



Decision Procedure for a Theory of String Sequences

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Abstract. The theory of sequences, supported by many SMT solvers, can model program data types including bounded arrays and lists. Sequences are parameterized by the element data type and provide operations such as accessing elements, concatenation, forming sub-sequences and updating elements. Strings and sequences are intimately related; many operations, e.g., matching a string according to a regular expression, splitting strings, or joining strings in a sequence, are frequently used in string-manipulating programs. Nevertheless, these operations are typically not directly supported by existing SMT solvers, which instead only consider the generic theory of sequences. In this paper, we propose a theory of string sequences and study its satisfiability. We show that, while it is undecidable in general, the decidability can be recovered by restricting to the straight-line fragment. This is shown by encoding each string sequence as a string, and each string sequence operation as a corresponding string operation. We provide pre-image computation for the resulting string operations with respect to automata, effectively casting it into the generic OSTRICH string constraint solving framework. We implement the new decision procedure as a tool OSTRICH^{SEQ}, and carry out experiments on benchmark constraints generated from real-world JavaScript programs, hand-crafted templates and unit tests. The experiments confirm the efficacy of our approach.

1 Introduction

Many real-world applications, such as web applications and programs processing data of type string, involve a multitude of complex operations that convert between strings and string sequences. Typical examples include *join* and *split*, which are present in almost all built-in libraries of modern programming languages. Reasoning about string sequences, also known as extendable string arrays [33], is therefore a crucial aspect of program analysis.

Satisfiability Modulo Theory (SMT [9]) solving provides an automatic way to verify software systems. At the moment, there is no standardized sequence theory in SMT-LIB [8]. Instead, in verification tools, often the theory of arrays is used, which is limited and does not provide even basic operations like sequence length. Fortunately, [14] proposed a basic format for a sequence theory, which has been implemented and extended by SMT solvers such as Z3 [25] and cvc5 [7] with practical operations. In particular, [43] presents a calculus in cvc5 for reasoning about string sequences, which extends the proposal [14] with **update** (aka **write**) operations. The implementation of sequences in Z3 has been documented in the Z3 guide [49] and includes operations such as **map** and **foldleft** (but no **update**).

Although existing SMT solvers are effective at reasoning about sequences in general, to the best of our knowledge, none of them directly support functions that convert between strings and string sequences such as **join** and **split**. While it is possible to implement these functions using existing ones for some SMT solvers (e.g., Z3 can use **map** and **foldleft** to implement the **join** function), usually the solvers do not provide decision procedures for those theories and fail to solve many instances. This limitation poses a significant challenge for program analysis that involves such operations. The purpose of our work is to fill this gap.

Contributions. We propose a logic (aka constraint language) **SeqStr** of string sequences, which extends the existing sequence theory in SMT solvers with dedicated operations of string sequences and strings. In addition to the standard string operations (**replace**/**replaceAll**, **reverse**, finite transducers, regular constraints, **length**, **indexOf**, **substring**, etc.), our logic emphasizes the interaction between strings and string sequences through operations such as **split**, **join**, **filter**, and **matchAll**. For instance, **split** splits a string into a sequence of substrings according to a regular expression; **join** concatenates all the strings in a sequence to obtain a single string. More details can be found in Sect. 4. Typically, when string sequences are considered, it is natural to consider the integer data type as well; for instance, one usually needs to refer to the length of a sequence or the index of a specific sequence element. As a result, **SeqStr** features three data types: integers, strings, and string sequences. In terms of operations, the logic subsumes, to our best knowledge, most string constraint languages that have been proposed in the literature.

Our main focus is the decision procedure for the satisfiability problem of **SeqStr**. Because of its expressiveness, it is perhaps not surprising that **SeqStr** is undecidable in general. To reinstate decidability, we consider the straight-line fragment, which imposes a syntactic restriction on the constraints written in the logic. Straight-line formulas naturally arise when verifying programs using bounded model checking or symbolic execution, which unroll loops in the programs up to a given depth and convert programs to static single assignment form (i.e. each variable is defined at most once). For example, a majority of the constraints in the standard Kaluza benchmarks [41] satisfy this condition. (Cf. Sect. 3 for a concrete example.)

The general strategy of our decision procedure is to encode each string sequence as a string in such a way that all operations for string sequences considered in **SeqStr** can be transformed into string operations, possibly involving

integers. As such, the decidability of (straight-line) **SeqStr** is reduced to string constraints with the integer data type, which was shown to be decidable [19]. Needless to say, this strategy requires overcoming certain technical challenges, as the string constraints resulting from the reduction typically encompass complex (and sometimes non-standard) string operations, which go well beyond the capacity of state-of-the-art string constraint solvers. Recall that for the decidability of string constraints with integers, cost-enriched finite automata (CEFA), a variant of cost-register automata, were utilized [19]. The crux of our technical contributions is thus to compute the backward images of CFAs under the fairly complex string operations (cf. Section 5.2), so the powerful string constraint solving framework **OSTRICH** [17, 20] can be harnessed.

We implement the decision procedure as a new solver **OSTRICH**^{SEQ} on top of **OSTRICH** [20] and **Princess** [39]. To evaluate the effectiveness of **OSTRICH**^{SEQ}, we curate two benchmark suites of constraints that are randomly generated from templates (only using the operations directly supported by some existing SMT solvers) and extracted from real-world JavaScript programs and unit tests (involving string sequence operations that existing SMT solvers do not natively support), respectively. On both benchmark suites, the experimental results show that **OSTRICH**^{SEQ} can solve considerably more constraints than SOTA string solvers including **cvc5**, **Z3**, **Z3-noodler** [22], **OSTRICH** and **Princess**^{ARR} [40], while being largely as efficient as most of them.

Organization. Section 2 gives the preliminaries. Section 3 presents a motivating example. Section 4 defines the logic **SeqStr** of string sequences. Section 5 presents the decision procedure. Section 6 presents the benchmarks and experiments. Section 7 discusses the related work. The paper is concluded in Sect. 8.

2 Preliminaries

Let \mathbb{N} denote the set of natural numbers. For $1 \leq n \in \mathbb{N}$, let $[n] := \{1, \dots, n\}$, and for $m < n \in \mathbb{N}$, let $[m, n] := \{j \mid m \leq j \leq n\}$. We also use standard quantifier-free/existential *linear integer arithmetic* (LIA) formulas, which are typically ranged over by ϕ, φ , etc. For an integer term t , we use $t[t_1/t_2]$ to denote the term obtained by replacing t_2 with t_1 in t , where t_1 and t_2 are integer terms.

Strings, Languages, and Transductions. We fix an alphabet Σ , i.e., a finite set of letters. A *string* over Σ is a finite sequence of letters from Σ . We use Σ^* to denote the set of strings over Σ and ε to denote the empty string. A *language* over Σ is a subset of Σ^* . We will use L_1, L_2, \dots to denote languages. For two languages $L_1, L_2 \subseteq \Sigma^*$, $L_1 \cup L_2$ denotes the union of L_1 and L_2 , and $L_1 \cdot L_2$ denotes the concatenation of L_1 and L_2 , that is, $\{u_1 \cdot u_2 \mid u_1 \in L_1, u_2 \in L_2\}$. For a language $L \subseteq \Sigma^*$, we define the complement of L as $\bar{L} = \{w \in \Sigma^* \mid w \notin L\}$, moreover, we define L^n for $n \in \mathbb{N}$, the *iteration* of L for n times, inductively as: $L^0 = \{\varepsilon\}$ and $L^n = L \cdot L^{n-1}$ for $n > 0$. We also use L^* to denote the iteration of L for arbitrarily many times, that is, $L^* = \bigcup_{n \in \mathbb{N}} L^n$, and let $L^+ = \bigcup_{n > 0} L^n$. A *transduction* over Σ is a binary relation over Σ^* , namely, a subset of $\Sigma^* \times \Sigma^*$.

Regular expressions over Σ are defined in a standard way, i.e.,

$$e \stackrel{\text{def}}{=} \emptyset \mid \varepsilon \mid a \mid e + e \mid e \cdot e \mid e^* \text{ where } a \in \Sigma.$$

The language $\mathcal{L}(e) \subseteq \Sigma^*$ of a regular expression e is defined inductively as: $\mathcal{L}(\emptyset) = \emptyset$, $\mathcal{L}(\varepsilon) = \{\varepsilon\}$, $\mathcal{L}(a) = \{a\}$, $\mathcal{L}(e_1 + e_2) = \mathcal{L}(e_1) \cup \mathcal{L}(e_2)$, $\mathcal{L}(e_1 \cdot e_2) = \mathcal{L}(e_1) \cdot \mathcal{L}(e_2)$, $\mathcal{L}(e_1^*) = (\mathcal{L}(e_1))^*$. A *regular language* is a language that can be defined by a regular expression.

Automata. A (*nondeterministic*) *Finite Automaton* (NFA) \mathcal{A} is a 5-tuple $(Q, \Sigma, \delta, I, F)$, where Q is a finite set of states, Σ is a finite alphabet, $\delta \subseteq Q \times \Sigma \times Q$ is the transition relation, $I, F \subseteq Q$ are the sets of initial and final states respectively. For readability, we write a transition $(q, a, q') \in \delta$ as $q \xrightarrow[a]{a} q'$ (or simply $q \xrightarrow{a} q'$). A *run* of \mathcal{A} on a string $w = a_1 \cdots a_n$ is a sequence of transitions $q_0 \xrightarrow{a_1} q_1 \cdots q_{n-1} \xrightarrow{a_n} q_n$ with $q_0 \in I$. The run is *accepting* if $q_n \in F$. A string w is accepted by an NFA \mathcal{A} if there is an accepting run of \mathcal{A} on w . In particular, the empty string ε is accepted by \mathcal{A} if $I \cap F \neq \emptyset$. The language of \mathcal{A} , denoted by $\mathcal{L}(\mathcal{A})$, is the set of strings accepted by \mathcal{A} . In addition, an NFA can be built to recognize the language of each given regular expression, and vice versa.

