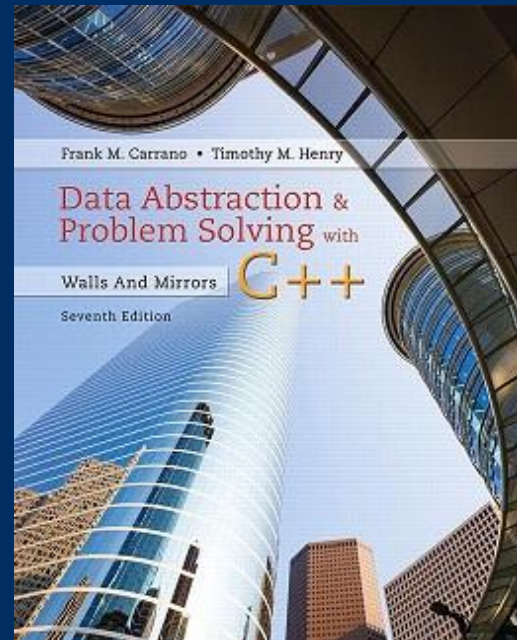


Chapter 19

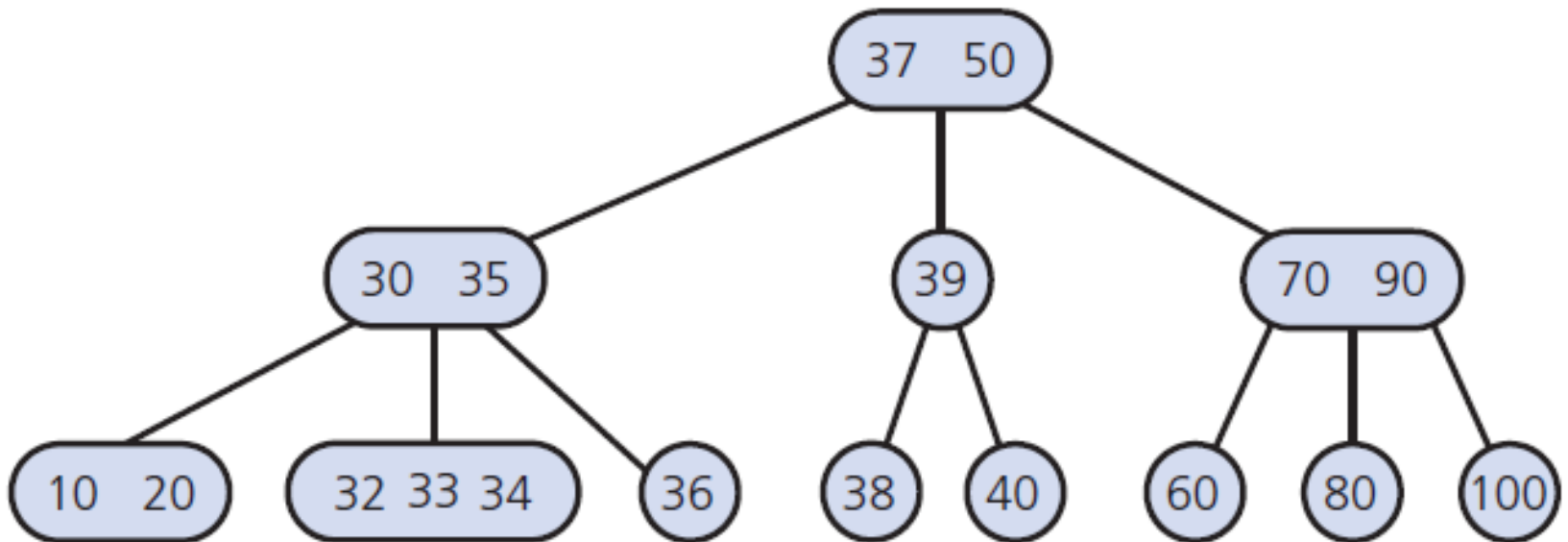
Balanced Search Trees

CS 302 - Data Structures

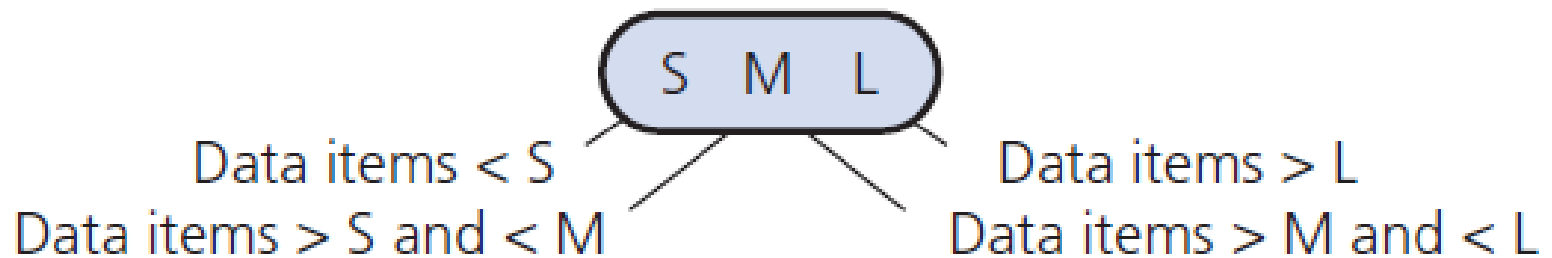
M. Abdullah Canbaz



2-3-4 Trees



- A 2-3-4 tree with the same data items as the 2-3 tree



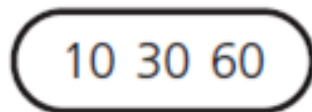
- A 4-node in a 2-3-4 tree

- Searching and traversing
 - Simple extensions of corresponding algorithms for a 2-3 tree
- Adding data
 - Like addition algorithm for 2-3 tree
 - Splits node by moving one data item up to parent node

