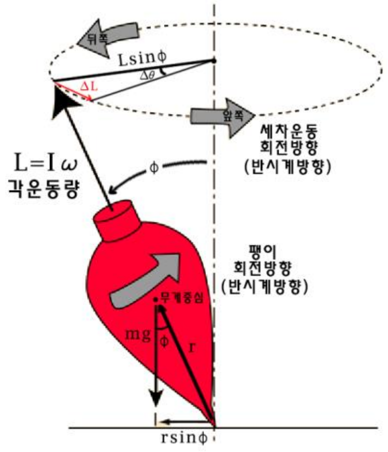
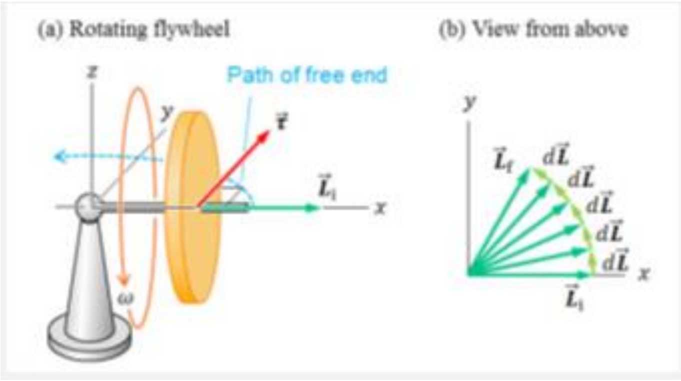
관성모멘트
=회전관성
=moment of inertia





송대근

```xml
<?xml version="1.0"?>
<robot xmlns:xacro="http://www.ros.org/wiki/xacro" >

    <!-- Specify some standard inertial calculations https://en.wikipedia.org/wiki/List_of_moments_of_inertia -->
    <!-- These make use of xacro's mathematical functionality -->

    <xacro:macro name="inertial_sphere" params="mass radius *origin">
        <inertial>
            <xacro:insert_block name="origin"/>
            <mass value="${mass}" />
            <inertia ixx="${(2/5) * mass * (radius*radius)}" ixy="0.0" ixz="0.0"
                    iyy="${(2/5) * mass * (radius*radius)}" iyz="0.0"
                    izz="${(2/5) * mass * (radius*radius)}" />
        </inertial>
    </xacro:macro>


    <xacro:macro name="inertial_box" params="mass x y z *origin">
        <inertial>
            <xacro:insert_block name="origin"/>
            <mass value="${mass}" />
            <inertia ixx="${(1/12) * mass * (y*y+z*z)}" ixy="0.0" ixz="0.0"
                    iyy="${(1/12) * mass * (x*x+z*z)}" iyz="0.0"
                    izz="${(1/12) * mass * (x*x+y*y)}" />
        </inertial>
    </xacro:macro>


    <xacro:macro name="inertial_cylinder" params="mass length radius *origin">
        <inertial>
            <xacro:insert_block name="origin"/>
            <mass value="${mass}" />
            <inertia ixx="${(1/12) * mass * (3*radius*radius + length*length)}" ixy="0.0" ixz="0.0"
                    iyy="${(1/12) * mass * (3*radius*radius + length*length)}" iyz="0.0"
                    izz="${(1/2) * mass * (radius*radius)}" />
        </inertial>
    </xacro:macro>

</robot>
```

관성
: 물체가 외부 힘을 받지 않는 한,
정지 또는 운동의 상태를 지속하려는 성질

뉴턴의 운동법칙

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a}$$

$$F = m\ a$$

$$\tau = I\ \alpha$$

이 식은 물체를 점으로 봤을 때 적용가능 하다..
하지만 물체는 점들의 집합으로 이루어져 있다.

즉, 부피를 가진 물체가 가진
관성이라는 성질을 운동학적으로 이해하려면,
아래의 과정이 선행되어야 한다.

