\le$ Income $\le$ 52K)} $\rightarrow$ {Cheat = No}

{Refund = No, (50K $\le$ Income $\le$ 60K)} $\rightarrow$ {Cheat = No}

- Given an itemset: $Z = \{z_1, z_2, \ldots, z_k\}$ and its generalization $Z' = \{z_1', z_2', \ldots, z_k'\}$

  P(Z): support of Z
  $E_{Z'}(Z)$: expected support of Z based on Z'

$$E_{Z'}(Z) = \frac{P(z_1)}{P(z_1')} \times \frac{P(z_2)}{P(z_2')} \times \cdots \times \frac{P(z_k)}{P(z_k')} \times P(Z')$$

  - Z is R-interesting w.r.t. Z' if $P(Z) \ge R \times E_{Z'}(Z)$

Interestingness Measure

- For S: $X \rightarrow Y$, and its generalization S': $X' \rightarrow Y'$

  P(Y|X): confidence of $X \rightarrow Y$
  P(Y'|X'): confidence of $X' \rightarrow Y'$
  $E_{S'}(Y|X)$: expected support of Z based on Z'

$$E(Y \mid X) = \frac{P(y_1)}{P(y_1')} \times \frac{P(y_2)}{P(y_2')} \times \cdots \times \frac{P(y_k)}{P(y_k')} \times P(Y' \mid X')$$

- Rule S is R-interesting w.r.t its ancestor rule S' if
  - Support, $P(S) \ge R \times E_{S'}(S)$ or
  - Confidence, $P(Y|X) \ge R \times E_{S'}(Y|X)$

Statistics-based Methods

- Example:

  Browser=Mozilla $\wedge$ Buy=Yes $\rightarrow$ Age: μ=23
- Rule consequent consists of a continuous variable, characterized by their statistics
  - mean, median, standard deviation, etc.
- Approach:
  - Withhold the target variable from the rest of the data
  - Apply existing frequent itemset generation on the rest of the data
  - For each frequent itemset, compute the descriptive statistics for the corresponding target variable
    - Frequent itemset becomes a rule by introducing the target variable as rule consequent
  - Apply statistical test to determine interestingness of the rule

Statistics-based Methods

- How to determine whether an association rule interesting?
  - Compare the statistics for segment of population covered by the rule vs segment of population not covered by the rule:

    $A \Rightarrow B: \mu$ versus $\bar{A} \Rightarrow B: \mu'$
  - Statistical hypothesis testing:

$$Z = \frac{\mu' - \mu - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

    - Null hypothesis: H0: $\mu' = \mu + \Delta$
    - Alternative hypothesis: H1: $\mu' > \mu + \Delta$
    - Z has zero mean and variance 1 under null hypothesis

Statistics-based Methods

- Example:

  r: Browser=Mozilla $\wedge$ Buy=Yes $\rightarrow$ Age: μ=23

  – Rule is interesting if difference between μ and μ' is greater than 5 years (i.e., Δ = 5)
  – For r, suppose n1 = 50, s1 = 3.5
  – For r' (complement): n2 = 250, s2 = 6.5

$$Z = \frac{\mu' - \mu - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{30 - 23 - 5}{\sqrt{\frac{3.5^2}{50} + \frac{6.5^2}{250}}} = 3.11$$

  – For 1-sided test at 95% confidence level, critical Z-value for rejecting null hypothesis is 1.64.
  – Since Z is greater than 1.64, r is an interesting rule

Min-Apriori (Han et al)

Document-term matrix:

TID	W1	W2	W3	W4	W5
D1	2	2	0	0	1
D2	0	0	1	2	2
D3	2	3	0	0	0
D4	0	0	1	0	1
D5	1	1	1	0	2

Example:

W1 and W2 tends to appear together in the same document

Min-Apriori

- Data contains only continuous attributes of the same "type"
  – e.g., frequency of words in a document

TID	W1	W2	W3	W4	W5
D1	2	2	0	0	1
D2	0	0	1	2	2
D3	2	3	0	0	0
D4	0	0	1	0	1
D5	1	1	1	0	2

- Potential solution:
  – Convert into 0/1 matrix and then apply existing algorithms
    - lose word frequency information
  – Discretization does not apply as users want association among words not ranges of words