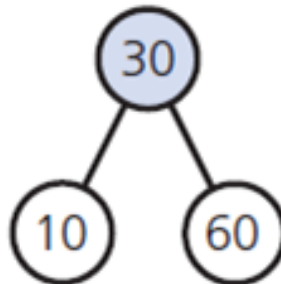
Adding Data to 2-3-4 Trees

- Adding 20 to a one-node 2-3-4 tree

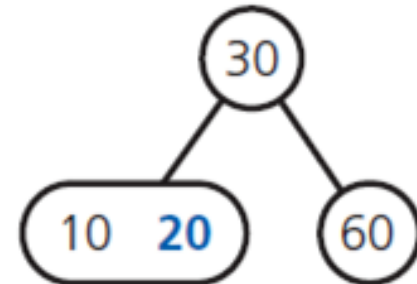
(a) The original tree

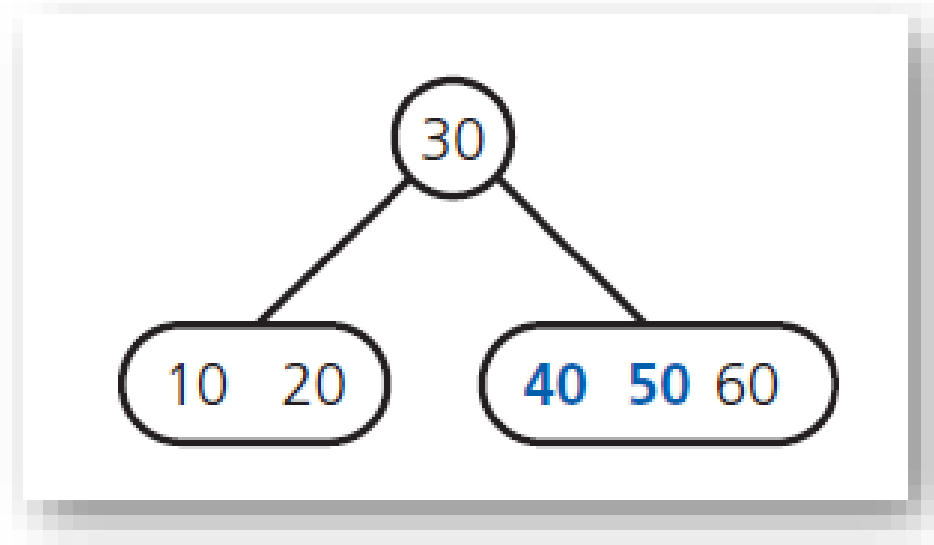


(b) After splitting the tree



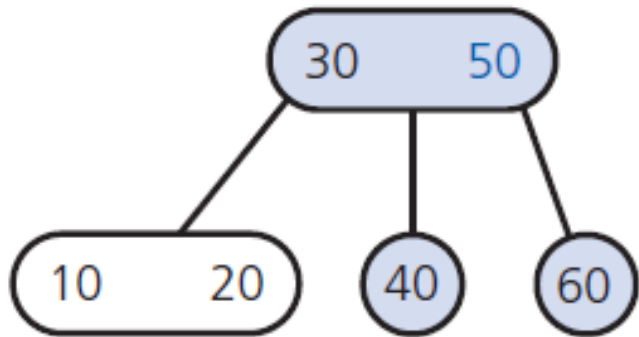
(c) After adding 20



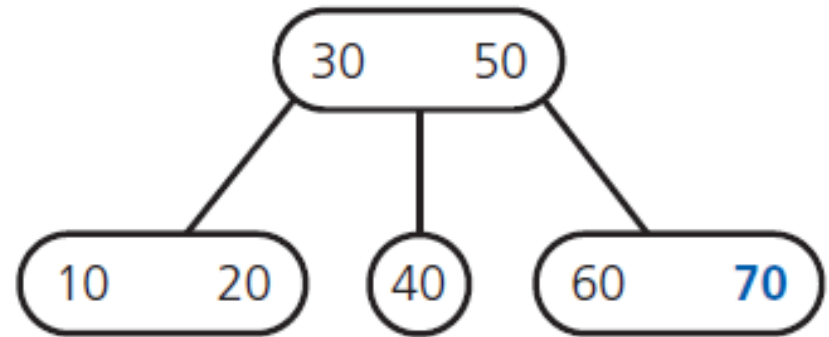


- After adding 50 and 40 to the tree

(a) After splitting the 4-node

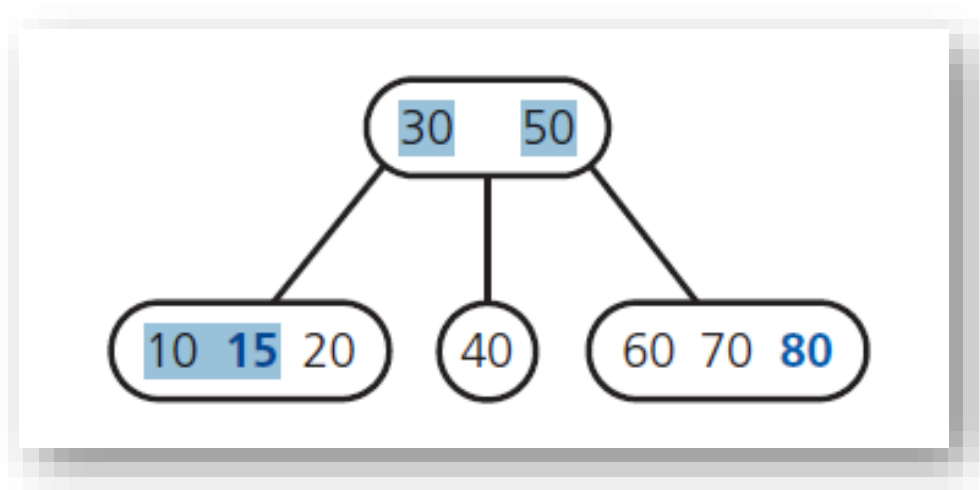


(b) After adding 70



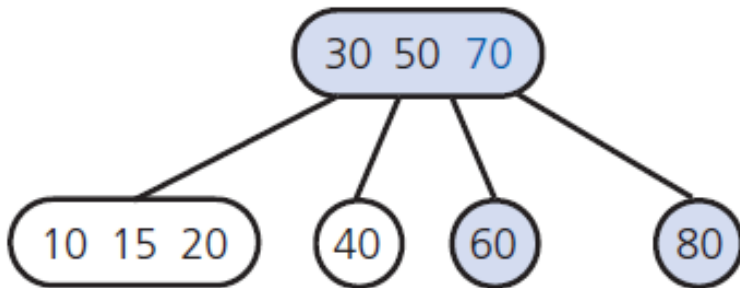
- The steps for adding 70 to the tree

Adding Data to 2-3-4 Trees

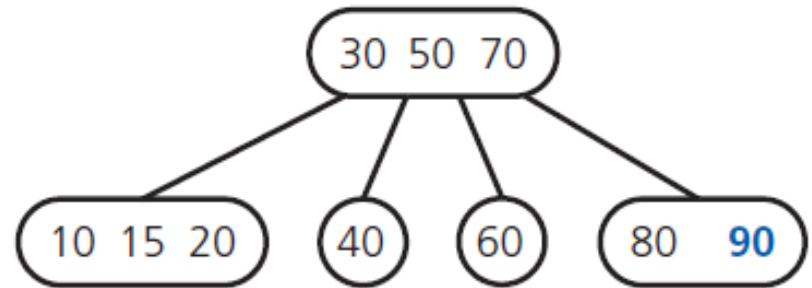


- After adding 80 and 15 to the tree

(a) After splitting the root's right child



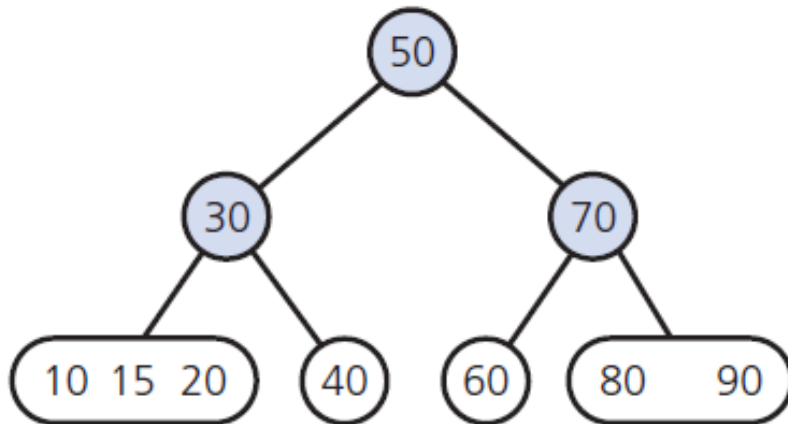
(b) After adding 90 to the root's right child