A (*nondeterministic*) *finite transducer* (NFT) \mathfrak{T} is an extension of NFA with outputs. Formally, an NFT \mathfrak{T} is a 5-tuple $(Q, \Sigma, \delta, I, F)$, where Q, Σ, I, F are the same as in NFA and the transition relation δ is a finite subset of $Q \times \Sigma \times Q \times \Sigma^*$. For readability, we write a transition $(q, a, q', u) \in \delta$ as $q \xrightarrow[\delta]{a, u} q'$ or $q \xrightarrow{a, u} q'$.

A run of \mathfrak{T} over a string $w = a_1 \cdots a_n$ is a sequence of transitions $q_0 \xrightarrow{a_1, u_1} q_1 \xrightarrow{a_2, u_2} q_2 \cdots q_{n-1} \xrightarrow{a_n, u_n} q_n$ where $q_0 \in I$. Similar to NFA, the run is *accepting* if $q_n \in F$. The string $u_1 \cdots u_n$ is called the *output* of the run. The transduction $\mathcal{T}(\mathfrak{T}) \subseteq \Sigma^* \times \Sigma^*$ defined by \mathfrak{T} is the set of string pairs (w, u) such that there is an accepting run of \mathfrak{T} on w , with the output u .

Definition 1 (Cost-enriched finite automata). A *cost-enriched finite automaton* (CEFA for short) \mathcal{A} is a 6-tuple $(R, Q, \Sigma, \delta, I, F)$ where

- $R = \{r_1, \dots, r_k\}$ is a finite set of registers,
- Q, I, F are the same as in NFA, i.e., the set of states, the set of initial states, and the set of final states, respectively,
- $\delta \subseteq Q \times \Sigma \times Q \times \mathbb{Z}^k$ is a transition relation, where \mathbb{Z}^k represents the values to update the registers in R .

We write $R_{\mathcal{A}}$ for the set of registers of \mathcal{A} and represent it as a vector (r_1, \dots, r_k) . Accordingly, updates $r_i := r_i + v_i$ for all $i \in [k]$ are simply identified as the vector $\vec{v} = (v_1, \dots, v_k)$, i.e., r_i is incremented by v_i for each $i \in [k]$. Typically, we write a transition $(q, a, q', \vec{v}) \in \delta$ as $q \xrightarrow[\vec{v}]{a} q'$.

Intuitively, CEFAs add write-only cost registers to finite automata, where “write-only” means that the cost registers can only be written/updated but cannot be read, i.e., they cannot be used in the guards of the transitions.

A run of the CEFA \mathcal{A} on a string $w = a_1 \cdots a_n$ is a sequence of transitions $q_0 \xrightarrow[\vec{v}_1]{a_1} q_1 \cdots q_{n-1} \xrightarrow[\vec{v}_n]{a_n} q_n$ such that $q_0 \in I$ and $q_{i-1} \xrightarrow[\vec{v}_i]{a_i} q_i$ for each $i \in [n]$. It is *accepting* if $q_n \in F$, and the vector $\vec{c} = \sum_{i \in [n]} \vec{v}_i$ is defined as the *cost* of the

run. (Note that all registers are initialized to zero.) We write $\vec{c} \in \mathcal{A}(w)$ if there is an accepting run of \mathcal{A} on w whose cost is \vec{c} . The language of a CEFA \mathcal{A} , denoted by $\mathcal{L}(\mathcal{A})$, is defined as the set of pairs $\{(w, \vec{c}) \in \Sigma^* \times \mathbb{Z}^{|R|} \mid \vec{c} \in \mathcal{A}(w)\}$. In particular, if $I \cap F \neq \emptyset$, then $(\varepsilon, \vec{0}) \in \mathcal{L}(\mathcal{A})$. We denote by $\mathcal{L}_1(\mathcal{A})$ the language $\{w \in \Sigma^* \mid \exists \vec{c} \in \mathbb{Z}^{|R|}. (w, \vec{c}) \in \mathcal{L}(\mathcal{A})\}$.

Pre-images Under String Functions. Consider a language $L \subseteq \Sigma^* \times \mathbb{Z}^{k_0}$ defined by a CEFA $\mathcal{A} = (R, Q, \Sigma, \delta, I, F)$ with the registers $R = (r_1, \dots, r_{k_0})$ and a function $f : (\Sigma^* \times \mathbb{Z}^{k_1}) \times \dots \times (\Sigma^* \times \mathbb{Z}^{k_l}) \rightarrow \Sigma^*$. For each $i \in [k_0]$, let r_1^i, \dots, r_l^i be the freshly introduced registers, and $\vec{t} = (t_1, \dots, t_{k_0})$ be a vector of LIA formulas such that t_i is a linear combination of r_1^i, \dots, r_l^i for each $i \in [k_0]$.

The pre-image of L under f with respect to \vec{t} , denoted by $f_{\vec{t}}^{-1}(L)$, is a relation

$$\mathcal{R} \subseteq (\Sigma^* \times \mathbb{Z}^{k_1+k_0}) \times \dots \times (\Sigma^* \times \mathbb{Z}^{k_l+k_0})$$

that comprises tuples of the form $((w_1, (\vec{c}_1, \vec{d}_1)), \dots, (w_l, (\vec{c}_l, \vec{d}_l)))$ such that

- for every $j \in [l]$, $\vec{c}_j \in \mathbb{Z}^{k_j}$ and $\vec{d}_j = (d_j^1, \dots, d_j^{k_0}) \in \mathbb{Z}^{k_0}$,
- let $w_0 = f((w_1, (\vec{c}_1, \vec{d}_1)), \dots, (w_l, (\vec{c}_l, \vec{d}_l)))$ and

$$\vec{d}' = \left(t_1 [d_1^1/r_1^1, \dots, d_l^1/r_l^1], \dots, t_{k_0} [d_1^{k_0}/r_1^{k_0}, \dots, d_l^{k_0}/r_l^{k_0}] \right),$$

it holds that $(w_0, \vec{d}') \in L$.

The pre-image $f_{\vec{t}}^{-1}(L)$ is *CEFA-definable* if $f_{\vec{t}}^{-1}(L) = \bigcup_{i=1}^n \mathcal{L}(\mathcal{A}_{i,1}) \times \dots \times \mathcal{L}(\mathcal{A}_{i,l})$ for some $n \geq 1$, where for all $i \in [n]$ and $j \in [l]$, $\mathcal{A}_{i,j}$ is a CEFA such that $\mathcal{L}(\mathcal{A}_{i,j}) \subseteq \Sigma^* \times \mathbb{Z}^{k_j}$. Finally, a pre-image of L under f , denoted by $f^{-1}(L)$, is a pair (\mathcal{R}, \vec{t}) such that $\mathcal{R} = f_{\vec{t}}^{-1}(L)$.

The core concept of *pre-image computation* for the function f involves determining the vector \vec{t} of LIA formulas and, for each CEFA \mathcal{A} , computing $f_{\vec{t}}^{-1}(\mathcal{L}(\mathcal{A}))$ represented as a finite set of CEFA's. Intuitively, we remove the string equalities of the form $y = f(x_1, \vec{it}_1, \dots, x_l, \vec{it}_l)$ (where $\vec{it}_1, \dots, \vec{it}_l$ are vectors of integer terms) one by one by computing the pre-images and adding the resulting CEFA membership constraints for x_1, \dots, x_l . In the end, the original string constraint is transformed into a conjunction of CEFA membership constraints and LIA formulas, whose satisfiability checking is known to be PSPACE-complete [19].

3 Motivating Example

To motivate and illustrate our approach, we consider a JavaScript snippet shown in Listing 1.1, which is extracted, with a slight adaptation, from a real-world example.¹ Here, the function `prepareVersionNo` extracts from the input string `version` a version number in the floating-point number format. Specifically, it splits `version` into a sequence/array `numbers` according to the non-numeric

¹ <https://github.com/hgoebl/mobile-detect.js/blob/master/mobile-detect.js>.

```

1  function prepareVersionNo(version) {
2    // precondition: version in [0-9]+([0-9a-zA-Z._ /-])*
3    numbers = version.split(/[a-z._ \\\-]/i);
4    if (numbers.length === 1) { // path-1
5      result = numbers[0];    }
6    if (numbers.length > 1) { // path-2
7      temp = numbers[0] + ".";
8      numbers1 = numbers.slice(1, numbers.length);
9      result = temp + numbers1.join("");    }
10   return Number(result);
11   // postcondition: result in [0-9]+(\.[0-9]*)?
12 }

```

Listing 1.1. JavaScript code snippet: The motivating example

```

1  ; precondition
2  (assert (str.in_re version preReg))
3  (assert (= numbers (str.splitre version splitReg)))
4  (assert (< 1 (seq.len numbers)))
5  (assert (= temp (str.++ (seq.nth numbers 0) ".")))
6  (assert (= numbers1
7    (seq.extract numbers 1 (- (seq.len numbers) 1))))
8  (assert (= result (str.++ temp (seq.join numbers1 ""))))
9  ; postcondition
10 (assert (not (str.in_re result postReg)))

```

Listing 1.2. SMT formula for path-2 in prepareVersionNo

characters. If `numbers` contains exactly one element, `numbers[0]` is returned; otherwise, the function returns the concatenation of `numbers[0]`, the dot, and the string obtained by concatenating the other elements of `numbers`. For instance, if `version = 12a56b23`, the output is 12.5623.

To verify the functional correctness of `prepareVersionNo`, we specify the following precondition and postcondition.

- The precondition `version ∈ [0-9]+([0-9a-zA-Z._ /-])*` requires that the input string starts with a decimal number, followed by a string of digits, letters, dot `.`, underline `_`, blank symbol, slash `/`, or hyphen `-`.
- The postcondition `result ∈ [0-9]+(\.[0-9]*)?` ensures that the output string starts with a decimal number, possibly followed by a dot `.` and a decimal number.