일단 물체의 운동을 분류하고,
분류된 운동에 대한 지배방정식을 살펴보자.

$$\Sigma F_x = m(a_G)_x$$
$$\Sigma F_y = m(a_G)_y$$

$$\Sigma M_G = I_G\alpha$$

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        <inertial>
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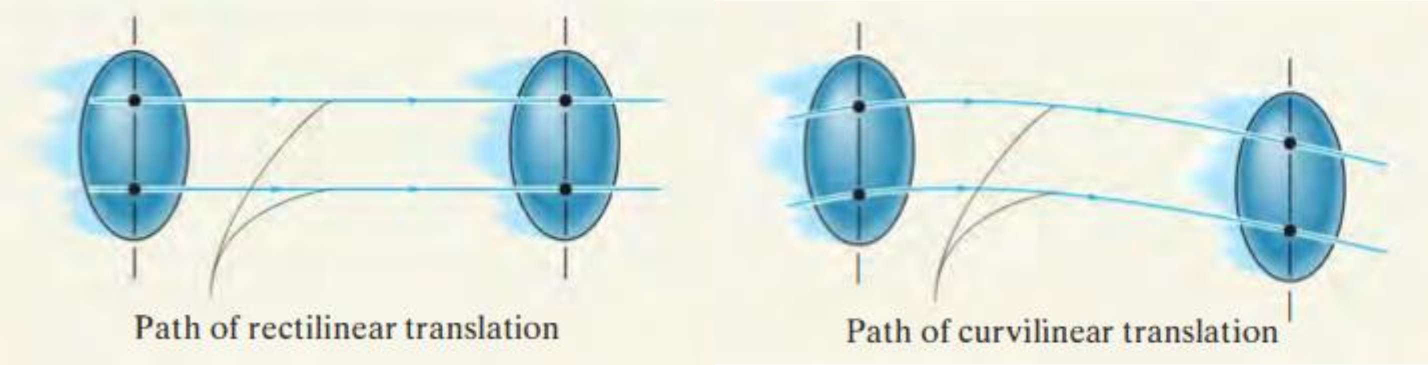
            <mass value="${mass}" />
            <inertia ixx="${(1/12) * mass * (3*radius*radius + length*length)}" ixy="0.0"
ixz="0.0"
                    iyy="${(1/12) * mass * (3*radius*radius + length*length)}" iyz="0.0"
                    izz="${(1/2) * mass * (radius*radius)}" />
        </inertial>
    </xacro:macro>