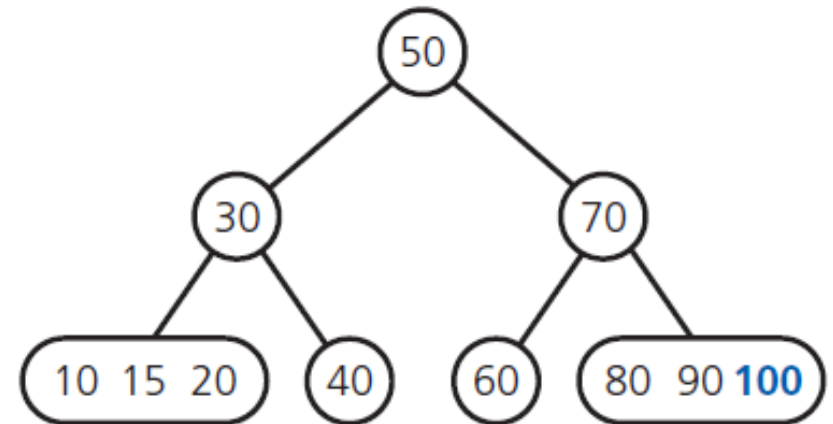
- The steps for adding 90 to the tree

Adding Data to 2-3-4 Trees

(a) After splitting the 4-node

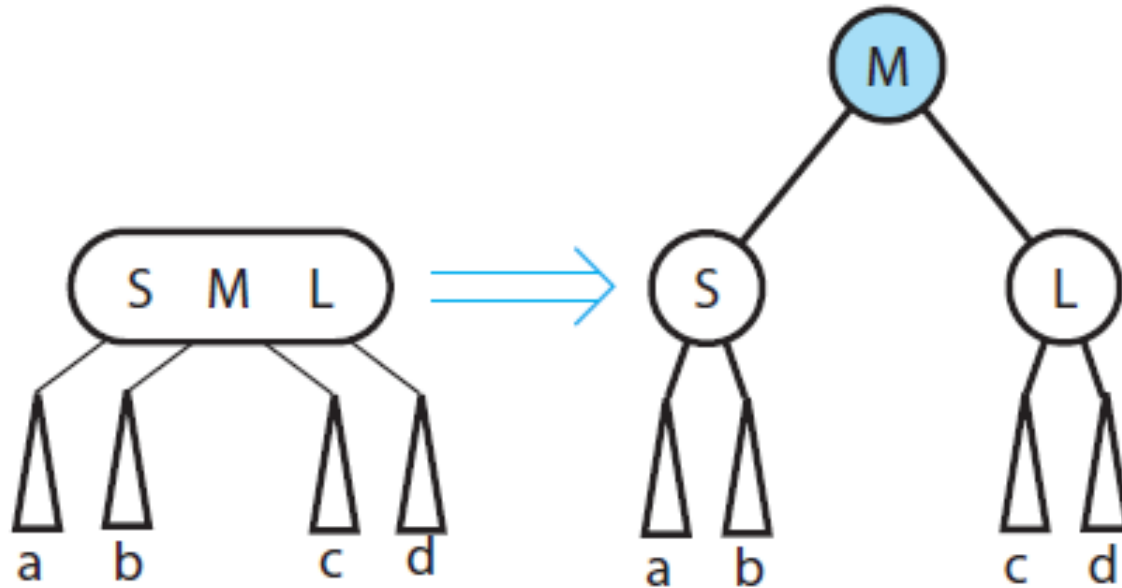


(b) After adding 100 to the rightmost leaf



- The steps for adding 100 to the tree

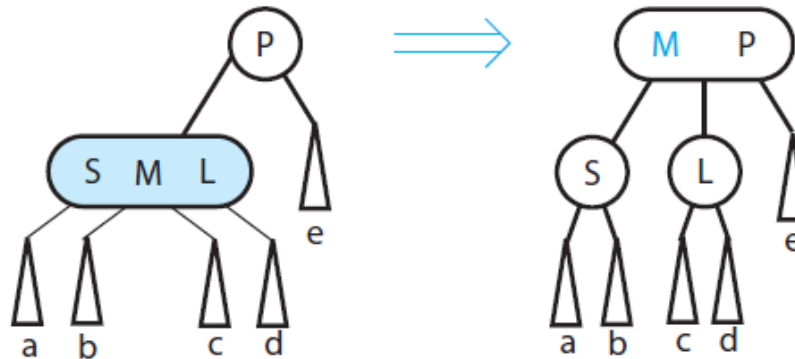
Adding Data to 2-3-4 Trees



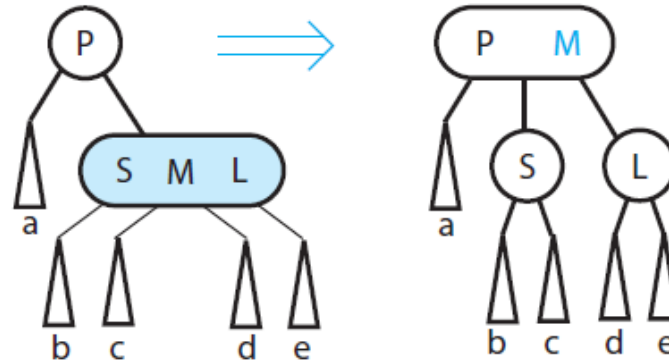
- Splitting a 4-node root when adding data to a 2-3-4 tree