We apply symbolic execution to verify `prepareVersionNo`. We enumerate its execution paths and check for each path that, provided the input string satisfies the precondition, the output string satisfies the postcondition after the executing the path. There are three execution paths: path-1 taking `numbers.length===1`, path-2 taking `numbers.length>1`, and path-3 where both conditions are false. As an example, we consider path-2, and the symbolic execution reduces to deciding the satisfiability of the SMT formula in Listing 1.2, where `preReg` and `postReg` denote the regular expressions in the precondition and postcon-

dition, i.e., $([0-9]+([0-9a-zA-Z_ -])^*)$ and $([0-9]+(\backslash.[0-9]^*)?)$, respectively, and `splitReg` represents the regular expression `[a-zA-Z_ -/-]`.

This SMT formula involves various operations of string sequences and strings, including sequence length `seq.len`, string split `str.splitre`, sequence read `seq.nth`, string concatenation `str.++`, sequence extract `seq.extract`, sequence join `seq.join`, as well as regular constraints `str.in_re`. While state-of-the-art SMT solvers such as Z3 and cvc5 can solve constraints in the generic sequence theory, they do not directly support complex operations that feature mutual transformations between strings and string sequences, such as `str.splitre` and `seq.join`. This motivates us to define an SMT theory of string sequences (`SeqStr`, cf. Sect. 4) and investigate its decision procedure (Sect. 5).

Due to the undecidability of `SeqStr`, we focus on its straight-line fragment (cf. Definition 2), into which the formula in Listing 1.2 falls. We encode string sequences as strings, and, accordingly, string sequence operations become string operations (Sect. 5.1). Then we transform the formula in Listing 1.2 into the following string constraint:

$$\begin{aligned}
 &\text{version} \in \text{preReg} \wedge \text{snumbers} = \text{splitstr}_{\text{splitReg}}(\text{version}) \wedge \\
 &1 < \text{seqlen}(\text{snumbers}) - 1 \wedge \text{temp} = \text{elem}(\text{snumbers}, 0) \cdot \text{'.'} \wedge \\
 &\text{snumbers1} = \text{subseq}[\text{snumbers}, 2, \text{seqlen}(\text{snumbers}) - 2] \wedge \\
 &\text{result} = \text{temp} \cdot \text{join}_\epsilon(\text{snumbers1}) \wedge \text{result} \notin \text{posReg}.
 \end{aligned} \tag{1}$$

where `snumbers`, `snumbers1` are string variables that encode the string sequences `numbers`, `numbers1` in Listing 1.2. Note that the resulting string constraint contains new string operations `splitstr`, `seqlen`, `elem`, \dots introduced in this paper (cf. Sect. 5.1). Subsequently, we employ the decision procedure outlined in [19], which *back propagates* the CEFA membership constraints by computing the pre-images under string functions (see Sect. 5.2).

In the sequel, we show how to solve the formula in (1).

1. We back propagate the regular membership constraint $\text{result} \in \overline{\text{posReg}}$ (where $\overline{\text{posReg}}$ denotes the complement of `posReg`) by computing the pre-images under the concatenation \cdot and join_ϵ , and adding the CEFA constraints for `temp` and `snumbers1`, say, $\text{temp} \in \mathcal{A}_1 \wedge \text{snumbers1} \in \mathcal{A}_2$ (where $\text{temp} \in \mathcal{A}_1$ means that the value of `temp` belongs to $\mathcal{L}_1(\mathcal{A}_1)$, similarly for $\text{snumbers1} \in \mathcal{A}_2$). Note that nondeterministic choices may be made here since the pre-images of $\mathcal{L}(\overline{\text{posReg}})$ under \cdot may be a union of products of CEFA's. After the propagation, the string equality $\text{result} = \text{temp} \cdot \text{join}_\epsilon(\text{snumbers1})$ is removed.
2. Then we back propagate the CEFA membership constraint $\text{temp} \in \mathcal{A}_1$ for the equality $\text{temp} = \text{elem}(\text{snumbers}, 0) \cdot \text{'.'}$ by computing the pre-image of $\mathcal{L}(\mathcal{A}_1)$ under \cdot as well as `elem` and adding some CEFA membership constraint for `snumbers`, say, $\text{snumbers} \in \mathcal{A}_3$. Similarly, we back propagate the CEFA membership constraint $\text{snumbers1} \in \mathcal{L}(\mathcal{A}_2)$ for the equality $\text{snumbers1} = \text{subseq}[\text{snumbers}, 2, \text{seqlen}(\text{snumbers}) - 1]$ by computing the

pre-image of $\mathcal{L}(\mathcal{A}_2)$ under `subseq` and adding some CEFA membership constraint for `snumbers`, say, `snumbers` $\in \mathcal{A}_4$. After these back-propagation operations, the two string equalities are removed.

3. Finally, we back propagate the CEFA membership constraint `snumbers` $\in \mathcal{A}_3 \cap \mathcal{A}_4$ for `snumbers` = `splitstrsplitReg(version)` by computing the pre-image of $\mathcal{L}(\mathcal{A}_3 \cap \mathcal{A}_4)$ under the function `splitstrsplitReg` and adding a CEFA membership constraint `version` $\in \mathcal{A}_5$. Moreover, the equality `snumbers` = `splitstrsplitReg(version)` is removed.
4. In the end, we obtain a string constraint that contains no string equalities, namely, a Boolean combination of CEFA membership constraints and LIA formulas, which are solvable using existing methods.

Note that here we prioritize the illustration of the main idea over the precision of the description; the technical details of the decision procedure are deferred to later sections.

Overall, the formula in Listing 1.2 can be transformed into a string constraint containing a complex combination of `str.replace_re_all`, `str.substr` and `str.len`, which cannot be solved by Z3 or cvc5 in a reasonable time limit (Z3 returns unknown as it does not support the `str.replace_re_all` function; cvc5 fails to solve the string constraint in 20h). In contrast, our new solver OSTRICH^{SEQ} solves it in less than 1 s.

4 A Theory of String Sequences (SeqStr)

A *sequence* over the set X is (a_0, \dots, a_{n-1}) , where $a_i \in X$ for each $i \in [0, n-1]$, and n is the *length* of the sequence. We shall focus on *string sequences*, namely, sequences over Σ^* . We use $()$ to denote the empty sequence.

Operations on string sequences and strings. We assume two string sequences $s_1 = (u_0, \dots, u_{m-1})$ and $s_2 = (v_0, \dots, v_{n-1})$.

- $s_1 \cdot s_2$ is the concatenation of s_1 and s_2 , i.e., $(u_0, \dots, u_{m-1}, v_0, \dots, v_{n-1})$.
- $\text{nth}(s_1, i)$ is u_i if $i \in [0, m-1]$, and undefined otherwise.
- $\text{join}_v(s_1)$ joins the elements in s_1 with v as the intermediate string, i.e., $\text{join}_v(s_1) = u_0 v u_1 v u_2 v \dots v u_{m-1}$.
- $s_1[i \rightarrow u]$ replaces u_i in s_1 by u if $i \in [0, m-1]$, and is s_1 otherwise, i.e.

$$[s_1[i \rightarrow u]] = \begin{cases} (u_0, \dots, u_{i-1}, u, u_{i+1}, \dots, u_{m-1}), & \text{if } i \in [0, m-1]; \\ s_1, & \text{otherwise.} \end{cases}$$

- $\text{filter}_e(s_1)$ filters out all elements in s_1 that do not match the regular expression e , i.e., $\text{filter}_e(s_1) = (u_{i_1}, \dots, u_{i_k})$ such that $0 \leq i_1 < \dots < i_k \leq m-1$ and for all $j \in [0, m-1]$, $u_j \in \mathcal{L}(e)$ iff $j \in \{i_1, \dots, i_k\}$. In particular, if none of u_0, \dots, u_{m-1} are in $\mathcal{L}(e)$, then $\text{filter}_e(s_1) = ()$.
- $\text{split}_e(u)$ splits a string u into a sequence of substrings according to the regular expression e , i.e., $\text{split}_e(u) = (u'_1, \dots, u'_k)$, where $u = u'_1 v_1 u'_2 v_2 \dots u'_{k-1} v_{k-1} u'_k$ such that (1) for each $i \in [k-1]$, v_i is the leftmost and longest substring of

- $u'_i v_i u'_{i+1} \cdots v_{k-1} u'_k$ that matches e , and (2) u'_k does not contain any substring that matches e . In particular, if u does not contain any substring that matches e , then $\text{split}_e(u) = (u)$.
- $s_1[i, j]$ is defined as follows: if $i \in [0, m-1]$ and $j \geq 1$, then $s_1[i, j]$ is the subsequence $(u_i, \dots, u_{\min(i+j-1, m-1)})$; if $i \in [0, m-1]$ and $j = 0$, then $s_1[i, j]$ is the empty sequence $()$; if $i \notin [0, m-1]$ or $j < 0$, then $s_1[i, j]$ is undefined.
 - $\text{matchAll}_e(u)$ extracts the substrings of u that match the regular expression e , i.e., $\text{matchAll}_e(u) = (v_1, \dots, v_{k-1})$ such that $u = u'_1 v_1 u'_2 v_2 \cdots u'_{k-1} v_{k-1} u'_k$ and (1) for each $i \in [k-1]$, v_i is the leftmost and longest substring of $u'_i v_i u'_{i+1} \cdots v_{k-1} u'_k$, and (2) u'_k has no substring that matches e .

Note that we choose the longest match semantics in $\text{split}_e(u)$ and $\text{matchAll}_e(u)$ just for illustrating our approach, where the shortest match semantics can be captured by adapting the automata construction in the pre-image computation.

The logic SeqStr of string sequences. SeqStr has three data types: Int (integer), Str (strings), and Seq (sequences). We use u, v, \dots (resp. $\mathbf{u}, \mathbf{v}, \dots$) to denote string constants (resp. variables); m, n, \dots (resp. $\mathbf{m}, \mathbf{n}, \dots$) to denote integer constants (resp. variables); s, t, \dots (resp. $\mathbf{s}, \mathbf{t}, \dots$) to denote sequence constants (resp. variables); and e to denote regular expressions.