</robot>
```

가정: rigid body

직선운동이나 원운동같은 운동경로와 무관

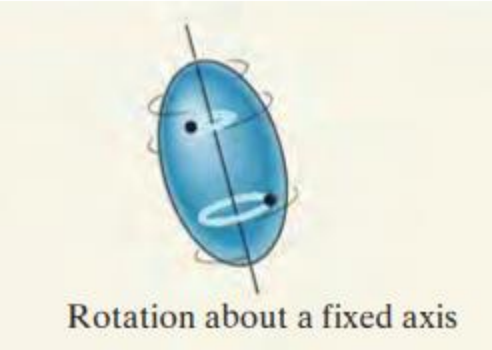
병진운동/선운동 (Translational motion)
- 물체의 모든 질점들이 동일한 변위로 운동



$$F = m\ a$$

가정: 회전축은 고정축

회전운동/각운동 (Rotational motion)
- 물체의 모든 질점들이 회전축 주위로 회전을 하는 운동



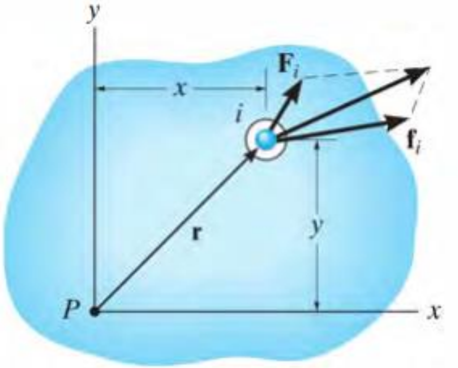
$$\tau = I\ \alpha$$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

Let, $\boldsymbol{\tau} = (\mathbf{M}_P)_i$

$(\mathbf{M}_P)_i = \mathbf{r} \times m_i \mathbf{a}_i$

$\mathbf{r} \times \mathbf{F}_i + \mathbf{r} \times \mathbf{f}_i = \mathbf{r} \times m_i \mathbf{a}_i$



$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$\begin{aligned}(\mathbf{M}_P)_i &= m_i \mathbf{r} \times (\mathbf{a}_P + \boldsymbol{\alpha} \times \mathbf{r} - \omega^2 \mathbf{r}) \\ &= m_i[\mathbf{r} \times \mathbf{a}_P + \mathbf{r} \times (\boldsymbol{\alpha} \times \mathbf{r}) - \omega^2(\mathbf{r} \times \mathbf{r})]\end{aligned}$$

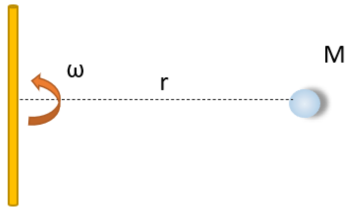
$$\begin{aligned}(M_P)_i\mathbf{k} &= m_i\{(x\mathbf{i} + y\mathbf{j}) \times [(a_P)_x\mathbf{i} + (a_P)_y\mathbf{j}] \\ &\quad + (x\mathbf{i} + y\mathbf{j}) \times [\alpha\mathbf{k} \times (x\mathbf{i} + y\mathbf{j})]\} \\ (M_P)_i\mathbf{k} &= m_i[-y(a_P)_x + x(a_P)_y + \alpha x^2 + \alpha y^2]\mathbf{k} \\ \circlearrowleft (M_P)_i &= m_i[-y(a_P)_x + x(a_P)_y + \alpha r^2]\end{aligned}$$