Adding Data to 2-3-4 Trees

(a) The 4-node is a left child



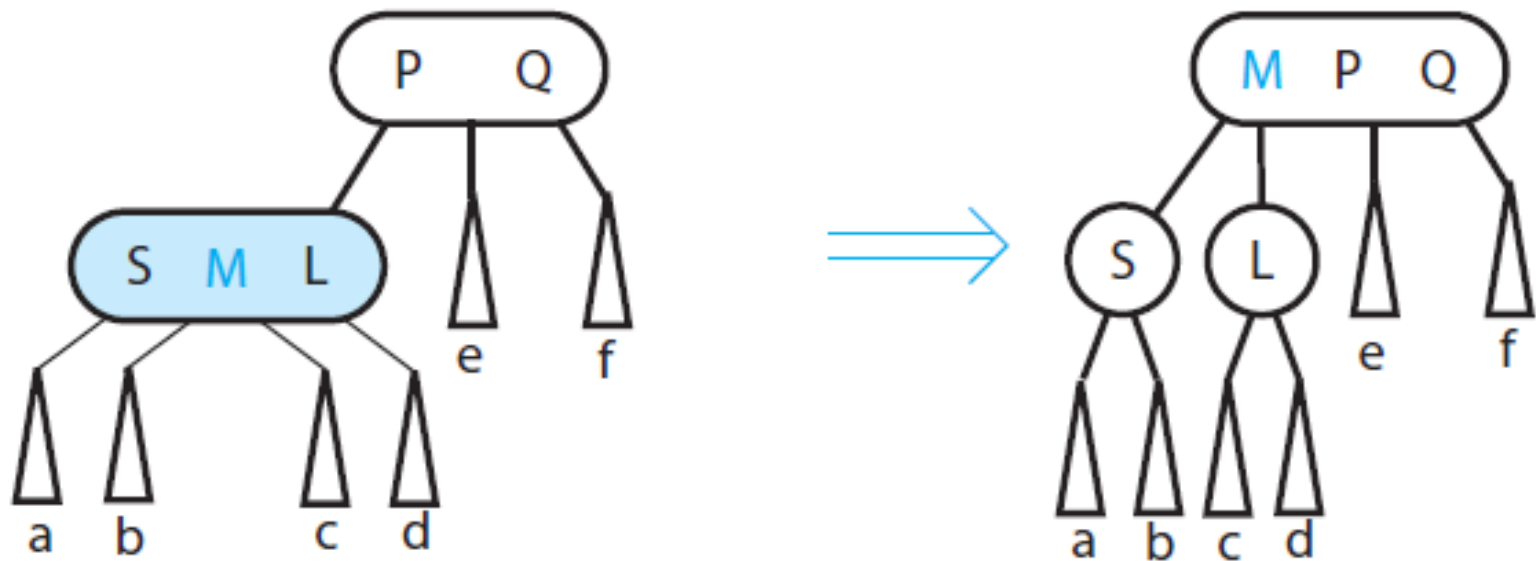
(b) The 4-node is a right child



- Splitting a 4-node whose parent is a 2-node when adding data to a 2-3-4 tree

Adding Data to 2-3-4 Trees

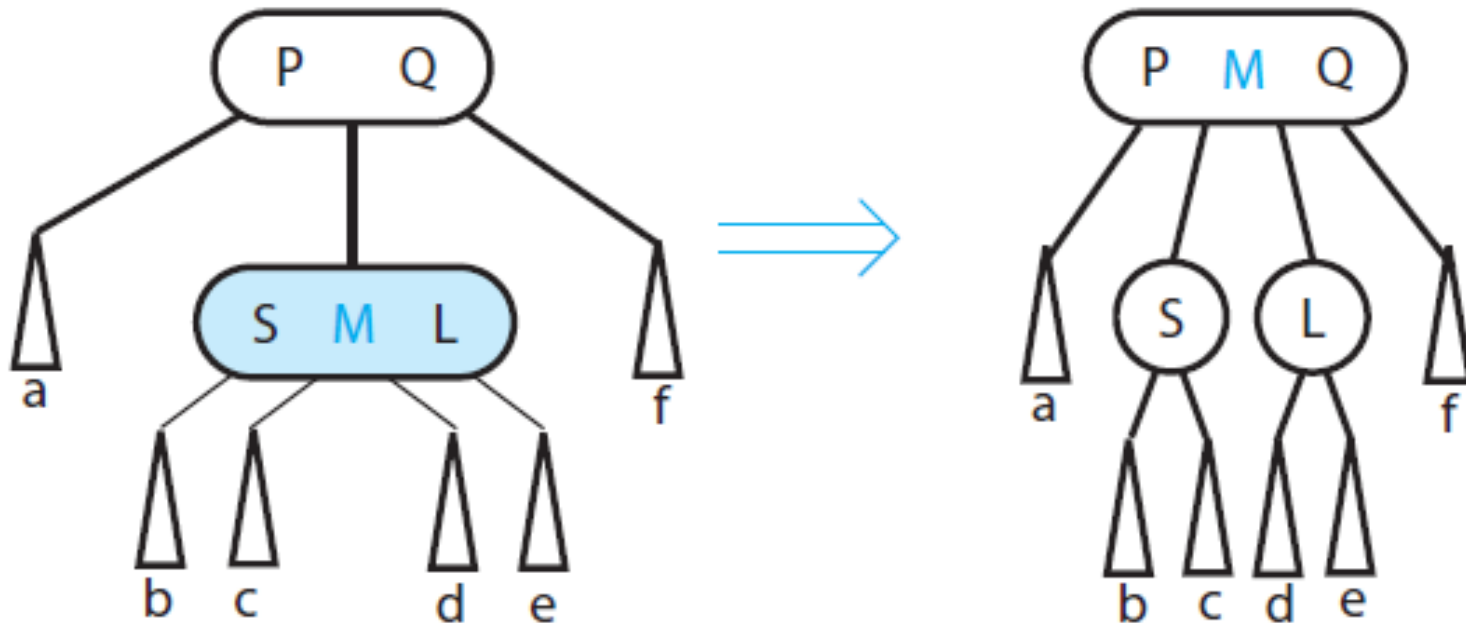
(a) The 4-node is a left child



- Splitting a 4-node whose parent is a 3-node when adding data to a 2-3-4 tree

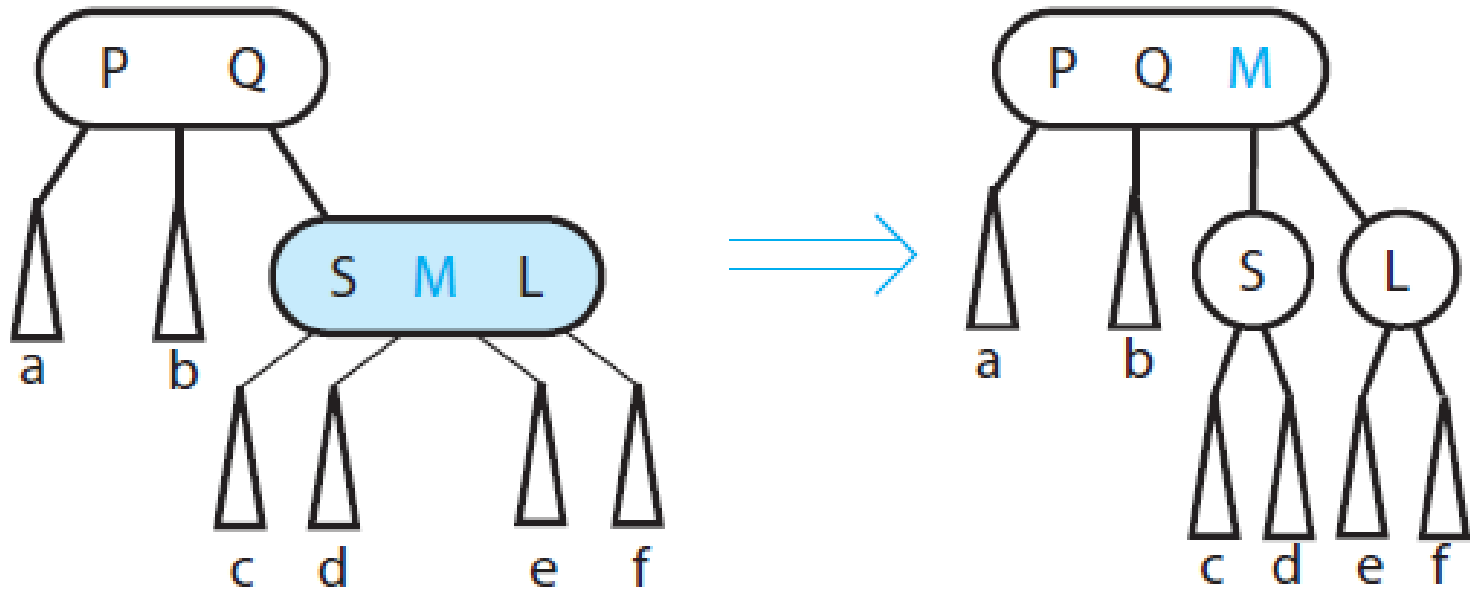
Adding Data to 2-3-4 Trees