The syntax of SeqStr is defined as follows:

$$\begin{array}{ll}
it & \stackrel{\text{def}}{=} m \mid \mathbf{m} \mid it + it \mid it - it \mid \text{strlen}(strt) \mid \text{seqlen}(seqt) & \text{integer terms} \\
strt & \stackrel{\text{def}}{=} u \mid \mathbf{u} \mid strt \cdot strt \mid \text{nth}(seqt, it) \mid \text{join}_u(seqt) & \text{string terms} \\
seqt & \stackrel{\text{def}}{=} (strt) \mid s \mid \mathbf{s} \mid seqt \cdot seqt \mid seqt[it \rightarrow strt] \mid \text{filter}_e(seqt) \mid \\
& \quad seqt[it, it] \mid \text{split}_e(strt) \mid \text{matchAll}_e(strt) & \text{sequence terms} \\
\varphi & \stackrel{\text{def}}{=} it \bowtie it \mid seqt = seqt \mid strt = strt \mid strt \in e \mid \varphi \wedge \varphi & \text{formulas}
\end{array}$$

where $\bowtie \in \{=, \neq, <, \leq, >, \geq\}$.

Here, it , $seqt$ and $strt$ denote integer, sequence and string terms, respectively. In particular, the integer terms are linear arithmetic expressions where $\text{seqlen}(\mathbf{s})$ (resp. $\text{strlen}(\mathbf{u})$) denotes the length of a sequence \mathbf{s} (resp. string \mathbf{u}); the sequence terms are constructed from a singleton sequence, sequence constants and sequence variables by applying the aforementioned sequence/string operations. A SeqStr formula is a conjunction of atomic formulas, each of which is of the form $it_1 \bowtie it_2$ where $\bowtie \in \{=, \neq, <, \leq, >, \geq\}$, a sequence equality $seqt_1 = seqt_2$, or a string equality $strt_1 = strt_2$. Note that sequence and string inequalities can be expressed as well. For instance (using disjunctions and quantifiers for the sake of presentation):

$$\begin{aligned}
\mathbf{x} \neq \mathbf{y} & \equiv \exists \mathbf{x}_1, \mathbf{x}_2, \mathbf{y}_2. \bigvee_{a, b \in \Sigma, a \neq b} (\mathbf{x} = \mathbf{x}_1 a \mathbf{x}_2 \wedge \mathbf{y} = \mathbf{x}_1 b \mathbf{y}_2) \vee \exists \mathbf{z}. \bigvee_{a \in \Sigma} (\mathbf{x} = \mathbf{y} a \mathbf{z} \vee \mathbf{y} = \mathbf{x} a \mathbf{z}) \\
\alpha \neq \beta & \equiv (\exists \alpha_1, \alpha_2, \beta_2, \mathbf{z}_1, \mathbf{z}_2. \alpha = \alpha_1 \cdot (\mathbf{z}_1) \cdot \alpha_2 \wedge \beta = \alpha_1 \cdot (\mathbf{z}_2) \cdot \beta_2 \wedge \mathbf{z}_1 \neq \mathbf{z}_2) \vee \\
& \quad \exists \gamma, \mathbf{z}. \alpha = \beta \cdot (\mathbf{z}) \cdot \gamma \vee \beta = \alpha \cdot (\mathbf{z}) \cdot \gamma.
\end{aligned}$$

Though rewriting inequalities produces disjunctions, they can still be handled in the DPLL(T) framework [29]. Hence, we focus on the disjunction-free fragment.

Proposition 1. *The satisfiability of SeqStr is undecidable.*

We define the *straight-line fragment* $\text{SeqStr}_{\text{SL}}$ of SeqStr . It is easy to see that by introducing fresh variables, we can rewrite a SeqStr formula to a form in which the left-hand side of each equality $\text{seqt}_1 = \text{seqt}_2$ (resp. $\text{strt}_1 = \text{strt}_2$) is a sequence (resp. string) variable. Note that the original and rewritten formulas are equisatisfiable. Henceforth, we assume that all sequence/string equalities satisfy this constraint unless otherwise stated.

Definition 2 (Straight-line fragment). *Given a SeqStr formula φ , the dependency graph G_φ of φ is a directed graph (V, E) such that V is the set of string and sequence variables in φ , and $(v_1, v_2) \in E$ iff $v_1 = \text{rhs}$ is a sequence or string equality in φ and v_2 occurs in rhs except for integer terms.*

φ is straight-line if φ is disjunction-free, and each string/sequence variable occurs as the left-hand side of equalities at most once; moreover, G_φ is acyclic.

Example 1. $\mathfrak{s} = \text{split}_e(u) \wedge \mathfrak{t} = \mathfrak{s}[1, \text{seqlen}(\mathfrak{s}) - 1]$ is straight-line, while $\mathfrak{s} = \mathfrak{s}[1, \text{seqlen}(\mathfrak{s}) - 1]$ is not, since there is a self-dependency on \mathfrak{s} .

5 The Decision Procedure

Theorem 1. *The satisfiability of $\text{SeqStr}_{\text{SL}}$ is decidable.*

The general idea of the decision procedure is to encode each string sequence as a string, based on which all string sequence operations in $\text{SeqStr}_{\text{SL}}$ can be transformed into string operations. It utilizes the framework [19] for solving straight-line string constraints with integer data type, as illustrated in Sect. 3. There are, however, certain technical challenges. For instance, $\text{seqt}[it \rightarrow \text{strt}]$ and $\text{seqt}[it, it]$ cannot be captured directly by standard string operations, so new ones have to be provided whose pre-images need to be computed.

5.1 From String Sequences to Strings

We fix a symbol $\dagger \notin \Sigma$ and let $\Sigma_\dagger := \Sigma \cup \{\dagger\}$. Each string sequence $s = (u_1, \dots, u_m)$ over Σ is encoded as a string $\text{enc}(s) := \dagger u_1 \dagger u_2 \dagger \dots \dagger u_m \dagger$ over Σ_\dagger . Note that the strings resulted from the encoding should match the regular expression $\dagger(\Sigma^*\dagger)^*$ and the string \dagger encodes the empty sequence.

With this encoding of string sequences as strings, each string sequence operation is also transformed into the corresponding *string operation*.

sequence concatenation $s_1 \cdot s_2$: string concatenation $\text{enc}(s_1) \cdot \text{enc}(s_2)$.

sequence write $s[i \rightarrow u]$: $\text{write}(\text{enc}(s), i + 1, u)$, which replaces the substring of $\text{enc}(s)$ between the $(i + 1)$ -th occurrence of \dagger and the $(i + 2)$ -th occurrence of \dagger , by u , if $i \in [0, n - 2]$, where n is the number of occurrences of \dagger in $\text{enc}(s)$, and is undefined otherwise (i.e. $i \notin [0, n - 2]$).

sequence filter operation $\text{filter}_e(s)$: $\text{filter}_e(\text{enc}(s))$, which removes every substring of $\text{enc}(s)$ between two consecutive occurrences of \dagger that does not match the regular expression e . (For instance, if $e = a^*b$ and $s = (ab, ac, aab)$, then $\text{filter}_e(\text{enc}(s)) = \dagger ab \dagger aab \dagger$.)

- subsequence operation** $s[i, j]$: $\text{subseq}(\text{enc}(s), i + 1, j)$, which keeps only the substring of $\text{enc}(s)$ between the $(i + 1)$ -th occurrence and the $\min(i + j, n - 1)$ -th occurrence of \dagger , if $i \in [0, n - 2]$, where n is the number of occurrences of \dagger in $\text{enc}(s)$, and is undefined otherwise (i.e. $i \notin [0, n - 2]$).
- split_e(u)**: string operation $\text{splitstr}_e(u)$ that transforms $u = u'_1 v_1 u'_2 \cdots u'_{k-1} v_{k-1} u'_k$ into $\dagger u'_1 \dagger u'_2 \cdots u'_{k-1} \dagger u'_k \dagger$, where for each $i \in [k - 1]$, v_i is the leftmost and longest matching of e in $u'_i v_i u'_{i+1} \cdots v_{k-1} u'_k$, and u'_k does not contain any substring that matches e .
- matchAll_e(u)**: $\text{matchAllstr}_e(u)$ that transforms $u = u'_1 v_1 u'_2 v_2 \cdots u'_{k-1} v_{k-1} u'_k$ into $\dagger v_1 \dagger \cdots \dagger v_{k-1} \dagger$, where for each $i \in [k - 1]$, v_i is the leftmost and longest occurrence of e in $u'_i v_i u'_{i+1} \cdots v_{k-1} u'_k$, moreover, u'_k does not contain any substring that matches e .
- sequence read operation** $\text{nth}(s, i + 1)$: $\text{elem}(\text{enc}(s), i + 1)$ that keeps only the substring between the $(i + 1)$ -th occurrence and the $(i + 2)$ -th occurrence of \dagger , if $i \in [0, n - 2]$, where n is the number of occurrences of \dagger in $\text{enc}(s)$, and is undefined otherwise (i.e. $i \notin [0, n - 2]$). (For instance, if $s = (ab, ac)$ and $i = 0$, then $\text{elem}(\text{enc}(s), 1) = \text{elem}(\dagger ab \dagger ac \dagger, 1) = ab$.)
- sequence join operation** $\text{join}_u(s)$: $\text{join}_u(\text{enc}(s))$ that removes the first and the last occurrences of \dagger and replaces all the remaining occurrences of \dagger by u .
- sequence length operation** $\text{seqlen}(s)$: $\text{seqlen}(\text{enc}(s)) - 1$ where $\text{seqlen}(\text{enc}(s))$ counts the number of occurrences of \dagger in $\text{enc}(s)$.

Hence, we obtain atomic string constraints of the following forms:

$$\begin{aligned} x &= u \mid x = \eta \mid x = \eta \cdot z \mid x = \mathfrak{T}(\eta) \mid z = \text{write}(x, it, \eta) \mid \eta = \text{filter}_e(x) \mid \\ \eta &= \text{splitstr}_e(x) \mid \eta = \text{subseq}(x, it_1, it_2) \mid \eta = \text{matchAllstr}_e(x) \mid \eta = \text{elem}(x, it) \mid \\ \eta &= \text{join}_u(x) \mid it = \text{strlen}(x) \mid it = \text{seqlen}(x), \end{aligned}$$

where \mathfrak{T} is a finite-state transducer.

