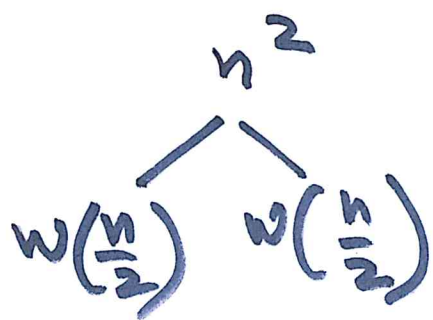
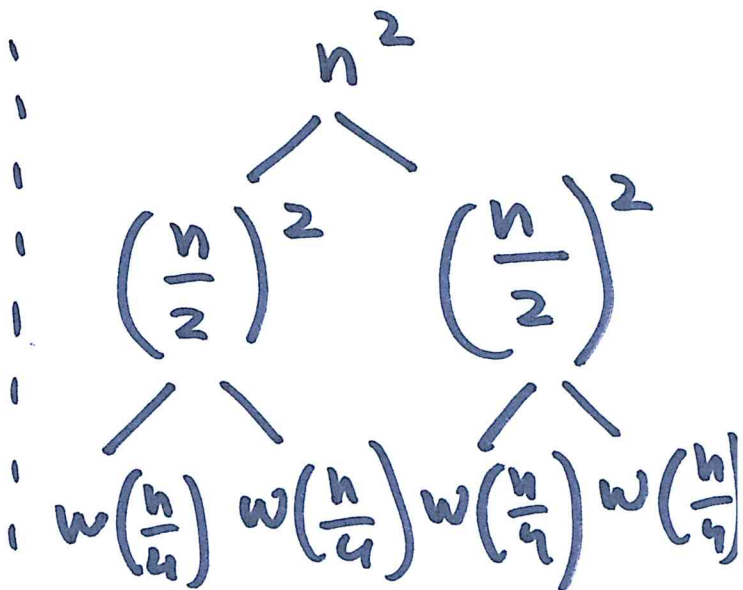


$$w(n) = 2w\left(\frac{n}{2}\right) + n^2$$



$$w\left(\frac{n}{2}\right) = 2w\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2$$



level 0:

$n^2$

pb. size    # pbs  
 $n$     1

level 1:

$\left(\frac{n}{2}\right)^2$      $\left(\frac{n}{2}\right)^2$

$\frac{n}{2}$

$2^1$

level 2:

$\left(\frac{n}{4}\right)^2$      $\left(\frac{n}{4}\right)^2$

$\frac{n}{4}$

$2^2$

level i:

$\left(\frac{n}{2^i}\right)^2$

$\frac{n}{2^i}$

$2^i$

level...  $w(1)$

$w(1)$

$$\frac{n}{2^i} = 1 \Rightarrow \boxed{i = \lg n}$$

$$W(n) = \sum_{i=0}^{\lg n - 1} 2^i \left(\frac{n}{2^i}\right)^2 + \sqrt{\lg n} \cdot W(1) =$$