(b) The 4-node is a middle child



- Splitting a 4-node whose parent is a 3-node when adding data to a 2-3-4 tree

(c) The 4-node is a right child



- Splitting a 4-node whose parent is a 3-node when adding data to a 2-3-4 tree



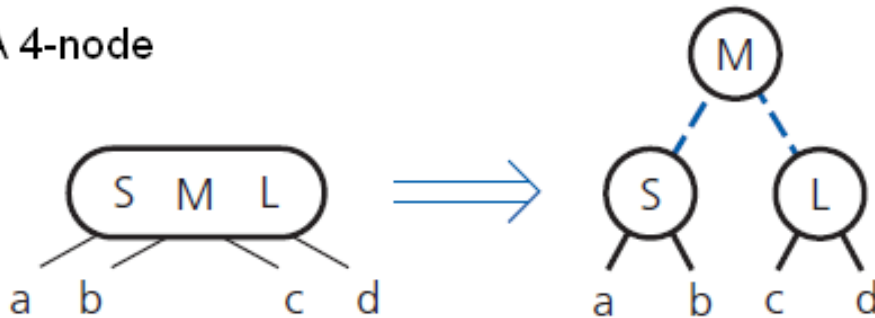
Removing Data from a 2-3-4 Tree

- Has same beginning as removal algorithm for a 2-3 tree
- Transform each 2-node into a 3-node or a 4-node
- Insertion and removal algorithms for 2-3-4 tree require fewer steps than for 2-3 tree