We denote by XStr the class of string constraints which are positive Boolean combinations (no negation) of the atomic string constraints, and XStr_{SL} denotes its straight-line fragment. For each formula φ in $\text{SeqStr}_{\text{SL}}$, $\text{enc}(\varphi)$ is the resulting formula in XStr . We have the following result.

Proposition 2. *For each constraint φ in SeqStr , φ and $\text{enc}(\varphi)$ are equisatisfiable. Moreover, if φ is in $\text{SeqStr}_{\text{SL}}$, then $\text{enc}(\varphi)$ is in XStr_{SL} .*

Example 2. Consider the constraint $\varphi := (\mathfrak{s}_1 = \mathfrak{s}_0 \cdot (u, v) \wedge \text{seqlen}(\mathfrak{s}_1) < 2)$ in $\text{SeqStr}_{\text{SL}}$ where \mathfrak{s}_0 is a string variable and u, v are string constants. Then, its string encoding is $\text{enc}(\varphi) := (\text{enc}(\mathfrak{s}_1) = \text{enc}(\mathfrak{s}_0) \cdot \dagger u \dagger v \dagger \wedge \text{seqlen}(\text{enc}(\mathfrak{s}_1)) - 1 < 2)$, which is unsatisfiable, as there are at least three occurrences of \dagger in $\text{enc}(\mathfrak{s}_1)$. \square

The satisfiability of $\text{SeqStr}_{\text{SL}}$ is now reduced to that of XStr_{SL} . The operations filter_e , splitstr_e , matchAllstr_e and join_u can all be represented by a finite-state transducer (see the full version of the paper), so the results in [19] are directly applicable. In the next section, we show that the pre-images under the remaining string operations, i.e., write , subseq , elem and seqlen , can be effectively computed.

5.2 Computing the Pre-images Under write, subseq, elem and seqen

We now show how to compute the pre-images under write, subseq, elem and seqen. Note that the pre-image computation utilizes an NFA \mathcal{A}_0 (resp. \mathcal{A}_1) to format the strings that represent the string sequences (resp. strings), namely, \mathcal{A}_0 (resp. \mathcal{A}_1) recognizes the language $\dagger(\Sigma\dagger)^*$ (resp. Σ^*).

Pre-image under write. Let $\mathcal{A} = (R, Q, \Sigma_{\dagger}, \delta, I, F)$ be a CEFA. $\text{write}^{-1}(\mathcal{L}(\mathcal{A}))$ is computed as the collection $((\mathcal{B}_{(p,q)} \cap \mathcal{A}_0, \mathcal{A}_{R_2/R}[p, q] \cap \mathcal{A}_1), t_{(p,q)})_{(p,q) \in Q \times Q}$, where $\mathcal{B}_{(p,q)}$, $\mathcal{A}_{R_2/R}[p, q]$ and $t_{(p,q)}$ are constructed as follows:

- $\mathcal{B}_{(p,q)}$ is constructed by the following procedure:
 - We create two copies of R , say R_1, R_2 . Moreover, we introduce a fresh cost register, say r' , to count the number of occurrences of \dagger .
 - We run \mathcal{A} on the input string and increase the cost register r' each time when \dagger is read. Moreover, the registers in R_1 , instead of R , are updated in the transitions.
 - When the current symbol is \dagger , we nondeterministically choose to stop increasing r' and pause the running of \mathcal{A} at the state p . Let us call this position as the pause position.
 - Finally, when reading the first \dagger after the pause position, we resume the run of \mathcal{A} at q , where the registers in R_1 (but not r') are updated.
- $\mathcal{A}_{R_2/R}[p, q]$ is obtained from $\mathcal{A}[p, q]$ by replacing each register in R with the corresponding copy in R_2 , where $\mathcal{A}[p, q]$ is the sub-automaton of \mathcal{A} that accepts the strings starting from the state p and ending at the state q .
- $t_{(p,q)} := (t_{(p,q),r})_{r \in R}$ and $t_{(p,q),r} = r^{(1)} + r^{(2)}$ where $r^{(1)}$ and $r^{(2)}$ are the copies of r for each $r \in R$.

Note that in the backward propagation of \mathcal{A} with respect to an equality $z = \text{write}(x, it, \eta)$, the constraint $r' = it$ is added to the LIA constraint to assert that r' is equal to the index it . Formally, $\text{write}^{-1}(\mathcal{L}(\mathcal{A}))$ is a finite collection of $(\mathcal{B}_{(p,q)} \cap \mathcal{A}_0, \mathcal{A}_{R_2/R}[p, q] \cap \mathcal{A}_1, t_{(p,q)})$, where $(p, q) \in Q \times Q$, and $\mathcal{B}_{(p,q)} = (R_1 \cup \{r'\}, Q', \Sigma_{\dagger}, \delta', I', F')$ such that $Q' = Q \times \{\text{pre}, \text{idle}, \text{post}\}$, $I' = I \times \{\text{pre}\}$, $F' = F \times \{\text{post}\}$, and δ' comprises:

- $((q_1, \text{pre}), a, (q_2, \text{pre}), (\vec{v}, 0))$ such that $(q_1, a, q_2, \vec{v}) \in \delta$ and $a \in \Sigma$ (thus $a \neq \dagger$),
- $((q_1, \text{pre}), \dagger, (q_2, \text{pre}), (\vec{v}, 1))$ such that $(q_1, \dagger, q_2, \vec{v}) \in \delta$,
- $((q_1, \text{pre}), \dagger, (p, \text{idle}), (\vec{v}, 1))$ such that $(q_1, \dagger, p, \vec{v}) \in \delta$,
- $((p, \text{idle}), a, (p, \text{idle}), (\vec{0}, 0))$ such that $a \in \Sigma$,
- $((p, \text{idle}), \dagger, (q_2, \text{post}), (\vec{v}, 0))$ such that $(q, \dagger, q_2, \vec{v}) \in \delta$,
- $((q_1, \text{post}), a, (q_2, \text{post}), (\vec{v}, 0))$ such that $(q_1, a, q_2, \vec{v}) \in \delta$ and $a \in \Sigma$,
- $((q_1, \text{post}), \dagger, (q_2, \text{post}), (\vec{v}, 0))$ such that $(q_1, \dagger, q_2, \vec{v}) \in \delta$.

From the construction of $\mathcal{B}_{(p,q)}$, we can observe that the pair (p, q) should satisfy that in \mathcal{A} , there is a \dagger -transition into p and a \dagger -transition out of q . Moreover, q should be reachable from p according to the construction of $\mathcal{A}_{R_2/R}[p, q] \cap \mathcal{A}_1$.

Example 3. Consider the constraint $\mathbf{v} = \text{write}(\mathbf{u}, 1, \mathbf{\eta}) \wedge \mathbf{v} \in \dagger(a^+\dagger)^* \wedge \text{strlen}(\mathbf{v}) \geq 4$. Let $\mathcal{A} = (\{r_1\}, \{q_0, q_1, q_2\}, \Sigma_\dagger, \delta, \{q_0\}, \{q_1\})$ be the CEFA, where the register r_1 is used to record the string length and δ comprises the transitions $q_0 \xrightarrow[(1)]{\dagger} q_1 \xrightarrow[(1)]{a} q_2 \xrightarrow[(1)]{a} q_2 \xrightarrow[(1)]{\dagger} q_1$. Let us consider the pairs (p, q) in \mathcal{A} satisfying that there is a \dagger -transition into p and a \dagger -transition out of q , moreover, q should be reachable from p . One can easily observe that there is exactly one such pair, that is, (q_1, q_2) . Therefore, $\text{write}^{-1}(\mathcal{L}(\mathcal{A}))$ comprises exactly one tuple $(\mathcal{B}_{(q_1, q_2)} \cap \mathcal{A}_0, \mathcal{A}_{R_2/R}[q_1, q_2] \cap \mathcal{A}_1, (r_1^{(1)} + r_1^{(2)}))$ such that $\mathcal{B}_{(q_1, q_2)} = (\{r_1^{(1)}, r'\}, Q', \Sigma_\dagger, \delta', I', F')$ where

- $Q' = \{q_0, q_1, q_2\} \times \{pre, idle, post\}$, $I' = \{(q_0, pre)\}$, $F' = \{(q_1, post)\}$, and
- δ' comprises the following transitions:
 - $((q_1, pre), a, (q_2, pre), (1, 0))$, $((q_2, pre), a, (q_2, pre), (1, 0))$,
 - $((q_0, pre), \dagger, (q_1, pre), (1, 1))$, $((q_2, pre), \dagger, (q_1, pre), (1, 1))$,
 - $((q_0, pre), \dagger, (q_1, idle), (1, 1))$ (since $(q_0, \dagger, q_1, 1) \in \delta$),
 - $((q_1, idle), a', (q_1, idle), (0, 0))$ such that $a' \in \Sigma$,
 - $((q_1, idle), \dagger, (q_1, post), (1, 0))$ (since $(q_2, \dagger, q_1, 1) \in \delta$),
 - $((q_1, post), a, (q_2, post), (1, 0))$ and $((q_2, post), a, (q_2, post), (1, 0))$,
 - $((q_2, post), \dagger, (q_1, post), (1, 0))$.

To illustrate how $\mathcal{B}_{(q_1, q_2)}$ and $\mathcal{A}_{R_2/R}[q_1, q_2]$ work, we can see that $(\dagger aa \dagger a \dagger, 6)$ is accepted by \mathcal{A} , moreover, for any $b \in \Sigma$,

- $(\dagger ab \dagger a \dagger, 4, 1)$ is accepted by $\mathcal{B}_{(q_1, q_2)}$, witnessed by

$$\begin{aligned} & (q_0, pre) \xrightarrow[(1,1)]{\dagger} (q_1, idle) \xrightarrow[(0,0)]{a} (q_1, idle) \xrightarrow[(0,0)]{b} (q_1, idle) \\ & \xrightarrow[(1,0)]{\dagger} (q_1, post) \xrightarrow[(1,0)]{a} (q_2, post) \xrightarrow[(1,0)]{\dagger} (q_1, post), \end{aligned}$$

- and $(aa, 2)$ is accepted by $\mathcal{A}_{R_2/R}[q_1, q_2]$, witnessed by $q_1 \xrightarrow[(1)]{a} q_2 \xrightarrow[(1)]{a} q_2$.

