Representing Negative Numbers in Binary

(“Two’s complement”)

8	4	2	1	
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
…				…

Sgn 4 2 1

0 1 0 1 5

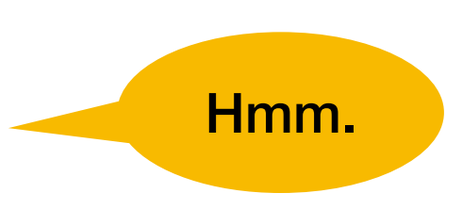
1 1 0 1 −5

If 8 bits, then
−5 = 10000101

–	4	2	1	
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

-	4	2	1	
1	1	1	1	-7
1	1	1	0	-6
1	1	0	1	-5
1	1	0	0	-4
1	0	1	1	-3
1	0	1	0	-2
1	0	0	1	-1
1	0	0	0	-0
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

−	4	2	1	
1	1	1	1	−7
1	1	1	0	−6
1	1	0	1	−5
1	1	0	0	−4
1	0	1	1	−3
1	0	1	0	−2
1	0	0	1	−1
1	0	0	0	−0
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7



-	4	2	1	
1	1	1	1	-7
1	1	1	0	-6
1	1	0	1	-5
1	1	0	0	-4
1	0	1	1	-3
1	0	1	0	-2
1	0	0	1	-1
1	0	0	0	-0
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

```
    5      0101
+ (-5)   + 1101
-------  -------
    2     10010
```

```
    6      0110
+ (-2)   + 1010
-------  -------
```

Let’s try something else … a different encoding.

One's Complement

−	4	2	1	
1	0	0	0	−7
1	0	0	1	−6
1	0	1	0	−5
1	0	1	1	−4
1	1	0	0	−3
1	1	0	1	−2
1	1	1	0	−1
1	1	1	1	−0
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

From 3, how do we get -3?

One's Complement

−	4	2	1	
1	0	0	0	−7
1	0	0	1	−6
1	0	1	0	−5
1	0	1	1	−4
1	1	0	0	−3
1	1	0	1	−2
1	1	1	0	−1
1	1	1	1	−0
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

From 3, how do we get -3?

```
    5        0101
+ (-5)    + 1010
-----    -------
   -0       11111
```

```
    5        0101
+ (-3)    + 1100
-----    -------
    1       10001
```

```
    6
+ (-2)
-----
```

One's Complement

−	4	2	1	
1	0	0	0	−7
1	0	0	1	−6
1	0	1	0	−5
1	0	1	1	−4
1	1	0	0	−3
1	1	0	1	−2
1	1	1	0	−1
1	1	1	1	−0
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

Two's Complement

−	4	2	1	
1	0	0	0	−8
1	0	0	1	−7
1	0	1	0	−6
1	0	1	1	−5
1	1	0	0	−4
1	1	0	1	−3
1	1	1	0	−2
1	1	1	1	−1
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

```
   5          0101
+ (-5)  ---->+ 1011
                10000
```

Invert the digits. Then add 1.

```
   6
+ (-2)
```

Two's Complement

− 4	2	1		
1	0	0	0	−8
1	0	0	1	−7
1	0	1	0	−6
1	0	1	1	−5
1	1	0	0	−4
1	1	0	1	−3
1	1	1	0	−2
1	1	1	1	−1
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

Two's Complement

How do you represent -3?

$$-8 + 4 + 1 = 3$$

$$-8\times\mathbf{1} + 4\times\mathbf{1} + 2\times\mathbf{0} + 1\times\mathbf{1} = 3$$

−8	4	2	1	
1	0	0	0	−8
1	0	0	1	−7
1	0	1	0	−6
1	0	1	1	−5
1	1	0	0	−4
1	1	0	1	−3
1	1	1	0	−2
1	1	1	1	−1
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7