Red-Black Trees

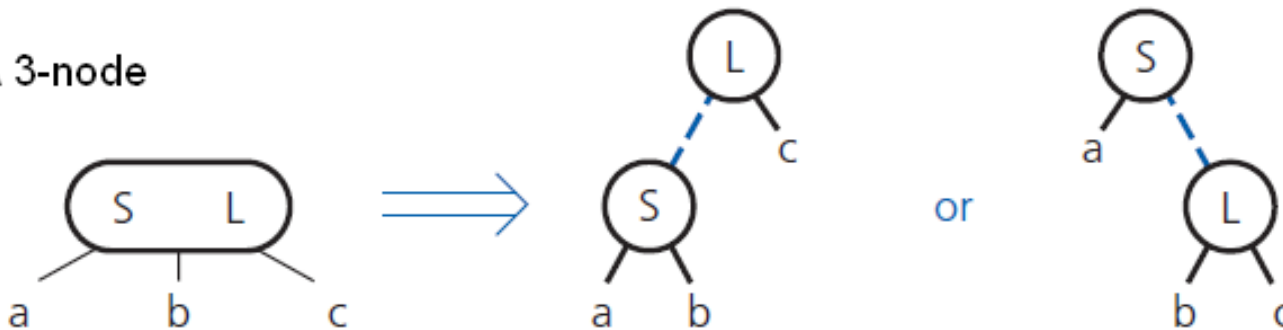
- A 2-3-4 tree requires more storage than binary search tree
- Red-black tree has advantages of a 2-3-4 tree but requires less storage
- In a red-black tree,
 - Red pointers link 2-nodes that now contain values that were in a 3-node or a 4-node

(a) A 4-node

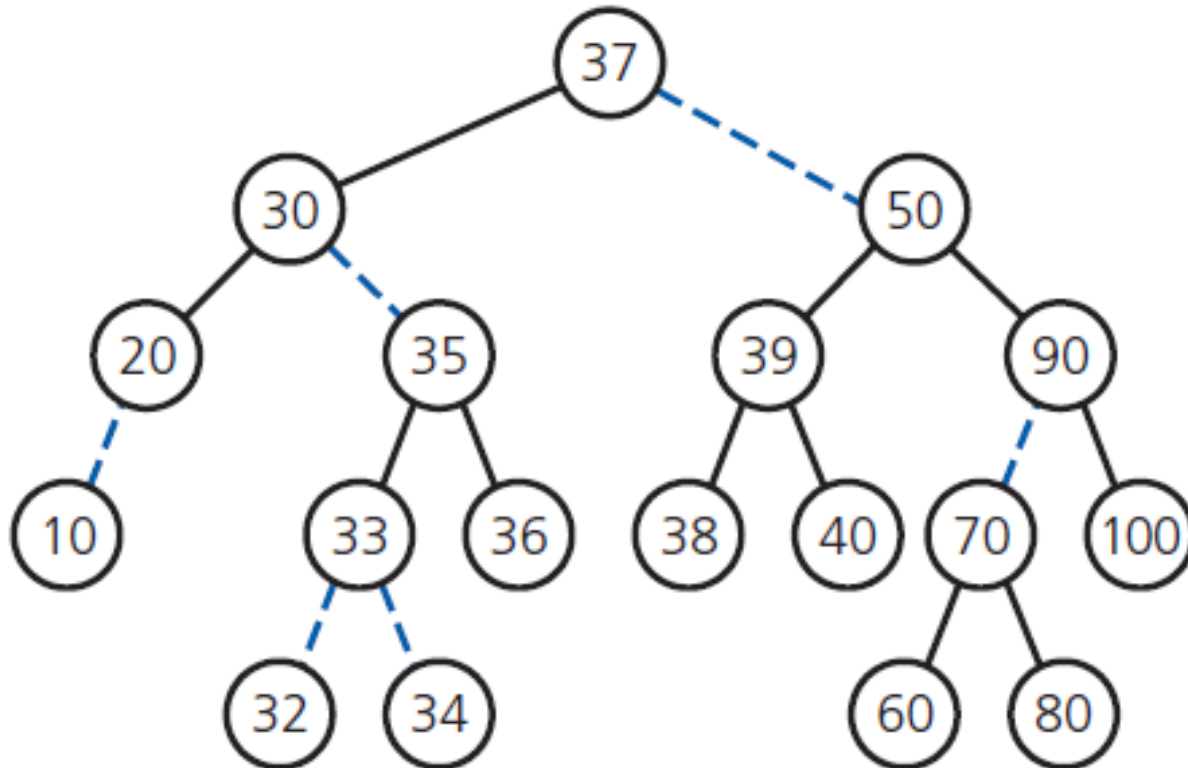


--- Red pointer
— Black pointer

(b) A 3-node



- Red-black representations of a 4-node and a 3-node



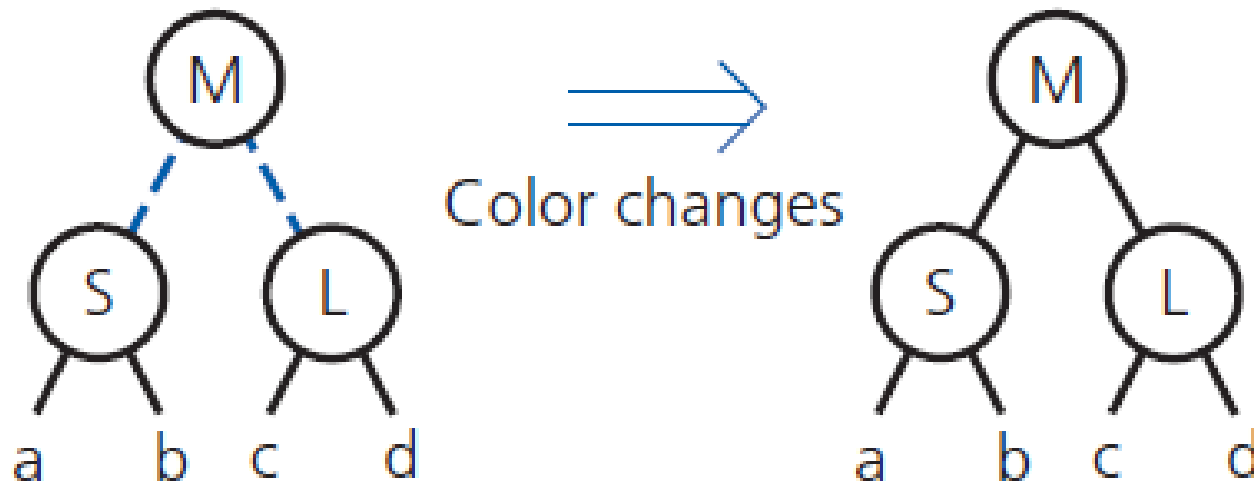
- A red-black tree that represents the 2-3-4 tree