Thus, \mathbf{u} and $\mathbf{\eta}$ in $\mathbf{v} = \text{write}(\mathbf{u}, 1, \mathbf{\eta})$ can be $\dagger ab \dagger a \dagger$ and aa , respectively. \square

Pre-image Under subseq. Let $\mathcal{A} = (R, Q, \Sigma_\dagger, \delta, I, F)$ be a CEFA. $\text{subseq}^{-1}(\mathcal{L}(\mathcal{A}))$ is computed as $(\mathcal{B} \cap \mathcal{A}_0, t)$ such that $t := \text{true}$ and \mathcal{B} introduces two fresh registers r'_1, r'_2 to count the two numbers of occurrences of \dagger that correspond to the starting position and the length of the subsequence, respectively. Moreover, \mathcal{B} simulates the run of \mathcal{A} on the substring representing the subsequence returned by subseq . Formally, $\text{subseq}^{-1}(\mathcal{L}(\mathcal{A}))$ is computed as $(\mathcal{B} \cap \mathcal{A}_0, \text{true})$, where $\mathcal{B} = (R \cup \{r'_1, r'_2\}, Q \cup \{pre, post\}, \Sigma_\dagger, \delta', \{pre\}, \{post\})$ such that $pre, post$ are two fresh states denoting the fact that \mathcal{B} is reading a symbol before and after the subsequence respectively and δ' comprises the following transitions:

- $(pre, a, pre, (\vec{0}, 0, 0))$ such that $a \in \Sigma$ (thus $a \neq \dagger$),
- $(pre, \dagger, pre, (\vec{0}, 1, 0))$,

- $(pre, \dagger, q, (\vec{v}, 1, 0))$ such that $(p, \dagger, q, \vec{v}) \in \delta$ for some $p \in I$,
- $(q, a, q', (\vec{v}, 0, 0))$ such that $(q, a, q', \vec{v}) \in \delta$ and $a \in \Sigma$,
- $(q, \dagger, q', (\vec{v}, 0, 1))$ such that $(q, \dagger, q', \vec{v}) \in \delta$,
- $(q, \dagger, post, (\vec{v}, 0, 1))$ such that $(q, \dagger, q', \vec{v}) \in \delta$ for some $q' \in F$,
- $(post, a, post, (\vec{0}, 0, 0))$ and $(post, \dagger, post, (\vec{0}, 0, 0))$ where $a \in \Sigma$.

Note that in the backward propagation of \mathcal{A} with respect to an equality $\mathfrak{z} = \text{subseq}(\mathfrak{x}, it_1, it_2)$, the constraint $r'_1 = it_1 \wedge r'_2 = it_2$ is added to the LIA constraint to assert that the value of r'_1 and r'_2 are equal to the beginning index and the length of the subsequence respectively.

Example 4. Let $\mathfrak{v} = \text{subseq}(\mathfrak{u}, 2, 1) \wedge \mathfrak{v} \in \dagger(a^+\dagger)^* \wedge \text{strlen}(\mathfrak{v}) \geq 4$ and $\mathcal{A} = (\{r_1\}, \{q_0, q_1, q_2\}, \Sigma_\dagger, \delta, \{q_0\}, \{q_1\})$ be the CEFA, where the register r_1 is used to record the string length and δ comprises the transitions $q_0 \xrightarrow[(1)]{\dagger} q_1 \xrightarrow[(1)]{a} q_2 \xrightarrow[(1)]{a} q_2 \xrightarrow[(1)]{\dagger} q_1$. Then $\text{subseq}^{-1}(\mathcal{L}(\mathcal{A}))$ is $(\mathcal{B} \cap \mathcal{A}_0, \text{true})$ such that

$$\mathcal{B} = (\{r_1^{(1)}, r'_1, r'_2\}, \{q_0, q_1, q_2, pre, post\}, \Sigma_\dagger, \delta', \{pre\}, \{post\})$$

where δ' comprises the following transitions:

- $(pre, a', pre, (0, 0, 0))$ such that $a' \in \Sigma$,
- $(pre, \dagger, pre, (0, 1, 0))$,
- $(pre, \dagger, q_1, (1, 1, 0))$ (since $(q_0, \dagger, q_1, 1) \in \delta$),
- $(q_1, a, q_2, (1, 0, 0), (q_2, a, q_2, (1, 0, 0))$,
- $(q_2, \dagger, q_1, (1, 0, 1))$,
- $(q_2, \dagger, post, (1, 0, 1))$ (since $(q_2, \dagger, q_1, 1) \in \delta$ and q_1 is an accepting state in \mathcal{A}),
- $(post, a', post, (0, 0, 0))$ and $(post, \dagger, post, (0, 0, 0))$ where $a' \in \Sigma$.

To illustrate how \mathcal{B} works, we can see that $(\dagger a \dagger a a \dagger, 4, 2, 1)$ is accepted by \mathcal{B} , witnessed by

$$pre \xrightarrow[(0,1,0)]{\dagger} pre \xrightarrow[(0,0,0)]{a} pre \xrightarrow[(1,1,0)]{\dagger} q_1 \xrightarrow[(1,0,0)]{a} q_2 \xrightarrow[(1,0,0)]{a} q_2 \xrightarrow[(1,0,1)]{\dagger} post.$$

Pre-image Under elem. Let $\mathcal{A} = (R, Q, \Sigma, \delta, I, F)$ be a CEFA. $\text{elem}^{-1}(\mathcal{L}(\mathcal{A}))$ is constructed similar to $\text{subseq}^{-1}(\mathcal{L}(\mathcal{A}))$. Nevertheless, the construction is slightly different since the output of elem is a string, instead of a sequence. Intuitively, $\text{elem}^{-1}(\mathcal{L}(\mathcal{A}))$ is computed as $(\mathcal{B} \cap \mathcal{A}_0, t)$ where $t := \text{true}$, a fresh register r' is introduced to count the number of occurrences of \dagger that corresponds to the position where the element is extracted. Formally, $\text{elem}^{-1}(\mathcal{L}(\mathcal{A}))$ is computed as $(\mathcal{B} \cap \mathcal{A}_0, \text{true})$ such that $\mathcal{B} = (R \cup \{r'\}, Q \cup \{pre, post\}, \Sigma_\dagger, \delta', \{pre\}, \{post\})$, where δ' comprises the following transitions:

- $(pre, a, pre, (\vec{0}, 0))$ such that $a \in \Sigma$ (thus $a \neq \dagger$),
- $(pre, \dagger, pre, (\vec{0}, 1))$,

- $(pre, \dagger, p, (\vec{v}, 1))$ such that $p \in I$,
- $(q, a, q', (\vec{v}, 0))$ such that $(q, a, q', \vec{v}) \in \delta$ and $a \in \Sigma$ (thus $a \neq \dagger$),
- $(q, \dagger, post, (\vec{0}, 0))$ such that $q \in F$,
- $(post, a, post, (\vec{0}, 0))$ and $(post, \dagger, post, (\vec{0}, 0))$ where $a \in \Sigma$.

Note that in the backward propagation of \mathcal{A} with respect to an equality $\mathfrak{z} = \text{elem}(\mathfrak{x}, it)$, the constraint $r' = it$ is added to the LIA constraint to assert that the value of r' is equal to the index it .

Finally, we remark that the operation $\text{seqlen}(\mathfrak{x})$ can be modeled as a CEFA, similarly to the CEFA for strlen in [19].

6 Implementation and Experiments

We implement a solver $\text{OSTRICH}^{\text{SEQ}}$ on top of the string solver OSTRICH [20] and the SMT solver Princess [39], using about 5,000 lines of Scala code. The core functionality, preimage computation for dedicated string operations such as write , subseq , elem and seqlen is implemented in about 2,000 lines of code and is integrated directly into the OSTRICH framework. To support the representation of sequences, we built a lightweight library of automata which treat the separator as a special transition. The library is implemented in about 1,000 lines of code.

6.1 Benchmarks

To evaluate the effectiveness of $\text{OSTRICH}^{\text{SEQ}}$, we curate two benchmark suites, SEQBASE and SEQEXT , where SEQBASE only uses operations that are directly supported by some existing SMT solvers while SEQEXT uses some string sequence operations that are not directly supported by any existing SMT solvers. SEQBASE . The SEQBASE benchmark suite comprises SeqStr formulas that contain only the generic sequence operations, that is, sequence operations that can be applied to the elements of any type (not necessarily string), such as sequence length seqlen , sequence concatenation \cdot , subsequence $\text{seqt}[it_1, it_2]$, sequence read $\text{nth}(\text{seqt}, it)$ and sequence write $\text{seqt}[it \rightarrow \text{strt}]$. SEQBASE is designed to facilitate a fair comparison with existing solvers since they do not support the string-specific sequence operations. SEQBASE contains 140 SeqStr constraints that are generated from three templates.

