$$T(n) = n + T(n-1)$$

$$= n + (n-1) + T(n-2)$$

$$= n + (n-1) + (n-2) + T(n-3)$$

$$\vdots$$

$$\qquad\qquad\qquad\qquad\qquad +1-1$$

$$= \underbrace{n + (n-1) + \dots + 3 + 2}_{} + T(1)$$

$$= \frac{n(n+1)}{2} - 1 + T(1)$$

$$= \Theta(n^2)$$

$$T(n) = c + T\left(\frac{n}{2}\right)$$

Guess $T(n) = O(\lg n)$

Goal: $T(n) \le d \lg n$

Assume: $T(k) \le d \lg k \,, \forall k < n$

$$k = \frac{n}{2} \Rightarrow T\left(\frac{n}{2}\right) \le d \cdot \lg \frac{n}{2}$$

Proof:

$$\boxed{T(n)} = c + T\left(\frac{n}{2}\right)$$

$$\le \boxed{c} + d \cdot \lg \frac{n}{2} =$$

$$= c + d \lg n - d \le d \lg n.$$

$$\cancel{\boxed{c \le d}}$$

$\cancel{n_0 = 1}$ ✓

$n_0 = 1$ : $T(1) = c \leq d \lg 1 = 0$

$\cancel{c \leq 0}$

$\boxed{n_0 = 2}$ : $T(2) = c + c = 2c$

$2c \leq d \lg 2 \Rightarrow \boxed{d \geq 2c}$

$T(n) = T(n-1) + n$

Guess: $T(n) = O(n^2)$ / $\Omega(n^2)$

Goal: $T(n) \leq c \cdot n^2$

Assume: $T(k) \leq ck^2, \forall k < n$

$\boxed{k = n-1}$ $T(n-1) \leq c(n-1)^2$

Proof:

$$T(n) = T(n-1) + n$$
$$\leq c(n-1)^2 + n =$$
$$= cn^2 - 2cn + c + n \leq cn^2$$

$$2cn - c - n \geq 0$$

$$c \geq \frac{n}{2n-1} = \frac{1}{2 - \frac{1}{n}} \qquad c \geq \frac{1}{2}$$

---

$n_0 = 1 \quad T(1) = 1 \leq c \cdot 1 \Rightarrow \boxed{c \geq 1}$

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

Guess : $T(n) = O(n \lg n)$

Goal: $T(n) \leq c \cdot n \lg n$

Assume: $T(k) \leq c \cdot k \cdot \lg k, \forall k < n$

$k = \frac{n}{2}$ : $\underbrace{T\left(\frac{n}{2}\right) \leq c \cdot \frac{n}{2} \cdot \lg \frac{n}{2}}$

Proof:

$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + n$.

$\leq \cancel{2} \cdot c \cdot \frac{n}{\cancel{2}} \cdot \lg \frac{n}{2} + n$

$= cn \lg n - cn + n \leq cn \lg n$.

$\boxed{c \geq 1}$

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$n_0 = 1$ : $T(1) = 1 \leq cn \lg n = c \cdot \lg 1 = 0$

$\boxed{n_0 = 2}$ : $T(2) = 2 + 2 = 4 \leq c \cdot 2 \cdot \lg 2$

$\Rightarrow \boxed{c \geq 2}$