Searching and Traversing a Red-Black Tree

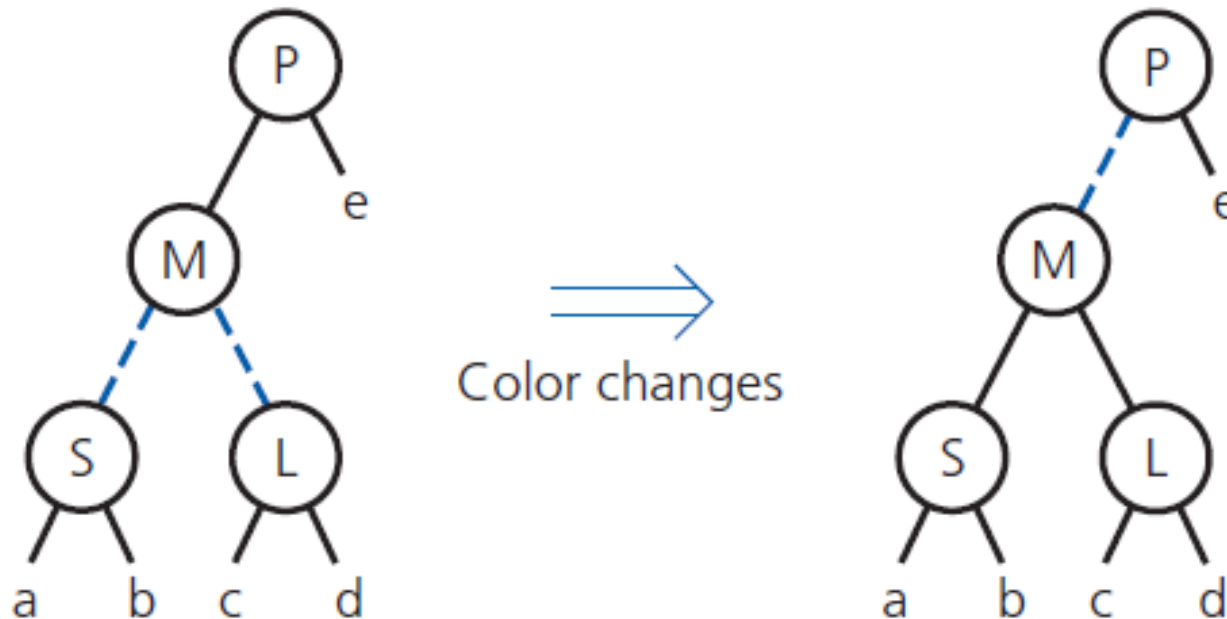
- A red-black tree is a binary search tree
- Thus, search and traversal
 - Use algorithms for binary search tree
 - Simply ignore color of pointers

- Red-black tree represents a 2-3-4 tree
 - Simply adjust 2-3-4 addition algorithms
 - Accommodate red-black representation
- Splitting equivalent of a 4-node requires simple color changes
 - Pointer changes called rotations result in a shorter tree



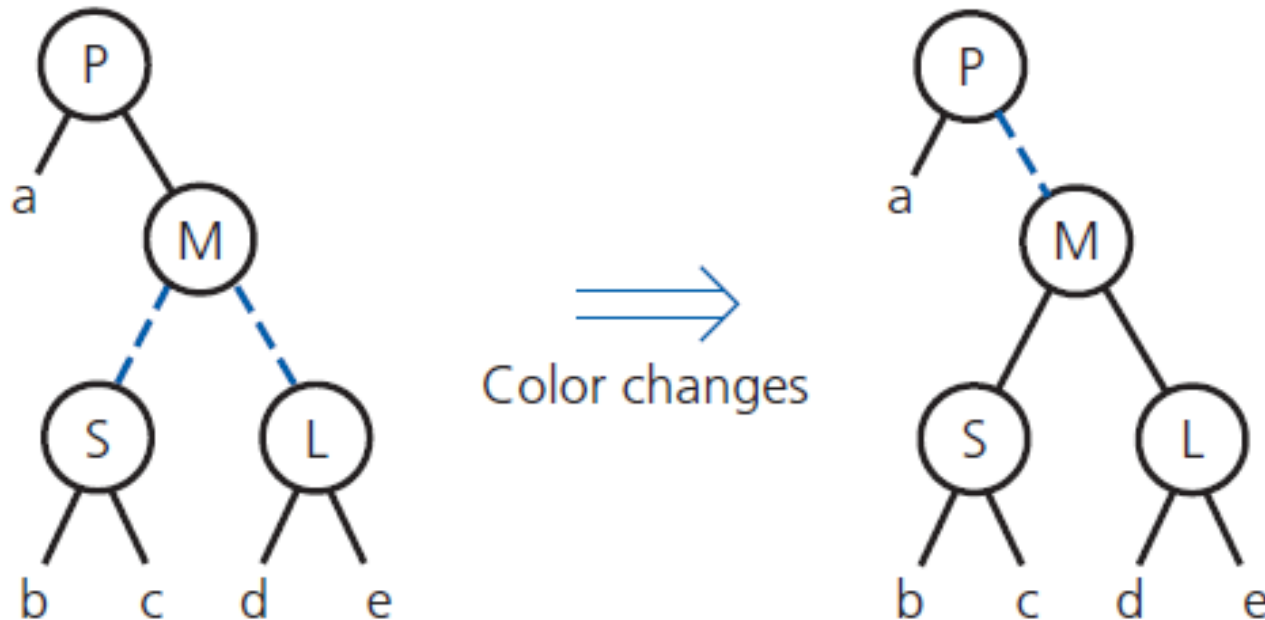
- Splitting a red-black representation of a 4-node root

