$2n+1 \leq 2^n$, all $n \geq 3$

<u>Base case:</u> $n = 3$

$$2 \cdot 3 + 1 \leq 2^3$$

$$7 \leq 8 \quad \checkmark$$

<u>Inductive step</u>:

<u>Assume</u>: $2n+1 \leq 2^n$ <u>true</u>

<u>Prove</u>: $\boxed{\underbrace{2(n+1)+1} \leq 2^{n+1}}$

$$= \underbrace{2n+1}_{\leq 2^n} + 2 \boxed{\leq} 2^n + \underbrace{2}_{\leq 2^n} \leq \underbrace{2^n + 2^n}_{= 2 \cdot 2^n} \leq 2^{n+1}$$

$$\sum_{i=1}^{n} (2i-1) = n^2 \quad \forall n \geq 1$$

<u>Base case</u>: $n=1$ $\quad 2 \cdot 1 - 1 = 1$ ✓

<u>Inductive case</u>:

Assume: $\sum_{i=1}^{n} (2i-1) = n^2$ true

Prove: $\boxed{\sum_{i=1}^{n+1} (2i-1)} = (n+1)^2$

$$= \sum_{i=1}^{n} (2i-1) + 2(n+1) - 1 =$$

$$= n^2 + 2n + 1 =$$

$$= \boxed{(n+1)^2}$$

$n! \geq 2^{n-1} \quad \forall n \geq 1$

Base case : $n = 1 \quad 1 \geq 2^0$ ✓

Inductive case :

Assume : $n! \geq 2^{n-1}$ true

Prove : $(n+1)! \geq 2^n$

$$\boxed{(n+1)! = (n+1) \cdot \underbrace{n!}_{\geq 2^{n-1}} \geq \underbrace{(n+1)}_{\geq 2} \cdot 2^{n-1} \geq 2 \cdot 2^{n-1} = 2^n}$$

$$T(n) = c + T\left(\frac{n}{2}\right)$$

$$T\left(\frac{n}{2}\right) = c + T\left(\frac{n}{4}\right)$$

$$T(n) = c + c + T\left(\frac{n}{4}\right)$$

$$T\left(\frac{n}{4}\right) = c + T\left(\frac{n}{8}\right)$$

$$T(n) = c + c + c + T\left(\frac{n}{8}\right)$$

$$= 3 \cdot c + T\left(\frac{n}{2^3}\right)$$

$$\vdots$$

$$= i \cdot c + T\left(\frac{n}{2^i}\right) \longrightarrow T(1)$$

Assume $n = 2^k$

$$\frac{n}{2^i} = 1 \Rightarrow \frac{2^k}{2^i} = 1$$

$$\boxed{i = k}$$

$$n = 2^k \,/\, \log$$

$$\log n = k$$

$$T(n) = k \cdot c + T(1) = c \cdot \log n + T(1)$$

$$= \Theta(\log n)$$

$$T(n) = n + 2\underbrace{T\left(\frac{n}{2}\right)}$$

$$\quad\quad\quad\quad \llcorner = \frac{n}{2} + 2 \cdot T\left(\frac{n}{4}\right)$$

$$T(n) = n + 2\left[\frac{n}{2} + 2 \cdot T\left(\frac{n}{4}\right)\right] \Rightarrow$$

$$T(n) = n + n + 4 \cdot \underbrace{T\left(\frac{n}{4}\right)}$$

$$\quad\quad\quad\quad \llcorner = \frac{n}{4} + 2 \cdot T\left(\frac{n}{8}\right)$$

$$T(n) = n + n + n + 8 \cdot T\left(\frac{n}{8}\right)$$

$$= 3n + 2^3 \cdot T\left(\frac{n}{2^3}\right)$$

$$\underset{i \text{ times}}{\cdots\cdots} = i \cdot n + 2^i \cdot T\left(\frac{n}{2^i}\right)$$

$$\frac{n}{2^i} = 1 \quad (\text{assume } n = 2^k)$$

$$i = k \qquad n = 2^k$$

$$\boxed{\log n = k}$$

$$T(n) = n \cdot \log n + n \cdot T(1)$$

$$= \Theta(n \log n)$$