- The first template $\mathfrak{u}_1 \in e_1 \wedge \mathfrak{s}_2 = \mathfrak{s}_1[n_1 \rightarrow \mathfrak{u}_1, n_2 \rightarrow \mathfrak{u}_1] \wedge \mathfrak{u}_2 = \text{nth}(\mathfrak{s}_2, m_1) \cdot \text{nth}(\mathfrak{s}_2, m_2) \wedge \mathfrak{u}_2 \in e_2 \wedge \text{seqlen}(\mathfrak{s}_2) < \text{strlen}(\mathfrak{u}_2)$ is utilized to curate 40 random instances, where \mathfrak{u}_1 is a string variable, $\mathfrak{s}_1, \mathfrak{s}_2$ are sequence variables, and e_1, e_2 are randomly generated regular expressions. In the first 20 instances, n_1, n_2, m_1, m_2 are integer variables while in the other 20 instances, they are random integers ranging from 0 to 10.
- The second template $\mathfrak{u}_1 \in e_1 \wedge \mathfrak{s}_2 = \mathfrak{s}_1[n_1 \rightarrow \mathfrak{u}_1, n_2 \rightarrow \mathfrak{u}_2] \wedge \mathfrak{s}_3 = \mathfrak{s}_2[n_3, \text{len}_1] \cdot \mathfrak{s}_2[n_4, \text{len}_2] \wedge \mathfrak{u}_3 = \text{nth}(\mathfrak{s}_3, m_1) \cdot \text{nth}(\mathfrak{s}_3, m_2) \cdot \text{nth}(\mathfrak{s}_3, m_3) \cdot \text{nth}(\mathfrak{s}_3, m_4) \wedge \mathfrak{u}_3 \in e_2 \wedge \text{seqlen}(\mathfrak{s}_3) < \text{strlen}(\mathfrak{u}_3) + 1$ is utilized to curate 40 random instances,

Table 1. Experimental results on SEQBASE.

	cvc5	Z3	Princess ^{ARR}	OSTRICH ^{SEQ}
sat	48	14	52	52
unsat	87	59	77	88
solved	135	73	129	140
unknown/timeout	5	67	11	0
avg. time (s)	3.53	24.36	6.84	3.23

where u_1, u_2 are string variables, s_1, s_2, s_3 are sequence variables, and e_1, e_2 are randomly generated regular expressions. Similar to the first template, n_i, m_i, len_1, len_2 for $i \in [1, 4]$ are integer variables in the first 20 instances but are random integers ranging from 0 to 10 in the other 20 instances.

- The third template $s_1 = s_0[m_1 \rightarrow u_1] \wedge \dots \wedge s_i = s_{i-1}[m_i \rightarrow u_i] \wedge u_1 = \text{nth}(s_1, n_1) \wedge \dots \wedge u_j = \text{nth}(s_j, n_j) \wedge u_1 \in e_1 \wedge \dots \wedge u_j \in e_j$ is utilized to curate 60 random instances, where i and j are randomly selected from the range 0 to 20, $s_0 \dots s_i$ are sequence variables, $u_1 \dots u_j$ are string variables, $e_1 \dots e_j$ are randomly generated regular expressions, and $m_1, \dots, m_i, n_1 \dots n_j$ are randomly generated integers constant ranging from 0 to 5. In this template, write ($s[\cdot \rightarrow \cdot]$) and read ($\text{nth}(s, \cdot)$) operations are performed randomly on a string sequence, and the strings read from the sequence are checked against some regular expressions.

SEQEXT. The SEQEXT benchmark suite comprises 60 SeqStr formulas that use specific string sequence operations, namely, filter_e , matchAll_e , split_e and join_u . Among them, 19 formulas are manually generated for the unit tests of OSTRICH^{SEQ}, and 41 formulas are generated from the real-world JavaScript programs collected from GitHub. Moreover, to compare with SMT solvers that do not support the considered string sequence operations, we rewrite them using the basic string operations whenever possible. (matchAll_e is not rewritten since it cannot be expressed using existing string operations.)

6.2 Experimental Results

We conduct experiments by comparing OSTRICH^{SEQ} with SOTA solvers, including cvc5, Z3, Z3-noodler, OSTRICH and Princess^{ARR} (referring to an array-theory-based solver implemented within Princess). SeCo [33] is excluded due to soundness issues. Additionally, foldleft and map operations are not used to simulate join , as most constraints return unknown under such usage in Z3. On SEQBASE, we evaluate OSTRICH^{SEQ} along with sequence solvers cvc5, Z3 and Princess^{ARR}. For SEQEXT, we rewrite the constraints using only the basic string operations and compare OSTRICH^{SEQ} with string solvers cvc5, Z3, Z3-noodler and OSTRICH.

Table 2. Experimental results on SEQEXT.

		cvc5	Z3	Z3-noodler	OSTRICH	OSTRICH ^{SEQ}
SEQEXT-N	sat	19	4	4	20	23
	unsat	21	6	4	18	25
	solved	40	10	8	38	48
	unknown/timeout	8	38	40	10	0
	avg. time (s)	10.83	0.13	0.05	15.92	3.53
SEQEXT-M	sat	—	—	—	—	12
	unsat	—	—	—	—	0
	solved	—	—	—	—	0
	unknown/timeout	—	—	—	—	0
	avg. time (s)	—	—	—	—	1.77

All experiments are run on a 24×3.1 GHz-core server with 185 GB RAM, where each solver is started as a single thread, with a 60 s time limit and without the memory limit.

The results on SEQBASE are reported in Table 1. We can see that OSTRICH^{SEQ} outperforms all the other solvers on SEQBASE in terms of both the number of solved instances and the solving time per instance. Specifically, OSTRICH^{SEQ} solves all the 140 instances in SEQBASE, while cvc5, Z3 and Princess^{ARR} only solve 135, 73 and 129 instances, respectively. Moreover, OSTRICH^{SEQ} is more efficient than cvc5, Z3 and Princess^{ARR} (OSTRICH^{SEQ} spends 3.23 seconds in solving each instance from SEQBASE on average, while cvc5, Z3 and Princess^{ARR} spend 3.25, 24.36, and 6.84 seconds, respectively).

The results on SEQEXT are reported in Table 2. Since the SOTA sequence solvers do not support filter_e , matchAll_e or split_e directly, the SeqStr constraints are rewritten to string constraints (instead of solving the SeqStr constraints directly) when we compare with them on SEQEXT. Moreover, while we can use $\text{str.replace_re_all}$ to encode filter_e and split_e , the encoding of matchAll_e requires finite-state transducers which have not been supported by most SOTA string solvers. As a result, we do *not* generate the string constraints for the 12 instances in SEQEXT that contain matchAll_e , which are denoted by SEQEXT-M, whereas the rest 48 instances are denoted by SEQEXT-N.

From Table 2, we can see that Z3 and Z3-noodler solve only 10 and 8 out of 48 constraints in SEQEXT-N, respectively, and cvc5 and OSTRICH solve 40 and 38 instances, respectively, indicating that the complexity of the string constraints generated from the SeqStr constraints. On the other hand, OSTRICH^{SEQ} solves all of them, demonstrating its superiority in solving the SeqStr constraints with string sequence operations. For efficiency, OSTRICH^{SEQ} spends 3.53 s per instance on average, significantly less than cvc5 and OSTRICH (10.83 and 15.92 s on average, respectively). For the constraints in SEQEXT-M, OSTRICH^{SEQ} successfully solves all 12 instances with an average time of 1.77 s. As aforementioned,

for the constraints in SEQEXT-M, we do not generate string constraints or compare against cvc5, Z3, Z3-noodler and OSTRICH, as they cannot be expressed using only the basic operations of string theory.

7 Related Work

String Constraint Solving. String constraint solving has received considerable attention in the literature. Amadini [5] provides a comprehensive survey of the literature up to 2021. In particular, prior approaches are classified into three main categories: automata-based (relying on finite automata to represent the domain of string variables and to handle string operations, e.g., [3, 12, 17, 22, 28, 31, 37]), word-based (algebraic approaches based on systems of word equations, e.g., [7, 11, 12, 22, 25, 36, 38, 45, 46, 50]), and unfolding-based approaches (explicitly reducing each string into a number of contiguous elements denoting its characters, e.g., [6, 24, 34, 35, 41, 42]).

From another perspective, there are two lines of work aiming for building practical string constraint solvers. (1) One could support as many operations as possible, but primarily resort to heuristics, offering no completeness/termination guarantees. Many string constraint solvers are implemented in SMT solvers, allowing combination with other theories, most commonly the theory of integers for string lengths. Some (non-exhaustive) examples include cvc4/5 [7, 36], Z3 [13, 25], Z3-str2/3/4 [11, 38, 50], S3(P) [45, 46], Trau [2] (or its variants Trau+ [4]), Slent [47], etc. More recent ones include Z3str3RE [12] and Z3-Noodler [22] (which is based on stabilization-based algorithm [15, 21] for solving word equations with regular constraints). (2) The second approach is to develop solvers for decidable fragments, including Norn [3], SLOTH [31] and OSTRICH [17, 20], which are usually based on complete decision procedures (e.g. [4, 28, 37]).

We also mention that there are solvers which emphasize certain aspects of string constraint solving by providing dedicated methods or optimization. These include, for instance, those for more expressive regular expressions [10, 18], regex-counting [32], string/integer conversion and flat regular constraints [48], Not-Substring Constraint [1], integer data type [19], etc.

Sequences as an Extension of Strings. Strings and sequences are closely related, but may exhibit in different forms. For instance, a string can be considered as a sequence over a finite alphabet. Jez et al. [33] recently studied sequence theories which are an extension of theories of strings with an infinite alphabet of letters, together with a corresponding alphabet theory (e.g. linear integer arithmetic). They provided decision procedures based on parametric automata/transducers, giving rise to a sequence constraint solver SeCo. Our work is different in that we essentially work on a sequence theory where the element is instantiated by the string type.

Theory of Arrays. There are many studies on solving formulas in the theory of arrays or its extensions, for instance, [16, 23, 26, 27, 30, 44], which are also related to the sequence theory. Note that in these studies, the elements are considered to be either of a generic type or of integer type. In other words, the theory of arrays

where the elements are of the string type has not been defined and investigated therein.

8 Conclusion

In this paper, we have proposed SeqStr, a logic of string sequences, which supports a wealth of string and sequence operations—especially their interactions—commonly in real-world string-manipulation programs. We have provided a decision procedure for the straight-line fragment, which is implemented as a new tool OSTRICH^{SEQ}. The experiments on both hand-crafted and real-world benchmarks demonstrate that OSTRICH^{SEQ} is an effective and promising tool for testing, analysis and verification of string-manipulation programs in practice.