(a) The 4-node is a left child



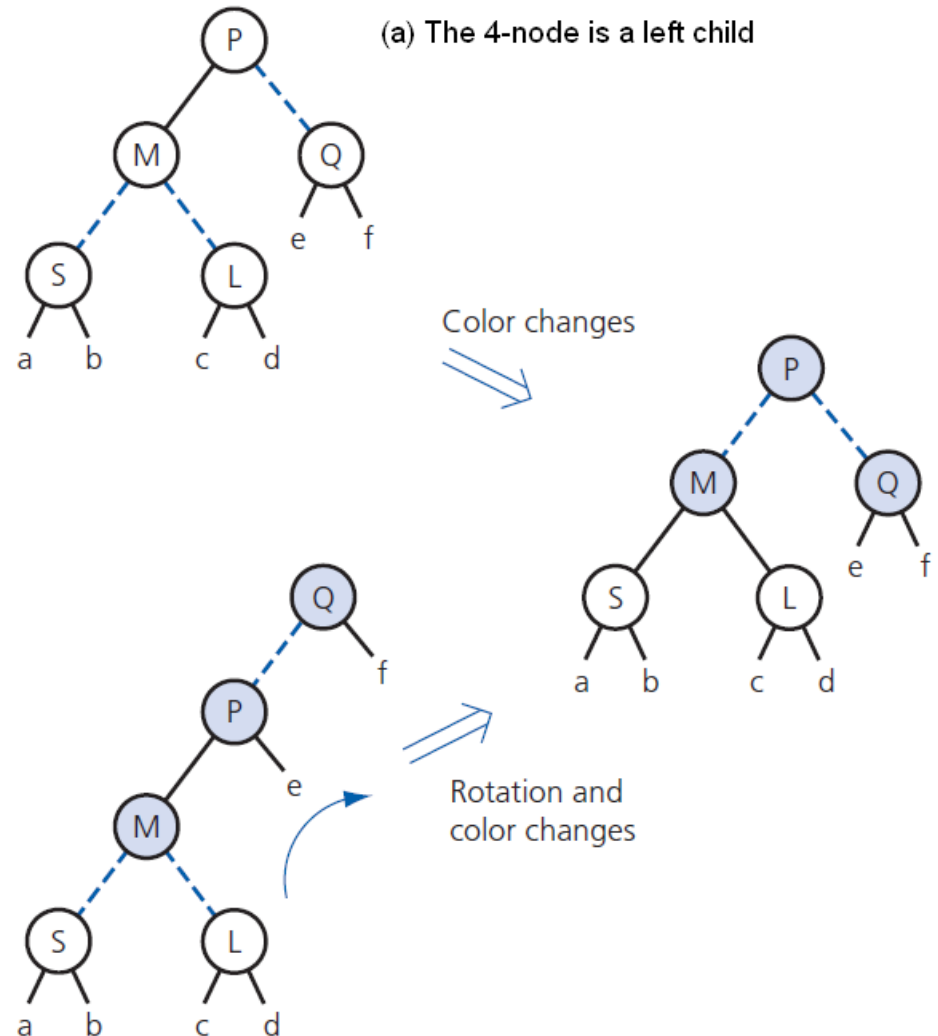
- Splitting a red-black representation of a 4-node whose parent is a 2-node

(b) The 4-node is a right child

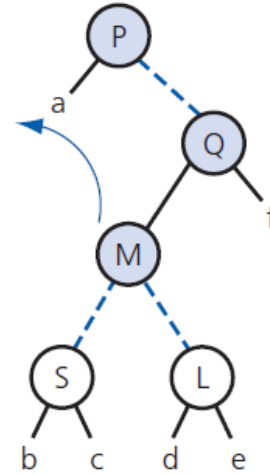


- Splitting a red-black representation of a 4-node whose parent is a 2-node

- Splitting a red-black representation of a 4-node whose parent is a 3-node

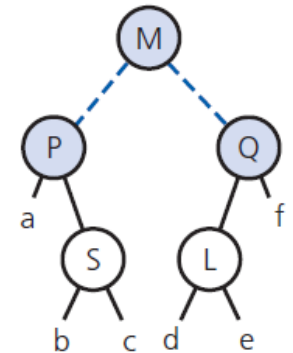


- Splitting a red-black representation of a 4-node whose parent is a 3-node

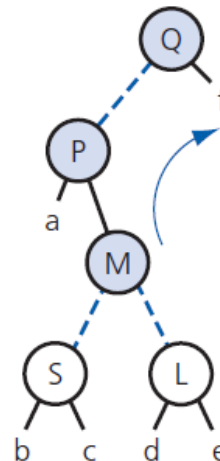


(b) The 4-node is a middle child

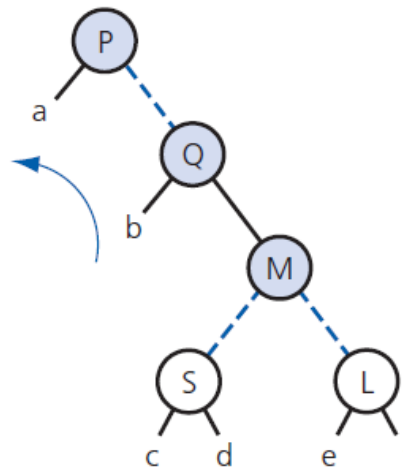
Rotation and color changes



Rotation and color changes

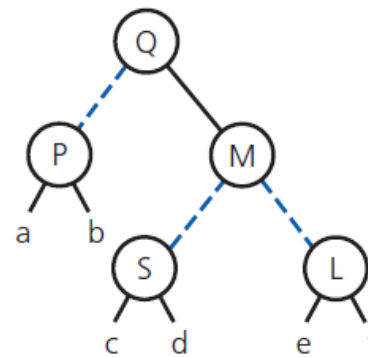


- Splitting a red-black representation of a 4-node whose parent is a 3-node



(c) The 4-node is a right child

Rotation and color changes



Color changes