Letting $m_i \rightarrow dm$

$$\circlearrowleft \Sigma M_P = -\left(\int_m y\, dm\right)(a_P)_x + \left(\int_m x\, dm\right)(a_P)_y + \left(\int_m r^2 dm\right)\alpha$$

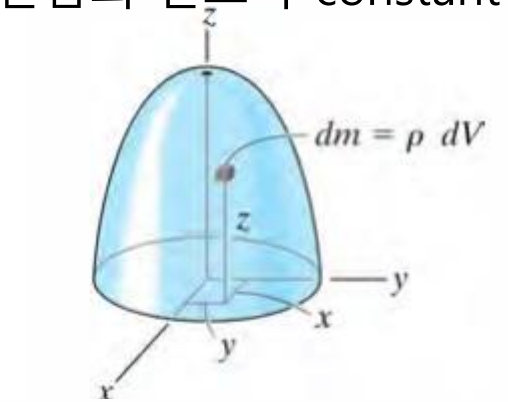
$\therefore$ $\boxed{\Sigma M_G = I_G \alpha}$ the mass center G for the body. $(\bar{x} = \bar{y} = 0)$

관성모멘트: $\boxed{I = \int_m r^2 \, dm}$



가정: 물체의 모든 질점의 밀도가 constant (ρ)

$$I = \rho \int_V r^2 \, dV$$

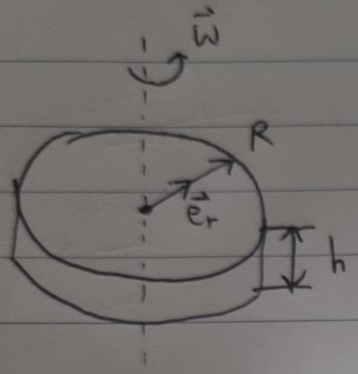


$$I = \int_m r^2 \, dm = \int_m [(d + x')^2 + y'^2] \, dm$$

$$= \int_m (x'^2 + y'^2) \, dm + 2d \int_m x' \, dm + d^2 \int_m dm$$

$$\boxed{I = I_G + md^2}$$

이론을 통한 관성모멘트 계산



$$I = \int_m r^2 \, dm \qquad // \; dm = \rho \, dV = \rho h \, dA = \rho h \, d(\pi r^2) = 2\pi r \rho h \, dr$$

$$= 2\pi\rho h \int_0^R r^3 \, dr$$

$$= 2\pi\rho h \times \frac{1}{4} R^4 \qquad // \; \pi R^2 h \rho = M$$

$$= \frac{1}{2} M R^2$$

물체의 복잡한 형상과 물성을 파악하고 계산하는 것은 정확하지만 어렵다..

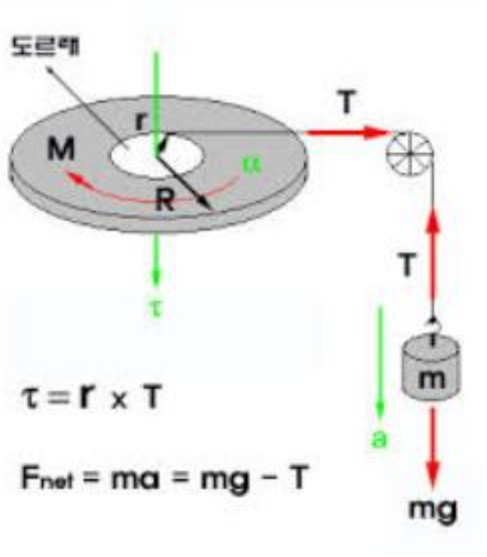
실험적 방법으로 측정하는 것이 효율적으로 보인다.

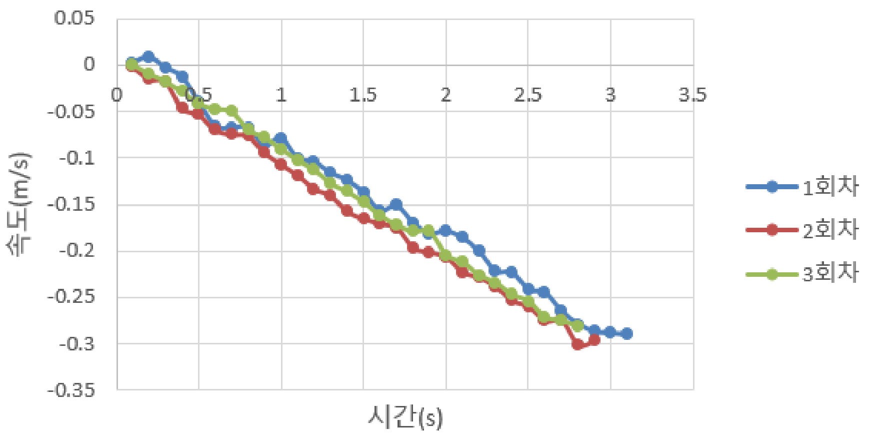
실험을 통한 관성모멘트 측정

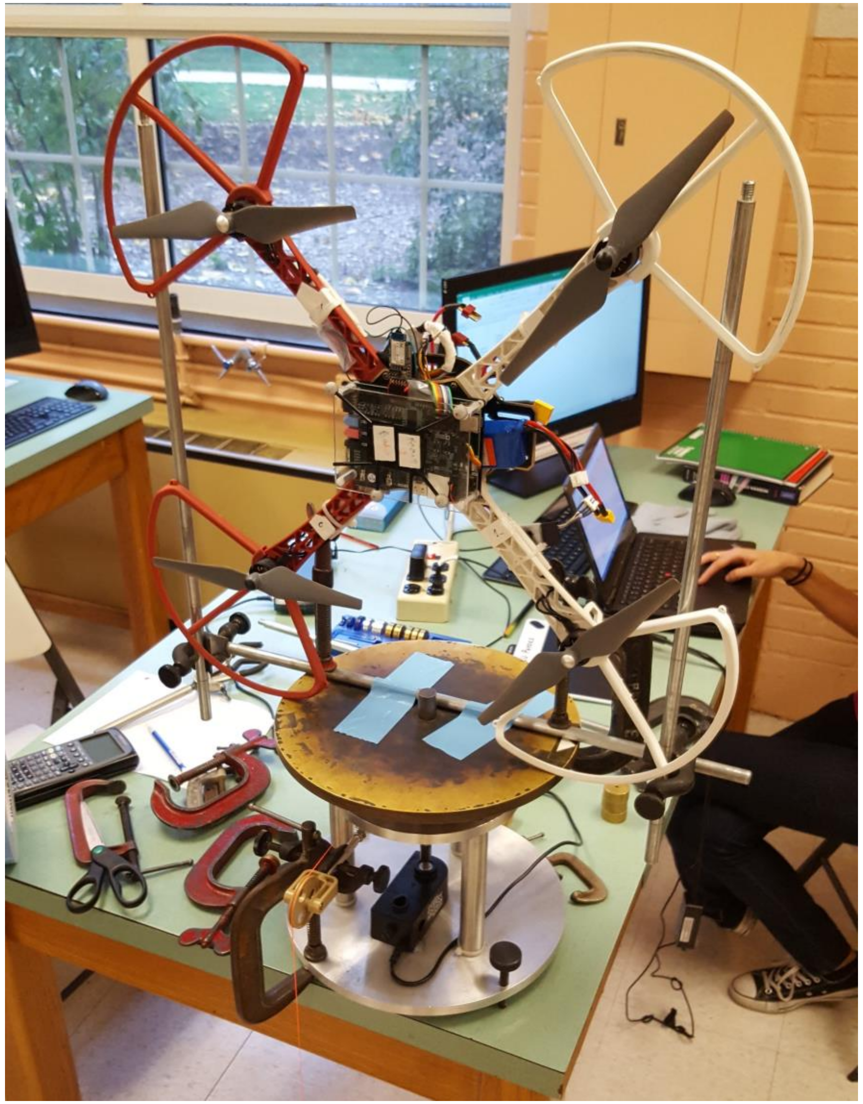
$$ma = mR\alpha = mg - T \cdots (1)$$

$$\tau = RT = I\alpha \cdots (2)$$

$$I = \frac{mgR^2}{a} - mR^2$$







참고자료

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