References

1. Abdulla, P.A., et al.: Solving not-substring constraint with flat abstraction. In: Asian Symposium on Programming Languages and Systems (APLAS), pp. 305–320 (2021)
2. Abdulla, P.A., et al.: Flatten and conquer: a framework for efficient analysis of string constraints. In: PLDI, pp. 602–617 (2017)
3. Abdulla, P.A., et al.: String constraints for verification. In: CAV, pp. 150–166 (2014)
4. Abdulla, P.A., Atig, M.F., Diep, B.P., Holík, L., Janku, P.: Chain-free string constraints. In: ATVA, pp. 277–293 (2019)
5. Amadini, R.: A survey on string constraint solving. *ACM Comput. Surv.* **55**(2), 161–1638 (2023)
6. Amadini, R., Gange, G., Stuckey, P.J.: Propagating LEX, FIND and REPLACE with dashed strings. In: van Hove, W.-J. (ed.) CPAIOR 2018. LNCS, vol. 10848, pp. 18–34. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-93031-2_2
7. Barbosa, H., et al.: cvc5: a versatile and industrial-strength SMT solver. In: Tools and Algorithms for the Construction and Analysis of Systems (TACAS), pp. 415–442 (2022)
8. Barrett, C., Fontaine, P., Tinelli, C.: The SMT-LIB Standard: Version 2.7. Technical report, Department of Computer Science, The University of Iowa (2025). www.SMT-LIB.org
9. Barrett, C., Tinelli, C.: Satisfiability Modulo Theories. In: Handbook of Model Checking, pp. 305–343. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-10575-8_11
10. Berzish, M., et al.: Towards more efficient methods for solving regular-expression heavy string constraints. *Theor. Comput. Sci.* **943**, 50–72 (2023)
11. Berzish, M., Ganesh, V., Zheng, Y.: Z3str3: a string solver with theory-aware heuristics. In: Formal Methods in Computer Aided Design (FMCAD), pp. 55–59 (2017)
12. Berzish, M., et al.: MAn SMT solver for regular expressions and linear arithmetic over string length. In: Computer Aided Verification (CAV), pp. 289–312 (2021)
13. Bjørner, N., Tillmann, N., Voronkov, A.: Path feasibility analysis for string-manipulating programs. In: TACAS, pp. 307–321 (2009)

14. Bjørner, N., Ganesh, V., Michel, R., Veanes, M.: An SMT-LIB format for sequences and regular expressions. In: *Proceedings of the 10th International Workshop on Satisfiability Modulo Theories (SMT)* (2012)
15. Blahoudek, F., et al.: Word equations in synergy with regular constraints. In: *Formal Methods (FM)*, pp. 403–423 (2023)
16. Bradley, A.R., Manna, Z., Sipma, H.B.: What’s decidable about arrays? In: *Verification, Model Checking, and Abstract Interpretation (VMCAI)*, pp. 427–442 (2006)
17. Chen, T., Chen, Y., Hague, M., Lin, A.W., Wu, Z.: What is decidable about string constraints with the replaceall function. *PACMPL* **2**(POPL), 31–329 (2018)
18. Chen, T., et al.: Solving string constraints with regex-dependent functions through transducers with priorities and variables. *Proc. ACM Program. Lang.* **6**(POPL), 1–31 (2022)
19. Chen, T., et al.: A decision procedure for path feasibility of string manipulating programs with integer data type. In: *Automated Technology for Verification and Analysis (ATVA)*, pp. 325–342 (2020)
20. Chen, T., Hague, M., Lin, A.W., Rümmer, P., Wu, Z.: Decision procedures for path feasibility of string-manipulating programs with complex operations. *Proc. ACM Program. Lang.* **3**(POPL)(2019)
21. Chen, Y., Chocholatý, D., Havlena, V., Holík, L., Lengál, O., Síc, J.: Solving string constraints with lengths by stabilization. *Proc. ACM Program. Lang.* **7**(OOPSLA2), 2112–2141 (2023)
22. Chen, Y., et al.: Z3-noodler: an automata-based string solver. In: *Tools and Algorithms for the Construction and Analysis of Systems (TACAS)*, pp. 24–33 (2024)
23. Daca, P., Henzinger, T.A., Kupriyanov, A.: Array folds logic. In: Chaudhuri, S., Farzan, A. (eds.) *CAV 2016*. LNCS, vol. 9780, pp. 230–248. Springer, Cham (2016). https://doi.org/10.1007/978-3-319-41540-6_13
24. Day, J.D., Ehlers, T., Kulczynski, M., Manea, F., Nowotka, D., Poulsen, D.B.: On solving word equations using SAT. In: Filiot, E., Jungers, R., Potapov, I. (eds.) *RP 2019*. LNCS, vol. 11674, pp. 93–106. Springer, Cham (2019). https://doi.org/10.1007/978-3-030-30806-3_8
25. de Moura, L., Bjørner, N.: Z3: an efficient SMT solver. In: *TACAS*, pp. 337–340 (2008)
26. Falke, S., Merz, F., Sinz, C.: Extending the theory of arrays: memset, memcpy, and beyond. In: *Verified Software: Theories, Tools, Experiments (VSTTE)*, pp. 108–128 (2013)
27. Farinier, B., David, R., Bardin, S., Lemerre, M.: Arrays made simpler: an efficient, scalable and thorough preprocessing. In: *Logic for Programming, Artificial Intelligence and Reasoning (LPAR)*, volume 57 of *EPiC Series in Computing*, pp. 363–380 (2018)
28. Ganesh, V., Minnes, M., Solar-Lezama, A., Rinard, M.C.: Word equations with length constraints: What’s decidable? In: *HVC*, pp. 209–226 (2012)
29. Ganzinger, H., Hagen, G., Nieuwenhuis, R., Oliveras, A., Tinelli, C.: DPLL(T): fast decision procedures. In: Alur, R., Peled, D.A. (eds.) *CAV 2004*. LNCS, vol. 3114, pp. 175–188. Springer, Heidelberg (2004). https://doi.org/10.1007/978-3-540-27813-9_14
30. Habermehl, P., Iosif, R., Vojnar, T.: What else is decidable about integer arrays? In: *Foundations of Software Science and Computational Structures (FOSSACS)*, pp. 474–489 (2008)

31. Holík, L., Janku, P., Lin, A.W., Rümmer, P., Vojnar, T.: String constraints with concatenation and transducers solved efficiently. *PACMPL* **2**(POPL), 41–432 (2018)
32. Hu, D., Wu, Z.: String constraints with regex-counting and string-length solved more efficiently. In: *Dependable Software Engineering. Theories, Tools, and Applications (SETTA)*, pp. 1–20 (2023)
33. Jez, A., Lin, A. W., Markgraf, O., Rümmer, P.: Decision procedures for sequence theories. In: *Computer Aided Verification (CAV)*, pp. 18–40 (2023)
34. Kiezun, A., Ganesh, V., Artzi, S., Guo, P.J., Hooimeijer, P., Ernst, M.D.: Hampi: a solver for word equations over strings, regular expressions, and context-free grammars. *ACM Trans. Softw. Eng. Methodol.* **21**(4) (2013)
35. Li, G., Ghosh, I.: PASS: string solving with parameterized array and interval automaton. In: Bertacco, V., Legay, A. (eds.) *HVC* 2013. LNCS, vol. 8244, pp. 15–31. Springer, Cham (2013). https://doi.org/10.1007/978-3-319-03077-7_2
36. Liang, T., Reynolds, A., Tinelli, C., Barrett, C., Deters, M.: A DPLL(T) theory solver for a theory of strings and regular expressions. In: *CAV*, pp. 646–662 (2014)
37. Lin, A.W., Barceló, P.: String solving with word equations and transducers: towards a logic for analysing mutation XSS. In: *Principles of Programming Languages (POPL)*, pp. 123–136 (2016)
38. Mora, F., Berzish, M., Kulczynski, M., Nowotka, D., Ganesh, V.: Z3str4: a multi-armed string solver. In: *Formal Methods (FM)*, pp. 389–406 (2021)
39. Rümmer, P.: A constraint sequent calculus for first-order logic with linear integer arithmetic. In: Cervesato, I., Veith, H., Voronkov, A. (eds.) *LPAR 2008. LNCS (LNAI)*, vol. 5330, pp. 274–289. Springer, Heidelberg (2008). https://doi.org/10.1007/978-3-540-89439-1_20
40. Rümmer, P.: Princess - the scala theorem prover (2024). <https://philipp.ruemmer.org/princess.shtml>. Latest release 08 Nov 2024
41. Saxena, P., Akhawe, D., Hanna, S., Mao, F., McCamant, S., Song, D.: A symbolic execution framework for javascript. In: *Security and Privacy (SP)*, pp. 513–528. IEEE Computer Society, USA (2010)
42. Scott, J.D., Flener, P., Pearson, J., Schulte, C.: Design and implementation of bounded-length sequence variables. In: Salvagnin, D., Lombardi, M. (eds.) *CPAIOR 2017. LNCS*, vol. 10335, pp. 51–67. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-59776-8_5
43. Sheng, Y., et al.: Reasoning about vectors using an smt theory of sequences. In: *Automated Reasoning*, pp. 125–143. Springer International Publishing, Cham (2022). https://doi.org/10.1007/978-3-031-10769-6_9
44. Stump, A., Barrett, C.W., Dill, D.L., Levitt, J.R.: A decision procedure for an extensional theory of arrays. In: *Logic in Computer Science (LICS)*, pp. 29–37. IEEE Computer Society (2001)
45. Trinh, M., Chu, D., Jaffar, J.: S3: a symbolic string solver for vulnerability detection in web applications. In: *CCS*, pp. 1232–1243 (2014)
46. Trinh, M., Chu, D., Jaffar, J.: Progressive reasoning over recursively-defined strings. In: *Computer Aided Verification (CAV)*, pp. 218–240 (2016)
47. Wang, H.-E., Chen, S.-Y., Yu, F., Jiang, J.-H.R.: A symbolic model checking approach to the analysis of string and length constraints. In: *ASE*, pp. 623–633. ACM (2018)
48. Wu, H., Chen, Y., Wu, Z., Xia, B., Zhan, N.: A decision procedure for string constraints with string/integer conversion and flat regular constraints. *Acta Informatica* **61**(1), 23–52 (2024)

49. Z3Prover. Sequences | online z3 guide (2025)
50. Zheng, Y., et al.: Z3str2: an efficient solver for strings, regular expressions, and length constraints. *Formal Methods Syst. Des.* **50**(2–3), 249–288 (2017)