$$= n^2 \sum_{i=0}^{\lg n - 1} \frac{1}{2^i} + n \cdot W(1)$$

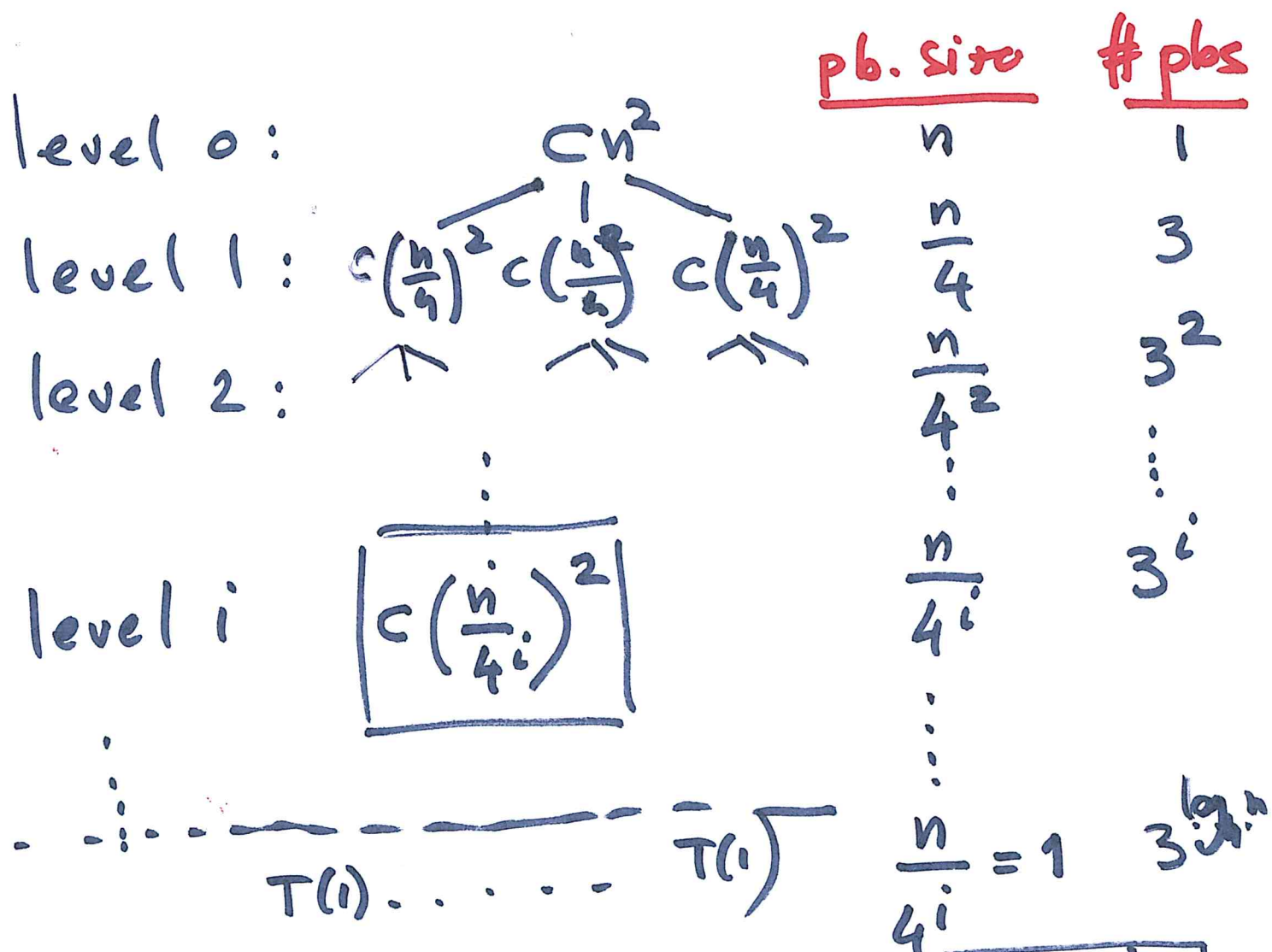
$$\leq n^2 \sum_{i=0}^{\infty} \frac{1}{2^i} + n \cdot W(1)$$

$$= n^2 \cdot \frac{1}{1 - 1/2} + n \cdot W(1)$$

$$= O(n^2)$$

not ~~θ~~

$$T(n) = 3T\left(\frac{n}{4}\right) + cn^2$$



$$\begin{aligned}
 T(n) &= \sum_{i=0}^{\log_4 n - 1} c\left(\frac{n}{4^i}\right)^2 \cdot 3^i + 3^{\log_4 n} \cdot T(1) = \\
 &= cn^2 \sum_{i=0}^{\log_4 n - 1} \left(\frac{3}{16}\right)^i + n^{\log_4 3} T(1) = \\
 &\leq cn^2 \cdot \frac{1}{1 - 3/16} + n^{\log_4 3} \cdot T(1) = O(n^2)
 \end{aligned}$$

$$T(n) = 3T\left(\frac{n}{4}\right) + cn^2$$

Guess:  $T(n) = O(n^2)$

Goal:  $T(n) \leq d \cdot n^2, d, n.$

Assume:  $T\left(\frac{n}{4}\right) \leq d \cdot \left(\frac{n}{4}\right)^2$

$$T(n) = 3T\left(\frac{n}{4}\right) + cn^2$$

$$\leq 3 \cdot d \left(\frac{n}{4}\right)^2 + cn^2$$

$$= \left[ \frac{3}{16} dn^2 + cn^2 \leq dn^2 \right]$$

$$d \geq \frac{16}{13} c$$

$$n = 1$$

Base case!



$$w(n) = w\left(\frac{n}{3}\right) + w\left(\frac{2n}{3}\right) + n$$

level 0:  $n$   $\xrightarrow{\text{Cost}}$   $n$

level 1:  $\frac{n}{3}$   $\frac{2n}{3}$   $\xrightarrow{\text{Cost}}$   $n$

level 2:  $\frac{n}{9}$   $\frac{2n}{9}$   $\frac{2n}{9}$   $\frac{4n}{9}$   $\xrightarrow{\text{Cost}}$   $n$

$\vdots$   
 $w(1)$   $\xrightarrow{\text{Cost}}$   $n$   
 $\circlearrowleft n$

level  $i$ :  $\left(\frac{2}{3}\right)^i n = 1 \Rightarrow \boxed{i = \log_{3/2} n}$

$w(n) \leq \underbrace{n + n + \dots + n}_{\text{all levels}} = n \cdot \log_{3/2} n = O(n \log n)$