Min-Apriori

- How to determine the support of a word?
  – If we simply sum up its frequency, support count will be greater than total number of documents!
    - Normalize the word vectors – e.g., using L_1 norm
    - Each word has a support equals to 1.0

TID	W1	W2	W3	W4	W5
D1	2	2	0	0	1
D2	0	0	1	2	2
D3	2	3	0	0	0
D4	0	0	1	0	1
D5	1	1	1	0	2

Normalize →

TID	W1	W2	W3	W4	W5
D1	0.40	0.33	0.00	0.00	0.17
D2	0.00	0.00	0.33	1.00	0.33
D3	0.40	0.50	0.00	0.00	0.00
D4	0.00	0.00	0.33	0.00	0.17
D5	0.20	0.17	0.33	0.00	0.33

Min-Apriori

- New definition of support:

$$\sup(C) = \sum_{i \in T} \min_{j \in C} D(i, j)$$

TID	W1	W2	W3	W4	W5
D1	0.40	0.33	0.00	0.00	0.17
D2	0.00	0.00	0.33	1.00	0.33
D3	0.40	0.50	0.00	0.00	0.00
D4	0.00	0.00	0.33	0.00	0.17
D5	0.20	0.17	0.33	0.00	0.33

Example:

Sup(W1,W2,W3)

= 0 + 0 + 0 + 0 + 0.17

= 0.17

Anti-monotone property of Support

TID	W1	W2	W3	W4	W5
D1	0.40	0.33	0.00	0.00	0.17
D2	0.00	0.00	0.33	1.00	0.33
D3	0.40	0.50	0.00	0.00	0.00
D4	0.00	0.00	0.33	0.00	0.17
D5	0.20	0.17	0.33	0.00	0.33

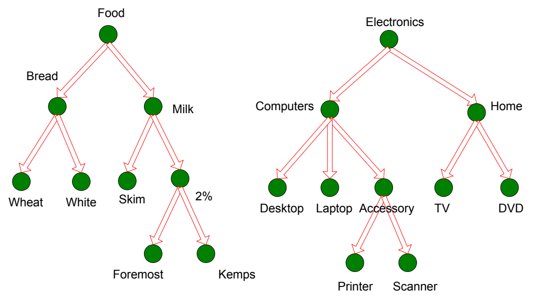
Example:

Sup(W1) = 0.4 + 0 + 0.4 + 0 + 0.2 = 1

Sup(W1, W2) = 0.33 + 0 + 0.4 + 0 + 0.17 = 0.9

Sup(W1, W2, W3) = 0 + 0 + 0 + 0 + 0.17 = 0.17

Multi-level Association Rules



Multi-level Association Rules

- Why should we incorporate concept hierarchy?
  - Rules at lower levels may not have enough support to appear in any frequent itemsets
  - Rules at lower levels of the hierarchy are overly specific
    - e.g., skim milk → white bread, 2% milk → wheat bread, skim milk → wheat bread, etc.

    are indicative of association between milk and bread

Multi-level Association Rules

- How do support and confidence vary as we traverse the concept hierarchy?
  - If X is the parent item for both X1 and X2, then $\sigma(X) \leq \sigma(X1) + \sigma(X2)$
  - If $\sigma(X1 \cup Y1) \geq minsup$,
    and X is parent of X1, Y is parent of Y1
    then $\sigma(X \cup Y1) \geq minsup$, $\sigma(X1 \cup Y) \geq minsup$
    $\sigma(X \cup Y) \geq minsup$
  - If $conf(X1 \Rightarrow Y1) \geq minconf$,
    then $conf(X1 \Rightarrow Y) \geq minconf$

Multi-level Association Rules

- Approach 1:
  - Extend current association rule formulation by augmenting each transaction with higher level items

  Original Transaction: {skim milk, wheat bread}

  Augmented Transaction:
  {skim milk, wheat bread, milk, bread, food}
- Issues:
  - Items that reside at higher levels have much higher support counts
    - if support threshold is low, too many frequent patterns involving items from the higher levels
  - Increased dimensionality of the data

Multi-level Association Rules

- Approach 2:
  - Generate frequent patterns at highest level first
  - Then, generate frequent patterns at the next highest level, and so on
- Issues:
  - I/O requirements will increase dramatically because we need to perform more passes over the data
  - May miss some potentially interesting cross-level association patterns